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PLANAR GRAPHS THAT HAVE NO SHORT CYCLES WITH A CHORD ARE 3-CHOOSABLE

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Abstract. In this paper we prove that every planar graph G is 3-choosable if it contains no cycle of length at most 10 with a chord. This generalizes a result obtained by Borodin [J. Graph Theory 21(1996) 183-186] and Sanders and Zhao [Graphs Combin. 11(1995) 91-94], which says that every planar graph G without k-cycles for all $4 \le k \le 9$ is 3-colorable.

1. INTRODUCTION

We only consider simple graphs in this paper unless otherwise stated. A *plane* graph is a particular drawing of a planar graph in the Euclidean plane. For a plane graph G, we denote its vertex set, edge set, face set, and minimum degree by $V(G)$, $E(G)$, $F(G)$, and $\delta(G)$, respectively. Let $d_G(x)$ (for short, $d(x)$) denote the degree of a vertex or a face x of G. We use $b(f)$ to denote the boundary of a face f, and write $f = [u_1u_2 \cdots u_n]$ if $u_1, u_2, \ldots, u_n$ are its boundary vertices in a cyclic order. A vertex (or face) of degree k is called a k-*vertex* (or k-*face*). We say that two cycles or faces of a plane graph are *adjacent* if they share at least one common (boundary) edge. For a face $f \in F(G)$, let $F_k(f)$ denote the set of k-faces adjacent to f, $V_i(f)$ the set of i-vertices incident to f, and $V_3^j(f)$ the set of 3-vertices in $V_3(f)$ each of which is incident to at least one j-face. Let $V_3'(f) = V_3(f) \setminus (V_3^3(f) \cup V_3^4(f) \cup V_3^5(f))$. For a graph G and a cycle $C \subseteq G$, an edge xy is called a *chord* of C if $xy \in E(G) \setminus E(C)$ but $x, y \in V(C)$. C is a *chordal-k-cycle* if C is of length k and has a chord in G. Let $c^*(G)$ denote the maximum integer k such that G contains no chordal-l-cycles for all $l \le k$. It is easy to see that $3 \le c^*(G) \le |V(G)|$ if G is a simple graph, and $c^*(G) < |V(G)|$ if $\delta(G) \ge 3$.

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A *coloring* of a graph G is a mapping ϕ from $V(G)$ to the set of colors $\{1, 2, \ldots, k\}$ such that $\phi(x) \neq \phi(y)$ for every edge xy of G. The *chromatic number* $\chi(G)$ is the smallest integer k such that G has a proper coloring into the set $\{1, 2, \ldots, k\}$. We say that L is an *assignment* for the graph G if it assigns a list $L(v)$ of possible colors to each vertex v of G. If G has a proper coloring ϕ such that $\phi(v) \in L(v)$ for all vertices v, then we say that G is L-colorable or ϕ is an L-coloring of G. The graph G is k*-choosable* if it is L-colorable for every assignment L satisfying $|L(v)| = k$ for all vertices v. The *choice number* or *list chromatic number* $\chi_\ell(G)$ of G is the smallest k such that G is k-choosable.

The concept of list coloring was introduced by Vizing [12] and independently by Erdös, Rubin and Taylor [5]. All 2-choosable graphs are completely characterized in [5]. Thomassen [9] proved that every planar graph is 5-choosable, whereas Voigt [13] presented an example of a planar graph which is not 4-choosable. Thomassen [10, 11] showed that every planar graph of girth greater than or equal to 5 is 3-choosable. Voigt [14] further constructed a non-3-choosable planar graph of girth 4. Alon and Tarsi [1] proved that every planar bipartite graph is 3-choosable. Other results on list coloring planar graphs are referred to [15-17]. In this paper, we investigate the 3-choosability of planar graphs G when $c^*(G)$ is sufficiently large.

2. Tructural Properties

The following lemma is an easy observation from the definition of $c^*(G)$.

Lemma 1. *Let G be a 2-connected plane graph with $\delta(G) \geq 3$. If f and f' are two adjacent faces, then $d(f) + d(f') \geq c^*(G) + 3$.*

A plane graph G is *normal* if it contains no vertex and face of degree less than 3. It was known [2, 7] that every normal plane graph contains an edge xy such that $d(x) + d(y) \leq 13$.

Lemma 2. *If G is a plane graph with $c^*(G) \geq 11$, then $\delta(G) \leq 2$.*

Proof. Suppose to the contrary that $\delta(G) \geq 3$. Then G^*, the dual of G, contains no face of degree less than 3. Since G is simple, G^* further contains no vertex of degree less than 3. Thus, G^* is a normal plane graph. By the previous result, G^* contains an edge xy such that $d_{G^*}(x) + d_{G^*}(y) \leq 13$. Let f_x and f_y denote the faces of G that correspond to x and y in G^*, respectively. Therefore, f_x is adjacent to f_y in G and $d_G(f_x) + d_G(f_y) \leq 13$. However, this contradicts Lemma 1, which asserts that $d_G(f_x) + d_G(f_y) \geq c^*(G) + 3 \geq 14$. This proves Lemma 2. ■

Lemma 2 is best possible in the sense that there exist plane graphs G having a cycle of length at most 11 with a chord such that $\delta(G) \geq 3$. Let H denote the graph obtained from the dodecahedron by sawing all its corners off. It is easy to see that H is a 3-regular plane graph containing cycles of length eleven with a chord.

For a fixed integer $N \geq 4$, it is easy to observe that if a graph G contains no k-cycles for all $4 \leq k \leq N$, then G contains no any cycle of length at most $N+1$ with a chord. This fact together Lemma 2 imply the following consequence.

Corollary 3. *If G is a plane graph without k-cycles for all $4 \leq k \leq 10$, then $\delta(G) \leq 2$.*

Lemma 4. *If G is a plane graph with $\delta(G) \geq 3$ and $c^*(G) = 10$, then G contains a cycle of length ten such that each of its vertices is of degree 3 in G.*

Proof. Suppose that the lemma is false. Let G be a connected counterexample. If G is not 2-connected, choose an end-block B of G that contains exactly one cut vertex u of G. Then B is 2-connected, $c^*(B) = 10$, $d_B(u) \geq 2$, and $d_B(x) \geq 3$ for all $x \in V(B) \setminus \{u\}$. Assume that u lies on the boundary of the infinite face of B, and let $v \neq u$ be a vertex of B lying on the infinite face. Take eleven copies $B_1, B_2, \cdots, B_{11}$ of B, and let u_i and v_i be the copies of u and v in B_i, respectively. Let H denote the graph constructed from $B_1, B_2, \cdots, B_{11}$ by identifying v_{11} with u_1, and v_i with u_{i+1} for $i = 1, 2, \cdots, 10$. It is easy to see that H is a 2-connected plane graph with $\delta(H) \geq 3$ and $c^*(H) = 10$ which contains no cycle of length ten with each vertex being of degree 3 in H. Hence H also is a counterexample to the lemma.

Thus we may now assume that G is 2-connected and hence all its facial walks are cycles. Using Euler's formula $|V(G)| - |E(G)| + |F(G)| = 2$ and the relations $\sum_{v \in V(G)} d(v) = \sum_{f \in F(G)} d(f) = 2|E(G)|$, we can derive the following identity.

$$\sum_{v \in V(G)} (d(v) - 6) + \sum_{f \in F(G)} (2d(f) - 6) = -12. \tag{1}$$

Let w denote a weight function defined on $V(G) \cup F(G)$ by $w(v) = d(v) - 6$ if $v \in V(G)$ and $w(f) = 2d(f) - 6$ if $f \in F(G)$. We shall discharge the face weight $w(x)$ to its incident vertices while keeping the total sum fixed so that the new weight $w'(x)$ is nonnegative for all $x \in V(G) \cup F(G)$. Hence

$$0 \leq \sum_{x \in V(G) \cup F(G)} w'(x) = \sum_{x \in V(G) \cup F(G)} w(x) = -12.$$

This is an obvious contradiction.

For a face f, let $m_3(f) = |F_3(f)|$, $n_i(f) = |V_i(f)|$, $n_3^j(f) = |V_3^j(f)|$, and $n_3'(f) = |V_3'(f)|$. Let $\alpha(f) = 2d(f) - 6 - \frac{3}{2}n_3^3(f) - \frac{5}{4}n_3^4(f) - \frac{11}{10}n_3^5(f)$, and

$\beta(f) = a(f)/(n'_3(f) + n_4(f) + n_5(f))$. With the aid of these notations, we define a discharging rule as follows.

(R) Every face f of degree at least 4 sends $\frac{3}{2}$ to each 3-vertex in $V_3^3(f)$, $\frac{5}{4}$ to each 3-vertex in $V_3^4(f)$, $\frac{11}{10}$ to each 3-vertex in $V_3^5(f)$, and $\beta(f)$ to each vertex in $V'_3(f) \cup V_4(f) \cup V_5(f)$ provided $n'_3(f) + n_4(f) + n_5(f) > 0$.

Let $w'(x)$ denote the new weight function for $x \in V(G) \cup F(G)$ once the discharging process is complete according to the rule (R). It remains to prove that $w'(x) \geq 0$ for all $x \in V(G) \cup F(G)$.

Claim. *Suppose that f is a face of degree at least 4. Then*

(1) $\alpha(f) \geq 0$.

(2) $\beta(f) \geq \beta_0$, *where* $\beta_0 = \frac{3}{2}$ *if* $d(f) \geq 12$; $\beta_0 = 1$ *if* $d(f) = 11$; $\beta_0 = \frac{w(f)}{d(f)}$ *if* $4 \leq d(f) \leq 9$; *and* $\beta_0 = 1$ *if* $d(f) = 10$ *with the following exceptions:*

(2.1) $\beta(f) = \frac{1}{2}$ *if* $n_3^3(f) = n_3(f) = 9$ *and* $n_4(f) + n_5(f) = 1$;

(2.2) $\beta(f) = \frac{3}{4}$ *if* $n_3^3(f) = 8$, $n_3^4(f) = 1$ *and* $n_4(f) + n_5(f) = 1$;

(2.3) $\beta(f) = \frac{9}{10}$ *if* $n_3^3(f) = 8$, $n_3^5(f) = 1$ *and* $n_4(f) + n_5(f) = 1$.

Remarks. For $1 \leq i \leq 3$, suppose that f_i is a 10-face satisfying Condition (2.i). Then f_i is incident to the unique vertex v of degree more than 3. And if $d(v) = 4$, then f_i is adjacent to some $(i+2)$-face f^* such that $v \in b(f_i) \cap b(f^*)$. However, v is not incident to any 3-face other than f^*.

Proof of the Claim. Let $T(f) = \frac{3}{2}n_3^3(f) + \frac{5}{4}n_3^4(f) + \frac{11}{10}n_3^5(f) + \beta_0(n'_3(f) + n_4(f) + n_5(f))$. It suffices to check that $T(f) \leq w(f)$.

If $d(f) \geq 12$, then $\beta_0 = \frac{3}{2}$ and $T(f) = \frac{3}{2}n_3^3(f) + \frac{5}{4}n_3^4(f) + \frac{11}{10}n_3^5(f) + \frac{3}{2}(n'_3(f) + n_4(f) + n_5(f)) \leq \frac{3}{2}(n_3(f) + n_4(f) + n_5(f)) \leq \frac{3}{2}d(f) \leq 2d(f) - 6 = w(f)$.

Suppose that $d(f) = 11$, then $w(f) = 16$ and $\beta_0 = 1$. If $n_3(f) \leq 10$, then $T(f) \leq \frac{3}{2}n_3(f) + n_4(f) + n_5(f) \leq \frac{3}{2}n_3(f) + (11 - n_3(f)) = 11 + \frac{1}{2}n_3(f) \leq 11 + 5 = 16$. Assume that $n_3(f) = 11$. It is easy to derive that $m_3(f) \leq 5$. If $m_3(f) \leq 4$, then $n_3^3(f) \leq 8$, and hence $T(f) \leq 8 \cdot \frac{3}{2} + 3 \cdot \frac{5}{4} = 15\frac{3}{4}$. If $m_3(f) = 5$, then some 3-vertex in $b(f)$ is not incident to any face of degree less than 6 by Lemma 1. This implies that $n_3^3(f) = 10$ and $n'_3(f) = 1$, hence $T(f) = 10 \cdot \frac{3}{2} + 1 = 16$.

Suppose that $d(f) = 9$. Then $\beta_0 = (2 \cdot 9 - 6)/9 = \frac{4}{3}$. By Lemma 1, f is not adjacent to any 3-face. Thus, $n_3^3(f) = 0$, and $T(f) \leq \frac{5}{4}n_3^4(f) + \frac{11}{10}n_3^5(f) + \frac{4}{3}(n'_3(f) + n_4(f) + n_5(f)) \leq \frac{4}{3}d(f) = 12 = w(f)$.

Suppose that $d(f) = 8$. Then $w(f) = 10$ and $\beta_0 = \frac{5}{4}$. It follows from Lemma 1 that f is not adjacent to any face of degree less than 5. Thus, $n_3^3(f) = n_3^4(f) = 0$, and $T(f) \leq \frac{11}{10}n_3^5(f) + \frac{5}{4}(n'_3(f) + n_4(f) + n_5(f)) \leq \frac{5}{4}d(f) = 10$.

Suppose that $4 \le d(f) \le 7$. Then $\beta_0 = w(f)/d(f)$. Lemma 1 asserts that f is not adjacent to any face of degree less than 6. Thus, $n_3^3(f) = n_3^4(f) = n_3^5(f) = 0$, and $T(f) \le \beta_0(n_3'(f) + n_4(f) + n_5(f)) \le \frac{w(f)}{d(f)} \cdot d(f) = w(f)$.

Finally, suppose that $d(f) = 10$. So, $w(f) = 14$. Since G contains no a 10-cycle with each boundary vertex being of degree 3, we know $n_3(f) \le 9$. If $n_3(f) \le 8$, then $T(f) \le 8 \cdot \frac{3}{2} + 2 \cdot 1 = 14$. Assume that $n_3(f) = 9$. It follows that $n_4(f) + n_5(f) \le 1$. If $n_4(f) + n_5(f) = 0$, then $T(f) \le 9 \cdot \frac{3}{2} = 13.5$. So suppose that $n_4(f) + n_5(f) = 1$. If $n_3^3(f) \le 7$, then $T(f) \le 7 \cdot \frac{3}{2} + 2 \cdot \frac{5}{4} + 1 = 14$. Assume that $n_3^3(f) = 8$. If $n_3^4(f) = 1$, then Case (2.2) holds and $T(f) = 8 \cdot \frac{3}{2} + \frac{5}{4} + \frac{3}{4} = 14$. If $n_3^5(f) = 1$, then Case (2.3) holds and $T(f) = 8 \cdot \frac{3}{2} + \frac{11}{10} + \frac{9}{10} = 14$. If $n_3^3(f) = 9$, then Case (2.1) holds and $T(f) = 9 \cdot \frac{3}{2} + \frac{1}{2} = 14$. This proves the Claim. ■

The statement (1) in the Claim implies that $w'(f) \ge 0$ for all $f \in F(G)$ with $d(f) \ge 4$. If $d(f) = 3$, then $w'(f) = w(f) = 0$.

Let $v \in V(G)$. Thus $d(v) \ge 3$ by $\delta(G) \ge 3$. If $d(v) \ge 6$, then $w'(v) = w(v) = d(v) - 6 \ge 0$. Assume that $d(v) = 5$, then $w(v) = -1$. By Lemma 1, v is incident to at most two 3-faces. Since each of the faces of degree at least 4 that are incident to v sends at least $\frac{1}{2}$ to v by the Claim, and hence $w'(v) \ge -1 + 3 \cdot \frac{1}{2} = \frac{1}{2}$. Assume that $d(v) = 3$, then $w(v) = -3$. Let f_1, f_2, f_3 be the incident faces of v that satisfies $d(f_1) \le d(f_2) \le d(f_3)$. If $d(f_1) = 3$, then $d(f_i) \ge 10$ by Lemma 1 and $\tau(f_i \to v) = \frac{3}{2}$ by (R) for $i = 2, 3$, thus $w'(v) \ge -3 + 2 \cdot \frac{3}{2} = 0$. If $d(f_1) = 4$, then $\tau(f_1 \to v) \ge \frac{1}{2}$ by the Claim. Since $d(f_i) \ge 9$ by Lemma 1 and $\tau(f_i \to v) = \frac{5}{4}$ by (R) for $i = 2, 3$, we have $w'(v) \ge -3 + \frac{1}{2} + 2 \cdot \frac{5}{4} = 0$. If $d(f_1) = 5$, then $\tau(f_1 \to v) \ge \frac{4}{5}$ by the Claim. Since $d(f_i) \ge 8$ by Lemma 1 and $\tau(f_i \to v) \ge \frac{11}{10}$ by (R) for $i = 2, 3$, we deduce $w'(v) \ge -3 + \frac{4}{5} + 2 \cdot \frac{11}{10} = 0$. Now assume $d(f_i) \ge 6$ for all $i = 1, 2, 3$. It is easy to note that f_i doesn't satisfy (2.1), (2.2), and (2.3) and thus $\tau(f_i \to v) \ge 1$ by the Claim. It turns out that $w'(v) \ge -3 + 3 \cdot 1 = 0$.

Suppose that $d(v) = 4$ and so $w(v) = -2$. Let f_1, f_2, f_3, f_4 denote the incident faces of v in clockwise direction with $d(f_1) = \min_{1 \le i \le 4}\{d(f_i)\}$. If $d(f_1) \ge 4$, then each of the faces f_i's sends at least $\frac{1}{2}$ to v by the Claim and therefore $w'(v) \ge -2 + 4 \cdot \frac{1}{2} = 0$. So suppose that $d(f_1) = 3$. By Lemma 1, $d(f_2), d(f_4) \ge 10$. If $d(f_3) = 3$, then it is easy to check that f_i for $i = 2, 4$ does not satisfy (2.1)-(2.3) whenever $d(f_i) = 10$. By the Claim, $\tau(f_i \to v) \ge 1$, and consequently $w'(v) \ge -2 + 1 + 1 = 0$.

Now assume $d(f_3) \ge 4$. First we see $\tau(f_3 \to v) \ge \frac{1}{2}$ by the Claim. If either $d(f_2) \ge 11$, or $d(f_2) = 10$ and f_2 does not satisfy (2.1)-(2.3), then $w'(v) \ge -2 + 2 \cdot \frac{1}{2} + 1 = 0$. Otherwise, the above Remarks implies that f_2 does not satisfy (2.2) and (2.3) because the 4-vertex v is incident to the 3-face f_1. Thus we may assume that $d(f_2) = 10$ and f_2 satisfies (2.1). Then $n_3^3(f_2) = n_3(f_2) = 9$ and $n_4(f_2) = 1$. Let $f_2 = [x_1 x_2 \ldots x_{10}]$ such that $v = x_1$, $x_1 x_2 \in b(f_1) \cap b(f_2)$, and $x_{10}x_1 \in b(f_3) \cap b(f_2)$. There exists a 3-face $f^* = [x_9 u x_{10}]$ adjacent to f_3. By Lemma 1, $d(f_3) \geq 10$. If $d(f_3) \geq 11$, then we similarly have $w'(v) \geq 0$. If $d(f_3) = 10$, f_3 does not satisfy (2.1)-(2.3) since the unique 4-vertex v is not on the common boundary of f_3 and some face of degree at most 5. We also derive that $w'(v) \geq 0$. ∎

3. 3-Choosability

In this section, we are ready to prove our main result. Every subgraph H of a planar graph G with $c^*(G) \geq 10$ is also a planar graph with $c^*(H) \geq 10$. Every subgraph of a list k-colorable graph is also list k-colorable. These straightforward facts are essential in carrying out the induction in the following proof.

Theorem 5. *Every plane graph G with $c^*(G) \geq 10$ is 3-choosable.*

Proof. We use induction on the vertex number $|V(G)|$. If $|V(G)| \leq 4$, the theorem is trivially true. Let G be a planar graph with $c^*(G) \geq 10$ and $|V(G)| \geq 5$. Let L denote an assignment for G such that $|L(v)| = 3$ for all $v \in V(G)$. If $\delta(G) \leq 2$, let u be a vertex of minimum degree in G. By the induction hypothesis, $G - u$ is L-colorable. Obviously, we can extend any L-coloring of $G - u$ into an L-coloring of G. If $\delta(G) \geq 3$, then $c^*(G) = 10$ by Lemma 2. Further, G contains a 10-cycle C such that each of its vertices is of degree 3 in G by Lemma 4. Since $c^*(G) = 10$, C is chordless in G. Thus, for every $x \in V(C)$, there exists a vertex $\overline{x} \in V(G) \setminus V(C)$ adjacent to x in G. By the induction hypothesis, $G - V(C)$ has an L-coloring ϕ. We define an assignment $L'(x) = L(x) \setminus \{\phi(\overline{x})\}$ for every $x \in V(C)$. It is easy to see that $|L'(x)| \geq |L(x)| - 1 = 3 - 1 = 2$. Thus C is L'-colorable. Consequently, G is 3-choosable. This proves Theorem 5. ∎

Steinberg ([6], p. 42) conjectured that every planar graph without 4- and 5-cycles is 3-colorable. This conjecture still remains open. Borodin [3], and independently Sanders and Zhao [8], proved that every planar graph without k-cycles for all $4 \leq k \leq 9$ is 3-colorable. Actually their result is an immediate corollary of our Theorem 5. The best known partial result on Steinberg's conjecture was obtained recently by Borodin et al.[4], where 9 is replaced by 7.

Remarks. Steinberg's conjecture cannot be extended to the chordal-cycle-free situation. Namely, a planar graph G without chordal-k-cycles for $4 \leq k \leq 5$ may not be 3-colorable. To construct such an example, let $\bar{H}$ be the plane graph obtained by adding the edges x_1x_3, x_2x_6, x_5x_7 to a 8-cycle $x_1x_2 \cdots x_8x_1$. Take a copy H' of $\bar{H}$ and let x_i' be the copy of x_i in H' for all $i = 1, 2, \cdots, 8$. Define the graph $\bar{G} = \bar{H} \cup H' \cup \{x_4x_4', x_8x_8', x_4x_8', x_8x_4'\}$. The graphs $\bar{H}$ and $\bar{G}$ are depicted in Fig. 1

and Fig. 2, respectively. Then $\bar{G}$ is a 2-connected planar graph with four 3-cycles, one 4-cycle, twelve 5-cycles, and without chordal-4-cycles and chordal-5-cycles. It is easy to show that $\chi_\ell(\bar{G}) = \chi(\bar{G}) = 4$.

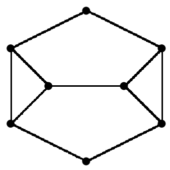

Fig. 1. The graph $\bar{H}$

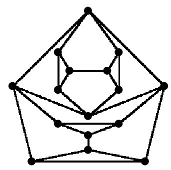

Fig. 2. The graph $\bar{G}$

Let γ denote the least integer k such that every planar graph G with $c^*(G) \geq k$ is 3-choosable. The graph $\bar{G}$ and Theorem 5 show that $6 \leq \gamma \leq 10$. We would like to propose the following conjecture which implies Steinberg's conjecture if established.

Conjecture 6. $\gamma = 6$.

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