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CONTINUITY AND BOUNDEDNESS FOR OPERATOR-VALUED MATRIX MAPPINGS

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Abstract. Let $E(X)$ and $F(Y)$ be vector-valued sequence spaces and A be an operator-valued infinite matrix which maps $E(X)$ into $F(Y)$. In this paper, we establish the continuity and boundedness results for matrix A which generalize the scalar results.

Let X, Y be Hausdorff topological vector spaces (TVS) and $L(X, Y)$ be the space of all continuous linear operators from X into Y. Let $S(X)$ be the vector space of all X-valued sequences, where the operations of addition and scalar multiplication are coordinatewise. Let $E(X)$ be a topological vector space which is a subspace of $S(X)$. If $x \in E(X)$, the kth coordinate of x will be denoted by x_k, i.e., $x = (x_k)$, and the coordinate function $x \mapsto x_k$ will be denoted by Q_k. We call $E(X)$ a $K(X)$-space if each Q_k is continuous; if X is the scalar field and the coordinate functionals are continuous, $E(X)$ is called a K-space.

If $x \in X$ and e_j is the scalar sequence with 1 in the jth coordinate and 0 elsewhere, we write $e_j \otimes x$ for the X-valued sequence with x in the jth coordinate and 0 elsewhere. Let c_{00}(X) be the linear span of $\{e_j \otimes x : j \in \mathbb{N},\ x \in X\}$ in $S(X)$, i.e., $c_{00}(X)$ is the subspace of all X-valued sequences with only a finite number of non-zero coordinates. For each n, let P_n be the section map $E(X) \to E(X)$ which sends $x = (x_1, x_2, \ldots)$ to $(x_1, x_2, \ldots, x_n, 0, \ldots)$. If X is the scalar field and $E(X)$ is a $K(X)$-space, then it is easily seen that each section map P_n is continuous.

Let $E(X)^{\beta Y}$ be the space of all sequences $T = (T_k) \subseteq L(X, Y)$ such that the series $\sum_{k=1}^{\infty} T_k x_k$ converges in Y for all $x = (x_k) \in E(X)$. We write $T \cdot x = \sum_{k=1}^{\infty} T_k x_k$ when $T \in E(X)^{\beta Y},\ x \in E(X)$. If X and Y are the scalar fields, we

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write $E(X) = E$ and $E(X)^{\beta Y} = E^\beta$. If $E \supseteq c_{00}$, E and E^β are in duality with respect to the bilinear pairing $y \cdot x, y \in E^\beta$, $x \in E$. We denote the weak (strong) topology on E from this pairing by $\sigma(E, E^\beta)(\beta(E, E^\beta))$; similar notation is used for the weak (strong) topology on E^β.

Let $A = [A_{ij}]$ be an infinite matrix with $A_{ij} \in L(X, Y)$. We say that the matrix A maps $E(X)$ into $F(Y)$ if $\sum\limits_{j=1}^{\infty} A_{ij} x_j$ converges for each $i \in \mathbb{N}$, $x \in E(X)$, and $Ax = \Big(\sum\limits_{j=1}^{\infty} A_{ij} x_j\Big) \in F(Y)$. We write $M(E(X),\ F(Y))$ for the vector space of all matrices which map $E(X)$ into $F(Y)$. If $A \in M(E(X),\ F(Y))$, then $A_i = (A_{i1}, A_{i2}, \ldots, A_{ij}, \ldots) \in E(X)^{\beta Y}$.

The classical Hellinger-Toeplitz Theorem asserts that a matrix which maps l^2 into l^2 is (norm) continuous. The result was extended to normal sequence spaces by Köthe and Toeplitz ([3], 30.7. (7), [4]) and to FK-spaces by Zeller [15]. Zeller's result was extended to vector-valued FK-spaces where the sequences have values in a Frechet space by Baric [1]. Recently, Swartz [12] established several continuity and boundedness results for matrix mappings between real-valued sequence spaces which serve as a complement to the results of Köthe, Toeplitz and Zeller. In this note, we consider continuity and boundedness conditions for operator-valued matrix mappings between vector-valued sequence spaces. Our vector results give generalizations of scalar results of Swartz [12].

If $i, j \in \mathbb{N}$ with $i \leq j$, let $[i, j] = \{k \in \mathbb{N} : i \leq k \leq j\}$ be the interval in $\mathbb{N}$ induced by i and j. If $\{I_j\}$ is a sequence of intervals in $\mathbb{N}$ with max $I_j < \min I_{j+1}$ for all j, we call $\{I_j\}$ an increasing sequence of intervals. If $\Delta \subseteq \mathbb{N}$, let C_Δ be the characteristic function of Δ, and if $x \in E(X)$, let $C_\Delta x$ be the pointwise product of C_Δ and x. Following ([5-6, 12]), $E(X)$ is said to have the zero Gliding Hump Property (0-GHP) if whenever $x^k \to 0$ in $E(X)$ and $\{I_k\}$ is an increasing sequence of intervals, there exists a subsequence $\{p_k\}$ such that $z = \sum\limits_{k=1}^{\infty} C_{I_{p_k}} x^{p_k} \in E(X)$, where the sum of the series is understood to be pointwise. There are many sequence spaces with 0-GHP. For example, $l^p (0 < p \leq \infty)$, s, c and c_0 have 0-GHP. Likewise, any FK-AB space has 0-GHP. Klis's example of a dense subspace of l^2 furnishes an example of a sequence space with 0-GHP which is not complete. $(l^1, \sigma(l^1, l^\infty))$ furnishes an example of a non-barrelled space with 0-GHP ([12-13]).

The example below shows that there are $x^k \to 0$ in $E(X)$ and an increasing sequence of intervals $\{I_k\}$ such that $\{C_{I_k} x^k\}$ is not bounded in $E(X)$.

Example 1. Let $E = c_{00}$ with the topology defined by the semi-norms as follows:

$$p_i(x) = |x_i| (i = 1, 2, \dots) \text{ and } q(x) = \sum_{i=1}^{\infty} |x_{2i} - x_{2i-1}|.$$

Pick $x^k = (1, 1, 2, 2, \dots, k, k, 0, 0, \dots)$ and $I_k = \{2k\}$. Then $\frac{x^k}{\sqrt{k}} \to 0$ in E, but $q\Big(C_{I_k} \frac{x^k}{\sqrt{k}}\Big) = \sqrt{k} \to \infty$. So $\Big\{C_{I_k} \frac{x^k}{\sqrt{k}}\Big\}$ is not bounded in $E(X)$.

It follows from Example 1 that the strong Gliding Hump Property (SGHP) in Theorem 1 and its Corollaries in [11] should be replaced by 0-GHP for the vector version. In fact, we can establish a much stronger result for spaces with 0-GHP.

Theorem 2. *Let $T \in E(X)^{\beta Y}$ and assume that $E(X)$ has 0-GHP. If $x^i \to 0$ in $E(X)$, then $\sum_{k=1}^{\infty} T_k x_k^i$ converges uniformly with respect to $i \in \mathbb{N}$.*

Proof. If not, there are a neighbourhood U of 0 in Y and two integer sequences $n_1 \le m_1 < n_2 \le m_2 < n_3 \le m_3 < \cdots$ and $i_1 < i_2 < \cdots$ such that

$$\sum_{k=n_l}^{m_l} T_k x_k^{i_l} \notin U, \quad l = 1, 2, \dots . \tag{1}$$

Let $I_l = \{k | k \in \mathbb{N} : n_l \le k \le m_l\}$. Then $T \cdot C_{I_l} x^{i_l} \notin U, l = 1, 2, \dots$. Since $E(X)$ has 0-GHP, there exists a subsequence $\{l_k\}$ such that $\sum_{k=1}^{\infty} C_{I_{l_k}} x^{i_{l_k}} \in E(X)$. Thus we have $T \cdot C_{I_{l_k}} x^{i_{l_k}} \to 0$. This contradicts (1).

Let $E(X) = S(X)$ and $S(X)$ take the product topology $X^{\mathbb{N}} = X \times X \times X \cdots$. Then $S(X)$ has 0-GHP. From Theorem 2 it follows that for each $T \in S(X)^{\beta Y}$ and $x^i \to 0$ in $S(X)$, the series $\sum_{k=1}^{\infty} T_k x_k^i$ converges uniformly with respect to $i \in \mathbb{N}$. In fact, we can show that $\sum_{k=1}^{\infty} T_k x_k$ converges uniformly with respect to all $x = (x_k) \in S(X)$ ([7], Th. 1). ■

Corollary 3. *Let X and E be Hausdaff topological vector spaces and E be a scalar sequence space which has 0-GHP, for example, $E = l^p (0 < p \le \infty)$, c_0, c. Let $\{y_k\} \subseteq X$. If for each $x = (x_k) \in E$, the series $\sum_{k=1}^{\infty} x_k y_k$ is convergent, then for each $x^i \to 0$ in E, the series $\sum_{k=1}^{\infty} x_k^i y_k$ converges uniformly with respect to $i \in \mathbb{N}$.*

Rolewicz ([10], III. 8) called a series $\sum_{i=1}^{\infty} x_i$ in a metric linear space Z a C-series if the series $\sum_{i=1}^{\infty} t_i x_i$ converges in Z for each $\{t_i\} \in c_0$. These series have been studied in detail in the case of normed spaces and it is known that a Banach space has the property that every C-series is (subseries) convergent if and only if the space contains no subspace (topologically) isomorphic to c_0 ([2]). For sequentially complete locally convex spaces, Li Ronglu and Bu Qingying proved that the conclusion is also true and, indeed, much more holds ([7], Th. 4).

Corollary 4. *Let $T \in E(X)^{\beta Y}$ and assume that $E(X)$ is a $K(X)$-space with 0-GHP. If $x^i \to 0$ in $E(X)$, then $T \cdot x^i \to 0$ in Y, i.e., T is sequentially continuous.*

Proof. For each neighbourhood U of 0 in Y, there exists a neighbourhood V of 0 in Y such that $V + V \subseteq U$. By Theorem 2, there exists $n_0 \in \mathbb{N}$ such that $\sum_{k=n_0+1}^{\infty} T_k x_k^i \in V$ holds for all $i \in \mathbb{N}$. Since $E(X)$ is a $K(X)$-space and $T_k \in L(X,Y)$, so there exists $i_0 \in \mathbb{N}$ such that whenever $i \geq i_0$ we have $\sum_{k=1}^{n_0} T_k x_k^i \in V$. It follows that whenever $i \geq i_0$ we have $\sum_{k=1}^{\infty} T_k x_k^i = \sum_{k=1}^{n_0} T_k x_k^i + \sum_{k=n_0+1}^{\infty} T_k x_k^i \in V + V \subseteq U$, i.e., T is sequentially continuous. ■

Now, we study the continuity and boundedness for operator-valued matrix mappings. Our proofs need a theorem on infinite matrices due to Antosik and Mikusinski. We state this result for the convenience of the reader.

Theorem 5. *Let G be a Hausdorff topological vector space and $x_{ij} \in G$ for $i, j \in \mathbb{N}$. If*

(I) $\lim_i x_{ij} = x_j$ *exists for each j and*

(II) *every increasing sequence of positive integers $\{m_j\}$ has a subsequence $\{n_j\}$ such that the sequence $\left\{\sum_{j=1}^{\infty} x_{in_j}\right\}_i$ converges,*

then $\lim_i x_{ij} = x_j$ uniformly for $j \in \mathbb{N}$. In particular, $\lim_i x_{ii} = 0$.

Theorem 5 has a great number of applications in functional analysis and measure theory ([9], [13-14]). For its proof, see ([8]). A matrix satisfying conditions (I) and (II) is called a **K**-matrix.

Let $X, Y \in TVS$. We will say that the pair (X, Y) has the weak Banach-Steinhaus Property if $\{T_k\} \subseteq L(X,Y)$ and $\lim_k T_k x = Tx$ for each $x \in X$ imply that $T \in L(X,Y)$. For example, if X is an F-space or if X is a barrelled locally convex space and Y is a locally convex space, then (X, Y) has the weak Banach-Steinhaus Property ([13]).

We say that the pair (X, Y) has the Uniform Boundedness (UB) if each pointwise bounded family Γ of $L(X, Y)$ is uniformly bounded on any bounded subset of X. For example, if X is an **A**-space or X is barrelled and Y is a locally convex space, then (X, Y) has the UB ([13]).

Theorem 6. *Let $E(X) \supseteq c_{00}(X)$ be a $K(X)$-space with 0-GHP and (X, Y) have the weak Banach-Steinhaus Property. If $A = [A_{ij}] \in M(E(X),\ F(Y))$, then for each $x^k \to 0$ in $E(X)$ and each $T \in F(Y)^{\beta Y}$, we have*

$$T \cdot Ax^k \to 0 \quad in \quad Y.$$

Proof. If not, there exist a neighbourhood U of 0 in Y, $x^k \to 0$ in $E(X)$ and $T \in F(Y)^{\beta Y}$ such that

$$T \cdot Ax^k \notin U \quad for\ each \quad k \in \mathbb{N}. \tag{2}$$

Take a neighbourhood V of 0 in Y such that $V + V \subseteq U$. Let $k_1 = 1$. We pick m_1 and n_1 such that

$$\sum_{i=1}^{m_1} T_i \sum_{j=1}^{n_1} A_{ij} x_j^{k_1} \notin U.$$

Since $E(X) \supseteq c_{00}(X)$, so for each j and x, $e_j \otimes x \in E(X)$ and hence, $(A_{ij}x)_i \in F(Y)$. Note that $T = (T_1, T_2, \ldots, T_i, \ldots) \in F(Y)^{\beta Y}$. It follows that the series $\sum_{i=1}^{\infty} T_i A_{ij} x$ is convergent. Since $T_i A_{ij} \in L(X, Y)$ and (X, Y) has the weak Banach-Steinhaus Property, it follows that $\sum_{i=1}^{\infty} T_i A_{ij} \in L(X, Y)$ for each $j \in \mathbb{N}$. So we have $\sum_{i=1}^{\infty} T_i \sum_{j=1}^{n_1} A_{ij} x_j^k = \sum_{j=1}^{n_1} \Big(\sum_{i=1}^{\infty} T_i A_{ij} \Big) x_j^k \to 0$. Therefore, there exists $k_2 > k_1$ such that

$$T \cdot AP_{n_1} x^{k_2} \in V. \tag{3}$$

From (2) and (3), $T \cdot A(x^{k_2} - P_{n_1} x^{k_2}) \notin V$. Pick $m_2 > m_1$ and $n_2 > n_1$ such that

$$\sum_{i=1}^{m_2} T_i \sum_{j=n_1+1}^{n_2} A_{ij} x_j^{k_2} \notin V.$$

Continuing this construction produces increasing sequences $\{k_p\}$, $\{m_p\}$ and $\{n_p\}$ such that

$$\sum_{i=1}^{m_p} T_i \sum_{j=n_{p-1}+1}^{n_p} A_{ij} x_j^{k_p} \notin V.$$

Let $I_p = \{j \in \mathbb{N} : n_{p-1} < j \leq n_p\}$. Then $\{I_p\}$ is an increasing sequence of intervals such that

$$\sum_{i=1}^{m_p} T_i \sum_{j \in I_p} A_{ij} x_j^{k_p} \notin V. \tag{4}$$

Consider the matrix $M = [m_{pq}] = \left[\sum\limits_{i=1}^{m_p} T_i \sum\limits_{j \in I_q} A_{ij} x_j^{k_q}\right]$. Since $AC_{I_q} x^{k_q} = \left(\sum\limits_{j \in I_q} A_{ij} x_j^{k_q}\right)_i \in F(Y)$, so $\sum\limits_{i=1}^{m_p} T_i \sum\limits_{j \in I_q} A_{ij} x_j^{k_q} \to T \cdot AC_{I_q} x^{k_q} (p \to \infty)$. Given any increasing sequence $\{r_q\}$, by 0-GHP, there exists a subsequence $\{s_q\}$ of $\{r_q\}$ such that $\tilde{x} = \sum\limits_{q=1}^{\infty} C_{I_{s_q}} x^{k_{s_q}} \in E(X)$. Therefore, $\lim_p \sum\limits_{i=1}^{m_p} T_i \sum\limits_{q=1}^{\infty} \sum\limits_{j \in I_{s_q}} A_{ij} x_j^{k_{s_q}} = \lim_p \sum\limits_{i=1}^{m_p} T_i P_{m_p} A\tilde{x} = T \cdot A\tilde{x}$. Hence, M is a $\mathbf{K}$-matrix, by Theorem 5, $\lim_p m_{pp} = \lim_p \sum\limits_{i=1}^{m_p} T_i \sum\limits_{j \in I_p} A_{ij} x_j^{k_p} = 0$. This contradicts (4). ∎

Corollary 7 ([12], Th. 4). *Let E, F be scalar sequence spaces and $(E,\ \tau) \supseteq c_{00}$ be a K-space with 0-GHP. If $A \in M(E, F)$, then A is $\tau - \sigma(F, F^\beta)$ sequentially continuous.*

Recall that $E(X)$ is said to be an AK space, if for each $x \in E(X)$ we have $P_n x \to x$ in $E(X)$.

Corollary 8. *If $E(X) \supseteq c_{00}(X)$ is an AK space with 0-GHP, (X, Y) has the weak Banach-Steinhaus Property and $A = [A_{ij}] \in M(E(X), F(Y))$, then for each $T = (T_i) \in F(Y)^{\beta Y}$ and $x = (x_j) \in E(X)$, we have $\left(\sum\limits_{i=1}^{\infty} T_i A_{ij}\right) \in E(X)^{\beta Y}$ and*

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} T_i A_{ij} x_j = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} T_i A_{ij} x_j.$$

Proof. From the weak Banach-Steinhaus Property and $E(X) \supseteq c_{00}(X)$, we infer that for each $j \in \mathbb{N}$, there exists $C_j \in L(X, Y)$ such that for each $x_0 \in X$ we have $\sum\limits_{i=1}^{\infty} T_i A_{ij}\, x_0 = C_j x_0$. Now, we show that $(C_j) \in E(X)^{\beta Y}$ and

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} T_i A_{ij} x_j = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} T_i A_{ij} x_j = \sum_{j=1}^{\infty} C_j x_j.$$

In fact, since $E(X)$ is an AK space, so $P_n x \to x$ in $E(X)$. Note that

$$\begin{aligned}
&\lim_n \sum_{j=1}^{n} C_j x_j = \lim_n \sum_{j=1}^{n} \sum_{i=1}^{\infty} T_i A_{ij} x_j \\
&= \lim_n \sum_{i=1}^{\infty} \sum_{j=1}^{n} T_i A_{ij} x_j = \lim_n \sum_{i=1}^{\infty} T_i \sum_{j=1}^{n} A_{ij} x_j \\
&= \lim_n T \cdot A P_n x = T \cdot Ax
\end{aligned}$$

(Theorem 6). So $(C_j) \in E(X)^{\beta Y}$ and

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} T_i A_{ij} x_j = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} T_i A_{ij} x_j = \sum_{j=1}^{\infty} C_j x_j. \qquad \blacksquare$$

We say that the subset D of $F(Y)^{\beta Y}$ is $\sigma(F(Y)^{\beta Y}, F(Y))$ bounded if for each $y = (y_j) \in F(Y)$, $\left\{ \sum_{j=1}^{\infty} T_j y_j | (T_j) \in D \right\}$ is a bounded subset of Y.

Theorem 9. *Let $E(X) \supseteq c_{00}(X)$ be a $K(X)$-space with 0-GHP and suppose that the section projections $P_n : F(Y)^{\beta Y} \to F(Y)^{\beta Y}$ are uniformly bounded on $\sigma(F(Y)^{\beta Y}, F(Y))$ bounded sets with respect to $\sigma(F(Y)^{\beta Y}, F(Y))$. If (X, Y) has the weak Banach -Steinhaus Property and the Uniform Boundedness and $A = [A_{ij}] \in M(E(X), F(Y))$, then for each bounded subset C of $E(X)$ and each bounded subset B of $(F(Y)^{\beta Y}, \sigma(F(Y)^{\beta Y}, F(Y)))$, $\{T \cdot Ax : x \in C,\ T \in B\}$ is a bounded subset of Y.*

Proof. If not, there exist a neighbourhood U of 0 in Y, a bounded subset C of $E(X)$ and $\{x^k\} \subseteq C$, $x^k \to 0$, $\{T^k\} \subseteq B$ and $t_k > 0$, $t_k \to 0$ such that

$$t_k T^k \cdot A x^k \notin U \quad \text{for all } k \in \mathbb{N}. \tag{5}$$

Take a neighbourhood V of 0 in Y such that $V + V \subseteq U$. Set $k_1 = 1$ and pick m_1, n_1 such that $t_{k_1} \sum_{i=1}^{m_1} T_i^{k_1} \sum_{j=1}^{n_1} A_{ij} x_j^{k_1} \notin U$. Since (X, Y) has the weak Banach-Steinhaus Property and Uniform Boundedness and $\{T^k\}$ is $\sigma(F(Y)^{\beta Y}, F(Y))$ bounded, so $\left\{ \sum_{i=1}^{\infty} T_i^k \sum_{j=1}^{n_1} A_{ij} x_j^k \right\} = \left\{ \sum_{j=1}^{n_1} \sum_{i=1}^{\infty} T_i^k A_{ij} x_j^k \right\}$ is a bounded subset of Y. Therefore, $\lim_k\ t_k \sum_{i=1}^{\infty} T_i^k \sum_{j=1}^{n_1} A_{ij} x_j^k = 0$. It follows that there exists $k_2 > k_1$ such that $t_{k_2} T^{k_2} \cdot A P_{n_1} x^{k_2} \in V$. Hence, $t_{k_2} T^{k_2} \cdot A(x^{k_2} - P_{n_1} x^{k_2}) \notin V$. Pick $m_2 > m_1, n_2 > n_1$ such that

$$t_{k_2} \sum_{i=1}^{m_2} T_i^{k_2} \sum_{j=n_1+1}^{n_2} A_{ij} x_j^{k_2} \notin V.$$

Continuing this construction produces increasing sequences $\{k_p\}, \{m_p\}$ and $\{n_p\}$ such that

$$t_{k_p} \sum_{i=1}^{m_p} T_i^{k_p} \sum_{j=n_{p-1}+1}^{n_p} A_{ij} x_j^{k_p} \notin V \quad \text{for all } p \in \mathbb{N}. \tag{6}$$

Let $I_p = \{j \in \mathbb{N} | n_{p-1} < j \leq n_p\}$. Then $\{I_p\}$ is an increasing sequence of intervals such that

$$t_{k_p} \sum_{i=1}^{m_p} T_i^{k_p} \sum_{j \in I_p} A_{ij} x_j^{k_p} \notin V.$$

Denote $M_1 = [m_{pq}] = \Big[t_{k_p} \sum_{i=1}^{m_p} T_i^{k_p} \sum_{j \in I_q} A_{ij} x_j^{k_q}\Big]$. From the fact that the section projections $P_n : F(Y)^{\beta Y} \to F(Y)^{\beta Y}$, uniformly bounded on $\sigma(F(Y)^{\beta Y}, F(Y))$ bounded sets with respect to $\sigma(F(Y)^{\beta Y}, F(Y))$, Uniform Boundedness and $E(X)$ with 0-GHP, it is not difficult to know that M_1 is a **K**-matrix. From Theorem 5 it follows that $\lim_p m_{pp} = \lim_p t_{k_p} \sum_{i=1}^{m_p} T_i^{k_p} \sum_{j \in I_p} A_{ij} x_j^{k_p} = 0$. This contradicts (6). ■

Corollary 10 ([12], Th. 10). *Let $(E, \tau) \supseteq c_{00}$ and F be scalar sequence spaces and E with the 0-GHP. Suppose that the section projections $P_n : F^\beta \to F^\beta$ are uniformly bounded on $\sigma(F^\beta, F)$ bounded sets with respect to $\sigma(F^\beta, F)$. Then $A \in M(E, F)$ is τ-$\beta(F, F^\beta)$ bounded.*

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