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ON DERIVATIONS OF CENTRALIZER NEAR-RINGS

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Abstract. It is proved that if a centralizer near-ring N has a nonzero derivation, then N is a near-field.

1. Introduction

An additively written group N equipped with a binary operation $\cdot : N\times N \to N$, $(x, y) \mapsto xy$, such that $(xy)z = x(yz)$ and $(y + z)x = yx + zx$ for all $x, y, z \in N$ is called a (*right*) *near-ring*. A near-ring N is said to be *zero-symmetric* if $x0 = 0$ for all $x \in N$. If $N^* = N \setminus \{0\}$ is a group, then N is said to be a *near-field*.

Let N be a near-ring and M a subnear-ring of N. An additive mapping $d : M \to N$ is said to be a *derivation* of M into N if $(xy)^d = xy^d + x^d y$ for all $x, y \in M$. Here x^d denotes the image of x under d. We refer the reader to the books of Clay [5], Meldrum [13] and Pilz [14] for basic results of near-ring theory and its applications.

In what follows, G is an additively written (not necessarily abelian) group and C is a fixed point free automorphism group of G (i.e. for all $x \in G$ and $\alpha \in C$ with $x \neq 0$ and $\alpha \neq 1$ we have $x\alpha \neq x$. Note that for clarity, we write the image of x under $\alpha \in C$ as $x\alpha$.) Next, set

$$M_C^0(G) = \{f : G \to G \mid f(0) = 0 \text{ and } f(x\alpha) = f(x)\alpha \text{ for all } x \in G \text{ and } \alpha \in C\}.$$

(Now, the image of x under $f \in M_C^0(G)$ is written as $f(x)$.) It is well-known that $M_C^0(G)$ is a zero-symmetric near-ring under the pointwise addition and composition of mappings. The near-ring $M_C^0(G)$ is usually referred to as the *centralizer near-ring* on G determined by C (see [5, 13, 14]). A transformation $f \in M_C^0(G)$ is said to be of *finite rank* if there exist finitely many elements $x_1, x_2, \ldots, x_n \in G$ such that $f(G) \subseteq \cup_{i=1}^n x_i C$, where, for each i, $x_i C = \{x_i\alpha \mid \alpha \in C\}$. When $C = \{1\}$, the

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near-ring $M_C^0(G)$ is denoted by $M_0(G)$ and is called *the transformation near-ring on G*.

Let F be a skew field and V a right vector space over F. Let $R = \mathrm{End}_F(V)$ be the ring of linear transformations of the vector space V. The concept of the centralizer near-ring $M_C^0(G)$ is a generalization of that of the ring of linear transformations of a vector space. It is well-known that if $d : R \to R$ (respectively, $\alpha : R \to R$) is a derivation (automorphism) of the ring R, then there exists a (bijective) additive transformation $T : V \to V$ such that $r^d = Tr - rT$ (respectively, $r^\alpha = TrT^{-1}$) for all $r \in R$ (see [11]). In 1974, Ramakotaiah [16] proved analogous results for automorphisms of transformation near-rings (see also [14, Theorem 7.39]). The study of derivations of near-rings was initiated by Bell and Mason [4] in 1987. Since then a number of research articles on the subject have been published [1, 2, 3, 4, 6, 7, 10, 19]. Researchers mainly studied different generalizations of Posner's [15] and Herstein's [8, 9] results into the context of near-rings. Recently Fong, Ke and Wang [7, Theorem 1.1] obtained the following result which was inspired by classical results on derivations of primitive rings with nonzero socle.

Theorem 1.1. *Let G be a nonzero additively written group, N a subnear-ring of $M_0(G)$ containing all the transformations with finite rank. Then there are no nonzero derivations of N into $M_0(G)$.*

In the present paper we continue the program of Fong, Ke and Wang in [7], and we shall prove following theorem.

Theorem 1.2. *Let G be a nonzero additively written group with a fixed point free automorphism group C, let M be a subnear-ring of $M_C^0(G)$ containing all the transformations of G which are of finite rank, and let $d : M \to M_C^0(G)$ be a nonzero derivation. Then*

(1) *G is the additive group of some near-field F, say;*

(2) *C is isomorphic to the multiplicative group $F^* = F \setminus \{0\} = G \setminus \{0\}$ that acts on G via right multiplications; and*

(3) *$M = M_C^0(G)$, and is isomorphic to G which acts on G via left multiplications.*

We note that Theorem 1.1 is a special case of Theorem 1.2 with $C = \{1\}$.

The question of whether there are nontrivial derivations on near-fields remains open. However, using the SONATA package of GAP [17, 18], we know that there are no nontrivial derivations on the 7 exceptional finite near-fields, and this is also the case for some small Dickson near-fields provided in SONATA. On the other hand, since the set of all distributive elements of a finite near-field is the center [14, Theorem 8.31], there are no nontrivial inner derivations on any finite near-field.

Finally, we remark that the question of when $M^0_C(G)$ is a near-field was considered in [12].

2. Proof of the Theorem

The following properties of a derivation on a zero-symmetric near-ring will be used throughout the proof.

Lemma 2.1. *Let N be a zero-symmetric near-ring, let $d : N \to N$ be a derivation and let $a, b, c \in N$. Then:*

(i) $(ab)^d = a^d b + ab^d$ [19, Proposition 1];

(ii) $(ab^d + a^d b)c = ab^d c + a^d bc$ *and* $(a^d b + ab^d)c = a^d bc + ab^d c$ [4, Lemma 1].

Now, we shall begin to prove Theorem 1.2. The proof will be achieved step by step in the form of lemmas.

In what follows, M is a subnear-ring of $M^0_C(G)$ containing all the transformations which are of finite rank, and $d : M \to M^0_C(G)$ is a nonzero derivation. We set $G^* = G \setminus \{0\}$. Given $x, y \in G^*$, and define the map $\delta_{x,y} : G \to G$ as follows:

$$\delta_{x,y}(z) = \begin{cases} x\alpha, & \text{if } z = y\alpha \text{ for some } \alpha \in C; \\ 0, & \text{if } z \notin yC. \end{cases}$$

Since C is a fixed point free group of automorphisms of G, $\delta_{x,y}$ is well-defined. It is also clear that $\delta_{x,y} \in M^0_C(G)$ and is of finite rank.

Denote by N the set of all finite sums of transformations of G of finite rank. Then N is a subnear-ring of M, and we have

$$M^0_C(G)N \subseteq N \quad \text{and} \quad NM^0_C(G) \subseteq N.$$

Given $x \in G^*$, we set $N(x) = \{f(x) \mid f \in N\}$.

Lemma 2.2. $N^d \subseteq N$, $N^d \neq 0$, *and for any* $x \in G^*$, $G = N(x)$.

Proof. First of all, we have $\delta_{x,y} \in N$ for all $x, y \in G^*$. Let f be a transformation of G of finite rank, say $f(G) \subseteq \cup_{i=1}^n x_i C$. Set $A = \sum_{i=1}^n \delta_{x_i,x_i} \in N$. Then $Af = f$, and so $f^d = (Af)^d = Af^d + A^d f$. Since $Af^d \in Nf^d \subseteq N$ and $A^d f \in A^d N \subseteq N$, we have $f^d \in N$. Therefore $N^d \subseteq N$.

Next, assume that $N^d = 0$. Pick $h \in M$ with $h^d \neq 0$, and take $x \in G$ with $h^d(x) \neq 0$. Since $\delta_{x,x} \in N$ and $h\delta_{x,x} \in N$, we have $\delta^d_{x,x} = 0$ and $(h\delta_{x,x})^d = 0$; thus

$$0 = (h\delta_{x,x})^d(x) = (h^d \delta_{x,x} + h\delta^d_{x,x})(x) = h^d(x) \neq 0,$$

which cannot be. Therefore, $N^d \neq 0$.

Finally, for any fixed $x \in G^{*}$, we have $\delta_{y,x}(x) = y$ and $\delta_{y,x} \in N$ for all $y \in G^{*}$, it follows at once that $G = N(x) = \{f(x) \mid f \in N\}$. ■

In view of the above lemma, we may assume, without loss of generality, that $M = N$.

Throughout the rest of this section, we shall fix an element $x_0 \in G^{*}$, and set $e = \delta_{x_0,x_0}$. From the fact that $f(x_0) = g(x_0)$ if and only if $f(y) = g(y)$ for all $y \in x_0C$ we conclude that

$$f(x_0) = g(x_0) \text{ if and only if } fe = ge. \tag{1}$$

Define a map $T : G \to G$ by the rule

$$\begin{aligned} T(f(x_0)) &= (fe)^d(x_0) = (f^d e + fe^d)(x_0) \\ &= f^d e(x_0) + fe^d(x_0) = f^d(x_0) + fe^d(x_0) \quad \text{for all } f \in N. \end{aligned}$$

Note that T is well-defined. For if $f, g \in N$ are such that $f(x_0) = g(x_0)$, then $fe = ge$ by (1), and so $T(f(x_0)) = (fe)^d(x_0) = (ge)^d(x_0) = T(g(x_0))$.

Lemma 2.3. *T is a nonzero endomorphism of G and $f^{\,d} = Tf - fT$ for all $f \in N$.*

Proof. Given $u, v \in G$, pick $f, g \in N$ with $f(x_0) = u$ and $g(x_0) = v$. Then

$$\begin{aligned} T(u + v) &= T(f(x_0) + g(x_0)) = T((f + g)(x_0)) = ((f + g)e)^d(x_0) \\ &= (fe + ge)^d(x_0) = (fe)^d(x_0) + (ge)^d(x_0) \\ &= T(f(x_0)) + T(g(x_0)) = T(u) + T(v), \end{aligned}$$

and so T is an endomorphism of G. Now, let $f \in N$ and $w \in G$. Pick $h \in N$ with $h(x_0) = w$. Then $he(x_0) = h(x_0) = w$. We have that

$$\begin{aligned} (Tf)(w) &= T(f(w)) = T(fh(x_0)) = (fhe)^d(x_0) = (f^d he + f(he)^d)(x_0) \\ &= (f^d he)(x_0) + (f(he)^d)(x_0) = f^d(he(x_0)) + f(he)^d(x_0) \\ &= f^d(w) + f(T(he(x_0))) = f^d(w) + (fT)(w) \end{aligned}$$

and so $(Tf - fT)(w) = f^d(w)$ for all $w \in G$. Therefore, $Tf - fT = f^d$ for all $f \in N$ as claimed. If $T = 0$, then for all $f \in N$, $f^d = Tf - fT = 0$, and so $d = 0$, a contradiction. Hence, T is a nonzero endomorphism of G. ■

As an endomorphism of G, T is a distributive element in $M_0(G)$. In particular,

$$T(f + g) = Tf + Tg \text{ and } (f + g)T = fT + gT \quad \text{for all } f, g \in N. \tag{2}$$

Lemma 2.4. *G is an abelian group.*

Proof. Let $f, g \in N$. Then

$$\begin{aligned} Tf + Tg - gT - fT &= T(f+g) - (f+g)T \\ &= (f+g)^d = f^d + g^d = Tf - fT + Tg - gT, \end{aligned}$$

and so $Tg - gT - fT = -fT + Tg - gT$. Thus,

$$g^d - fT = -fT + g^d.$$

Set $H = G \setminus \ker(T)$, where $\ker(T)$ is the kernel of T. Then $H \neq \varnothing$. Take any $u \in G$ and $v \in H$. Since $T(v) \neq 0$, there is an $f \in N$ such that $-fT(v) = u$. Thus

$$\begin{aligned} g^d(v) + u &= g^d(v) - fT(v) = (g^d - fT)(v) \\ &= (-fT + g^d)(v) = -fT(v) + g^d(v) = u + g^d(v). \end{aligned}$$

This shows that

$$g^d(H) \subseteq Z(G) \quad \text{for all } g \in N, \tag{3}$$

where $Z(G)$ denotes the center of G. As $Z(G)C = Z(G)$, we also have

$$g^d(HC) \subseteq g^d(H)C \subseteq Z(G)C = Z(G) \quad \text{for all } g \in N. \tag{4}$$

We claim that $g^d(H) \neq 0$ for some $g \in N$. Assume on the contrary that $g^d(H) = 0$ for all $g \in N$. Since $d \neq 0$, there exists a $g \in N$ with $g^d \neq 0$. Therefore $g^d(y) \neq 0$ for some $y \in G^*$. Take $z \in H$ and $f \in N$ such $f(z) = y$. Then $(gf)^d(z) = 0$ and $f^d(z) = 0$, and so

$$g^d(y) = g^d f(z) = g^d f(z) + g f^d(z) = (gf)^d(z) = 0,$$

a contradiction. This proves our claim.

Take $g \in N$ and $a \in H$ with $g^d(a) \neq 0$. Set

$$x = g(a), \quad y = T(a), \quad \text{and} \quad z = g(y) = gT(a).$$

Note that $y \neq 0$, and we cannot have both $x = 0$ and $z = 0$ because this will lead to the contradiction that

$$0 \neq g^d(a) = Tg(a) - gT(a) = T(x) - z = T(0) - 0 = 0.$$

Moreover, if $x = 0$, then $y \notin aC$. For if $y = a\alpha$ for some $\alpha \in C$, then

$$z = g(y) = g(a\alpha) = g(a)\alpha = 0\alpha = 0,$$

which cannot be. On the other hand, if $y = a\alpha \in aC$, then we have $g(y) = g(a\alpha) = g(a)\alpha = x\alpha$.

Now, define

$$h = \begin{cases} \delta_{z,y} & \text{if } x = 0 \text{ (hence } y \notin aC \text{ and } z \neq 0), \\ \delta_{x,a} + \delta_{z,y} & \text{if } x \neq 0,\ y \notin aC, \text{ and } z \neq 0, \\ \delta_{x,a} & \text{if } x \neq 0,\ y \notin aC, \text{ and } z = 0, \\ \delta_{x,a} & \text{if } x \neq 0 \text{ and } y \in aC. \end{cases}$$

It is easy to check that $h(a) = x = g(a)$ and $h(y) = z = g(y)$, and so

$$\begin{aligned} h^d(a) &= Th(a) - hT(a) = Th(a) - h(y) \\ &= Tg(a) - g(y) = Tg(a) - gT(a) = g^d(a). \end{aligned}$$

Therefore, it follows that $h^d(a) \neq 0$. Thus,

(5) either $\delta^d_{x,a}(a) \neq 0$ (hence $x \neq 0$), or $\delta^d_{z,y}(a) \neq 0$ (hence $z \neq 0$).

Next we are going to show that $\delta^d_{u,v}(H) \neq 0$ for some $u, v \in H$. Assume on the contrary that $\delta^d_{u,v}(H) = 0$ for all $u, v \in H$. Then

$$T(u) - \delta_{u,v}T(v) = T\delta_{u,v}(v) - \delta_{u,v}T(v) = (T\delta_{u,v} - \delta_{u,v}T)(v) = \delta^d_{u,v}(v) = 0$$

and so $T(u) = \delta_{u,v}T(v)$ for all $u, v \in H$. Since $T(u) \neq 0$, we conclude from the definition of $\delta_{u,v}$ that $T(v) \in vC$ for all $v \in H$. For each $u \in H$, let $\alpha_u \in C$ be such that $T(u) = u\alpha_u$. But then for any $u, v \in H$, we have

$$u\alpha_u = T(u) = \delta_{u,v}T(v) = \delta_{u,v}(v\alpha_v) = u\alpha_v,$$

and so $\alpha_u = \alpha_v$ as they are fixed point free. Thus $\alpha_u = \alpha_v$ for all $u, v \in H$. Set $\alpha = \alpha_v$. We have $T(u) = u\alpha$ for all $u \in H$. Take $w \in \ker(T)$. Then $a + w \in H$ and so $a\alpha = T(a) = T(a + w) = (a + w)\alpha = a\alpha + w\alpha$ which forces $w\alpha = 0$. Again, since α is fixed point free, $w = 0$. Therefore, $\ker(T) = 0$, and $H = G^*$. Now we conclude from (5) that

(1) either $x, a \in H$ such that $\delta^d_{x,a}(a) \neq 0$, a contradiction to the assumption;

(2) or $y, z \in H$ such that $\delta^d_{x,a}(a) = 0$, again a contradiction to the assumption.

Therefore, we must have $\delta^d_{u,v}(H) \neq 0$ for some $u, v \in H$.

Pick $u, v, w \in H$ with $s = \delta^d_{u,v}(w) \neq 0$. Let $t \in G^*$. Using (4), we have $\delta^d_{t,s}\delta_{u,v}(H) \subseteq Z(G)$ because $\delta_{u,v}(H) \subseteq \{0\} \cup uC \subseteq \{0\} \cup HC$. Furthermore,

$$\begin{aligned} (\delta_{t,s}\delta_{u,v})^d(w) &= \delta^d_{t,s}\delta_{u,v}(w) + \delta_{t,s}\delta^d_{u,v}(w) \\ &= \delta^d_{t,s}\delta_{u,v}(w) + \delta_{t,s}(s) \\ &= \delta^d_{t,s}\delta_{u,v}(w) + t \end{aligned}$$

and so (3) and (4) imply that $t = -\delta^d_{t,s}\delta_{u,v}(w) + (\delta_{t,s}\delta_{u,v})^d(w) \in Z(G)$. Therefore $G = Z(G)$ and hence is abelian. This completes that proof. ■

Proof. [Proof of Theorem 1.2]. For $f, g \in N$, we have

$$\begin{aligned} Tfg - fgT = (fg)^d = f^d g + fg^d &= (Tf - fT)g + f(Tg - gT) \\ &= Tfg - fTg + f(Tg - gT), \end{aligned}$$

and so

(6) $$fTg - fgT = f(Tg - gT) \quad \text{for all } f, g \in N.$$

Set $\widehat{C} = C \cup \{\mathbf{0}\}$ where $y\mathbf{0} = 0$ for all $y \in G$. We now claim that

(7) $$T(y) \in y\widehat{C} \quad \text{for all } y \in G.$$

Suppose this is not the case and let $y \in G$ be such that $T(y) \notin y\widehat{C}$. Then

$$y \neq 0, \quad T(y) \neq 0, \quad \text{and } T(y) \neq y.$$

Let $g = \delta_{T(y)-y,T(y)} + \delta_{y,y} \in N$. Then $g(y) = y$ and $gT(y) = T(y) - y$. Therefore $Tg(y) - gT(y) = T(y) - (T(y) - y) = y$. Taking $f = \delta_{y,y}$ and using (6), we have

$$\begin{aligned} y = \delta_{y,y}(y) = \delta_{y,y}(Tg(y) - gT(y)) &= \delta_{y,y}Tg(y) - \delta_{y,y}gT(y) \\ &= \delta_{y,y}T(y) - \delta_{y,y}(T(y) - y) = -\delta_{y,y}(T(y) - y) \end{aligned}$$

and so $T(y) - y \in yC$. Say $T(y) - y = y\beta$ where $\beta \in C$. Then

$$y = -\delta_{y,y}(T(y) - y) = -y\beta$$

forcing

$$-y = \delta_{y,y}(T(y) - y) = y\beta = T(y) - y,$$

and hence $T(y) = 0$, a contradiction. Therefore (7) holds.

Given $\alpha \in \widehat{C}$, we set

$$G_\alpha = \{y \in G \mid T(y) = y\alpha\}.$$

Since T is an endomorphism of G, each G_α is a subgroup of G. What we have just shown in (7) was that

(8) $$G = \cup_{\alpha \in \widehat{C}}\, G_\alpha.$$

Assume that $G = G_\alpha$ for some $\alpha \in \widehat{C}$. Then for any $f \in N$ and $y \in G_\alpha$, we have

$$f^d(y) = (Tf - fT)(y) = Tf(y) - fT(y) = f(y)\alpha - f(y\alpha) = 0$$

which means $d = 0$, a contradiction. Therefore $G \neq G_\alpha$ for all $\alpha \in \widehat{C}$. Set $\mathrm{supp}(G) = \{\alpha \in \widehat{C} \mid G_\alpha \neq 0\}$. Since a group cannot be the union of two proper subgroups, we conclude that $|\mathrm{supp}(G)| > 2$.

Take $\alpha_1, \alpha_2 \in \mathrm{supp}(G)$ with $\alpha_1 \neq \alpha_2$, and let $y \in G_{\alpha_1} \setminus \{0\}$ and $z \in G_{\alpha_2} \setminus \{0\}$. Then $y\alpha_1 - y\alpha_2 \neq 0$ since α_1 and α_2 are fixed point free automorphisms of G. Using (6), we obtain that

$$\begin{aligned} y\alpha_1 - y\alpha_2 &= \delta_{y,y} T \delta_{y,z}(z) - \delta_{y,y}\delta_{y,z}T(z) \\ &= \delta_{y,y}(T\delta_{y,z}(z) - \delta_{y,z}T(z)) = \delta_{y,y}(y\alpha_1 - y\alpha_2), \end{aligned}$$

and so

$$y\alpha_1 - y\alpha_2 = y\beta_{y,z} \tag{9}$$

for some $\beta_{y,z} \in C$. Note that $y - z \in G_{\alpha_3}$ for some $\alpha_3 \in C$. Since $z \in G_{\alpha_2}$, $y \notin G_{\alpha_2}$, and G_{α_2} is a subgroup of G, we see that $\alpha_3 \neq \alpha_2$. We now get from (9) that

$$\begin{aligned} (y-z)\alpha_3 &= T(y-z) = T(y) - T(z) = y\alpha_1 - z\alpha_2 \\ &= y\alpha_1 - y\alpha_2 + y\alpha_2 - z\alpha_2 = y\beta_{y,z} + (y-z)\alpha_2 \end{aligned}$$

and so $(y-z)\alpha_3 - (y-z)\alpha_2 = y\beta_{y,z}$. On the other hand, by (9) there is a $\beta_{y-z,z} \in C$ such that $(y-z)\alpha_3 - (y-z)\alpha_2 = (y-z)\beta_{y-z,z}$ and so $(y-z)\beta_{y-z,z} = y\beta_{y,z}$. Therefore

$$y - z = y\beta_{y,z}\beta_{y-z,z}^{-1} \in yC. \tag{10}$$

Note that $-z \in G_{\alpha_2}$ and so substituting $-z$ for z in (10) we get $y + z \in yC$. Similarly $y + z \in zC$. Thus, yC and zC are orbits of the fixed point free automorphism group C having nontrivial intersection, and so $yC = zC$. Summarizing, we have that

if $\alpha_1 \neq \alpha_2$ and $y, z \in G^*$ are such that $y \in G_{\alpha_1}$ and $z \in G_{\alpha_2}$, then $yC = zC$.

Since $G = \cup_{\alpha \in \widehat{C}} G_\alpha$, we conclude that $G^* = yC$ for each $y \in G^*$. In particular, $G^* = x_0 C$ and $G = x_0 \widehat{C}$.

Define a multiplication by the rule $yz = y\alpha$ when $z = x_0\alpha$, $\alpha \in \widehat{C}$. This turns G into a near-field because C is a fixed point free group on G. We shall denote this near-field by F. From the definition of the multiplication, one sees that the group C can be identified with the multiplicative group $F^* = F \setminus \{0\}$ that acts on $F = G$ via right multiplications. Finally, the rule $f \mapsto f(x_0)$, $f \in M_C^0(G)$, allows us to identify $M_C^0(G)$ with left multiplications by elements of F. Indeed, given $z = x_0\alpha$, $\alpha \in \widehat{C}$, we have that $f(x_0)z = f(x_0)\alpha = f(x_0\alpha) = f(z)$. The proof is now complete. ■

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