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WELL-POSEDNESS OF SYSTEMS OF EQUILIBRIUM PROBLEMS

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Abstract. In this paper we introduce the concepts of well-posedness and generalized well-posedness for a system of equilibrium problems. We derive a metric characterization of well-posedness by considering the diameter of approximating solution set and a Furi-Vignoli type characterization of generalized well-posedness by considering the Kuratowski noncompactness measure of approximating solution set. Under suitable conditions, we prove that the well-posedness of a system of equilibrium problems is equivalent to the existence and uniqueness of its solution.

1. INTRODUCTION

The concept of well-posedness was first introduced by Tykhonov [37] for a minimization problem, which has been known as Tykhonov well-posedness. The Tykhonov well-posedness of a minimization problem means the existence and uniqueness of minimizers, and the convergence of every minimizing sequence toward the unique minimizer. The importance of Tykhonov well-posedness for a minimization problem has been widely recognized by researchers from theoretical and practical fields. For details, we refer the readers to [7, 12, 19, 31, 36] and the references therein. The concept of well-posedness has been generalized to several problems related to minimization problems for past decades. As known, under convexity and differentiability assumptions, a minimization problem is equivalent to a variational inequality problem. This fact motivates researchers to study the well-posedness of

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various variational inequality problems. Lucchetti and Patrone [31] introduced the first notion of well-posedness for a variational inequality in the literature. Since then, some concepts of well-posedness have been introduced and studied for several classes of variational inequality problems (see, e.g., [1, 9, 10, 14, 15, 28, 30]). The concept of well-posedness has also been generalized to Nash equilibrium problems (see, e.g., [29, 32-35]) and fixed point problems (see, e.g., [26, 27]). Blum and Oettli [8] introduced and studied the equilibrium problem, which provides a unifying formulation of minimization problems, variational inequality problems, complementarity problems, Nash equilibrium problems and fixed point problems. Recently, Fang et al [15] further considered the well-posedness of equilibrium problems.

On the other hand, some new contributions have been given to the theories of variational inequalities and equilibrium problems. Some authors introduced and studied systems of variational inequalities (see, e.g., [5, 6, 16, 20, 21, 38]). The motivations originate from the fact that under suitable conditions, a Nash equilibrium problem is equivalent to a system of variational inequalities (see, e.g., [22]). Some authors further introduced and studied systems of various equilibrium problems (see, e.g., [2, 3, 4, 17, 18, 23]). The purpose of this paper is to investigate the well-posedness of a system of equilibrium problems. We generalize the concept of well-posedness to a system of equilibrium problems and derive some metric characterizations of well-posedness. Under suitable conditions, we further prove that the well-posedness of a system of equilibrium problems is equivalent to the existence and uniqueness of its solution.

2. Preliminaries and Notations

Let E and K be two Banach spaces and let $D \subset E$ and $K \subset X$ be two nonempty sets. Let $f : D \times K \times K \to R$ and $g : K \times D \times D \to R$ be two functions. The system of equilibrium problems is formulated as follows: find $(x^*, u^*) \in D \times K$ such that

$$(SEP): \quad \begin{cases} f(x^*, u^*, u) \geq 0, & \forall u \in K, \\ g(u^*, x^*, x) \geq 0, & \forall x \in D. \end{cases}$$

The system of equilibrium problems includes as special cases systems of variational inequalities considered in [38, 5, 20, 21].

In the sequel we recall some concepts.

Definition 2.1. [8]. A bifunction $\varphi : K \times K \to R$ is said to be monotone if

$$\varphi(u, v) + \varphi(v, u) \leq 0, \quad \forall u, v \in K.$$

Definition 2.2. [8]. Let K be convex. A bifunction $\varphi : K \times K \to \bar{R}$ is said to be hemicontinuous if for each $x, y \in K$,

$$\limsup_{t \to 0+} \varphi(x + t(y-x), y) \le \varphi(x, y).$$

The following Minty type lemma plays a very important role in the theory of equilibrium problems.

Lemma 2.1. [8]. *Let K be convex and let $\varphi : K \times K \to R$ be a monotone and hemicontinuous bifunction. Assume that*

(i) $\varphi(u, u) \ge 0$ *for all* $u \in K$.

(ii) *for every* $u \in K$, $\varphi(u, \cdot)$ *is convex.*

Then for given $u^* \in K$,

$$\varphi(u^*, v) \ge 0, \quad \forall v \in K$$

if and only if

$$\varphi(v, u^*) \le 0, \quad \forall v \in K.$$

To derive a Furi-Vignoli type characterization of generalized well-posedness, we need the following concepts.

Definition 2.3. (see [25]). Let A be a nonempty subset of E. The measure of noncompactness μ of the set A is defined by

$$\mu(A) = \inf\{\epsilon > 0 : A \subset \cup_{i=1}^{n} A_i, \mathrm{diamA_i} < \epsilon, \mathrm{i} = 1, 2, \cdots, \mathrm{n}\},$$

where diam means the diameter of a set.

Definition 2.4. Let A, B be nonempty subsets of E. The Hausdorff metric $\mathcal{H}(\cdot, \cdot)$ between A and B is defined by

$$\mathcal{H}(A, B) = \max\{e(A, B), e(B, A)\},$$

where $e(A, B) = \sup_{a \in A} d(a, B)$ with $d(a, B) = \inf_{b \in B} \|a - b\|$. Let $\{A_n\}$ be a sequence of nonempty subsets of E. We say that A_n converges to A in the sense of Hausdorff metric if $\mathcal{H}(A_n, A) \to 0$. It is easy to see that $e(A_n, A) \to 0$ if and only if $d(a_n, A) \to 0$ for all selection $a_n \in A_n$. For more details on this topic, we refer the readers to [24, 25].

3. Well-posedness and Metric Characterizations

In this section we introduce the concepts of well-posedness and generalized well-posedness for (SEP) and give some metric characterizations of well-posedness and generalized well-posedness.

Definition 3.1. A sequence $\{(x_n, u_n)\}$ is called an approximating sequence for (SEP) if $(x_n, u_n) \in D \times K$ for all $n \in N$ and there exists $0 < \epsilon_n \to 0$ such that for all $n \in N$,

$$\begin{cases} f(x_n, u_n, u) + \epsilon_n \geq 0, & \forall u \in K, \\ g(u_n, x_n, x) + \epsilon_n \geq 0, & \forall x \in D. \end{cases}$$

Definition 3.2. We say that (SEP) is well-posed if (SEP) has a unique solution and every approximating sequence for (SEP) converges strongly to the unique solution; We say that (SEP) is generalized well-posed if the solution set S of (SEP) is nonempty and every approximating sequence for (SEP) has a subsequence which converges strongly to some point of S.

To give some metric characterizations of well-posedness and generalized well-posedness, we consider the following approximating solution set:

$$\Omega(\epsilon) = \{(x, u) \in D \times K : f(x, u, v) + \epsilon \geq 0, \forall v \in K \text{ and } g(u, x, y) + \epsilon \geq 0, \forall y \in D\}.$$

Obviously, $\Omega(\epsilon_1) \leq \Omega(\epsilon_2)$ whenever $0 \leq \epsilon_1 \leq \epsilon_2$.

The following theorem derives a metric characterization of well-posedness of (SEP).

Theorem 3.1. *Let D and K be nonempty, closed and convex subsets of Banach spaces E and X respectively. Assume that $f : D \times K \times K \to R$ and $g : K \times D \times D \to R$ satisfy the following conditions:*

(i) *for every $(x, u) \in D \times K$, $f(x, u, u) \geq 0$ and $g(u, x, x) \geq 0$.*

(ii) *for every $(x, u) \in D \times K$, $f(x, \cdot, \cdot)$ and $g(u, \cdot, \cdot)$ are monotone and hemicontinuous.*

(iii) *for every $(x, u) \in D \times K$, $f(\cdot, u, \cdot)$ and $g(\cdot, x, \cdot)$ are convex and lower semicontinuous.*

Then (SEP) is well-posed if and only if

$$\Omega(\epsilon) \neq \emptyset, \forall \epsilon > 0, \text{ and } diam\, \Omega(\epsilon) \to 0 \text{ as } \epsilon \to 0.$$

Proof. Suppose that (SEP) is well-posed. Then there exists unique $(x^*, u^*) \in D \times K$ such that

$$\begin{cases} f(x^*, u^*, u) \geq 0, & \forall u \in K, \\ g(u^*, x^*, x) \geq 0, & \forall x \in D. \end{cases}$$

Obviously $\Omega(\epsilon) \neq \emptyset$ since $(x^*, u^*) \in \Omega(\epsilon)$ for all $\epsilon > 0$. If $diam\ \Omega(\epsilon) \not\to 0$ as $\epsilon \to 0$, then there exist $l > 0$, $0 < \epsilon_n \to 0$, and $(x_n, u_n), (y_n, v_n) \in \Omega(\epsilon_n)$ such that

$$\|(x_n, u_n) - (y_n, v_n)\| > l, \quad \forall n \in N.$$

Clearly (x_n, u_n) and (y_n, v_n) are approximating sequences for (SEP). By the well-posedness of (SEP), they have to converge strongly to the unique solution (x^*, u^*) of (SEP), a contradiction.

Conversely, suppose that $\Omega(\epsilon) \neq \emptyset$ for all $\epsilon > 0$ and $diam\ \Omega(\epsilon) \to 0$ as $\epsilon \to 0$. Let $\{(x_n, u_n)\}$ be an approximating sequence for (SEP). Then there exists $0 < \epsilon_n \to 0$ such that for all $n \in N$,

$$\begin{cases} f(x_n, u_n, u) + \epsilon_n \geq 0, & \forall u \in K, \\ g(u_n, x_n, x) + \epsilon_n \geq 0, & \forall x \in D. \end{cases} \tag{1}$$

This means $(x_n, u_n) \in \Omega(\epsilon_n)$ for all $n \in N$. Taking into account $diam\ \Omega(\epsilon) \to 0$ as $\epsilon \to 0$, $\{(x_n, u_n)\}$ is a Cauchy sequence and so it converges strongly to $(\bar{x}, \bar{u}) \in D \times K$. From (1) and conditions (ii)-(iii), we get

$$\begin{aligned} f(\bar{x}, v, \bar{u}) &\leq \liminf_{n\to\infty} f(x_n, v, u_n) \\ &\leq \liminf_{n\to\infty}\{-f(x_n, u_n, v)\} \leq \liminf_{n\to\infty} \epsilon_n = 0, \forall v \in K \end{aligned} \tag{2}$$

and

$$\begin{aligned} g(\bar{u}, y, \bar{x}) &\leq \liminf_{n\to\infty} g(u_n, y, x_n) \\ &\leq \liminf_{n\to\infty}\{-g(u_n, x_n, y)\} \leq \liminf_{n\to\infty} \epsilon_n = 0, \forall y \in D. \end{aligned} \tag{3}$$

By assumptions, it is easy to see that $f(\bar{x}, \cdot, \cdot)$ and $g(\bar{u}, \cdot, \cdot)$ satisfy all the assumptions of Lemma 2.1 respectively. It follows from (2)-(3) and Lemma 2.1 that

$$f(\bar{x}, \bar{u}, v) \geq 0, \forall v \in K \text{ and } g(\bar{u}, \bar{x}, y) \geq 0, \forall y \in D.$$

Thus $(\bar{x}, \bar{u})$ solves (SEP).

To complete the proof, it is sufficient to prove that the solution of (SEP) is unique. This follows directly from $diam\ \Omega(\epsilon) \to 0$ as $\epsilon \to 0$. ■

When (SEP) has more than one solutions, the diameter of Ω does not tend to zero. In this case, we consider the Kuratowski noncompactness measure of approximating solution set instead of the diameter and obtain a Furi-Vignoli type characterization of generalized well-posedness.

Theorem 3.2. *Let D and K be nonempty, closed and convex subsets of real Banach spaces E and X respectively. Let $f : D \times K \times K \to R$ and $g : K \times D \times D \to R$ be such that for every $(x, u) \in D \times K$, $f(\cdot, \cdot, u)$ and $g(\cdot, \cdot, x)$ are upper semicontinuous. Then (SEP) is generalized well-posed if and only if*

$$\Omega(\epsilon) \neq \emptyset, \forall \epsilon > 0, \text{ and } \mu(\Omega(\epsilon)) \to 0 \text{ as } \epsilon \to 0.$$

Proof. Suppose that (SEP) is generalized well-posed. Then the solution set S of (SEP) is nonempty compact. Clearly $\Omega(\epsilon) \neq \emptyset$ since $S \subset \Omega(\epsilon)$ for all $\epsilon > 0$. If $\mu(\Omega(\epsilon)) \not\to 0$ as $\epsilon \to 0$, then there exists $\delta > 0$ such that

$$\mu(\Omega(\epsilon)) > \delta > 0, \qquad \forall \epsilon > 0. \tag{4}$$

By the compactness of S, there exists a finite set $\{w_1, w_2, \cdots, w_m\} \subset S$ such that

$$S \subset \cup_{k=1}^{m} B(w_k, \frac{\delta}{2}), \tag{5}$$

where $B(w_k, \frac{\delta}{2})$ denotes the open ball centered at w_k with radius $\frac{\delta}{2}$. Clearly, $w_k \in \Omega(\epsilon)$ for every $\epsilon > 0$. By (4), for every $n = 1, 2, \cdots, k$, there exists $z_n = (x_n, u_n) \in \Omega(1/n)$ such that

$$z_n \notin \cup_{i=1}^{k} B(w_i, \frac{\delta}{2}) \tag{6}$$

since $\Omega(1/n)$ cannot be covered by finitely many sets with diameter less that δ. It is easy to see that $\{z_n\}$ is an approximating sequence for (SEP). By the generalized well-posedness of (SEP), $\{z_n\}$ has a subsequence converging strongly to some point of S. But as is seen in (5)-(6), $\{z_n\}$ has no subsequence converging to some point of S, a contradiction.

Conversely, assume that

$$\Omega(\epsilon) \neq \emptyset, \forall \epsilon > 0, \text{ and } \mu(\Omega(\epsilon)) \to 0 \text{ as } \epsilon \to 0.$$

We first show that $\Omega(\epsilon)$ is closed for all $\epsilon > 0$. Let $(x_n, u_n) \in \Omega(\epsilon)$ with $(x_n, u_n) \to (\hat{x}, \hat{u})$. It follows that

$$\begin{cases} f(x_n, u_n, u) + \epsilon \geq 0, & \forall u \in K, \\ g(u_n, x_n, x) + \epsilon \geq 0, & \forall x \in D. \end{cases}$$

Since $f(\cdot, \cdot, u)$ and $g(\cdot, \cdot, x)$ are upper semicontinuous,

$$\begin{cases} f(\hat{x}, \hat{u}, u) + \epsilon \geq 0, & \forall u \in K, \\ g(\hat{u}, \hat{x}, x) + \epsilon \geq 0, & \forall x \in D. \end{cases}$$

This yields $(\hat{x}, \hat{u}) \in \Omega(\epsilon)$ and so $\Omega(\epsilon)$ is closed. Observe that

$$S = \cap_{\epsilon > 0} \Omega(\epsilon).$$

Since

$$\mu(\Omega(\epsilon)) \to 0,$$

Theorem on p.412 in [25] can be applied and one concludes that S is nonempty, compact, and

$$e(\Omega(\epsilon), S) = \mathcal{H}(\Omega(\epsilon), S) \to 0 \text{ as } \epsilon \to 0. \tag{7}$$

Let $\{(x_n, u_n)\} \subset D \times K$ be an approximating sequence for (SEP). Then there exists $0 < \epsilon_n \to 0$ such that

$$\begin{cases} f(x_n, u_n, u) + \epsilon_n \geq 0, & \forall u \in K, \\ g(u_n, x_n, x) + \epsilon_n \geq 0, & \forall x \in D. \end{cases}$$

This means that $(x_n, u_n) \in \Omega(\epsilon_n)$. It follows from (7) that

$$d((x_n, u_n), S) \leq e(\Omega(\epsilon_n), S) \to 0.$$

Taking into account the compactness of S, there exists $(\bar{x}_n, \bar{u}_n) \in S$ such that

$$\|(x_n, u_n) - (\bar{x}_n, \bar{u}_n)\| = d((x_n, u_n), S) \to 0.$$

Again from the compactness of S, $\{(\bar{x}_n, \bar{u}_n)\}$ has a subsequence $\{(\bar{x}_{n_k}, \bar{u}_{n_k})\}$ converging strongly to $(\bar{x}, \bar{u}) \in S$. Hence the corresponding subsequence $\{(x_{n_k}, u_{n_k})\}$ of $\{(x_n, u_n)\}$ converges strongly to $(\bar{x}, \bar{u})$. Thus (SEP) is generalized well-posed. ∎

4. Uniqueness and Well-posedness

A classical result on the well-poseness of a minimization problem is that under suitable conditions, the well-posedness is equivalent to the existence and uniqueness of its solution. In this section we shall establish an analogous result for the well-posedness of systems of equilibrium problems.

Theorem 4.1. *Let D and K be nonempty, closed and convex subsets of finite-dimensional Banach spaces E and X respectively. Assume that $f : D \times K \times K \to R$ and $g : K \times D \times D \to R$ satisfy the following conditions:*

(*i*) *for every $(x, u) \in D \times K$, $f(x, u, u) \geq 0$ and $g(u, x, x) \geq 0$.*

(*ii*) *for every $(x, u) \in D \times K$, $f(x, \cdot, \cdot)$ and $g(u, \cdot, \cdot)$ are monotone and hemicontinuous.*

(*iii*) *for every $(x, u) \in X \times K$, $f(\cdot, u, \cdot)$ and $g(\cdot, x, \cdot)$ are convex and lower semicontinuous.*

Then (SEP) is well-posed if and only if it has a unique solution.

Proof. The necessity holds trivially. For the sufficiency, suppose that (SEP) has a unique solution (x^*, u^*). It follows that

$$\begin{cases} f(x^*, u^*, u) \geq 0, \quad \forall u \in K, \\ g(u^*, x^*, x) \geq 0, \quad \forall x \in D. \end{cases}$$

Since $f(x^*, \cdot, \cdot)$ and $g(u^*, \cdot, \cdot)$ are monotone, we get

$$\begin{cases} f(x^*, u, u^*) \leq -f(x^*, u^*, u) \leq 0, \quad \forall u \in K, \\ g(u^*, x, x^*) \leq -g(u^*, x, x^*) \leq 0, \quad \forall x \in D. \end{cases} \tag{8}$$

Let $\{(x_n, u_n)\}$ be an approximating sequence for (SEP). Then there exists $0 < \epsilon_n \to 0$ such that

$$\begin{cases} f(x_n, u_n, u) + \epsilon_n \geq 0, \quad \forall u \in K, \\ g(u_n, x_n, x) + \epsilon_n \geq 0, \quad \forall x \in D. \end{cases} \tag{9}$$

Since $f(x_n, \cdot, \cdot)$ and $g(u_n, \cdot, \cdot)$ are monotone, from (9) we get

$$f(x_n, u, u_n) \leq -f(x_n, u_n, u) \leq \epsilon_n, \quad \forall u \in K \tag{10}$$

and

$$g(u_n, x, x_n) \leq -g(u_n, x_n, x) \leq \epsilon_n, \quad \forall x \in D. \tag{11}$$

We assert that $\{(x_n, u_n)\}$ is bounded. Indeed, if $\{(x_n, u_n)\}$ is unbounded, without loss of generality, we can suppose that $\|(x_n, u_n)\| \to +\infty$. Set

$$t_n = \frac{1}{\|(x_n, u_n) - (x^*, u^*)\|} \text{ and } (y_n, v_n) = (1 - t_n)(x^*, u^*) + t_n(x_n, u_n).$$

Without loss of generality, we can suppose that $t_n \in (0, 1)$ and $(y_n, v_n) \to (\bar{y}, \bar{v})$ with $(\bar{y}, \bar{v}) \neq (x^*, u^*)$ since $E \times X$ is finite-dimensional. It follows from (8), (10)-(11) and conditions (ii)-(iii) that

$$\begin{aligned} f(\bar{y}, u, \bar{v}) &\leq \liminf_{n\to\infty} f(y_n, u, v_n) \\ &\leq \liminf_{n\to\infty} \{t_n f(x_n, u, u_n) + (1 - t_n) f(x^*, u, u^*)\} \\ &\leq \liminf_{n\to\infty} t_n \epsilon_n = 0, \quad \forall u \in K \end{aligned} \tag{12}$$

and

$$
\begin{aligned}
g(\bar{v}, x, \bar{y}) &\leq \liminf_{n\to\infty} g(v_n, x, y_n) \\
&\leq \liminf_{n\to\infty} \{t_n g(u_n, x, x_n) + (1-t_n) g(u^*, x, x^*)\} \\
&\leq \liminf_{n\to\infty} t_n \epsilon_n = 0, \quad \forall x \in D. \qquad (13)
\end{aligned}
$$

Lemma 2.1 together with (12)-(13) implies that

$$
\begin{cases} f(\bar{y}, \bar{v}, u) \geq 0, & \forall u \in K, \\ g(\bar{v}, \bar{y}, x) \geq 0, & \forall x \in D. \end{cases}
$$

Thus $(\bar{y}, \bar{v})$ is also a solution of (SEP), a contradiction to the uniqueness of solution.

Thus $\{(x_n, u_n)\}$ is bounded. Let $\{(x_{n_k}, u_{n_k})\}$ be any subsequence of $\{(x_n, u_n)\}$ such that $(x_{n_k}, u_{n_k}) \to (\bar{x}, \bar{u})$. Since $f(\cdot, u_n, \cdot)$ and $g(\cdot, x_n, \cdot)$ are convex and lower semicontinuous, from (10)-(11) we get

$$
f(\bar{x}, u, \bar{u}) \leq \liminf_{k\to\infty} f(x_{n_k}, u, u_{n_k}) \leq 0, \quad \forall u \in K
$$

and

$$
g(\bar{u}, x, \bar{x}) \leq \liminf_{k\to\infty} g(u_{n_k}, x, x_{n_k}) \leq 0, \quad \forall x \in D.
$$

This together with Lemma 2.1 yields

$$
\begin{cases} f(\bar{x}, \bar{u}, u) \geq 0, & \forall u \in K, \\ g(\bar{u}, \bar{x}, x) \geq 0, & \forall x \in D. \end{cases}
$$

Since (x^*, u^*) is the unique solution of (SEP), we get $(\bar{x}, \bar{u}) = (x^*, u^*)$ and (x_n, u_n) converges to (x^*, u^*). Therefore, (SEP) is well-posed. ■

Theorem 4.2. *Let D and K be nonempty, closed and convex subsets of finite-dimensional Banach spaces E and X respectively. Assume that $f : D\times K\times K \to R$ and $g : K \times D \times D \to R$ satisfy the following conditions:*

(*i*) *for every $(x, u) \in D \times K$, $f(x, u, u) \geq 0$ and $g(u, x, x) \geq 0$.*

(*ii*) *for every $(x, u) \in D\times K$, $f(x, \cdot, \cdot)$ and $g(u, \cdot, \cdot)$ are monotone and hemicontinuous.*

(*iii*) *for every $(x, u) \in X \times K$, $f(\cdot, u, \cdot)$ and $g(\cdot, x, \cdot)$ are convex and lower semicontinuous.*

If there exists some $\epsilon > 0$ such that $\Omega(\epsilon)$ is nonempty bounded, then (SEP) is generalized well-posed.

Proof. Let $\{(x_n, u_n)\}$ be an approximating sequence for (SEP). Then there exists $0 < \epsilon_n \to 0$ such that

$$\begin{cases} f(x_n, u_n, u) + \epsilon_n \geq 0, & \forall u \in K, \\ g(u_n, x_n, x) + \epsilon_n \geq 0, & \forall x \in D. \end{cases}$$

Clearly $\{(x_n, u_n)\} \in \Omega(\epsilon)$ for all sufficiently large n. Taking into account the boundedness of $\Omega(\epsilon)$, there exists a subsequence $\{(x_{n_k}, u_{n_k})\}$ of $\{(x_n, u_n)\}$ such that $(x_{n_k}, u_{n_k}) \to (\bar{x}, \bar{u})$ as $k \to \infty$. By same arguments as in Theorem 4.1, $(\bar{x}, \bar{u})$ is a solution of (SEP). Therefore, (SEP) is generalized well-posed. ■

We note that Theorem 4.2 says nothing but that under suitable conditions, the generalized well-posedness of (SEP) is equivalent to the existence of solutions.

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