TAIWANESE JOURNAL OF MATHEMATICS
Vol. 2, No. 3, pp. 287-295, September 1998

GROWTH CONDITIONS AND BISHOP'S PROPERTY (β)

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Abstract. We show that a certain logarithmic growth condition on a bounded linear operator on a complex Banach space implies Bishop's property (β), and discuss several applications of this result in local spectral theory.

1. Local Spectral Properties Revisited

A bounded linear operator $T \in L(X)$ on a complex Banach space X is said to have *property* (β) if, for every open subset U of the complex plane $\mathbb{C}$ and every sequence of analytic functions $f_n : U \to X$ for which $(T - \lambda)f_n(\lambda) \to 0$ as $n \to \infty$ locally uniformly on U, it follows that $f_n(\lambda) \to 0$ as $n \to \infty$, again locally uniformly on U. In a slightly different, but equivalent version, this condition was introduced by Bishop [5], in an attempt to develop a general duality theory for operators on Banach spaces that should capture some of the important features of the spectral theory of normal operators on Hilbert spaces. The present note is to build on the recent progress in local spectral theory related to Bishop's property (β) and to exhibit some natural classes of operators with this property.

Let $H(U, X)$ denote the Fréchet space of all X-valued analytic functions on an open subset U of $\mathbb{C}$. It is easily seen that an operator $T \in L(X)$ has property (β) if and only if, for each open set $U \subseteq \mathbb{C}$, the operator $T_U : H(U, X) \to H(U, X)$ given by $(T_U f)(\lambda) := (T - \lambda)f(\lambda)$ for all $f \in H(U, X)$ and $\lambda \in U$ is injective and has closed range. This characterization intimates that property (β) should be quite useful in connection with topological tensor products and duality theory, but the precise role of this condition in local spectral was determined only recently by Albrecht and Eschmeier [2].

Received April 28, 1997; revised September 25, 1997.
Communicated by S.-Y. Shaw.
1991 *Mathematics Subject Classification*: Primary 47A11, Secondary 47B40.
Key words and phrases: Local spectral theory, decomposable operator, Bishop's property (β), isometry, Cesàro operator.

To describe the recent development, we need to recall a few notions. An operator $T \in L(X)$ is said to be *decomposable* if, for every open cover $\{U, V\}$ of $\mathbb{C}$, there exist T-invariant closed linear subspaces Y and Z of X for which $Y + Z = X$, $\sigma(T \,|\, Y) \subseteq U$, and $\sigma(T \,|\, Z) \subseteq V$, where σ denotes the spectrum. This simple definition of decomposability is equivalent to the slightly more complicated original definition due to Foiaş. We refer to Colojoară and Foiaş [6], Lange and Wang [13], and Vasilescu [20] for thorough discussions of the theory of decomposable operators.

Given an operator $T \in L(X)$ and a closed set $F \subseteq \mathbb{C}$, let $X_T(F)$ consist of all $x \in X$ for which there exists an analytic function $f : \mathbb{C} \setminus F \to X$ with the property that $(T - \lambda)f(\lambda)X = x$ for all $\lambda \in \mathbb{C} \setminus F$. Evidently, $X_T(F)$ is a linear subspace of X, but need not be closed. The operator T is said to have the *decomposition property* (δ) if $X = X_T(\overline{U}) + X_T(\overline{V})$ for every open cover $\{U, V\}$ of $\mathbb{C}$. As observed in [3], an operator is decomposable if and only if it has both properties (β) and (δ).

More importantly, Albrecht and Eschmeier [2] have recently established that an operator has property (β) precisely when it is similar to the restriction of a decomposable operator to one of its closed invariant subspaces, that property (δ) characterizes the quotients of decomposable operators by closed invariant subspaces, and that (β) and (δ) are, in a natural way, dual to each other: an operator $T \in L(X)$ has property (β) if and only if its adjoint $T^* \in L(X^*)$ on the topological dual space X^* has property (δ), and the same equivalence holds when the roles of (β) and (δ) are interchanged. These characterizations require rather sophisticated tools from the theory of topological tensor products, and complete Bishop's quest for duality results in the axiomatic spectral theory of operators on Banach spaces.

An obvious combination of the preceding results shows that an operator $T \in L(X)$ is decomposable precisely when both T and T^* have property (β). This remarkable characterization of decomposability in purely analytic terms was obtained earlier by Eschmeier and Putinar [9], and, in the reflexive case, also by Lange [12]. It will be a useful tool in some of our applications.

To underline the significance of Bishop's property (β), we mention the important connections to sheaf theory and the spectral theory of several commuting operators from the recent monograph by Eschmeier and Putinar [10]. There are also interesting applications to invariant subspaces [10], harmonic analysis [8], and the theory of automatic continuity [15].

Unfortunately, but perhaps not surprisingly, the direct verification of property (β) in concrete cases tends to be a difficult task. It is therefore desirable to have sufficient conditions for property (β) which are easier to handle. In the following sections, we show that a certain natural growth condition implies property (β), and discuss several consequences.

2. A Logarithmic Growth Condition

Theorem 1. *Let $T \in L(X)$ be an operator on a Banach space X, let D be a closed disc that contains the spectrum $\sigma(T)$ and has non-empty interior, and let V be an open neighbourhood of D. Suppose that there exist a totally disconnected compact subset E of the boundary of D, a locally bounded function $\omega : V \setminus E \to (0, \infty)$, and an increasing function $\gamma : (0, \infty) \to (0, \infty)$ such that $\log \circ \gamma$ has an integrable singularity at zero and*

$$\gamma(\operatorname{dist}(\lambda, \partial D))\|x\| \leq \omega(\lambda)\|(T - \lambda)x\| \quad \textit{for all } x \in X \textit{ and } \lambda \in V \setminus \partial D.$$

Then T has property (β).

Since the increasing function $\log \circ \gamma$ is Riemann integrable on each compact interval in $(0, \infty)$, the integrability assumption of Theorem 1 means that the improper Riemann integral $\int_0^c \log \gamma(t)\, dt$ converges for each $c > 0$. This condition is obviously fulfilled by a polynomial of the form $\gamma(t) = t^m$ for any integer $m \in \mathbb{N}$, and also by an exponential function of the form $\gamma(t) = \exp(-t^{\alpha})$ for any $\alpha \in (-1, 0)$.

The role of the exceptional set E and the function ω is to relax the growth condition near certain points in the boundary of the spectrum. If E is empty, then ω will be a positive constant.

For simplicity, we restrict ourselves, in Theorem 1, to the case of discs, but it should be evident that the method of the following proof works also in more general cases. The case of operators with real spectrum will be treated in Theorem 3 below.

Our main tool will be Jensen's inequality from classical complex analysis: if the complex-valued function h is continuous on the closed unit disc and analytic on the open unit disc, then

$$\log|h(z)| \leq \frac{1}{2\pi} \int_{-\pi}^{\pi} \log\left|he^{it})\right| \operatorname{Re}\frac{e^{it} + z}{e^{it} - z}\, dt \quad \text{for all } z \in \mathbb{C} \text{ with } |z| < 1\,.$$

This estimate follows easily from the Poisson-Jensen formula, and expresses the fact that $\log|h|$ is subharmonic; see Rudin [18] for details. We shall also need the following well-known elementary properties of the Poisson kernel:

$$0 < \frac{1 - |z|}{1 + |z|} \leq \operatorname{Re}\frac{e^{it} + z}{e^{it} - z} \leq \frac{1 + |z|}{1 - |z|} \qquad \text{for } |z| < 1 \text{ and } -\pi \leq t \leq \pi\,.$$

3. Proof of Theorem 1

Without loss of generality, we may assume that D is the closed unit disc. Consider now an open set $U \subseteq \mathbb{C}$ and a sequence of functions $f_n \in H(U,X)$ for which $T_U f_n \to 0$ as $n \to \infty$ locally uniformly on U. Since the resolvent function is locally bounded on the resolvent set $\rho(T) := \mathbb{C} \setminus \sigma(T)$, and since

$$\|f_n(\lambda)\| \leq \left\|(T-\lambda)^{-1}\right\| \|(T-\lambda)f_n(\lambda)\| \qquad \text{for all } \lambda \in U \setminus D,$$

we conclude that $f_n \to 0$ as $n \to \infty$ locally uniformly on $U \setminus D$. Similarly, from the conditions on ω and γ and from

$$\|f_n(\lambda)\| \leq \gamma(\mathrm{dist}(\lambda, \partial D))^{-1}\omega(\lambda)\, \|(T-\lambda)f_n(\lambda)\| \quad \text{for all } \lambda \in U \cap \mathrm{int}\ D,$$

we infer that $f_n \to 0$ locally uniformly also on $U \cap \mathrm{int}\ D$. Hence it remains to be seen that every point in $U \cap \partial D$ has a neighbourhood on which the sequence $(f_n)_{n\in\mathbb{N}}$ converges uniformly to zero.

Given any $\zeta \in U \cap \partial D$, we choose a $\delta \in (0,1)$ for which the closed disc $\nabla(\zeta,\delta) := \{\lambda \in \mathbb{C} : |\lambda - \zeta| \leq \delta\}$ is contained in $U \cap V$, and define $\varepsilon_n := \sup\{\|(T-\lambda)f_n(\lambda)\| : \lambda \in \nabla(\zeta,\delta)\}$ for all $n \in \mathbb{N}$. Then clearly $\varepsilon_n \to 0$ as $n \to \infty$. Also, let $W := \{z \in \mathbb{C} : |e^{iz} - \zeta| \leq \delta\}$, and choose a real number a so that $\zeta = e^{ia}$.

Since E is a totally disconnected compact subset of ∂D, there exists an $r \in (0,1)$ for which $\nabla(a,r) \subseteq W$ and $e^{i(a+r)}$, $e^{i(a-r)} \notin E$. Note that this condition ensures that $e^{iz} \notin E$ for all $z \in \partial\nabla(a,r)$. Because ω is locally bounded on $V \setminus E$, we obtain a constant $c > 0$ with the property that $\omega(e^{iz}) \leq c$ for all $z \in \partial\nabla(a,r)$. Also, by the definition of ε_n, it is clear that $\|(T - e^{iz})f_n(e^{iz})\| \leq \varepsilon_n$ for all $z \in \partial\nabla(a,r)$ and $n \in \mathbb{N}$.

Finally, since an elementary calculation shows that $|u| \leq e\,|e^u - 1|$ for all $u \in [-1,1]$, we obtain, for every $t \in [-\pi/2, \pi/2]$, the estimates

$$\mathrm{dist}(e^{i(a\pm r\, e^{it})}, \partial D) = \Big|\, |\, e^{i(a\pm r\, e^{it})}\,| - 1\Big| = |e^{\mp r \sin t} - 1| \geq \frac{r}{e}\,|\sin t| \geq k\,|t|$$

with the constant $k := (2\,r)/(\pi e)$. For arbitrary $n \in \mathbb{N}$, we may therefore conclude from the main assumption of Theorem 1 that

$$\left\|f_n(e^{i(a\pm r\, e^{it})})\right\| \leq c\,\varepsilon_n\,\gamma\,(k\,|t|)^{-1} \qquad \text{for all non-zero } t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

Now, fix an integer $n \in \mathbb{N}$ for which $\varepsilon_n < 1/c$, and consider an arbitrary functional $\varphi \in X^*$ with $\|\varphi\| \leq 1$. Since Jensen's inequality applies to the function h_n given by $h_n(z) := \varphi(f_n(\mathrm{e}^{i(a+r\,z)}))$ for all $z \in \nabla(0,1)$, we obtain

$$\log\left|\varphi(f_n(e^{i(a+r\,z)}))\right| \leq \frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left\|f_n(e^{i(a+r\,e^{it})})\right\| \mathrm{Re}\frac{e^{it}+z}{e^{it}-z}\,dt$$

for all $z \in \mathbb{C}$ with $|z| < 1$. To proceed further, we restrict z to the condition that $|z| \leq 1/2$, and split the integral on the right-hand side of the last inequality into two parts.

For $|t| \leq \pi/2$, we obtain directly from the preceding estimates that

$$\begin{aligned}
\int_{-\pi/2}^{\pi/2} & \log \quad \left\| f_n(e^{i(a+r\,e^{it})}) \right\| \mathrm{Re}\frac{e^{it}+z}{e^{it}-z}\,dt \\
&\leq \int_{-\pi/2}^{\pi/2} (\log(c\,\varepsilon_n) - \log\gamma\,(k\,|t|))\ \mathrm{Re}\frac{e^{it}+z}{e^{it}-z}\,dt \\
&\leq \frac{1-\frac{1}{2}}{1+\frac{1}{2}}\,\pi\,\log(c\,\varepsilon_n) + \frac{1+\frac{1}{2}}{1-\frac{1}{2}}\,2\int_0^{\pi/2} |\log\gamma\,(k\,t)|\,dt \\
&\leq \frac{\pi}{3}\,\log(c\,\varepsilon_n) + 6L\,,
\end{aligned}$$

where L stands for the last integral. For the integral over the union of the remaining intervals $[-\pi, -\pi/2]$ and $[\pi/2, \pi]$, a simple change of variables leads to exactly the same estimates. We conclude that

$$\log\left|\varphi(f_n(e^{i(a+r\,z)}))\right| \leq \frac{1}{3}\,\log(c\,\varepsilon_n) + 2L \qquad \text{for all } z \in \nabla(0,1/2)\,,$$

and therefore, by the Hahn-Banach theorem, that

$$\left\|f_n(e^{i(a+r\,z)})\right\| \leq c^{1/3}\,\varepsilon_n^{1/3}\,e^{2L} \qquad \text{for all } z \in \nabla(0,1/2).$$

It follows that $f_n \to 0$ uniformly on the neighbourhood $\{e^{iz} : |z-a| \leq r/2\}$ of ζ, and hence that $f_n \to 0$ locally uniformly on U, as desired. ∎

4. Examples and Applications

Theorem 1 entails that all isometries have Bishop's property (β). Indeed, if $T \in L(X)$ is an isometry on a Banach space X, then

$$|1-|\lambda||\,\|x\| \leq \|(T-\lambda)x\| \qquad \text{for all } \lambda \in \mathbb{C} \text{ and } x \in X.$$

This inequality is immediate when $|\lambda| \leq 1$, and follows from a straightforward norm estimate for the geometric series of the resolvent function when $|\lambda| > 1$. Thus Theorem 1 applies with the choice $\gamma(t) := t$ for all $t > 0$, $E = \emptyset$, and $\omega(\lambda) := 1$ for all $\lambda \in \mathbb{C}$. This simple special case of Theorem 1 is essentially due to Tornehave, and may be found in [14]. For an alternative proof, we note that, by a result of Douglas [7], every isometry may be extended to an invertible isometry on some larger Banach space, and that every invertible isometry is, by Proposition 5.1.4 of [6], generalized scalar, and therefore decomposable.

This shows that every isometry is the restriction of a decomposable operator, and hence has property (β).

As a more substantial application, we mention the Cesàro operator C_p on the classical Hardy space $H^p(\mathbb{D})$ for $1 \leq p < \infty$, where $\mathbb{D}$ denotes the open unit disc. This operator is given by

$$(C_p f)(\lambda) := \frac{1}{\lambda} \int_0^\lambda \frac{f(\zeta)}{1-\zeta}\, d\zeta \qquad \text{for all } f \in H^p(\mathbb{D}) \text{ and } \lambda \in \mathbb{D}\,,$$

and has been studied, in this or equivalent versions, by many authors. The work of Siskakis [19] contains the basic facts about the spectral properties of the Cesàro operator. In [16], the results of Siskakis [19] are combined with a certain special case of Theorem 1 to show that the Cesàro operator C_p has property (β) whenever $1 < p < \infty$. Here the exceptional set E plays a crucial role, while the growth function is simply $\gamma(t) = t$ for all $t > 0$. In the classical Hilbert space case, $p = 2$, it is known from Kriete and Trutt [11] that the Cesàro operator C_2 is, in fact, subnormal. However, rather different methods are needed to settle property (β) for C_p when $p \neq 2$. Moreover, it remains an intriguing open problem whether this result extends to the case where $p = 1$.

We now address the question under which conditions the assumptions of Theorem 1 will actually entail decomposability. It turns out that a certain strengthened version of decomposability is appropriate in this context. A decomposable operator $T \in L(X)$ is said to be *strongly decomposable* if the restriction $T\,|\,X_T(F)$ is decomposable for every closed set $F \subseteq \mathbb{C}$; see for instance [13]. This definition makes sense, since it is well-known that, for every decomposable operator $T \in L(X)$ and every closed set $F \subseteq \mathbb{C}$, the space $X_T(F)$ is a T-invariant closed linear subspace of X. An example due to Albrecht [1] shows that, in general, decomposable operators need not be strongly decomposable. On the other hand, the work of Bacalu [4] reveals that, for large classes of operators, these two notions coincide.

Theorem 2. *Under the assumptions of Theorem* 1, *the following assertions are equivalent*:
(a) T *is strongly decomposable;*
(b) T *is decomposable;*
(c) $\sigma(T) \subseteq \partial D$;
(d) $D \cap \rho(T)$ *is non-empty.*

Proof. Evidently, the growth condition of Theorem 1 ensures that $T - \lambda$ is bounded below for all $\lambda \in V \setminus \partial D$, and therefore that the approximate point spectrum $\sigma_{ap}(T)$ is contained in ∂D. Hence the implication (b) $\Rightarrow$ (c) is immediate from the fact that the spectrum of any decomposable operator coincides with its approximate point spectrum; see Corollary 2.1.4 of [6].

Moreover, since $\partial\sigma(T)$ is always contained in $\sigma_{ap}(T)$, we see that either $\sigma(T) \subseteq \partial D$ or $\sigma(T) = D$. This establishes the equivalence (c) $\Leftrightarrow$ (d).

To verify that (c) implies (b), suppose that $\sigma(T) \subseteq \partial D$. Then the growth condition assumes the form

$$\gamma(\mathrm{dist}(\lambda, \partial D))\, \|(T-\lambda)^{-1}\| \le \omega(\lambda) \qquad \text{for all } \lambda \in V \setminus \partial D\,.$$

Moreover, the adjoint operator T^* satisfies $\sigma(T^*) = \sigma(T)$ and

$$\gamma(\mathrm{dist}(\lambda, \partial D))\, \|(T^*-\lambda)^{-1}\| \le \omega(\lambda) \qquad \text{for all } \lambda \in V \setminus \partial D\,.$$

Hence Theorem 1 ensures that both T and T^* have property (β). By the result of Eschmeier and Putinar [9] mentioned in Section 1, we conclude that T is decomposable. Thus (c) $\Rightarrow$ (b).

It remains to show that the equivalent conditions (b), (c), and (d) imply that T is actually strongly decomposable. To this end, consider an arbitrary closed set $F \subseteq \mathbb{C}$, and observe that the restriction $T\,|\,X_T(F)$ satisfies the assumption of Theorem 1. Moreover, it follows from Proposition 1.3.8 of [6] that $\sigma(T\,|\,X_T(F)) \subseteq \sigma(T) \subseteq \partial D$. Consequently, the preceding part of the proof yields the decomposability of $T\,|\,X_T(F)$. Thus T is indeed strongly decomposable. ■

Theorem 2 applies directly to the case of isometries and shows that a non-invertible isometry cannot be decomposable, although we have seen above that all isometries have Bishop's property (β).

We conclude with the canonical counterpart of Theorems 1 and 2 for operators with real spectrum. The same approach may be used to handle the more general case of operators whose spectrum is contained in a Jordan curve, but we leave the details to the interested reader.

Theorem 3. *Let $T \in L(X)$ be an operator on a Banach space X with real spectrum, and consider a totally disconnected compact subset E of $\sigma(T)$ and an open neighbourhood V of $\sigma(T)$ in $\mathbb{C}$. Suppose that $\omega : V \setminus E \to (0,\infty)$ is a locally bounded function and that $\gamma : (0,\infty) \to (0,\infty)$ is an increasing function such that $\log \circ \gamma$ has an integrable singularity at zero and*

$$\gamma(|\mathrm{Im}\ \lambda|)\, \|(T-\lambda)^{-1}\| \le \omega(\lambda) \qquad \textit{for all } \lambda \in V \textit{ with } \mathrm{Im}\ \lambda \ne 0\,.$$

Then T is strongly decomposable.

Proof. As in the proof of Theorem 1, an application of Jensen's inequality implies that T has property (β). Since $\gamma(|\mathrm{Im}\ \lambda|)\, \|(T^*-\lambda)^{-1}\| \le \omega(\lambda)$ for all

$\lambda \in V$ with $\mathrm{Im}\,\lambda \neq 0$, we infer that also T^* has property (β). Again by the result of Eschmeier and Putinar [9], it follows that T is decomposable. Since the same reasoning applies to the restriction $T \mid X_T(F)$ for an arbitrary closed set $F \subseteq \mathbb{C}$, we conclude that T is actually strongly decomposable. ■

Theorems 2 and 3 are related to Theorems 5.3.6 and 5.4.3 of Colojoară and Foiaş [6], but our methods are rather different. In fact, in the classical approach, growth conditions are often employed to construct a functional calculus on a suitable algebra of functions with partitions of unity. Details of the latter method may be found in [6] and [20]. Here, our main intention is to show that Bishop's property (β) plays a natural role in this context, while keeping the technicalities at a minimal level. For further results on growth conditions and decomposability, we refer, for instance, to Radjabalipour [17]. Related results in the perturbation theory of operators on Hilbert spaces are contained in the monograph [13].

We finally note that the theory of generalized scalar operators is dominated by quite restrictive polynomial growth conditions [6]. It is therefore appropriate that Theorem 1 requires only a logarithmic growth condition, but it remains open to which extent this condition can be further weakened.

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