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STABILITY OF n-JORDAN $*$-DERIVATIONS IN C^*-ALGEBRAS AND JC^*-ALGEBRAS

Shahram Ghaffary Ghaleh and Khatereh Ghasemi

Abstract. In this paper, we investigate superstability and the Hyers-Ulam stability of n-Jordan $*$-derivations in C^*-algebras and JC^*-algebras for the generalized Jensen-type functional equation

$$rf(\frac{a+b}{r}) + rf(\frac{a-b}{r}) = 2f(a).$$

1. Introduction and Preliminaries

Let $n \in \mathbb{N} - \{1\}$ and let A be a ring and B be an A-module. An additive map $D : A \to B$ is called n-Jordan derivation (n-ring derivation) if

$$D(a^n) = D(a)a^{n-1} + aD(a)a^{n-2} + \ldots + a^{n-2}D(a)a + a^{n-1}D(a),$$

for all $a \in A$.

$$D(\prod_{i=1}^{n} a_i) = D(a_1)a_2 \ldots a_n + a_1D(a_2)a_3 \ldots a_n + a_1a_2 \ldots a_{n-1}D(a_n)$$

for all $a_1, a_2, \ldots, a_n \in A$). The concept of n-jordan derivations was studied by Eshaghi Ghordji [4]. (see also [5, 6, 11]).

The stability of functional equations was first introduced by Ulam [26] in 1940. More precisely, he proposed the following problem: Given a group H_1, a metric group (H_2, d) and $\epsilon > 0$, does there exist a $\delta > 0$ such that if a mapping $f : H_1 \to H_2$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in H_1$, then there exists a homomorphism $T : H_1 \to H_2$ such that $d(f(x), T(x)) < \epsilon$ for all $x \in H_1$? As mentioned above, when this problem has a solution, we say that the homomorphisms from H_1 to H_2 are stable. In 1941, Hyers [13] gave a partial solution of $Ulam's$

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problem for the case of approximate additive mappings under the assumption that H_1 and H_2 are Banach spaces. In 1950, Aoki [1] generalized the Hyers' theorem for approximately additive mappings. In 1978, Th.M. Rassias [24] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences.

During the last decades several stability problems of functional equations have been investigated by many mathematicians (see [2, 3, 7, 8, 9, 14, 15, 16, 18, 17, 20, 21, 22, 23, 25]).

Definition 1.1. Let A , B be C^*-algebras. A $\mathbb{C}$-linear mapping $D : A \to B$ is called n-Jordan $*$-derivation if

$$D(a^n) = D(a)a^{n-1} + aD(a)a^{n-2} + \ldots + a^{n-2}D(a)a + a^{n-1}D(a),$$
$$D(a^*) = D(a)^*$$

for all $a \in A$.

A C^*-algebra A, endowed with the Jordan product $a \circ b = \frac{ab+ba}{2}$ on A is called a JC^*-algebra(see [20])

Definition 1.2. Let A , B be JC^*-algebras. A $\mathbb{C}$-linear mapping $D : A \to B$ is called n-Jordan $*$-derivation if

$$D(a^n) = D(a) \circ a^{n-1} + a \circ D(a) \circ a^{n-2} + \ldots + a^{n-2} \circ D(a) \circ a + a^{n-1} \circ D(a),$$
$$D(a^*) = D(a)^*$$

for all $a \in A$.

Throughout this paper, in section 2, we investigate the Hyers-Ulam stability of n-Jordan $*$-derivations in C^*-algebras associated with the following functional inequality

$$\|rf(\frac{a+b}{r}) + rf(\frac{a-b}{r})\| \leq 2\|f(a)\|. \tag{1.1}$$

We moreover prove the the Hyers-Ulam stability of n-Jordan $*$-derivations in C^*-algebras associated with the following functional equation

$$rf(\frac{a+b}{r}) + rf(\frac{a-b}{r}) = 2f(a). \tag{1.2}$$

In section 3, we investigate n-Jordan $*$-derivations in JC^*-algebras associated with the functional inequality (1.1), and prove the generalized Hyers-Ulam stability of n-Jordan $*$-derivations in JC^*-algebras associated with the functional equation (1.2).

In this paper, assume that n is an integer greater than 1.

2. n-Jordan $*$-Derivations on C^*-Algebras

Lemma 2.1. *Let A, B be C^*-algebras, and let $D : A \to B$ be a mapping such that*

$$\|rD(\frac{a+b}{r})+rD(\frac{a-b}{r})\|_B \leq 2\|D(a)\|_B \tag{2.1}$$

for all $a, b \in A$. Then D is additive.

Proof. Letting $a=b=0$ in (2.1), we get

$$\|2rD(0)\|_B \leq 2\|D(0)\|_B$$

So $D(0)=0$.

Letting $a=b$ in (2.1), we get

$$rD(\frac{2a}{r}) = 2D(a) \tag{2.2}$$

for all $a \in A$. Hence

$$rD(\frac{a+b}{r})+rD(\frac{a-b}{r}) = D(a+b)+D(a-b) = 2D(a) \tag{2.3}$$

for all $a, b \in A$. Replace a by $a+b$ in (2.3), we have

$$D(a+2b)+D(a) = 2D(a+b) \tag{2.4}$$

for all $a, b \in A$. By using (2.2), we have

$$2D(\frac{a+2b}{2})+2D(\frac{a}{2}) = 2D(a+b) \tag{2.5}$$

for all $a, b \in A$. Hence,

$$D(\frac{a}{2}+b)+D(\frac{a}{2}) = D(a+b)$$

for all $a, b \in A$. Put $x=\frac{a}{2}$, $y=\frac{a}{2}+b$ in above equation, we get

$$D(x)+D(y) = D(x+y)$$

for all $x, y \in A$. Hence, D is Cauchy additive. ■

We prove the superstability of n-Jordan $*$-derivations.

Theorem 2.2. *Let A, B be C^*-algebras, and let $r, p > 1$ and θ be nonnegative real numbers. Let $f : A \to B$ be a mapping such that*

$$\begin{aligned}\|r\mu f(\frac{a+b}{r}) &+ r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} \\ &+ \ldots + c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \leq \theta\end{aligned} \tag{2.6}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $a, b, c, d \in A$. Then the mapping $f : A \to B$ is an n-Jordan $*$-derivation.

Proof. Let $\mu = 1$, $c = d = 0$ in (2.6). By Lemma 2.1, the mapping $f : A \to B$ is additive. Letting $a = b$, $c = d = 0$ in (2.6), we get

$$\|\mu f(2a) - 2f(\mu a)\|_B \leq \theta$$

for all $a \in A$ and all $\mu \in \mathbb{T}^1$. So

$$\mu f(2a) = 2f(\mu a)$$

for all $a \in A$ and all $\mu \in \mathbb{T}^1$. Hence $f(\mu a) = \mu f(a)$ for all $a \in A$ and all $\mu \in \mathbb{T}^1$. By [20, Theorem 2.1], the mapping $f : A \to B$ is $\mathbb{C}$-linear.

Letting $a = b = d = 0$ in (2.6), we get

$$f(c^n) = f(c)c^{n-1} + cf(c)c^{n-2} + \ldots + c^{n-2}f(c)c + c^{n-1}f(c)$$

for all $c \in A$. And by letting $a = b = c = 0$ in (2.6), we have

$$f(d^*) = f(d)^*$$

for all $d \in A$. Hence the mapping $f : A \to B$ is an n-Jordan $*$-derivation. ■

Theorem 2.3. *Let A, B be C^*-algebras, and let $p, r < 1$ and θ be nonnegative real numbers. Let $f : A \to B$ be a mapping satisfying* (2.6). *Then the mapping $f : A \to B$ is an n-Jordan $*$-derivation.*

Proof. The proof is similar to the proof of Theorem 2.2. ■

Now we prove the Hyers-Ulam stability of n-Jordan derivations in C^*-algebras.

Theorem 2.4. *Let A, B be C^*-algebras. Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A \times A \times A \times A \to \mathbb{R}^+$ such that*

$$\psi(a, b, c, d) = \sum_{i=0}^{\infty} r^{-i}\varphi(r^i a, r^i b, r^i c, r^i d) < \infty, \tag{2.7}$$

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} \\ &+ \ldots + c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \leq \varphi(a, b, c, d) \end{aligned} \tag{2.8}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \psi(a, a, 0, 0) \tag{2.9}$$

for all $a \in A$.

Proof. Letting $\mu = 1$, $b = c = d = 0$ and replacing a by ra in (2.8), we get

$$\|f(a) - r^{-1}f(ra)\|_B \leq \frac{1}{2r}\varphi(ra, 0, 0, 0) \tag{2.10}$$

for all $a \in A$. Using the induction method, we have

$$\|f(a) - r^{-n}f(r^n a)\|_B \leq \frac{1}{2r^n}\sum_{i=1}^{n}\varphi(r^i a, 0, 0, 0) \tag{2.11}$$

for all $a \in A$. Replace a by a^m in (2.10) and then divide by r^m, we have

$$\|f(a^m) - r^{-n-m}f(r^{n+m}a)\|_B \leq \frac{1}{2r^{n+m}}\sum_{i=m}^{m+n}\varphi(r^i a, 0, 0, 0)$$

for all $a \in A$. Hence, $\{r^{-n}f(r^n a)\}$ is a Cauchy sequence. Since A is complete, then

$$D(a) = \lim_n r^{-n}f(r^n a)$$

exists for all $a \in A$. By (2.8) one can show that

$$\begin{aligned} &\|rD(\frac{a+b}{r}) + rD(\frac{a-b}{r}) - 2D(a)\|_B \\ &= lim_n \frac{1}{r^n}\|rf(r^{n-1}(a+b)) + rf(r^{n-1}(a-b)) - 2f(r^n a)\|_B \\ &\leq \lim_n \frac{1}{r^n}\varphi(r^n a, r^n b, 0, 0) \end{aligned} \tag{2.12}$$

for all $a, b \in A$. So

$$rD(\frac{a+b}{r}) + rD(\frac{a-b}{r}) = 2D(a)$$

for all $a, b \in A$. Put $x = \frac{a+b}{r}$, $y = \frac{a-b}{r}$ in above equation, we have

$$r(D(x) + D(y)) = 2rD(\frac{x+y}{2})$$

for all $x, y \in A$. Hence, D is Cauchy additive. On the other hand, we have

$$D(\mu a) - \mu D(a) = \lim_n \frac{1}{r^n}\|f(\mu r^n a) - \mu f(r^n a)\|_B \leq \lim_n \frac{1}{r^n}\varphi(r^n a, r^n a, 0, 0) = 0$$

for all $\mu \in \mathbb{T}^1$, and all $a \in A$. So it is easy to show that D is linear. It follow from (2.8) that

$$\begin{aligned} &\|D(c^n) - D(c)c^{n-1} + cD(c)c^{n-2} + \ldots + c^{n-2}D(c)c + c^{n-1}D(c)\|_B \\ &= \lim_m \|\frac{1}{r^{mn}}f((r^m c)^n) - \frac{1}{r^{mn}}(f(r^m r^{m(n-1)}c) + f(r^{2m}r^{m(n-2)}c) \\ &+ f(r^{3m}r^{m(n-3)}c))^n + \ldots f(r^{m(n-1)}r^m c)\|_B \leq \lim_m \frac{1}{r^{mn}}\varphi(0, 0, 0, r^m c) \\ &\leq \lim_m \frac{1}{r^m}\varphi(0, 0, 0, r^m c) \\ &= 0 \end{aligned} \tag{2.13}$$

for all $c \in A$. and we have

$$\begin{aligned} \|D(d^*) - D(d)^*\|_B &= lim_n \|\frac{1}{r^n} f(r^n d^*) - \frac{1}{r^n}(f(r^n d))^*\|_B \\ &\le \lim_n \frac{1}{r^{mn}} \varphi(0,0,0,r^n d) \\ &= 0 \end{aligned} \tag{2.14}$$

for all $d \in A$. Hence $D : A \to B$ is a unique n-Jordan $*$-derivation. ■

Corollary 2.5. *Let A, B be C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \ge 0$ and $p_1, p_2, p_3, p_4 \in (-\infty, 1)$ such that*

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c)c^{n-1} \\ &+cf(c)c^{n-2} + \ldots + c^{n-2} f(c)c + c^{n-1} f(c) + f(d^*) - f(d)^*\|_B \\ &\le \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4}) \end{aligned} \tag{2.15}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \le \frac{r\theta\|a\|_A^{p_1}}{r^{1-p_1} - 1} \tag{2.16}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c, d) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4})$ in Theorem 2.4, we have

$$\|f(a) - D(a)\|_B \le \frac{r\theta\|a\|_A^{p_1}}{r^{1-p_1} - 1}$$

for all $a \in A$, as desired. ■

Theorem 2.6. *Let A, B be C^*-algebras. Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A \times A \times A \times A \to \mathbb{R}^+$ such that*

$$\psi(a, b, c, d) = \sum_{i=0}^{\infty} r^i \varphi(r^{-i}a, r^{-i}b, r^{-i}c, r^{-i}d) < \infty, \tag{2.17}$$

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c)c^{n-1} + cf(c)c^{n-2} \\ &+ \ldots + c^{n-2} f(c)c + c^{n-1} f(c) + f(d^*) - f(d)^*\|_B \le \varphi(a, b, c, d) \end{aligned} \tag{2.18}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \le \psi(a, a, 0, 0) \tag{2.19}$$

for all $a \in A$.

Proof. Letting $\mu = 1$, $b = c = d = 0$ in (2.18), we get

$$\|f(a) - rf(r^{-1}a)\|_B \leq \frac{1}{2}\varphi(a, 0, 0, 0) \tag{2.20}$$

for all $a \in A$. Using the induction method, we have

$$\|f(a) - r^n f(r^{-n}a)\|_B \leq \frac{1}{2}\sum_{i=1}^{n} r^i \varphi(r^{-i}a, 0, 0, 0) \tag{2.21}$$

for all $a \in A$. Replace a by a^m in (2.20) and then divide by r^m, we have

$$\|f(a^m) - r^{n+m} f(r^{-n-m}a)\|_B \leq \frac{1}{2}\sum_{i=m}^{m+n} r^i \varphi(r^{-i}a, 0, 0, 0)$$

for all $a \in A$. Hence, $\{r^n f(r^{-n}a)\}$ is a Cauchy sequence. Since A is complete, then

$$D(a) = \lim_n r^n f(r^{-n}a)$$

exists for all $a \in A$. By (2.18) one can show that

$$\begin{aligned} &\|rD(\frac{a+b}{r}) + rD(\frac{a-b}{r}) - 2D(a)\|_B \\ &= lim_n r^n \|rf(r^{-n-1}(a+b)) + rf(r^{-n-1}(a-b)) - 2f(r^{-n}a)\|_B \\ &\leq \lim_n r^n \varphi(r^{-n}a, r^{-n}b, 0, 0) \end{aligned} \tag{2.22}$$

for all $a, b \in A$. So

$$rD(\frac{a+b}{r}) + rD(\frac{a-b}{r}) = 2D(a)$$

for all $a, b \in A$. Put $x = \frac{a+b}{r}$, $y = \frac{a-b}{r}$ in above equation, we have

$$r(D(x) + D(y)) = 2rD(\frac{x+y}{2})$$

for all $x, y \in A$. Hence, D is Cauchy additive.

The rest of proof is similar to the proof of Theorem 2.4. ■

Corollary 2.7. *Let A, B be C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \geq 0$ and $p_1, p_2, p_3, p_4 \in (-\infty, 1)$ such that*

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c)c^{n-1} \\ &+cf(c)c^{n-2} + \ldots + c^{n-2}f(c)c + c^{n-1}f(c) + f(d^*) - f(d)^*\|_B \\ &\leq \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4}) \end{aligned} \tag{2.23}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \frac{r\theta\|a\|_A^{p_1}}{r^{1-p_1} - 1} \tag{2.24}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c, d) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4})$ in Theorem 2.6, we have

$$\|f(a) - D(a)\|_B \leq \frac{r\theta\|a\|_A^{p_1}}{r^{1-p_1} - 1}$$

for all $a \in A$, as desired. ■

3. n-JORDAN $*$-DERIVATIONS ON JC^*-ALGEBRAS

Theorem 3.1. *Let A, B be JC^*-algebras, and let $r, p > 1$ and θ be nonnegative real numbers. Let $f : A \to B$ be a mapping such that*

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c) \circ c^{n-1} + c \circ f(c) \circ c^{n-2} \\ &+ \ldots + c^{n-2} \circ f(c) \circ c + c^{n-1} \circ f(c) + f(d^*) - f(d)^*\|_B \leq \theta \end{aligned} \tag{3.1}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $a, b, c, d \in A$. Then the mapping $f : A \to B$ is an n-Jordan $*$-derivation.

Proof. By the same reasoning as in the proof of Theorem 2.2, the mapping $f : A \to B$ is $\mathbb{C}$-linear.

Letting $a = b = d = 0$ in (3.1), we get

$$f(c^n) = f(c) \circ c^{n-1} + c \circ f(c) \circ c^{n-2} + \ldots + c^{n-2} \circ f(c) \circ c + c^{n-1} \circ f(c)$$

for all $c \in A$. And by letting $a = b = c = 0$ in (3.1), we have

$$f(d^*) = f(d)^*$$

for all $d \in A$. Hence the mapping $f : A \to B$ is an n-Jordan $*$-derivation. ■

Theorem 3.2. *Let A, B be JC^*-algebras, and let $p, r < 1$ and θ be nonnegative real numbers. Let $f : A \to B$ be a mapping satisfying* (2.6) *and* (3.1). *Then the mapping $f : A \to B$ is an n-Jordan $*$-derivation.*

Proof. The proof is similar to the proofs of Theorems 2.2 and 3.1. ■

We prove the generalized Hyers-Ulam stability of n-Jordan derivations in JC^*-algebras.

Theorem 3.3. *Suppose that A, B be JC^*-algebras. Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A \times A \times A \times A \to \mathbb{R}^+$ satisfying* (2.7) *such that*

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) \\ &-f(c) \circ c^{n-1} + c \circ f(c) \circ c^{n-2} + \ldots + c^{n-2} \circ f(c) \circ c \\ &+c^{n-1} \circ f(c) + f(d^*) - f(d)^*\|_B \leq \varphi(a,b,c,d) \end{aligned} \tag{3.2}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \psi(a, a, 0, 0)$$

for all $a \in A$.

Proof. By the same reasoning as in the proof of Theorem 2.4. there exists a unique $\mathbb{C}$-linear $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \psi(a, a, 0, 0)$$

for all $a \in A$. The mapping $D : A \to B$ is given by

$$D(a) = \lim_n r^{-n} f(r^n a)$$

for all $a \in A$. Letting $\mu = 1$, $b = c = d = 0$ and replacing a by ra in (3.3), we get

$$\|f(a) - r^{-1} f(ra)\|_B \leq \frac{1}{2r} \varphi(ra, 0, 0, 0)$$

for all $a \in A$. The rest of proof is similar to the proof of Theorem 2.4. ∎

Corollary 3.4. *Let A, B be JC^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \geq 0$ and $p_1, p_2, p_3, p_4 \in (-\infty, 1)$ such that*

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c) \circ c^{n-1} \\ &\quad +c \circ f(c) \circ c^{n-2} + \ldots + c^{n-2} \circ f(c) \circ c + c^{n-1} \circ f(c) + f(d^*) - f(d)^*\|_B \\ &\leq \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4}) \end{aligned}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \frac{r\theta \|a\|_A^{p_1}}{r^{1-p_1} - 1}$$

for all $a \in A$.

Proof. Letting $\varphi(a,b,c,d) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4})$ in Theorem 3.3, we obtain the result. ■

Theorem 3.5. *Let A, B be JC*-algebras. Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A \times A \times A \times A \to \mathbb{R}^+$ satisfying* (2.17) *and* (3.2)*. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that*

$$\|f(a) - D(a)\|_B \leq \psi(a,a,0,0)$$

for all $a \in A$.

Proof. The rest of the proof is similar to the proofs of Theorems 2.4 and 3.3. ■

Corollary 3.6. *Let A, B be JC*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \geq 0$ and $p_1, p_2, p_3, p_4 \in (-\infty, 1)$ such that*

$$\begin{aligned} &\|r\mu f(\frac{a+b}{r}) + r\mu f(\frac{a-b}{r}) - 2f(\mu a) + f(c^n) - f(c)\circ c^{n-1} \\ &+c\circ f(c)\circ c^{n-2} + \ldots + c^{n-2}\circ f(c)\circ c + c^{n-1}\circ f(c) + f(d^*) - f(d)^*\|_B \\ &\leq \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4}) \end{aligned}$$

for all $a, b, c, d \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-derivation $D : A \to B$ such that

$$\|f(a) - D(a)\|_B \leq \frac{r\theta\|a\|_A^{p_1}}{r^{1-p_1} - 1}$$

for all $a \in A$.

Proof. Letting $\varphi(a,b,c,d) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3} + \|d\|^{p_4})$ in Theorem 3.5, we obtain the result. ■

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Shahram Ghaffary Ghaleh
Department of Mathematics
Payame Noor University of Zahedan
Zahedan, Iran
E-mail: shahram.ghaffary@gmail.com

Khatereh Ghasemi
Department of Mathematics
Payame Noor University of Khash
Khash, Iran
E-mail: khatere.ghasemi@gmail.com