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ON THE RECURSIVE SEQUENCE $x_{n+1} = x_{n-1}/g(x_n)$

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Abstract. In [5] the following problem was posed. Is there a solution of the following difference equation

$$x_{n+1} = \frac{\beta x_{n-1}}{\beta + x_n}, \quad x_{-1}, x_0 > 0,\ \beta > 0, \quad n = 0, 1, 2, ...$$

such that $x_n \to 0$ as $n \to \infty$.

We prove a result which, as a special case, solves the above problem.

1. Introduction

Recently there has been a lot of interest in studying the global attractivity, the boundedness character and the periodic nature of nonlinear difference equations. For some recent results concerning, among other problems, the periodic nature of scalar nonlinear difference equations see, for example, [1-6] and [8]. In [3] and [7] two closely related global convergence results were established which can be applied to nonlinear difference equations in proving that every solution of these difference equations converges to a period-two solution (which is not the same for all solutions).

The following question is posed in [5].

Open problem. *Is there a solution of the following difference equation*

$$x_{n+1} = \frac{\beta x_{n-1}}{\beta + x_n}, \quad x_{-1}, x_0 > 0,\ \beta > 0, \quad n = 0, 1, 2, \ldots \tag{1}$$

such that $x_n \to 0$ *as* $n \to \infty$.

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Note that we can assume that $\beta = 1$, i.e. we can consider the equation

$$x_{n+1} = \frac{x_{n-1}}{1 + x_n}, \quad x_{-1}, x_0 > 0, \quad n = 0, 1, 2, \dots \ . \tag{2}$$

We independently arrived at a problem which was similar to a problem studied by the authors in [4]. They examined the behaviour of the following sequence:

$$x_{n+1} = x_{n-1} e^{-x_n}, \quad x_{-1}, x_0 > 0, \quad n = 0, 1, 2, \dots \ .$$

This similarity motivated us to consider a class of sequences which generalize their sequence and the sequence of Eq. (1).

In this paper we give an affirmative ansver to the Open problem. Moreover, we generalize this result to the equation of the following form:

$$x_{n+1} = \frac{x_{n-1}}{g(x_n)}, \quad x_{-1}, x_0 > 0, \quad n = 0, 1, 2, \dots \ . \tag{3}$$

2. On The Recursive Sequence $x_{n+1} = x_{n-1}/(1 + x_n)$

In this section we consider Eq. (2).

Theorem 1. *Consider the difference equation* (2). *Then the following statements are true.*

(a) *The sequences* (x_{2n}) *and* (x_{2n+1}) *are decreasing and there exist* $p, q \geq 0$ *such that*

$$\lim_{n\to\infty} x_{2n} = p \quad and \quad \lim_{n\to\infty} x_{2n+1} = q.$$

(b) $(q, p, q, p, ...)$ *is a solution of Eq.* (2) *of period two.*

(c) $pq = 0$.

(d) *If there exists* $n_0 \in \mathbf{N}$ *such that* $x_n \geq x_{n+1}$ *for all* $n \geq n_0$, *then* $\lim_{n\to\infty} x_n = 0$.

(e) *The following formulae*

$$\begin{aligned} x_{2n} &= x_0 \left(1 - x_1 \sum_{j=1}^{n} \prod_{i=1}^{2j-1} \frac{1}{1 + x_i} \right) \\ x_{2n+1} &= x_{-1} \left(1 - \frac{x_0}{1 + x_0} \sum_{j=0}^{n} \prod_{i=1}^{2j} \frac{1}{1 + x_i} \right) \end{aligned}$$

hold.

(f) *If* $x_0 + x_0^2 \quad x_{-1}$ *then* $x_{2n} \to p \neq 0$ *and* $x_{2n+1} \to 0$ *as* $n \to \infty$.

(g) *If a solution of Eq.* (2) *converges to zero it must be decreasing.*

Proof. (a)-(c) Since $x_{n+1} < x_{n-1}$, $n \in \mathbf{N}$, we obtain that there exist $\lim_{n\to\infty} x_{2n} = p$ and $\lim_{n\to\infty} x_{2n+1} = q$. Hence $p = p/(1+q)$ or $q = q/(1+p)$, and consequently $pq = 0$, as desired.

(d) If there exists $n_0 \in \mathbf{N}$ such that $x_n \geq x_{n+1}$ for all $n \geq n_0$, then $0 \quad p \quad q \quad p$. Since $pq = 0$ we obtain the result.

(e) Subtracting x_{n-1} from the left and right-hand sides in Eq. (2) we obtain

$$x_{n+1} - x_{n-1} = -\frac{x_{n-1}x_n}{1+x_n}.$$

Since

$$x_n + x_{n-1}x_n = x_{n-2}$$

we get

$$x_{n+1} - x_{n-1} = \frac{1}{1+x_n}(x_n - x_{n-2}).$$

From that we have that the signum of $x_n - x_{n-2}$ remains the same for all $n \geq 2$. Also the following formula

$$x_{n+1} - x_{n-1} = (x_1 - x_{-1})\prod_{i=1}^{n}\frac{1}{1+x_i} \tag{4}$$

holds.

Replacing n by $2j$ in (4) and summing from $j=0$ to $j=n$ we obtain

$$x_{2n+1} - x_{-1} = (x_1 - x_{-1})\sum_{j=0}^{n}\prod_{i=1}^{2j}\frac{1}{1+x_i}.$$

From that the second formula in (e) follows. Proof of the first formula is similar and will be omitted.

(f) Suppose that $p = q = 0$. By (e) we have

$$\frac{1}{x_1} = \sum_{j=1}^{\infty}\prod_{i=1}^{2j-1}\frac{1}{1+x_i} \quad \text{and} \quad \frac{1+x_0}{x_0} = \sum_{j=0}^{\infty}\prod_{i=1}^{2j}\frac{1}{1+x_i}. \tag{5}$$

Since

$$\frac{1+x_0}{x_{-1}} = \frac{1}{x_1} = \sum_{j=1}^{\infty}\prod_{i=1}^{2j-1}\frac{1}{1+x_i} > \sum_{j=1}^{\infty}\prod_{i=1}^{2j}\frac{1}{1+x_i} = \frac{1+x_0}{x_0} - 1 = \frac{1}{x_0},$$

we arrive at a contradiction.

(g) By shifting we obtain that if $x_{n_0+1}+x_{n_0+1}^2 \quad x_{n_0}$ for some $n_0 \in \mathbf{N}$ then $p=0$ and $q \neq 0$, or $q=0$ and $p \neq 0$. Therefore $x_n < x_{n+1}+x_{n+1}^2$ for each $n \in \mathbf{N}$ which is equivalent to

$$x_{n+2}=\frac{x_n}{1+x_{n+1}}<x_{n+1}, \quad \text{for all} \quad n \in \mathbf{N}.$$

Remark 1. By Theorem 1 we see that showing whether or not a solution of Eq. (2) converges to zero is equivalent to showing that (5) holds.

It is clear that we have to consider the following functional sequence $(x_n(u,v))$, $u,v \in (0,\infty)$, where $x_n(u,v)$ denotes the n-th term of the solution of Eq. (2) with initial conditions $x_{-1}=u$ and $x_0=v$. For our purpose we can consider $x_n(u,v)$ as a function of the argument v, i.e., we will take $u>0$ to be fixed, and use simply the notation $x_n(v)$.

According to Theorem 1 (e) and (f), the following sets play a fundamental role in solving the open problem (see also [4]).

$$G_n=\left\{v \in (0,+\infty) : \begin{array}{c} x_k(v)<x_{k+1}(v)(1+x_{k+1}(v)) \text{ for } k=-1,0,1,...,n-1 \\ \text{and} \\ x_n(v) \geq x_{n+1}(v)(1+x_{n+1}(v)) \end{array}\right.$$

and

$$H=\{v \in (0,+\infty) : x_k(v)<x_{k+1}(v)(1+x_{k+1}(v)) \quad \text{for all} \quad k=-1,0,1,\dots\}.$$

3. On The Recursive Sequence $x_{n+1}=x_{n-1}/g(x_n)$

In this section we consider Eq. (3) where the function $g(x)$ satisfies the following conditions

(a) $g \in C^1(\mathbf{R}_+)$;

(b) $g(0)=1$;

(c) $g'(x)>0, \quad$ for $\quad x \in \mathbf{R}_+$.

Hence, $g(x)>1$ for $x \in \mathbf{R}_+ \setminus \{0\}$ and consequently the equation $x=x/g(x)$ has only solution $x=0$. Therefore $x=0$ is the only non-negative equilibrium solution of Eq. (3).

Note that for the case of Eq. (2), $g(x)=1+x$.

It is clear that if $x_{-1} = x_0 = 0$, then $x_n = 0$ for all $n \in \mathbf{N}$. On the other hand, if $x_{-1} = 0$ and $x_0 \neq 0$, or $x_{-1} \neq 0$ and $x_0 = 0$, we obtain that (x_n) is a two-periodic solution

$$(x_{-1}, x_0, x_{-1}, x_0, x_{-1}, x_0, ...).$$

Finally if $x_{-1}, x_0 > 0$, then $x_n > 0$ for all $n \in \mathbf{N}$.

Let $(x_n(v))$, with $v \in (0, \infty)$, denote the solution of Eq. (3) with initial conditions $x_{-1} = u$ and $x_0 = v$.

Motivated by the previous section we introduce the following sets.

$$\mathcal{G}_n = \left\{ v \in (0, +\infty) : \begin{array}{c} x_k(v) < x_{k+1}(v)g(x_{k+1}(v)) \quad \text{for} \quad k = -1, 0, 1, ..., n-1 \\ \text{and} \\ x_n(v) \geq x_{n+1}(v)g(x_{n+1}(v)) \end{array} \right.$$

and

$$\mathcal{H} = \{v \in (0, +\infty) : x_k(v) < x_{k+1}(v)g(x_{k+1}(v)) \quad \text{for all} \quad k = -1, 0, 1, ...\}.$$

It is clear that the following statements are true.

1. $\mathcal{G}_i \cap \mathcal{G}_j = \emptyset$ for all $i, j \in \{-1, 0, ...\}$ with $i \neq j$.
2. $\mathcal{G}_i \cap \mathcal{H} = \emptyset$ for all $i \in \{-1, 0, ...\}$.
3. $(\cup_{n=0}^{\infty} \mathcal{G}_{2n}) \cup (\cup_{n=0}^{\infty} \mathcal{G}_{2n-1}) \cup \mathcal{H} = (0, \infty)$.

Let

$$\mathcal{U} = \cup_{n=0}^{\infty} \mathcal{G}_{2n-1} \quad \text{and} \quad \mathcal{V} = \cup_{n=0}^{\infty} \mathcal{G}_{2n}.$$

One can easily prove the following theorem.

Theorem 2. *Suppose that (x_n) is a solution of Eq. (3) with $x_{-1}, x_0 > 0$. Then the following statements are true.*

(a) *The sequences (x_{2n}) and (x_{2n+1}) are decreasing and there exist $p, q \geq 0$ suchthat*

$$\lim_{n \to \infty} x_{2n} = p \quad \text{and} \quad \lim_{n \to \infty} x_{2n+1} = q.$$

(b) $(q, p, q, p, ...)$ is a solution of Eq. (3) of period two.

(c) $pq = 0$.

(d) If there exists $n_0 \in \mathbf{N}$ such that $x_n \geq x_{n+1}$ for all $n \geq n_0$, then $\lim_{n \to \infty} x_n = 0$.

Theorem 3. *Suppose that* (x_n) *is a solution of Eq.* (3) *with* $x_{-1}, x_0 > 0$. *Assume that there exists* $n_0 \in \mathbf{N}$ *such that*

$$x_{n_0-1} \geq x_{n_0} g(x_{n_0}). \tag{6}$$

Then for all $n \geq 0$,

$$x_{n_0+2n} < x_{n_0+2n+1} g(x_{n_0+2n+1})$$

and

$$x_{n_0+2n+1} > x_{n_0+2n+2} g(x_{n_0+2n+2}).$$

Proof. It suffices to show that

$$x_{n_0} < x_{n_0+1} g(x_{n_0+1})$$

and

$$x_{n_0+1} > x_{n_0+2} g(x_{n_0+2}).$$

From (3), (6) and the fact that $g(x)$ is increasing, we obtain

$$\begin{aligned} x_{n_0+1} g(x_{n_0+1}) &= \frac{x_{n_0-1}}{g(x_{n_0})} g(x_{n_0+1}) \\ &\geq x_{n_0} g(x_{n_0+1}) \\ &> x_{n_0} g(0) = x_{n_0}, \end{aligned}$$

and consequently

$$x_{n_0+1} > x_{n_0+2}. \tag{7}$$

Applying (7), (3) and (6), consecutively, we obtain

$$x_{n_0+2} g(x_{n_0+2}) < x_{n_0+2} g(x_{n_0+1}) = x_{n_0} \quad \frac{x_{n_0-1}}{g(x_{n_0})} = x_{n_0+1},$$

as desired.

Theorem 4. *Let* $n \in \{0, 1, ...\}$. *Then the following statements are true.*

(a) *Suppose* $v \in \mathcal{G}_{2n-1}$. *Then* $\lim_{n\to\infty} x_{2n}(v) = 0$.

(b) *Suppose* $v \in \mathcal{G}_{2n}$. *Then* $\lim_{n\to\infty} x_{2n+1}(v) = 0$.

(c) *Suppose* $v \in \mathcal{H}$. *Then* $x_{-1} = u, x_0 g(x_0) > u, x_0 > x_1 > x_2 > ...$ *and* $\lim_{n\to\infty} x_n(v) = 0$.

(d) $\mathcal{U} \neq \emptyset$.

(e) $\mathcal{V} \neq \emptyset$.

(f) *$\mathcal{U}$ and $\mathcal{V}$ are open subset of* $(0, \infty)$.

Proof. (a) Suppose $v \in \mathcal{G}_{2n-1}$. We know that $x_{2n-1}(v) \geq x_{2n}(v)g(x_{2n}(v))$. By Theorem 3 it follows that

$$x_{2k+1}(v) \geq x_{2k+2}(v)g(x_{2k+2}(v)) > x_{2k+2}(v) \quad \text{for all} \quad k \geq n-1.$$

By Theorem 2 (c) the result follows.

(b) Suppose $v \in \mathcal{G}_{2n}$. We know that $x_{2n}(v) \geq x_{2n+1}(v)g(x_{2n+1}(v))$. By Theorem 3 it follows that

$$x_{2k}(v) \geq x_{2k+1}(v)g(x_{2k+1}(v)) > x_{2k+1}(v) \quad \text{for all} \quad k \geq n.$$

By Theorem 2 (c) the result follows.

(c) Suppose $v \in \mathcal{H}$. We know that $x_n(v) < x_{n+1}(v)g(x_{n+1}(v))$ for all $n = -1, 0, 1, \ldots$ and therefore

$$x_{n+2}(v) < x_{n+1}(v) \quad \text{for all} \quad n = -1, 0, 1, \ldots$$

By Theorem 2 (d) the result follows.

(d) We need to find $c \in (0, \infty)$ such that $u = cg(c)$. For a fixed $u \in (0, \infty)$, since the function $w(x) = xg(x)$ is continuous and increasing and $w(0) = 0$ and $w(\infty) = \infty$, we obtain that there is a unique solution $x = c$ of the equation $u = w(x)$. Let $x_{-1} = u$ and $x_0 = c$. Since $x_{-1} = u = cg(c) = x_0 g(x_0)$, we have $c \in \mathcal{G}_0$.

(e) We need to find $d \in (0, \infty)$ such that $dg(d) = ug(u/g(d))$. For a fixed $u \in (0, \infty)$, consider the function

$$w_1(x) = xg(x) - ug(u/g(x)).$$

This function is continuous and increasing and $w_1(0) = -ug(u) < 0$ and $w(\infty) = \infty$. Hence we obtain that there is a unique solution $x = d$ of the equation $w_1(x) = 0$. Let $x_{-1} = u$ and $x_0 = d$. Then

$$x_0 g(x_0) = dg(d) = ug(u/g(d)) > u = x_{-1}.$$

On the other hand

$$x_1 g(x_1) = \frac{u}{g(d)} g\left(\frac{u}{g(d)}\right) = d = x_0.$$

Hence $d \in \mathcal{G}_1$.

(f) Let us prove that $\mathcal{U}$ is open. The proof that $\mathcal{V}$ is open is similar and will be omitted.

Choose $v \in \mathcal{U}$. It suffices to show that there exists $\varepsilon > 0$ such that if $\omega \in (0, \infty)$ and $|\omega - v| < \varepsilon$, then $\omega \in \mathcal{U}$.

There exists $n_0 \geq 0$ such that $v \in \mathcal{G}_{2n_0-1}$, and so

$$x_k(v) < x_{k+1}(v) g(x_{k+1}(v)) \quad \text{for} \quad k = -1, 0, 1, \ldots, 2n_0 - 2$$

and

$$x_{2n_0-1}(v) \geq x_{2n_0}(v) g(x_{2n_0}(v)). \tag{8}$$

It follows by Theorem 3 that for $n \geq 0$,

$$x_{2n_0+2n}(v) < x_{2n_0+2n+1}(v) g(x_{2n_0+2n+1}(v)).$$

and

$$x_{2n_0+2n+1}(v) > x_{2n_0+2n+2}(v) g(x_{2n_0+2n+2}(v)).$$

For $n = -1, 0, \ldots$, let $f_n : (0, \infty) \to (0, \infty)$ be defined as follows:

$$f_{-1}(\omega) = u \quad \text{for} \quad \omega \in (0, \infty)$$

$$f_0(\omega) = \omega \quad \text{for} \quad \omega \in (0, \infty)$$

and for $n \geq 1$

$$f_n(\omega) = \frac{f_{n-2}(\omega)}{g(f_{n-1}(\omega))} \quad \text{for} \quad \omega \in (0, \infty).$$

Then for each $n \geq 1$, $f_n(\omega) \in C^1(\mathbf{R}_+)$, and

$$x_n(\omega) = f_n(\omega), \quad \text{for} \quad n = -1, 0, \ldots .$$

Note that for $\omega \in (0, \infty)$

$$f_{-1}(\omega) = u \quad \text{and} \quad f'_{-1}(\omega) = 0 \quad 0$$

$$f_0(\omega) = \omega \quad \text{and} \quad f'_0(\omega) = 1 > 0$$

and for $n \geq 1$,

$$f'_{2n-1}(\omega) = \frac{f'_{2n-3}(\omega) g(f_{2n-2}(\omega)) - g'(f_{2n-2}(\omega)) f'_{2n-2}(\omega) f_{2n-3}(\omega)}{g^2(f_{2n-2}(\omega)} < 0$$

and

$$f'_{2n}(\omega) = \frac{f'_{2n-2}(\omega)g(f_{2n-1}(\omega)) - g'(f_{2n-1}(\omega))f'_{2n-1}(\omega)f_{2n-2}(\omega)}{g^2(f_{2n-1}(\omega)} > 0.$$

For $n = -1, 0, \dots$, let $h_n : (0, \infty) \to (0, \infty)$ be a C^1 function given by

$$h_n(\omega) = f_n(\omega) - f_{n+1}(\omega)g(f_{n+1}(\omega)) \quad \text{for} \quad \omega \in (0, \infty).$$

That is

$$h_n(\omega) = x_n(\omega) - x_{n+1}(\omega)g(x_{n+1}(\omega)).$$

By (8), we have the following two cases to consider.

Case 1. Suppose that $x_{2n_0-1}(v) > x_{2n_0}(v)g(x_{2n_0}(v))$.

Then it follows by the continuity of $h_{-1}, h_0, \dots , h_{2n_0-1}$ that there exists $\varepsilon > 0$ such that if $\omega \in (0, \infty)$ and $|\omega - v| < \varepsilon$, then

$$h_i(\omega) < 0 \quad \text{for} \quad i = -1, 0, \dots , 2n_0 - 2$$

while

$$h_{2n_0-1}(\omega) > 0,$$

and so if $\omega \in (0, \infty)$ and $|\omega - v| < \varepsilon$, we see that $\omega \in \mathcal{G}_{2n_0-1} \subset \mathcal{U}$.

Case 2. Suppose that $x_{2n_0-1}(v) = x_{2n_0}(v)g(x_{2n_0}(v))$.

Note that for $\omega \in (0, \infty)$,

$$h'_{2n_0-1}(\omega) = f'_{2n_0-1}(\omega) - g(f_{2n_0}(\omega))f'_{2n_0}(\omega) - g'(f_{2n_0}(\omega))f'_{2n_0}(\omega)f_{2n_0}(\omega) < 0,$$

and so it follows, by the continuity of $h_{-1}, h_0, ..., h_{2n_0-2}$, the differentiability of h_{2n_0-1}, and continuity of h_{2n_0} and h_{2n_0+1}, that there exists $\varepsilon > 0$ such that if $\omega \in (0, \infty)$ and $|\omega - v| < \varepsilon$, then

$$h_i(\omega) < 0 \quad \text{for} \quad i = -1, 0, ..., 2n_0 - 2,$$

$h_{2n_0-1}(\omega) > 0$ if $v - \varepsilon < \omega < v$ and $h_{2n_0-1}(\omega) < 0$ if $v < \omega < v + \varepsilon$,

$$h_{2n_0}(\omega) < 0 \quad \text{and} \quad h_{2n_0+1}(\omega) > 0.$$

It follows that $\omega \in \mathcal{G}_{2n_0-1} \subset \mathcal{U}$ if $v - \varepsilon < \omega < v$ and $\omega \in \mathcal{G}_{2n_0+1} \subset \mathcal{U}$ if $v < \omega < v + \varepsilon$.

We are now in a position to formulate and prove the main result.

Theorem 5. *Let $u \in (0, \infty)$. Then there exists a solution (x_n) of Eq. (3) with $x_{-1} = u$ and $x_0 g(x_0) > u$ such that $x_0 > x_1 > x_2 > ...$ and $\lim_{n\to\infty} x_n = 0$.*

Proof. Since

$$(\cup_{n=0}^{\infty}\mathcal{G}_{2n}) \cup (\cup_{n=0}^{\infty}\mathcal{G}_{2n-1}) \cup \mathcal{H} = (0,\infty),$$

$\mathcal{U}$ and $\mathcal{V}$ are open subsets of $(0,\infty)$, and $(0,\infty)$ is connected, we must have $\mathcal{H} \neq \emptyset$.

The following corollary solves the open problem.

Corollary 1. *Let* $u \in (0,\infty)$. *Then there exists a solution* (x_n) *of Eq.* (2) *with* $x_0(1+x_0) > x_{-1}$, *such that* $x_0 > x_1 > x_2 > \ldots$ *and* $\lim_{n\to\infty} x_n = 0$.

In particular, we have the following.

Corollary 2. *Let* $u \in (0,\infty)$. *Then there exists a solution* (x_n) *of Eq.* (2) *such that* (5) *holds.*

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