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CRITICAL EXPONENT FOR A SYSTEM OF SLOW DIFFUSION EQUATIONS WITH BOTH REACTION AND ABSORPTION TERMS

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Abstract. Let Ω is a bounded domain in R^N with a smooth boundary $\partial\Omega$, $m, n > 1$ and p, q, r, s, a, b are positive constants. For the initial and boundary value problem

$$\begin{aligned} u_t = \triangle u^m + v^p - au^r, &\quad x \in \Omega, \quad t > 0, \\ v_t = \triangle v^n + u^q - bv^s, &\quad x \in \Omega, \quad t > 0, \\ u = v = 0, &\quad x \in \partial\Omega, \quad t > 0, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), &\quad x \in \Omega, \end{aligned}$$

we prove that all solutions are globally bounded if $pq < \max\{n, r\} \max\{n, s\}$; while there are finite time blowing up solutions if $pq < \max\{n, r\} \max\{n, s\}$ and the initial data are sufficiently large. For the critical case $pq = \max\{m, r\}$ $\max\{n, s\}$, the existence or nonexistence of global solutions depends on the relation between the exponents m, n, r, s, and also the range of the parameters a, b.

1. Introduction

In this paper, we consider the following degenerate parabolic system

$$u_t = \triangle u^m + v^p - au^r, \quad x \in \Omega, \quad t > 0, \tag{1.1}$$

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(1.2) $$v_t = \triangle v^n + u^q - bv^s, \quad x \in \Omega, \quad t > 0,$$

with boundary condition

(1.3) $$u = v = 0, \quad x \in \partial\Omega, \quad t > 0,$$

and initial condition

(1.4) $$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad x \in \Omega,$$

where Ω is a bounded domain in R^N with a smooth boundary $\partial\Omega$, $m, n > 1$, p, q, r, s, a, b are positive constants. It is well-known that when $m, n > 1$ the problem (1.1)-(1.4) admits solutions only in some weak sense. Since we are interested only in nonnegative solutions, we therefore assume that the initial functions u_0 and v_0 are nonnegative and $u_0, v_0 \in L^\infty(\Omega)$.

The system (1.1)-(1.4) can be used to model as the cooperative reaction of two species in an ecological system. The presence of the u-population species encourages the growth of the v-population species but reduces its own growth and vice versa. The choice of $m, n > 1$ describes the density dependent diffusion phenomenon.

We say that the solution (u, v) of the problem (1.1)-(1.4) blows up in finite time if there exists a finite time $T > 0$ such that the solution is defined in $(0, T)$, and

$$\limsup_{t \nearrow T} \{||u(\cdot,t)||_\infty + ||v(\cdot,t)||_\infty\} = +\infty.$$

The motivation of this study is from [2] in which Bedjaoui and Souplet studied (1.1)-(1.4) for the case $m = n = 1$. By comparing with some suitable supersolutions and subsolutions, they obtained optimal conditions on the exponents p, q, r, s for the existence of blowing up solutions and the global boundedness of all solutions, respectively.

The main purpose of this paper is to study the existence or nonexistence of global solutions for the problem (1.1)-(1.4) when $m, n > 1$. Indeed, we show that all solutions are globally bounded if $pq < \max\{m, r\}\max\{n, s\}$; while there are finite time blowing up solutions if $pq > \max\{m, r\}\max\{n, s\}$ and the initial data are sufficiently large. For the critical case $pq = \max\{m, r\}\max\{n, s\}$, the existence or nonexistence of global solutions depends on the relation between the exponents m, n, r, s, and also the range of the parameters a, b.

We mention here that the problem (1.1)-(1.4) with $a = b = 0$ has been investigated by many authors, see for example [1, 9] and the reference cited therein. The case for a slow diffusion equation with absorption of the type $u_t = \triangle u^m - au^r$, we refer to, for example, [3-8] and the reference cited therein.

This paper is organized as follows. We study the case $pq \neq \max\{m,r\}\max\{n,s\}$ in §2 and the critical case $pq = \max\{m,r\}\max\{n,s\}$ in §3.

2. The Case $pq \neq \max\{m,r\}\max\{n,s\}$

For convenience, we denote $\Omega\times(0,T)$ and $\partial\Omega\times(0,T)$ by Q_T and S_T, respectively. Based on [9], we use the following definition of (weak) solution throughout this paper.

Definition 2.1. A pair of functions (u,v) is called a solution of (1.1)-(1.4) in Q_T, $0<T<\infty$, if

(i) $u, v \in L^\infty(Q_T)\bigcap C([0,T];L^2(\Omega))$ and $u^m, v^n \in L^2(0,T;H^1(\Omega))$,

(ii) u and v satisfy the identities

$$\begin{aligned}&\int_\Omega \varphi(x,T)u(x,T)dx + \iint_{Q_T} \triangledown\varphi\cdot\triangledown u^m dxdt\\ =&\iint_{Q_T}[\varphi(v^p-au^r)+\varphi_t u]dxdt + \int_\Omega \varphi(x,0)u_0(x)dx,\\ &\int_\Omega \varphi(x,T)v(x,T)dx + \int\int_{Q_T} \triangledown\varphi\cdot\triangledown v^n dxdt\\ =&\iint_{Q_T}[\varphi(u^q-bv^s)+\varphi_t v]dxdt + \int_\Omega \varphi(x,0)v_0(x)dx,\end{aligned}$$

for any $\varphi\in C^1(\overline{Q_T})$ such that $\varphi = 0$ on S_T.

(iii) $u^m = 0, v^n = 0$ on S_T in the trace sense.

Definition 2.2. A pair of functions $(\overline{u},\overline{v})$ is called a (weak) supersolution of (1.1)-(1.3) in Q_T, $0<T<\infty$, with initial data $(\overline{u}_0,\overline{v}_0)$, if

(i) $\overline{u},\overline{v}\in L^\infty(Q_T)\bigcap C([0,T];L^2(\Omega))$ and $\overline{u}^m,\overline{v}^n\in L^2(0,T;H^1(\Omega))$,

(ii) $\overline{u}$ and $\overline{v}$ satisfy the inequalities

$$\begin{aligned}&\int_\Omega \varphi(x,T)\overline{u}(x,T)dx + \iint_{Q_T} \triangledown\varphi\cdot\triangledown \overline{u}^m dxdt\\ \geq&\iint_{Q_T}[\varphi(\overline{v}^p-a\overline{u}^r)+\varphi_t \overline{u}]dxdt + \int_\Omega \varphi(x,0)\overline{u}_0(x)dx,\\ &\int_\Omega \varphi(x,T)\overline{v}(x,T)dx + \iint_{Q_T} \triangledown\varphi\cdot\triangledown \overline{v}^n dxdt\\ \geq&\iint_{Q_T}[\varphi(\overline{u}^q-b\overline{v}^s)+\varphi_t \overline{v}]dxdt + \int_\Omega \varphi(x,0)\overline{v}_0(x)dx,\end{aligned}$$

for any nonnegative function $\varphi \in C^1(\overline{Q_T})$ such that $\varphi = 0$ on S_T.

(iii) $\overline{u}^m \geq 0, \overline{v}^n \geq 0$ on S_T in the trace sense.

A (weak) subsolution is defined by replacing $\geq$ in (ii) and (iii) by $\leq$.

We shall use the following comparison principle to prove the existence or nonexistence of global solutions. The proof can be found in [9] and we omit it.

Lemma 2.1. *Let $(\overline{u}, \overline{v})$ and $(\underline{u}, \underline{v})$ be a supersolution and a subsolution of (1.1)-(1.3) in Q_T, $T > 0$, with initial data satisfying $\overline{u}(x, 0) \geq \underline{u}(x, 0)$ and $\overline{v}(x, 0) \geq \underline{v}(x, 0)$. Then $\overline{u} \geq \underline{u}$ and $\overline{v} \geq \underline{v}$ in Q_T.*

We remark here that the global existence of solutions for (1.1)-(1.4) when $p < m$ and $q < n$ has been obtained in [9]. Using the super-sub-solution method, we can also recover this known results. Indeed, we have the following theorem for the subcritical case.

Theorem 2.2. *Suppose that $pq < \max\{m, r\} \max\{n, s\}$. Then all solutions of (1.1)-(1.4) are globally bounded.*

Proof. Take $R > 0$ such that $\Omega \subset B(0; R)$. Let $(\overline{u}, \overline{v}) = (C_1 e^{-L_1|x|^2}, C_2 e^{-L_2|x|^2})$, where L_1, L_2, C_1, C_2 are positive constants satisfying

$$L_1 \leq N/(4mR^2),\ L_2 \leq N/(4nR^2),\ C_1 \geq e^{L_1R^2}|u_0|_\infty,\ C_2 \geq e^{L_2R^2}|v_0|_\infty,$$

and

$$C_2^{pq} e^{q|rL_1 - pL_2|R^2} \leq a^q C_1^{qr} \leq a^q b^r e^{-r|qL_1 - sL_2|R^2} C_2^{rs}, \text{ if } pq < rs,$$

$$C_2^{pq} e^{q|mL_1 - pL_2|R^2} \leq (NmL_1)^q C_1^{mq} \leq (NmL_1)^q (NnL_2)^m e^{-m|qL_1 - nL_2|R^2} C_2^{mn}, \text{ if } pq < mn,$$

$$C_2^{pq} e^{q|rL_1 - pL_2|R^2} \leq a^q C_1^{qr} \leq a^q (NnL_2)^r e^{-r|qL_1 - nL_2|R^2} C_2^{nr}, \text{ if } pq < nr,$$

$$C_2^{pq} e^{q|mL_1 - pL_2|R^2} \leq (NmL_1)^q C_1^{mq} \leq b^m (NmL_1)^q e^{-m|qL_1 - sL_2|R^2} C_2^{ms}, \text{ if } pq < ms.$$

It is easy to check that $(\overline{u}, \overline{v})$ is a supersolution of (1.1)-(1.3) with $\overline{u}(x, 0) \geq u_0(x)$ and $\overline{v}(x, 0) \geq v_0(x)$. Thus, by Lemma 2.1, we obtain that $u \leq \overline{u}$ and $v \leq \overline{v}$ as long as the solution (u, v) exists. Therefore, (u, v) is globally bounded. This completes the proof. ∎

Borrowing an idea from [2] and [10], we will construct a self-similar subsolution to prove the existence of blowing up solutions.

Theorem 2.3. *Suppose that* $pq > \max\{m, r\}\max\{n, s\}$. *Then the solution of the problem* $(1.1) - (1.4)$ *blows up in finite time for initial data large enough.*

Proof. We first consider the case $m \le r$ and $n \le s$. Without loss of generality, we may assume that $0 \in \Omega$. Since $pq > rs$, we have either $s/q < (p+1)/(q+1)$ or $r/p < (q+1)/(p+1)$. Therefore, we may without loss of generality assume that $s/q < (p+1)/(q+1)$ (otherwise, we exchange the roles of u and v).

We choose λ and β such that

$$\frac{s}{q} < \lambda < \min\left\{\frac{p+1}{q+1}, \frac{p}{r}\right\}, \quad \frac{1}{\lambda q - 1} < \beta < \frac{1}{s-1}.$$

Set $\alpha = \lambda\beta$. Then it is easy to check that

$$\beta p > \alpha + 1,\ \beta p > \alpha r \ge \alpha m,\ \alpha q > \beta + 1 > \beta s \ge \beta n. \tag{2.1}$$

Pick a positive number l such that

$$l < \frac{1}{2}\min\{\beta p - \alpha m, \beta + 1 - \beta n, \alpha q - \beta n\}. \tag{2.2}$$

We seek a subsolution of the following form

$$\underline{u}(x,t) = (T-t)^{-\alpha} U\left(\frac{|x|}{(T-t)^l}\right), \quad \underline{v}(x,t) = (T-t)^{-\beta} V\left(\frac{|x|}{(T-t)^l}\right),$$

where $U(y) = (A^2 - y^2)_+^{1/m}$, $V(y) = (A^2 - K^{-2}y^2)_+^{1/n}$, $K \in (1, \sqrt{(\beta n + 2l)/(\beta n)})$ is a constant, and A, $T > 0$ are constants to be determined later. To show $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3), it suffices to show that

$$\begin{aligned} &(T-t)^{-(\alpha+1)}\{\alpha U(y) + lyU'(y)\} + a(T-t)^{-\alpha r}U^r(y) \\ &-(T-t)^{-(\alpha m+2l)}[(U^m)''(y) + (N-1)/y(U^m)'(y)] \\ \le\ &(T-t)^{-\beta p}V^p(y) \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} &(T-t)^{-(\beta+1)}\{\beta V(y) + lyV'(y)\} + b(T-t)^{-\beta s}V^s(y) \\ &-(T-t)^{-(\beta n+2l)}[(V^n)''(y) + (N-1)/y(V^n)'(y)] \\ \le\ &(T-t)^{-\alpha q}U^q(y) \end{aligned} \tag{2.4}$$

hold pointwisely for $y > 0$, $y \neq A, KA$.

For $y > A$, it is clear that (2.3) holds. For $0 < y < A$, (2.3) is equivalent to

$$\begin{aligned}(2.5)\qquad &(T-t)^{-(\alpha+1)}\left\{\alpha(A^2-y^2)^{1/m}-\frac{2l}{m}y^2(A^2-y^2)^{1/m-1}\right\}\\ &+2N(T-t)^{-(\alpha m+2l)}+a(T-t)^{-\alpha r}(A^2-y^2)^{r/m}\\ &\leq (T-t)^{-\beta p}(A^2-y^2/K^2)^{p/n}.\end{aligned}$$

Since $A^2 - y^2/K^2 \geq A^2(1-1/K^2) > 0$, it follows from (2.1) and (2.2) that (2.5) holds provided that T is sufficiently small.

For $y > KA$, it is clear that (2.4) holds. Let $\theta \in (\sqrt{(\beta n)/(\beta n + 2l)}K, 1)$ be fixed. For $\theta A < y < KA$, (2.4) holds provided that

$$\begin{aligned}(2.6)\qquad &\left\{\beta(A^2-y^2/K^2)-\frac{2l}{n}(y^2/K^2)\right\}\\ &+2N(T-t)^{\beta+1-\beta n-2l}(A^2-y^2/K^2)^{1-1/n}\\ &+b(T-t)^{\beta+1-\beta s}(A^2-y^2/K^2)^{1-1/n+s/n}\leq 0.\end{aligned}$$

Since $\beta(A^2-y^2/K^2)-2l/n(y^2/K^2) \leq A^2(\beta-\theta^2/K^2(\beta+2l/n)) < 0$, it follows from (2.1) and (2.2) that (2.6) holds provided that T is sufficiently small. For $0 < y < \theta A$, (2.4) is equivalent to

$$\begin{aligned}(2.7)\qquad &(T-t)^{-(\beta+1)}\left\{\beta(A^2-y^2/K^2)^{1/n}-\frac{2l}{n}(y^2/K^2)(A^2-y^2/K^2)^{1/n-1}\right\}\\ &+2N(T-t)^{-\beta n-2l}+b(T-t)^{-\beta s}(A^2-y^2/K^2)^{s/n}\\ &\leq (T-t)^{-\alpha q}(A^2-y^2)^{q/m}.\end{aligned}$$

Since $A^2 - y^2 \geq A^2(1-\theta^2) > 0$, it follows from (2.1) and (2.2) that (2.7) holds provided that T is sufficiently small.

Now, we fix T so that $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3). For any $t \in [0, T)$, supp $\underline{u}(\cdot, t) \subset$ supp $\underline{v}(\cdot, t) \subset \overline{B(0; KAT^l)} \subset \Omega$ if $A > 0$ is sufficiently small. Hence it follows from Lemma 2.1 that the solution (u, v) of (1.1)-(1.4) blows up in finite time if $u_0 \geq \underline{u}(x, 0)$ and $v_0 \geq \underline{v}(x, 0)$.

For $m > r$ or $n > s$, we shall only consider the case $m > r$ and $n \leq s$ since the proof for the other two cases is similar. Since $\eta^r \leq \eta^m + 1$ if $\eta \geq 0$, the solution of (1.1)-(1.3) is a supersolution of

$$(2.8)\qquad u_t = \triangle u^m + v^p - a(u^m+1), \quad x \in \Omega, \quad t > 0,$$

(2.9) $$v_t = \triangle v^n + u^q - bv^s, \quad x \in \Omega, \quad t > 0,$$

(2.10) $$u = v = 0, \quad x \in \partial\Omega, \quad t > 0.$$

Proceeding all steps in the previous case with a slight modification, we can show that $(\underline{u}, \underline{v})$ is a subsolution of (2.8)-(2.10) and that the solution (u, v) of (1.1)-(1.4) blows up in finite time if $u_0 \geq \underline{u}(x, 0)$ and $v_0 \geq \underline{v}(x, 0)$. Hence the proof is complete. ∎

3. The Case $pq = \max\{m, r\} \max\{n, s\}$

Following the proofs of Theorems 2.2 and 2.3 with a slight modification, we get the following results for the critical case.

Theorem 3.1. *Let* $pq = \max\{m, r\} \max\{n, s\}$.

(i) *Suppose that* $r > m$ *and* $s > n$ *and suppose also that* a *and* b *are sufficiently small. Then the solution of the problem (1.1)-(1.4) blows up in finite time for initial data large enough.*

(ii) *Suppose that* $r \geq m$, $s \geq n$, *and suppose also that* $a^q b^r \geq 1$. *Then all solutions of (1.1)-(1.4) are globally bounded.*

Proof. (i) Without loss of generality, we may assume that $0 \in \Omega$. Since $pq = rs$, we have either $s/q \leq (p + 1)/(q + 1)$ or $r/p \leq (q + 1)/(p + 1)$. Therefore, we may without loss of generality assume that $s/q \leq (p + 1)/(q + 1)$ (otherwise, we exchange the roles of u and v).

Set $\alpha = s/[(s - 1)q]$ and $\beta = 1/(s - 1)$. It is easy to check that

(3.1) $$\beta p \geq \alpha + 1, \ \beta p = \alpha r > \alpha m, \ \alpha q = \beta + 1 = \beta s > \beta n.$$

Pick a positive number l such that

(3.2) $$l \leq \frac{1}{2} \min\{\beta p - \alpha m, \beta + 1 - \beta n, \alpha q - \beta n\}.$$

We seek a subsolution of the following form

$$\underline{u}(x, t) = C_1 (T - t)^{-\alpha} U \left(\frac{|x|}{(T - t)^l} \right), \quad \underline{v}(x, t) = C_2 (T - t)^{-\beta} V \left(\frac{|x|}{(T - t)^l} \right),$$

where $U(y) = (A^2 - y^2)_+^{1/m}$, $V(y) = (A^2 - K^{-2}y^2)_+^{1/n}$, K, C_1, C_2 are positve constants satisfying

$$\begin{aligned} &1 \le K \le \sqrt{\frac{\beta n + 2l}{\beta n}}, \\ &\alpha A^{2/m} C_1 < [A^2(1 - \frac{1}{K^2})]^{p/n} C_2^p \\ &< (\beta A^{2/n})^{-p} [A^2(1 - \frac{1}{K^2}]^{p/n} [A^2(1-\theta^2)]^{pq/m} C_1^{pq} \end{aligned} \tag{3.3}$$

and A, $T > 0$ are constants to be determined later. To show $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3), it suffices to show that

$$\begin{aligned} &C_1(T-t)^{-(\alpha+1)}\{\alpha U(y) + lyU'(y)\} - C_1^m (T-t)^{-(\alpha m + 2l)} \triangle (U^m)(y) \\ &+ aC_1^r (T-t)^{-\alpha r} U^r(y) \\ &\le C_2^p (T-t)^{-\beta p} V^p(y) \end{aligned} \tag{3.4}$$

and

$$\begin{aligned} &C_2(T-t)^{-(\beta+1)}\{\beta V(y) + lyV'(y)\} - C_2^n (T-t)^{-(\beta n + 2l)} \triangle (V^n)(y) \\ &+ bC_2^s (T-t)^{-\beta s} V^s(y) \\ &\le C_1^q (T-t)^{-\alpha q} U^q(y) \end{aligned} \tag{3.5}$$

hold pointwisely for $y > 0$, $y \ne A, KA$, where $\triangle = d^2/dy^2 + (N-1)/y d/dy$.

For $y > A$, it is clear that (3.4) holds. For $0 < y < A$, (3.4) is equivalent to

$$\begin{aligned} &C_1(T-t)^{-(\alpha+1)} \left\{ \alpha (A^2 - y^2)^{1/m} - \frac{2l}{m} y^2 (A^2 - y^2)^{1/m - 1} \right\} \\ &+ 2NC_1^m (T-t)^{-(\alpha m + 2l)} + aC_1^r (T-t)^{-\alpha r} (A^2 - y^2)^{r/m} \\ &\le C_2^p (T-t)^{-\beta p} (A^2 - y^2/K^2)^{p/n}. \end{aligned} \tag{3.6}$$

Since $A^2 - y^2/K^2 \ge A^2(1 - 1/K^2) > 0$, it follows from (3.1), (3.2), and (3.3) that (3.6) holds provided that a and T are sufficiently small.

For $y > KA$, it is clear that (3.5) holds. Let $\theta \in (\sqrt{(\beta n)/(\beta n + 2l)} K, 1)$ be fixed. For $\theta A < y < KA$, (3.5) holds provided that

$$\begin{aligned} &C_2 \left\{ \beta (A^2 - y^2/K^2) - \frac{2l}{n} (y^2/K^2) \right\} \\ &+ 2NC_2^n (T-t)^{\beta + 1 - \beta n - 2l} (A^2 - y^2/K^2)^{1 - 1/n} \\ &+ bC_2^s (T-t)^{\beta + 1 - \beta s} (A^2 - y^2/K^2)^{1 - 1/n + s/n} \le 0. \end{aligned} \tag{3.7}$$

Since $\beta(A^2-y^2/K^2)-2l/n(y^2/K^2) \le A^2(\beta-\theta^2/K^2(\beta+2l/n)) < 0$, it follows from (3.1), (3.2), and (3.3) that (3.7) holds provided that b and T are sufficiently small.

For $0 < y < \theta A$, (3.5) is equivalent to

$$\begin{aligned}
&C_2(T-t)^{-(\beta+1)}\left\{\beta(A^2-y^2/K^2)^{1/n}-\frac{2l}{n}(y^2/K^2)(A^2-y^2/K^2)^{1/n-1}\right\}\\
&+2NC_2^n(T-t)^{-\beta n-2l}+bC_2^s(T-t)^{-\beta s}(A^2-y^2/K^2)^{s/n}\\
&\le C_1^q(T-t)^{-\alpha q}(A^2-y^2)^{q/m}.
\end{aligned} \tag{3.8}$$

Since $A^2-y^2 \ge A^2(1-\theta^2) > 0$, it follows from (3.1), (3.2), and (3.3) that (3.8) holds provided that b and T is sufficiently small.

Now, we fix T so that $(\underline{u}, \underline{v})$ is a subsolution of (1.1)-(1.3). For any $t \in [0, T)$, supp $\underline{u}(\cdot, t) \subset$ supp $\underline{v}(\cdot, t) \subset \overline{B(0; KAT^l)} \subset \Omega$ if $A > 0$ is sufficiently small. Hence it follows from Lemma that the solution (u, v) of (1.1)-(1.4) blows up in finite time if $u_0 \ge \underline{u}(x, 0)$ and $v_0 \ge \underline{v}(x, 0)$.

(ii) Let $(\overline{u}, \overline{v}) = (C_1, C_2)$, where C_1 and C_2 are positive constants satisfying $C_1 > |u_0|_\infty$, $C_2 > |v_0|_\infty$, and $C_2^{pq} \le a^q C_1^{qr} \le a^q b^r C_2^{rs}$. It is easy to check that $(\overline{u}, \overline{v})$ is a supersolution of (1.1)-(1.3) with $\overline{u}(x, 0) \ge u_0(x)$ and $\overline{v}(x, 0) \ge v_0(x)$. Thus, by Lemma , we obtain that $u \le \overline{u}$ and $v \le \overline{v}$ as long as the solution (u, v) exists. Therefore, (u, v) is globally bounded. This completes the proof. ■

We remark that the remaining of the critical case can be treated if one can construct suitable supersolutions or subsolutions. However, we are unable to construct such one for these cases. So we left them as open problems.

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