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GRAPHS WITH ISOMORPHIC NEIGHBOR-SUBGRAPHS

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Abstract. A graph G is said to be H-regular if for each vertex $v \in V(G)$, the graph induced by $N_G(v)$ is isomorphic to H. A graph H is a feasible neighbor-subgraph if there exists an H-regular graph, otherwise H is a forbidden neighbor-subgraph. In this paper, we obtain some classes of graphs H which are forbidden and then we focus on searching H-regular graphs especially those graphs of smaller order.

1. INTRODUCTION

A graph G is said to be H-regular if for each vertex $v \in V(G)$, the graph induced by $N_G(v)$ is isomorphic to H. Since for each vertex in an H-regular graph its neighbor induces H, an H-regular must be a regular graph. A bit of reflection, such graphs do exist. For example, the complete graphs, balanced complete multipartite graphs and triangle-free regular graphs are H-regular for some H respectively. On the other hand, it is not difficult to realize that H can not be a star with at least two edges. For convenience, we say a graph H is feasible if there exists an H-regular graph, otherwise H is forbidden.

In this paper, by using several results on finding forbidden graphs and feasible graphs we are able to characterize all feasible graphs of order at most 5. We also include four graphs in Appendix which are C_6, C_7, P_6 and P_7-regular respectively. From these graphs, we expect that to characterize all feasible graphs in general is going to be very difficult. To conclude, we also present a strongly regular graph which is not an H-regular graph for some H, this supports our expectation.

2. FORBIDDEN GRAPHS

We start the study with the existence of forbidden graphs. For the graph terms, we refer to the textbook written by D.B. West [3]. The following lemma shows that there are quite a few connected graphs which are forbidden.

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Proposition 2.1. *Let H be a graph with $|V(H)| \geq 3$. If there exist two vertices x and y such that x, $y \in V(H)$, $d_H(x) = |V(H)| - 1$ and $d_H(y) = 1$, then H is a forbidden graph.*

Proof. Suppose not. Let G be an H-regular graph and we consider an arbitrary vertex v in G. By the definition of an H-regular graph, $N_G(v)$ induces a graph G' which is isomorphic to H. Let $u \in N_G(v)$ such that $d_{G'}(u) = |V(H)| - 1$ and $w \in N_G(v)$ such that $d_{G'}(w) = 1$. Now, since $w \in V(G)$, $N_G(w)$ also induces a graph G'' which is isomorphic to H. But, by the fact that $w \in N_G(v)$ and $d_{G'}(w) = 1$, $V(G'')$ contains exactly $|V(H)| - 2$ vertices which are not in $V(G') \cup \{v\}$, moreover $\{u, v\} \subseteq V(G'')$. Now, since $d_G(u) = d_G(v) = |V(H)|$, uv is an independent edge in G''. By assumption that H is connected, G'' is not isomorphic to H. Therefore, G can not be an H-regular graph. This concludes the proof. ■

Corollary 2.2. *Let H be a graph with $|V(H)| \geq 3$. If there exist two vertices x and y in H such that $d_H(x) = |V(H)| - 1$ and $d_H(y) = 1$. Then $H \cup O_t$ is a forbidden graph for each $t \geq 1$.*

Proof. The proof follows by a similar argument. ■

If the connected graph we consider in Proposition 2.1 is a tree, then we can lower down the maximum degree.

Proposition 2.3. *Let H be a tree of order n and $x \in V(H)$ such that $d_H(x) > (2n - 2)/3$. Then H is a forbidden graph.*

Proof. Suppose not. Let G be an H-regular graph and v is an arbitrary vertex of $N_G(v)$. By assumption $G[N_G(v)] = H$. Let $u \in N_G(v)$ be the vertex of degree k larger than $(2n-2)/3$ in $G[N_G(v)]$ and $A = N_G(v) \setminus N_G[u]$, $B = N_G(u) \setminus N_G[v]$. If any vertex in $A \bigcup B$ is adjacent to two vertices in $N_G(v) \bigcap N_G(u)$, then we will find a C_4 in $G[N_G(v)]$ or $G[N_G(u)]$ which are not trees. Since $|A| + |B| < 2[n - 1 - \frac{(2n-2)}{3}] = \frac{(2n-2)}{3}$, there exists a vertex w such that only u and v are adjacent to w in $N_G[u]$ (similarly in $N_G[v]$). If w is adjacent to any vertex of $N_G(v) \bigcap N_G(u)$ in G, then there is a C_3 in $N_G(v)$. So, uv is an independent edge in $G[N_G(w)]$. By assumption that H is connected, $G[N_G(w)]$ is not isomorphic to H. Therefore, G can not be an H-regular graph. ■

Proposition 2.4. *If $H = K_n - P_s$, then H is a forbidden graph for $n \geq 3$ and $2 \leq s \leq n - 1$.*

Proof. Suppose G is a $(K_n - P_s)$-regular graph for some $2 \leq s \leq n-1$ and v is an arbitrary vertex of G. By assumption, $G[N_G(v)] = K_n - P_s$. Let $H = K_n - P_s$.

Then there exist $x, y, z \in V(H)$ such that $d_H(x) = n-1, d_H(y) = n-2$, and $d_H(z) = n-2$. Let $H_1 = H \cup \{v\}$. Now, we consider two cases. Case 1. $s = 2$. Consider the vertex y. Because y is adjacent to v, so $d_{H_1}(y) = n-2+1 = n-1$. Since $n-1$ neighbors of y which are of full degrees, $G[N_G(y)] \neq K_n - P_2$. Case 2. $3 \leq s \leq n-1$. Let $G_1 = G[N_{H_1}(y)]$ and consider the vertex y. Because y is adjacent to v, so $d_{H_1}(y) = n-2+1 = n-1$, $d_{G_1}(v) = 2+n-4 = n-2$, $d_{G_1}(x) = 2+n-4 = n-2$, and the vertices of $G_1 - \{v, x\}$ are of degree at most $n-2$ in G_1. Since y is adjacent to z and $d_{H_1}(y) = n-2+1 = n-1$, there exists a vertex w which is not in H_1, and w is adjacent to z. As to the vertex $u \in G[N_G(y)]$, $d_{G[N_G(y)]}(u) \leq n-2$. Now, consider the vertex w. Since $d_G(y) = d_G(z) = n$, $G[N_G(w)] \neq K_n - P_s$. Both cases lead to a contradiction. Hence, the proof is concluded. ■

Proposition 2.5. *If $H = K_{m,n}$ and $m \neq n$, then H is a forbidden graph.*

Proof. Suppose not. Let G be an H-regular graph and v be an arbitrary vertex of G. By assumption, $G[N_G(v)] = H$. Suppose that H consists of X and Y, where $|X| = m, |Y| = n$ and $m > n$. Let $G_1 = G[N_G(v)]$. Then $d_{G_1}(x) = n+1$ for all $x \in X$ and $G[N_{G_1}(x)] = K_{1,n}$. Since X is an independent part, $N_G(v) \bigcap N_G(x) = Y$. By the fact that $G[N_G(x)]$ is isomorphic to H, each vertex of A joins to each vertex of Y, where $A = N_G(x) \setminus (Y \cup \{v\})$. But $d_G(y) = (m+1) + (m-1) = 2m > m+n$ for all $y \in Y$, this leads to a contradiction. Hence, the proof is concluded. ■

Corollary 2.6. *If $H = K_{n_1, n_2, \cdots, n_r}$ and $n_i \neq n_j$, for some $i \neq j$, then H is a forbidden graph.*

Proof. The proof follows by a similar argument. ■

3. Constructions of H-Regular Graphs

In this section, we will use operations of graphs to discuss the structure of H-regular graphs.

Proposition 3.1. *If G is an H-regular graph, then $G \vee G$ is a $(G \vee H)$-regular graph.*

Proof. Let v be an arbitrary vertex of $G \vee G$. Then $G[N_{G \vee G}(v)] = G \vee G[N_G(v)] = G \vee H$. ■

Corollary 3.2. *$C_n \vee C_n$ is a K_5-regular graph for $n = 3$ and it is a $(C_n \vee O_2)$-regular graph for all $n \geq 4$.*

Proof. By Proposition 3.1, since C_3 is an P_2-regular graph, $C_3 \vee C_3$ is a $(C_3 \vee P_2)$ -regular graph, i.e., K_5-regular graph. On the other hand, C_n is an O_2-regular graph, for all $n \geq 4$, $C_n \vee C_n$ is a $(C_n \vee O_2)$-regular graph, for all $n \geq 4$. ■

Proposition 3.3. *If G_1 is an H_1-regular graph and G_2 is an H_2-regular graph, then the Cartesian product $G_1 \square G_2$ is an ($H_1 \cup H_2$)-regular graph.*

Proof. Choose a vertex $x \in V(G_1 \square G_2)$. By definition of Cartesian product, $N_{G_1 \square G_2}(x) = N_{G_1}(x) \cup N_{G_2}(x)$. Hence $G[N_{G_1 \square G_2}(x)] = G[N_{G_1}(x) \cup N_{G_2}(x)] = H_1 \cup H_2$. ■

Corollary 3.4. *If H-regular graphs exist, then $(H \cup O_t)$-regular graphs exist for $t \geq 1$.*

Proof. Let G be an H-regular graph. Because $K_{t,t}$ is an O_t-regular graph for each $t \geq 1$, by Proposition 3.3, $G \square K_{t,t}$ is an $(H \cup O_t)$-regular graph. ■

Proposition 3.5. *If G is an H-regular graph, then G^t is a $(\bigcup^t H)$-regular graph for each $t \geq 1$, where $\bigcup^t H$ is $H \cup H \cup \cdots \cup H$ (t tuple).*

Proof. By Proposition 3.3, G^t is an $(\bigcup^t H)$-regular graph for each $t \geq 1$. ■

Corollary 3.6. *$(K_3)^t$ is an M_t-regular graph for each $t \geq 1$.*

Proof. Because K_3 is an M_1-regular graph, by Proposition 3.5, we conclude that $(K_3)^t$ is an M_t-regular graph. ■

Corollary 3.7. *If G is an H-regular graph, then $G \square (K_3)^t$ is an $(H \cup M_t)$-regular graph.*

Proof. Because $(K_3)^t$ is an M_t-regular graph, by Proposition 3.3, we get $G \square (K_3)^t$ is an $(H \cup M_t)$-regular graph. ■

4. H-Regular Graphs of Small Orders

We shall consider the graphs H with order ≤ 5.

Proposition 4.1. *C_n-regular graph exists for $n = 3, 4, 5$.*

Proof. The followings are easy to check.

- $n = 3$ Tetrahedron is a C_3-regular graph.

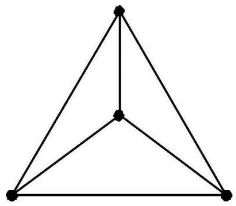

Fig. 1. C_3-regular graph.

- $n = 4$ Octahedron is a C_4-regular graph.

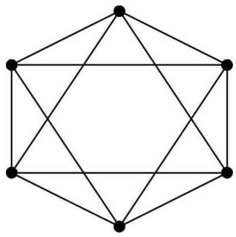

Fig. 2. C_4-regular graph.

- $n = 5$ Icosahedron is a C_5-regular graph.

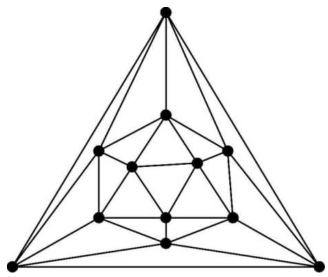

Fig. 3. C_5-regular graph.

Proposition 4.2. *A P_n-regular graph exists for $n = 2, 4, 5$.*

Proof. The followings are easy to check,

- $n = 2$ C_3 is a P_2-regular graph.
- $n = 3$ No P_3-regular graph, by Proposition 2.4.

- $n = 4$

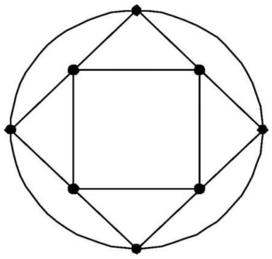

Fig. 4. P_4-regular graph.

- $n = 5$

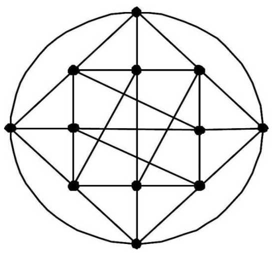

Fig. 5. P_5-regular graph.

Proposition 4.3. *For each graph of order 2, H, there exists an H-regular graph.*

Proof. Since H is of order 2, $H = P_2$ or O_2. The proof follows by letting the H-regular graphs be K_3 and C_4 respectively. ■

Proposition 4.4. *There exists an H-regular graph for each graph H of order 3 except $H = P_3$.*

Proof.
- $H = O_3$ $K_{3,3}$ is an O_3-regular graph.
- $H = P_2 \cup O_1$ Since K_3 is an P_2-regular graph, by Proposition 3.3, $K_3 \square K_2$ is a $P_2 \cup O_1$-regular graph.
- $H = P_3$ Because $P_3 = K_3 - P_2$, by Proposition 2.4, no P_3-regular graphs exist.
- $H = K_3$ K_4 is a K_3-regular graph. ■

Proposition 4.5. *There exists an H-regular graph for the graphs H of order 4 except $H = K_4 - P_2, K_4 - P_3, S_3$ or $P_3 \cup O_1$.*

Proof.

- $H = O_4$ $K_{4,4}$ is a a O_4-regular graph.
- $H = P_2 \cup O_2$ Since $K_3 \square K_2$ is a $P_2 \cup O_1$-regular graph, by Proposition 3.3, $(K_3 \square K_2) \square K_2$ is a $P_2 \cup O_2$-regular graph.
- $H = M_2$ $(K_3)^2$ is an M_2-regular graph. (Corollary 3.6.)
- $H = C_3 \cup O_1$ Since K_4 is a C_3-regular graph, by Proposition 3.3, $K_4 \square K_2$ is a $C_3 \cup O_1$-regular graph.
- $H = P_4$ or C_4 By Proposition 4.1 and Proposition 4.2.
- $H = K_4$ K_5 is a K_4-regular graph.
- $H = K_4 - P_2$ or $K_4 - P_3$ By Proposition 2.4, no $(K_4 - P_2)$-regular graphs and $(K_4 - P_3)$-regular graphs exist.
- $H = S_3$ or $P_3 \cup O_1$ By Proposition 2.1 and Corollary 2.2, no S_3-regular graphs and $(P_3 \cup O_1)$-regular graphs exist. ■

Proposition 4.6. *Let H be a graph of order 5. Then an H-regular graph exists if and only if $H = G_1, G_2, G_4, G_5, G_7, G_8, G_{10}, G_{13}, G_{14}, G_{20}, G_{21}, G_{24}, G_{25}, G_{34}$, see Figure 6.*

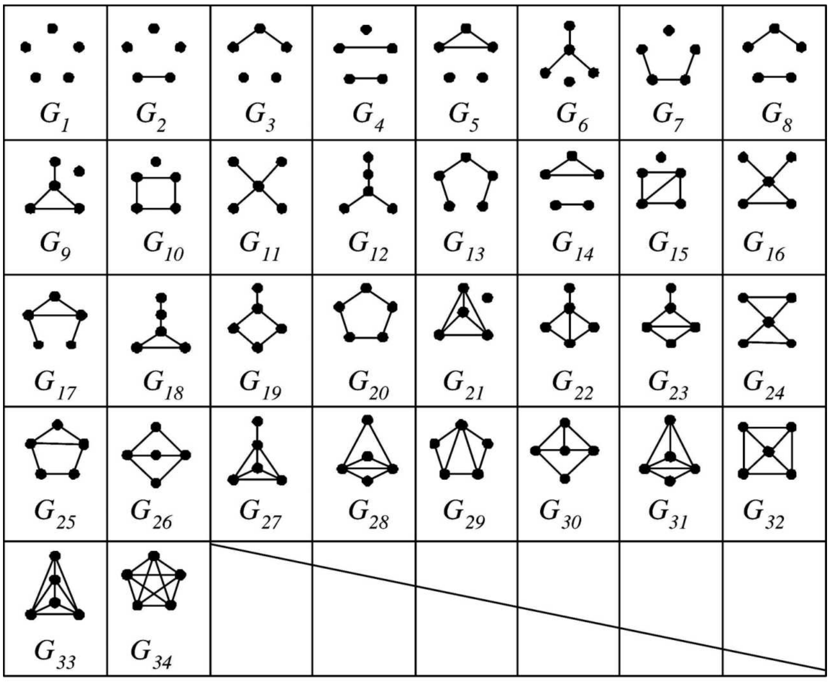

Fig. 6. All graphs of order 5 [2].

Proof.

- $H = G_1$ and G_{34} $K_{5,5}$ is a G_1-regular graph and K_6 is a G_{34}-regular graph.
- $H = G_2, G_4, G_5, G_7, G_{10}, G_{14}$ and G_{21} By Corollary 3.4, $G_2, G_4, G_5, G_7, G_{10}, G_{14}$ and G_{21}-regular graphs exist respectively.
- $H = G_{24}$ and G_{25} $D_1 = (Z_8, E_1)$ where $uv \in E_1$ if and only if $min\{8 - |u-v|, |u-v|\} \in \{1, 3, 4\}$ is a G_{24}-regular graph. $D_2 = (Z_8, E_2)$ where $uv \in E_2$ if and only if $min\{8 - |u-v|, |u-v|\} \in \{1, 2, 4\}$ is a G_{25}-regular graph.
- $H = G_8$ It was obtained by D.G. Hoffman first. Here, we present a G_8-regular graph with smaller order.

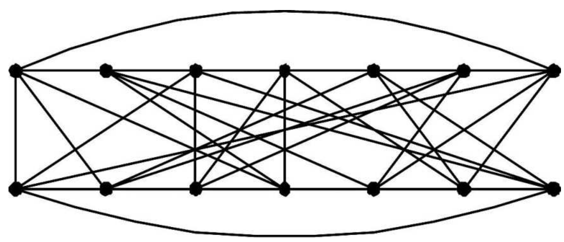

Fig. 7. $(P_3 \cup P_2)$-regular graph.

- $H = G_{13}$ and G_{20} By Proposition 4.1 and Proposition 4.2, G_{13} and G_{20}-regular graphs exist respectively.
- $H = G_3, G_6$ and G_{11} By Corollary 2.2, G_3, G_6 and G_{11} are forbidden graphs.
- $H = G_{12}$ By Proposition 2.3, G_{12} is a forbidden graph.
- $H = G_9, G_{15}, G_{29}, G_{31}$ and G_{33} By Corollary 2.2 and Proposition 2.4, $G_9, G_{15}, G_{29}, G_{31}$ and G_{33} are forbidden graphs.
- $H = G_{16}, G_{22}$ and G_{27} By Proposition 2.1, G_{16}, G_{22} and G_{27} are forbidden graphs.
- $H = G_{26}$ Because G_{26} is $K_{3,2}$. By Proposition 2.5, no G_{26}-regular graphs exist.
- $H = G_{28}$ By Proposition 2.6, no G_{28}-regular graphs exist.

For the followings cases, we shall use similar technique to prove the nonexistence of an H-regular graph for $H = G_{17}, G_{18}, G_{19}, G_{23}, G_{30}$ and G_{32}. Since their proofs are similar, we show the proofs of the first two cases, and omit the others.

- $H = G_{17}$ Let G be a G_{17}-regular graph and $v \in V(G)$ such that $G[N_G(v)] = G_{17}$. Let $N_G(v) = \{x, y, z, w, u\}$ such that $x \sim y$, $x \sim z$, $x \sim w$, $z \sim w$ and $w \sim u$. By assumption $G[N_G(z)] = G_{17}$, there exist two vertices p and q which are not in $N_G(v)$ such that $p \sim x$ and $q \sim w$. It's easy to see that p is not incident to q by consider $G[N_G(x)]$. Since $d_G(v)$ and $d_G(x)$ are both of degree 5, xv is an independent edge in $G[N_G(y)]$. Hence, $G[N_G(y)] \neq G_{17}$. This is a contradiction and thus G_{17} is forbidden.
- $H = G_{18}$ Let G be a G_{18}-regular graph and $v \in V(G)$ such that $G[N_G(v)] = G_{18}$. Let $N_G(v) = \{x, y, z, w, u\}$ such that $x \sim y$, $x \sim w$, $x \sim u$, $y \sim z$ and $w \sim u$. By assumption $G[N_G(x)] = G_{18}$, there exists a vertex p which is not in $N_G(v)$ such that $p \sim x$ and $p \sim y$. Consider $G[N_G(y)]$. Since $d_G(v)$ and $d_G(x)$ are of degree 5, $G[N_G(y)] \neq G_{18}$. This is a contradiction. Hence, G_{18} is forbidden. ■

5. Concluding Remark

The study of neighbor-regular graphs has just begun. So far, not much is known. In this paper, we manage to obtain several classes of graphs which are forbidden and for quite a few graphs H we construct an H-regular graph. But, we also realize the difficulty of obtaining general results. For example, we can construct H-regular graphs for $H = C_n$ or P_n whenever $n \leq 7$ (Figure 8, 9, 10, 11). How about $n \geq 8$? On the other hand, we are able to say something about forbidden graphs, but there are quite a few forbidden graphs remained unknown. To conclude this paper, we would like to present an example to show the differences between H-regular graphs and strongly regular graphs, see Appendix.

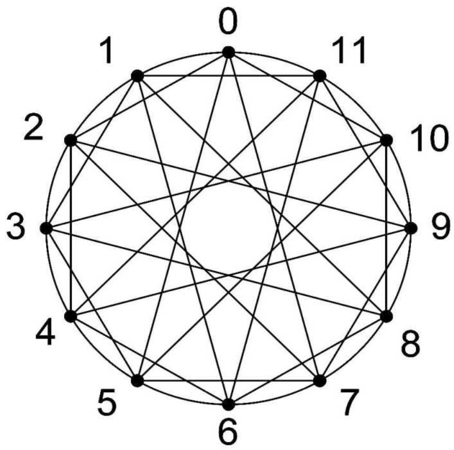

Fig. 8. C_6-regular graph.

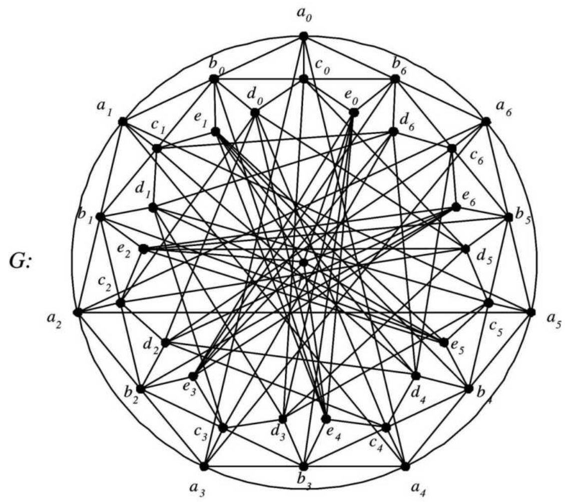

Fig. 9. C_7-regular graph.

$V(G) = \{a_i, b_i, c_i, d_i, e_i, f| \ i \in Z_7\}$, and edges of G are :
$a_i \sim [a_{i+1}, a_{i+3}, a_{i+4}, a_{i+6}, b_i, c_i, b_{i+6}]; b_i \sim [a_i, c_i, c_{i+1}, a_{i+1}, d_i, e_{i+1}, e_{i+4}];$
$c_i \sim [a_i, b_i, b_{i+6}, d_i, d_{i+3}, d_{i+5}, e_i]; d_i \sim [b_i, c_i, c_{i+2}, c_{i+4}, d_{i+2}, d_{i+5}, e_{i+4}];$
$e_i \sim [b_{i+3}, b_{i+6}, c_i, d_{i+3}, e_{i+3}, e_{i+4}, f]; f \sim [e_0, e_3, e_6, e_2, e_5, e_1, e_4].$
Note : $x \sim [\alpha_1, \alpha_2, \ldots, \alpha_k] =_{def} \{x \sim \alpha_i| \ i = 1, 2, \ldots, k\}.$

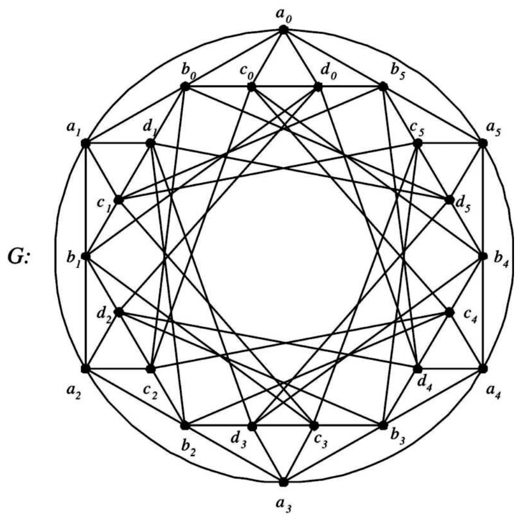

Fig. 10. P_6-regular graph.

$V(G) = \{a_i, b_i, c_i, d_i| \ i \in Z_6\}$, and edges of G are :
$a_i \sim [a_{i+1}, b_i, c_i, d_i, b_{i+5}, a_{i+5}]; b_i \sim [c_{i+2}, c_i, a_i, a_{i+1}, d_{i+1}, d_{i+5}];$
$c_i \sim [d_i, a_i, b_i, c_{i+2}, c_{i+4}, b_{i+4}]; d_i \sim [c_i, a_i, b_{i+5}, d_{i+4}, d_{i+2}, b_{i+1}].$

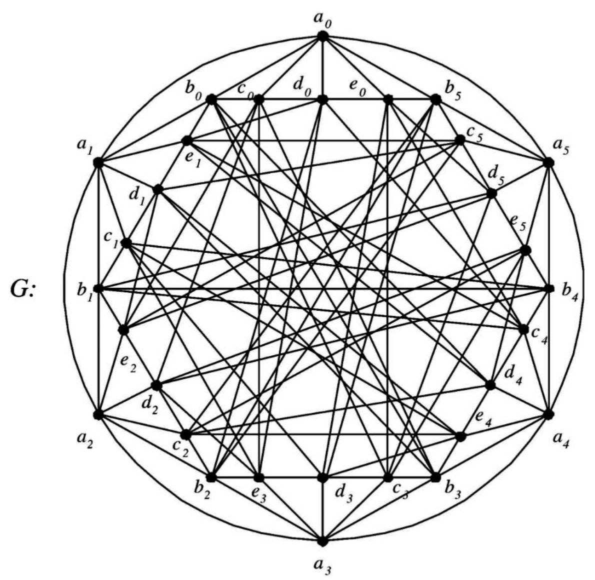

Fig. 11. P_7-regular graph.

$V(G) = \{a_i, b_i, c_i, d_i, e_i |\ i \in Z_6\}$, and edges of G are :

$a_i \sim [a_{i+1}, b_i, c_i, d_i, e_i, b_{i+5}, a_{i+5}\ ]; b_i \sim [d_{i+4}, e_{i+1}, a_{i+1}, a_i, c_i, b_{i+3}, c_{i+3}];$

$c_i \sim [e_{i+2}, d_{i+2}, e_{i+3}, d_i, a_i, b_i, b_{i+3}]; d_i \sim [e_{i+1}, c_{i+4}, e_i, a_i, c_i, e_{i+3}, b_{i+2}];$

$e_i \sim [c_{i+4}, d_i, a_i, b_{i+5}, d_{i+3}, c_{i+3}, d_{i+5}].$

Appendix

A strongly regular graph which is not an H-regular graph for some H. Let G be a strongly regular graph with 17 vertices and parameters $(k, \lambda, \mu) = (8, 3, 4)$, and the adjacent matrix of G is

$$
\begin{array}{c}
\\
v_1\\ v_2\\ v_3\\ v_4\\ v_5\\ v_6\\ v_7\\ v_8\\ v_9\\ v_{10}\\ v_{11}\\ v_{12}\\ v_{13}\\ v_{14}\\ v_{15}\\ v_{16}\\ v_{17}
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccccccccc}
v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} & v_{17}
\end{array}\\
\begin{pmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0\\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0\\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1\\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0\\
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1\\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1\\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1\\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0\\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0\\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1\\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1\\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1\\
0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1\\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

where $v_i \in V(G)$ for all $i = 1, 2, 3, \ldots, 17$. Consider the neighbors of v_1 and v_5, then we get $G[N_G(v_1)]$ is not isomorphic to $G[N_G(v_5)]$.

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