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NULL BOUNDARY CONTROLLABILITY FOR A FOURTH ORDER PARABOLIC EQUATION

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Abstract. We study the null boundary controllability for a one-dimensional fourth order parabolic equation. We show that if the initial data is continuous then the fourth order parabolic equation is controllable.

1. Introduction

The aim of this work is to study the null boundary controllability problem for an one-dimensional fourth order parabolic equation. We consider the following initial boundary value problem for a fourth order equation

$$w_t + w_{xxxx} = 0 \quad \text{on } (0,1) \times (0,\infty) \tag{1.1}$$

$$w(0,t) = 0, w_x(0,t) = 0 \quad \text{for } t \geq 0, \tag{1.2}$$

$$w(x,0) = w_0(x) \quad \text{for } x \in [0,1], \tag{1.3}$$

$$w(1,t) = g(t), w_x(1,t) = h(t) \quad \text{for } t \geq 0, \tag{1.4}$$

The problem of null boundary controllability for (1.1)–(1.4) can be stated as follows. Given $T > 0$, is it possible to find corresponding controllers $g(t)$ and $h(t)$ so that the solution of the resulting problem (1.1)-(1.4) satisfies $w(x,T) \equiv 0$ for $x \in [0,1]$ for every initial data $w_0(x)$ in an appropriate space?

The method we use here is based on the work of Y.-J. L. Guo and W. Littman [6] in which the control problem is converted to two well-posed problems. For our case, the method proceeds roughly as follows:

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(1) Extend the domain of the initial data w_0 to be $[0, 2]$ so that the property of w_0 is maintained and $w_0(x) \equiv 0$ in a neighborhood of 2.

(2) With the new modified initial data $w_0(x)$, solve the initial-boundary value problem:

$$v_t + v_{xxxx} = 0 \quad \text{on } (0, 2) \times (0, \infty), \tag{1.5}$$

$$v(0, t) = 0, v_x(0, t) = 0 \quad \text{for } t \geq 0, \tag{1.6}$$

$$v(2, t) = 0, v_x(2, t) = 0 \quad \text{for } t \geq 0, \tag{1.7}$$

$$v(x, 0) = w_0(x) \quad \text{for } x \in (0, 2), \tag{1.8}$$

(3) Let ψ be a cut-off function satisfying $\psi(t) = 1$ for $t \cdot\ T/2$ and $\psi(t) = 0$ for $t \geq T$. Let

$$\xi(t) = v_{xx}(0, t) \cdot \psi(t), \quad \zeta(t) = v_{xxx}(0, t) \cdot \psi(t)$$

where v is the solution in (2).

(4) Solve the Cauchy problem

$$u_{xxxx} = -u_t \quad \text{for } t \geq T_0, x > 0, \tag{1.9}$$

$$u(0, t) = 0, u_x(0, t) = 0, u_{xx}(0, t) = \xi(t), u_{xxx}(0, t) = \zeta(t) \quad \text{for } t \geq T_0, \tag{1.10}$$

in the x-direction to get a solution which vanishes for $t \geq T$ and equals the solution v for $t \cdot\ T/2$ where T_0 is a positive constant.

(5) The boundary functions are obtained by setting $g(t) = u(1, t)$ and $h(t) = u_x(1, t)$.

The initial-boundary value problem (1.5)-(1.8) can be solved by the standard method. To solve the cauchy problem (1.9)-(1.10), we use the nonlinear Cauchy-Kowalevski Theorem. If the solution $u(x, t)$ of (1.9)-(1.10) exists beyond $x = 1$, we obtain controllers by reading the values of the derivatives of $v(x, t)$ and $u(x, t)$ at $x = 1$ where $v(x, t)$ and $u(x, t)$ are solutions of (1.5)-(1.8) and (1.9)-(1.10) respectively. To estimate the length of the maximal x-interval of existence for the solution $u(x, t)$, we shall check the constants in the proof of the nonlinear Cauchy-Kowalevski Theorem. In [6], the authors consider the control problem for semilinear heat equations and the result of the null boundary controllability for semilinear heat equations is obtained for continuously differentiable and sufficiently small initial data. The smallness condition on initial data is imposed to ensure that the maximal interval of existence for the problem similar to problem (1.9)-(1.10) is greater than 1. In [5], the author consider linear heat equations with time dependent coefficients and assume that the initial data are continuous without imposing the smallness condition. The linearity of differential equation and the Gevrey class 2 property for the coefficients of the equation will give us clues to show that the x-interval of existence for the problem similar to problem (1.9)-(1.10) is greater than 1. In this work, we consider a linear fourth order parabolic equation. The linearity and the simplicity of the coefficients of the equation will guarantee that the maximal interval of existence of the solution of (1.9)-(1.10) is greater than 1. We will show that the equation is controllable if the initial data is continuous.

A great deal of developments in the controllability theory of the linear heat equation were initiated by Fattorini and Russell. These results have been presented in numerous articles (see, e.g. [1], [2]). Most of the results obtained are for parabolic equations. For the controllability of second order semilinear parabolic equation, the readers may consult [5]. Here we consider the case for fourth order equation.

The paper is organized as follows. In Section 2, we use the nonlinear Cauchy-Kowalevski Theorem to solve the Cauchy problem (1.9)-(1.10). Since we need to estimate the interval of existence, we state the nonlinear Cauchy-Kowalevski Theorem in detail in this section. In Section 3, we obtain the result for the null boundary controllability.

2. SOLUTIONS OF THE CAUCHY PROBLEM IN THE x-DIRECTION

In this section, we shall apply the nonlinear Cauchy-Kowalevski Theorem to solve the following Cauchy problem:

$$u_{xxxx} = -u_t \quad \text{for} \quad x > 0, t \geq T_0, \tag{2.1}$$

$$u(0,t) = 0, u_x(0,t) = 0, u_{xx}(0,t) = \xi(t), u_{xxx}(0,t) = \zeta(t), \text{ for } t \geq T_0, \tag{2.2}$$

where $\xi(t)$ and $\zeta(t)$ are Gevrey class 2 functions in t and T_0 is a positive constant. We shall prove that the solution of (2.1), (2.2) exists and the x-inteval of existence is greater than 1.

The nonlinear Cauchy-Kowalevski Theorem originally due to Ovcyannikov is exploited in a number of ways to obtain existence results for the nonlinear abstract Cauchy problem

$$\begin{aligned} &\frac{du}{dx} = F(u,x), \quad |x| < \eta, \quad \eta > 0, \\ &u(0) = u_0. \end{aligned}$$

Here the solutions are in the form, as functions of the variable x, in a scale of Banach space $\{X_s\}$. The nonlinear Cauchy-Kowalevski Theorem is a generalization of the well-known Cauchy-Kowalevski Theorem and is reduced to the Cauchy-Kowalevski Theorem when all data are real analytic.

We shall use the same method as used in [6] to solve problem (2.1)-(2.2). Since we shall estimate the parameters in the nonlinear Cauchy-Kowalevski Theorem to obtain the interval of existence, we shall restate the theorem here. We begin by considering a 1-parameter family of Banach spaces $\{X_s\}$ where the parameter s is allowed to vary in [0, 1].

Definition 2.1 $\{X_s\}_{0 \le s \le 1}$ is a scale of Banach spaces if for any $s \in [0, 1]$, X_s is a linear subspace of X_0 and if $s' \le s$ then $X_s \subset X_{s'}$ and the natural injection of X_s into $X_{s'}$ has norm less than or equal to 1.

We denote by $\|\cdot\|_s$ the norm of X_s.

For each i, $i = 1, \cdots, m$, let $\{X_s^i\}_{0 \le s \le 1}$ be a scale of Banach spaces with norm $\|\cdot\|_s^i$. Consider the system of differential equations

$$\frac{du_i}{dx} = F_i(u_1, u_2, \cdots, u_m, x), \quad |x| < \eta,\ \eta > 0,\ i = 1, \cdots, m, \tag{2.3}$$

$$u_i(0) = u_{i,0} \quad i = 1, \cdots, m, \tag{2.4}$$

where the u_i, as functions of the variable x, are in X_s^i, $i = 1, \cdots, m$.

We need the following assumptions.

(H1) $u_{i,0} \in X_s^i$ for every $s \in [0, 1]$ and satisfies

$$\|u_{i,0}\|_s \le R_{i,0},$$

for some $R_{i,0} < \infty$ for $i = 1, \cdots, m$.

(H2) There are $R_i > R_{i,0} > 0$, $i = 1, \cdots, m$, $\eta > 0$, such that for every pair of numbers s, s' with $0 \le s' < s \le 1$ the mapping $F_i(u_1, \cdots, u_m, x)$, $i = 1, \cdots, m$, is continuous from the set

$$\{u_1 \in X_s^1 \mid \|u_1\|_s < R_1\} \times \cdots \times \{u_m \in X_s^m \mid \|u_m\|_s < R_m\}$$
$$\times \{x \mid |x| < \eta\}$$

into $X_{s'}^i$.

(H3) There are constants C_i, $i = 1, \cdots, m$, such that for every pair of numbers s, s' with $0 \leq s' < s \leq 1$, for all $\|u_j\|_s < R_j$, $\|v_j\|_s < R_j$, $j = 1, \cdots, m$, and for all x, $|x| < \eta$, we have

$$\|F_i(u_1, u_2, \cdots, u_m, x) - F_i(v_1, v_2, \cdots, v_m, x)\|_{s'}$$
$$\leq \frac{C_i}{(s-s')^{\alpha_i}}[\vartheta_i^1 \|u_1 - v_1\|_s + \cdots + \vartheta_i^m \|u_m - v_m\|_s],$$
$$i = 1, \cdots, m,$$

where the number ϑ_i^j is set to be zero if F_i is independent of u_j and to be one otherwise, for some parameters $\alpha_i \geq 0$, $i = 1, \cdots, m$, such that for any collection of m^2 numbers c_i^j, the degree of $P(\lambda, \mu)$ with respect to λ, μ is at most m, where the expression $P(\lambda, \mu)$ of two variables λ, μ is defined by

$$P(\lambda, \mu) = det(\lambda I - [\mu^{\alpha_i} \vartheta_i^j c_i^j]),$$

with I the $m \times m$ identity matrix and the degree is defined to be the highest degree among all monomials in $P(\lambda, \mu)$.

(H4) $F_i(0, \cdots, 0, x)$ is a continuous function of x, $|x| < \eta$, with values in X_s^i for every $s < 1$ and satisfies

$$\|F_i(0, \cdots, 0, x)\|_s \leq \frac{K_i}{(1-s)^{\alpha_i}}, \quad 0 \leq s < 1,$$

for some constants K_i, $i = 1, \cdots, m$, with α_i defined in (H3).

Then we have the following existence and uniqueness theorem for solutions of (2.3)-(2.4).

Theorem 2.1 [6]. *Under the preceding hypotheses* (H1)–(H4), *there is a positive constant* ρ *such that the Cauchy problem* (2.3)-(2.4) *has a unique solution* $\{u_i(x), i = 1, \cdots, m\}$, *which are continuously differentiable functions of* x, $|x| < \rho(1-s)$, *with values in* X_s^i, $\|u_i(x)\|_s < R_i$, *for every* $s < 1/2$.

In order to apply Theorem 2.1 to solve the Cauchy problem (2.1)-(2.2), we choose the following scale of Banach spaces.

Definition 2.2. Let K be a compact interval and let θ_0 and θ_1 be two positive constants such that $\theta_0 < \theta_1 < \infty$. Given $s \in [0, 1]$, we define the space $B_s(K)$ to be the set of all $C^\infty(K)$ functions ϕ satisfying

$$\|\phi\|_s \equiv \sup_{n \geq 0} \max_{t \in K} \frac{\tilde{n}^4 \theta(s)^n}{\lambda(2n)!} |\phi^{(n)}(t)| < \infty,$$

where $1/\theta(s) = (1-s)/\theta_0 + s/\theta_1$, $\tilde{n} = \max(n,1)$, and λ is any positive constant satisfying

$$\lambda \cdot \ \ 1/\left[2 + 2^4 \sum_{k=1}^{\infty} (1/k)^4\right].$$

It is easy to check that $\{B_s(K)\}_{0 \ \ s \ \ 1}$ is a scale of Banach spaces.

The Gevrey class 2 functions which play an important role in this paper are defined as follows.

Definition 2.3. Let be a subset of $\mathbf{R}^n$ and $\delta > 0$. A C^∞ function f in is said to be of Gevrey class δ in (in short, $f \in \gamma^\delta(\ \ )$) if there exist positive constants C and H such that

$$|D_x^\alpha f(x)| \cdot \ \ CH^{|\alpha|}(\delta|\alpha|)!,$$

for all multi-indices α and for all $x \in$ where $\alpha! = \Gamma(\alpha+1)$ and Γ is the usual gamma function.

It is clear that any function which is of Gevrey class δ in is bounded.

The following relationship between the spaces $B_s(K)$ and the Gevrey class 2 functions can be found in [6, Proposition 4.4].

(a) The space $B_s(K)$ is contained in γ^2 for all $s \in [0,1]$.

(b) Suppose $\phi : \mathbf{R} \to \mathbf{R}$ is an infinitely differentiable function defined in K and there are positive constants C and H such that

$$|\phi^{(j)}(t)| \cdot \ \ CH^j(2j)!,$$

for all t and for all $j = 1, 2, \cdots$. If the constant θ_1 in defining $B_s(K)$ satisfying $\theta_1 < 1/H$, then $\phi \in B_s(K)$ for all $s \in [0,1]$.

Furthermore, by [6, Proposition 4.2], the partial differentiation $\partial/\partial t$ defines a bounded linear operator from $B_s(K)$ into $B_{s'}(K)$ for $0 \cdot \ \ s' < s \cdot \ \ 1$ with norm less than or equal to $C/(s-s')^2$, where C is a positive constant which can be taken as $(4/e)^2\theta_0/(\theta_1-\theta_0)^2$. We note that the constant C can be made as small as we wish by taking the constant θ_0 sufficiently small while keeping the constant θ_1 fixed in the definition of $B_s(K)$.

Now, we are ready to prove the main result of this section as follows.

Theorem 2.2. *Suppose that* $\xi(t), \zeta(t) \in \gamma^2([T_0,\infty))$ *with support* $[T_0, T]$, $T > T_0$. *Then a classical solution* $u(x,t)$ *of* (2.1), (2.2) *exists and the* x*-interval of existence is greater than* 1.

Proof. In order to apply Theorem 2.1, we convert the problem (2.1)–(2.2) to a first order system of differential equations by introducing the variables $u_1 = u$, $u_2 = u_x$, $u_3 = u_{xx}$, $u_4 = u_{xxx}$ and $u_5 = u_t$. Then (2.1)–(2.2) can be rewritten as

$$\begin{aligned}
\frac{du_1}{dx}(x,\cdot) &= u_2(x,\cdot),\\
\frac{du_2}{dx}(x,\cdot) &= u_3(x,\cdot),\\
\frac{du_3}{dx}(x,\cdot) &= u_4(x,\cdot),\\
\frac{du_4}{dx}(x,\cdot) &= -u_5(x,\cdot),\\
\frac{du_5}{dx}(x,\cdot) &= \frac{\partial}{\partial t}u_2(x,\cdot),
\end{aligned}$$

with the Cauchy data

$$u_1(0,\cdot) = 0,\ u_2(0,\cdot) = 0,\ u_3(0,\cdot) = \xi(\cdot), u_4(0,\cdot) = \zeta(\cdot), u_5(0,\cdot) = 0.$$

Let $K = [T_0, T+\epsilon]$ and $D = [0,2]\times K$ where ϵ is any finite positive number. Since $\xi(t), \zeta(t) \in \gamma^2(D)$, there exist positive constants M_i, H_i, $i = 1,2$ such that

$$|\partial_t^j \xi(t)| \leq M_1 H_1^j (2j)!,$$

and

$$|\partial_t^j \zeta(t)| \leq M_2 H_2^j (2j)!,$$

for all $t \in K$ and any nonnegative integers j. Let θ_0, θ_1 be two constants satisfing $0 < \theta_0 < \theta_1 < min(1/H_1, 1/H_2)$ and $\{B_s\}_{0 \leq s \leq 1}$ be the scale of Banach spaces as defined in Definition 2.2 with constants θ_0 and θ_1. Then it is easy to check that $\xi(t), \zeta(t) \in B_s(K)$ for all $s \in [0,1]$ and all hypotheses (H1)-(H4) of Theorem 2.1 are satisfied with $C_i = 1$, for $i = 1,2,3,4$, and $C_5 = (4/e)^2\theta_0/(\theta_1-\theta_0)^2$ which can be made as small as we wish by taking the constant θ_0 sufficiently small while keeping the constant θ_1 fixed in the definition of $B_s(K)$. By Theorem 2.1, there exists a constant $\rho > 0$ such that (2.1)-(2.2) has a solution $u(x,\cdot) \in B_0$ for $|x| < \rho$.

According to the proof of the nonlinear Cauchy-Kowalevski Theorem in [6], the length of the x-interval of existence ρ is any constant satisfying

$$\frac{123(2\rho)^3}{1-r}[R_{3,0}+16\rho R_{4,0}] < \frac{R_1}{2},$$
$$\frac{16(2\rho)^2}{1-r}[R_{3,0}+16\rho R_{4,0}] < \frac{R_2}{2},$$
$$\frac{8192(2\rho)^5 C_5}{1-r}[R_{3,0}+16\rho R_{4,0}] < \frac{R_3-R_{3,0}}{2},$$
$$\frac{2048(2\rho)^4 C_5}{1-r}[R_{3,0}+16\rho R_{4,0}] < \frac{R_4-R_{4,0}}{2},$$
$$\frac{2048(2\rho)^2 C_5}{1-16r}[R_{3,0}+16\rho R_{4,0}] < \frac{R_5}{2},$$

where $r = 4096(2\rho)^2 C_5$ and $R_{i,0}$ is the bound for the $\|\cdot\|_s$-norm of the Cauchy data for every $s \in [0,1]$ and R_i is any constant greater than $R_{i,0}$ for $i = 1, 2, \cdots, 5$.

By choosing R_i large enough for $i = 1, 2, \cdots, 5$ and taking the constant C_5 small enough, the x-interval of existence ρ can be greater than 1. ■

3. Existence of Boundary Controller

In this section, we shall prove the existence of the boundary controllers $g(t)$ and $h(t)$ that steer a prescribed initial data w_0 to the zero for the problem (1.1)–(1.4). The controllers $g(t)$ and $h(t)$ will be continuously differentiable on a finite time duration $0 \cdot \ t \cdot \ T$ with $T > 0$.

The proof of the following theorem is similar to that of Theorem 2.1 in the paper of D. Kinderlehrer and L. Nirenberg[**?**] with some modification. We omit the proof. Also see [10] for more details for a second order parabolic equation.

Theorem 3.1. *Let $v(x,t) \in C^\infty([0,1]\times[0,1])$ be a solution of the problem*

$$v_t + v_{xxxx} = 0 \quad \textit{on } 0 < x < 1, t > 0,$$
$$v(0,t) = 0, v_x(0,t) = 0 \quad \textit{for } t > 0.$$

Then for each σ, $0 < \sigma < \frac{1}{2}$, $v(x,t)$ is of Gevrey class 2 in x and t in

$$\{(x,t) : 0 \cdot \ \ x < 1-\sigma, \sigma < t < 1\},$$

that is, the derivatives of v satisfy

$$|\partial_x^k \partial_t^j v| \cdot \ \ CH^{2k+2j}(2k+2j)!,$$

for some positive constants C, H *and for all* $k = 0, 1, 2, \cdots$, *and* $j = 0, 1, 2, \ldots$.

Now, we state the principal result of this section.

Theorem 3.2. *Let the initial data* $w_0(x)$ *be a continuous function in* $[0, 1]$ *and vanish at* 0. *Then for any finite time* $T > 0$, *there exist controllers* $g(t)$, $h(t) \in C^\infty((0, \infty)) \cap C([0, \infty))$ *such that the solution* $w(x, t)$ *of* (1.1)–(1.4) *satisfies* $w(x, T) \equiv 0$ *for* $x \in [0, 1]$.

Proof. We organize the proof in the following steps.

Step 1. Extend the domain of the initial data $w_0(x)$ to be $[0, 2]$ so that $w_0(x)$ is continuous and $w_0(x) \equiv 0$ in a neighborhood of 2.

Step 2. We solve the initial-boundary value problem with the new modified initial condition:

$$w_t + w_{xxxx} = 0 \quad \text{on } (0, 2) \times (0, \infty), \tag{3.1}$$

$$w(0, t) = 0, w_x(0, t) = 0 \quad \text{for } t \geq 0, \tag{3.2}$$

$$w(2, t) = 0, w_x(2, t) = 0 \quad \text{for } t \geq 0, \tag{3.3}$$

$$w(x, 0) = w_0(x) \quad \text{for } x \in (0, 2). \tag{3.4}$$

It is well-known that the solution $w(x, t)$ exists [3, 9]. Let $T > 0$ be any given finite time and $\epsilon < T$ be any small positive number. Then it is clear that the solution $w(x, t)$ is a C^∞ function for $0 \le x \le 1$ and $\epsilon \le t \le T$.

Step 3. We claim that $w_{xx}(0, t)$ and $w_{xxx}(0, t)$ belong to Gevrey class 2 in t for $\epsilon \le t \le T$ where $w(x, t)$ is the solution obtained in Step 2.

Let $u_0(x) = w(x, \epsilon)$, where $\epsilon < T$ is any small positive number as in the Step 2. Since $w(x, t)$ is a $C^\infty([0, 1] \times [\epsilon, T])$ solution of the problem

$$w_t + w_{xxxx} = 0 \quad \text{on } (0, 1) \times (\epsilon, T],$$
$$w(0, t) = 0 \quad \text{for } \epsilon \le t \le T,$$
$$w(x, \epsilon) = u_0(x) \quad \text{for } x \in (0, 1),$$

it follows from Theorem 3.1 that $w(x, t)$ is of Gevrey class 2 in t for $0 \le x \le \epsilon$ and $\epsilon \le t \le T$. Thus $w_{xx}(0, t)$ and $w_{xxx}(0, t)$ belong to the Gevrey class 2 in t for $\epsilon \le t \le T$.

Step 4. Next, we modify $w_{xx}(0,t)$ and $w_{xxx}(0,t)$ to be functions $w_{xx}(0,t)\psi(t)$ and $w_{xxx}(0,t)\psi(t)$ with support in $[0,T]$. Here $\psi(t) \in C^{\infty}[0,\infty)$ satisfying

$$\begin{aligned} &0 \cdot \ \psi(t) \cdot \ 1, \\ &\psi(t) = 0 \quad \text{for } t \geq T, \\ &\psi(t) = 1 \quad \text{for } 0 \cdot \ t \cdot \ T/2. \end{aligned}$$

With some cares we choose $\psi(t)$ to be of Gevrey class 2, cf. [7].

Let

$$\xi(t) = \begin{cases} w_{xx}(0,t)\psi(t) & \text{for } \epsilon \cdot \ t \cdot \ T, \\ 0 & \text{for } t \geq T, \end{cases}$$

and

$$\zeta(t) = \begin{cases} w_{xxx}(0,t)\psi(t) & \text{for } \epsilon \cdot \ t \cdot \ T, \\ 0 & \text{for } t \geq T. \end{cases}$$

Since the Gevrey class of functions forms an algebra which is closed under multiplication, $\xi(t), \zeta(t) \in \gamma^2$ in t for $t \geq \epsilon$ and vanish for $t \geq T$.

Step 5. In this step, we solve the Cauchy problem:

$$u_{xxxx} = -u_t \quad \text{on } (0,2) \times (\epsilon,\infty), \tag{3.5}$$

$$u(0,t) = 0, u_x(0,t) = 0, u_{xx}(0,t) = \xi(t), u_{xxx}(0,t) = \zeta(t) \quad \text{for } t \geq \epsilon. \tag{3.6}$$

It follows from Theorem 2.2 that there exist a constant $\rho > 1$ and a classical solution $u(x,t)$ of (3.5)–(3.6) for $0 < x < \rho$, $t \geq \epsilon$. This solution vanishes for $t \geq T$ by Nirenberg's Theorem [11].

Step 6. By L. Nirenberg's Theorem [11], it is easy to derive that $w(x,t)$ and $u(x,t)$ are identical on $[0,1] \times [\epsilon, T/2]$. Now, the required boundary controllers $g(t)$ and $h(t)$ are defined as $g(t) = w(1,t)$, $h(t) = w_x(1,t)$ for $0 \cdot \ t \cdot \ \epsilon$ and $g(t) = u(1,t)$, $h(t) = u_x(1,t)$ for $t \geq \epsilon$.

This proves the theorem. ∎

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