TAIWANESE JOURNAL OF MATHEMATICS
Vol. 11, No. 1, pp. 151-160, March 2007
This paper is available online at http://www.math.nthu.edu.tw/tjm/

ON THE MULTIPLIERS OF THE INTERSECTION OF WEIGHTED FUNCTION SPACES

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Abstract. In this paper we are interested in the problem of multipliers for the intersection of weighted $L^p(G)$-spaces. We prove theorem by the different characterization of multipliers, which include the results of Murthy and Unni(1973) as particular case.

1. Introduction

Let $(A, \|.\|_A)$ be a Banach algebra, a Banach space $(V, \|.\|_V)$ is called a Banach $A-$module, if V is a module in the algebraic sense satisfying $\|av\|_V \leq \|a\|_A \|v\|_V$ for all $a \in A$ and $v \in V$. A Banach $A-$module is called essential if the closed linear span of AV coincides vith V. If the Banach algebra $(A, \|.\|_A)$ contains a bounded approximate identity, i.e., a bounded net $(e_\alpha)_{\alpha \in I}$ such that $\lim_\alpha \|e_\alpha a - a\|_A = 0$ for all $a \in A$ then a Banach $A-$module V is an essential one, by Cohen's factorization theorem, if and only if $\lim_\alpha \|e_\alpha v - v\|_V = 0$ for all $v \in V$ (Doran-Wichmann, [3]), (Hewitt-Ross, [7]).

Let V and W be a Banach $A-$module then $Hom_A(V, W)$ denotes the Banach space of all continuous $A-$module homomorphisms from V to W with the operator norm. The elements of $Hom_A(V, W)$ are traditionally called multipliers from V to W.

Let $V \otimes_\pi W$ denote the projective tensor product of V and W as Banach space for the norm $\|v \times w\| = \inf \left\{ \sum_{i=1}^{\infty} \|v_i\|_V \|w_i\|_W \mid v \times w = \sum_{i=1}^{\infty} v_i \otimes w_i \right\}$. (Dunford-Schwartz, [4]), (Grothendieck, [6]), (Bonsall-Duncan, [1]), (Schatten, [14]), (Rieffel, [13]). Then the Banach algebra of all bounded operators from V to W^*, the dual of W, denoted by $B(V, W^*)$ identifies with the dual space $V \otimes_\pi W$ and naturally, if A is a subalgebra of $B(V, W^*)$, then

Received October 8, 2004, revised June 25, 2005.
Communicated by Sen-Yen Shaw.
2000 *Mathematics Subject Classification*: Primary 43A22.
Key words and phrases: Multipliers, Tensor Product, Weighted Spaces.

(1.1) $$Hom_A(V,W) \cong (V \otimes_\pi W \diagup A)^* = (V \otimes_A W)^*.$$

Let G be a locally compact abelian group with Haar measure dx and ω be a non negative continuous function on G, $L^p_\omega(G) = \{f \mid f\omega \in L^p(G)\}$ denote the Banach space under the natural norm $\|f\| = \|f\omega\|_{p,\omega}$, $1 \leq p \leq \infty$. Then its dual space is $L^{p'}_{\omega^{-1}}(G)$ where $\frac{1}{p} + \frac{1}{p'} = 1$, $1 \leq p < \infty$. Moreover if $1 < p < \infty$, $L^p_\omega(G)$ is a reflexive Banach space. $C_{\infty,\omega}(G)$ denotes a Banach subspace of $L^\infty_\omega(G)$ such that $f\omega \in C_0(G)$, the space of all continuous, complex valued functions on G which vanish at infinity. $C_C(G)$ is the space of all continuous functions on G with compact support.

Let $1 < p_1,\ p_2 < \infty$, $S\,(p_1, p_2, \omega)$ be the set of all (classes of) measurable, complex valued functions g which can be written as

$$g = g_1 + g_2 with\,(g_1, g_2) \in L^{p_1}_\omega(G) \times L^{p_2}_\omega(G).$$

We define a norm on $S\,(p_1, p_2, \omega)$ by

$$\|g\|_S = \inf\left\{\|g_1\|_{p_1,\omega} + \|g_2\|_{p_2,\omega}\right\},$$

where the infimum is taken over all such decompositions of g. $S\,(p_1, p_2, \omega)$ is a Banach space under this norm.

Similarly, if $D(p_1, p_2, \omega)$ denotes the set of all (classes of) measurable, complex valued functions defined on G which are in $L^{p_1}_\omega(G) \cap L^{p_2}_\omega(G)$, we introduce a norm by

$$\|f\|_D = \max\left(\|f\|_{p_1,\omega}, \|f\|_{p_2,\omega}\right).$$

Then $D(p_1, p_2, \omega)$ is also a Banach under this norm.

If ω is a weight function, i.e., a continuous function satisfying $\omega(x) \geq 1$, $\omega(x+y) \leq \omega(x)\omega(y)$ for all $x,\ y \in G$. Then the space $L^1_\omega(G)$ is a Banach algebra with respect to convolution. It is called a Beurling algebra (Reiter, [12]). It follows that $L^p_\omega(G)$ is an essential Banach $L^1_\omega(G)-$module.

It is not hard to prove that $D(p_1, p_2, \omega)$ and $S\,(p_1, p_2, \omega)$ are reflexive Banach $L^1_\omega(G)$-modules and the following duality relations hold:

$$D(p_1, p_2, \omega)^* \cong S\left(p_1', p_2', \omega^{-1}\right),$$
$$D(p_1, p_2, \omega^{-1})^* \cong S\left(p_1', p_2', \omega\right)$$

where $\frac{1}{p_i} + \frac{1}{p_i'} = 1$, $(i = 1, 2)$, (Murthy-Unni, [11]), (Liu-Wang, [9]), (Liu-Rooij [10]).

So, if the relation (1.1) applied to the $L^p_\omega(G)$ becomes

$$Hom_{L^1_\omega(G)}(L^p_\omega(G), L^{q'}_{\omega^{-1}}(G)) \cong (L^p_\omega(G) \otimes_{L^1_\omega(G)} L^q_\omega(G))^*$$

for $1 \leq p \leq \infty$ and $1 \leq q < \infty$.

We remark that the relation (1.1) does not immediately apply to the case of $Hom_{L^1_\omega(G)}(L^p_\omega(G), L^1_\omega(G))$, since $L^1_\omega(G)$ is not a dual space. (Gaudry, [5]) showed that $Hom_{L^1_\omega(G)}(L^1_\omega(G), L^1_\omega(G)) \cong M(\omega)$,the space of Radon measure μ on G for which $\|\mu\|_\omega < \infty$. However, when $p = q = \infty$, using the similar approach of (Larsen, [8]), we get the following proposition. We shall denote by $L^{\infty,w}_{\omega^{-1}}(G)$ the space $L^\infty_{\omega^{-1}}(G)$ considered with the weak* topology induced by elements of $L^1_\omega(G)$.

Proposition 1.1. *Let G be a locally compact abelian group and suppose $T : L^{\infty,w}_{\omega^{-1}}(G) \to L^{\infty,w}_{\omega^{-1}}(G)$ is a linear transformation. Then the following are equivalent*

(1) $T \in M(L^{\infty,w}_{\omega^{-1}}(G), L^{\infty,w}_{\omega^{-1}}(G))$,

(2) *There exists a unique $\mu \in M(\omega)$ such that $Tf = \mu * f$ for each $f \in L^{\infty,w}_{\omega^{-1}}(G)$.*

It is well known that if G is non-compact and $p > q$ then $M(L^p, L^q) = \{0\}$, for the weighted L^p spaces we can assume hereafter that $p_i > 1$ and $q_i > 1, (i = 1, 2)$ with $p_i \leq q_i$.

In section 2, the function space $\Lambda^D_S(G)$ is defined as in (Rieffel, [13]) and the basic properties are studied. In section 3 and 4 the multipliers spaces $Hom_{L^1_\omega(G)}(L^{p_1}_\omega(G) \cap L^{p_2}_\omega(G), L^{q_1}_\omega(G) \cap L^{q_2}_\omega(G))$ and $Hom_{L^1_\omega(G)}(L^1_\omega(G), \Lambda^D_S(G))$ are also considered.

2. The Space $\Lambda^D_S(G)$ and Some Properties

Throughout this section we will assume that G is a locally compact abelian group and ω is a symmetric weight function on G.

Proposition 2.1. *If $1 < p, q' < \infty$, $\frac{1}{p} + \frac{1}{q'} = \frac{1}{r} + 1$ and $\frac{1}{p} + \frac{1}{q'} \geq 1$ then $L^p_\omega(G) * L^{q'}_{\omega^{-1}}(G) \subset L^r_{\omega^{-1}}(G)$*

Proposition 2.2. *If $1 < p_i, q_i' < \infty$, $\frac{1}{p_i} + \frac{1}{q_i'} = \frac{1}{r_i} + 1$ and $\frac{1}{p_i} + \frac{1}{q_i'} \geq 1$ $(i = 1, 2)$ then $f * g \in S\left(r_1, r_2, \omega^{-1}\right)$ for any $f \in D(p_1, p_2, \omega)$, $g \in S\left(q_1', q_2', \omega^{-1}\right)$ and*

$$\|f * g\|_S \leq \|f\|_D \|g\|_S$$

Proof. For each $f \in D(p_1, p_2, \omega)$ and $g \in S\left(q_1', q_2', \omega^{-1}\right)$, $g = g_1 + g_2$, where $g_1 \in L_{\omega^{-1}}^{q_1'}(G)$, $g_2 \in L_{\omega^{-1}}^{q_2'}(G)$, from Proposition 2.1, $f * g_1 \in L_{\omega^{-1}}^{r_1}(G)$, $f * g_2 \in L_{\omega^{-1}}^{r_2}(G)$ and so,

$$\|f * g\|_S \leq \|f\|_D \|g\|_S .$$

In view of Proposition 2.2 we can define a bilinear map b from $D(p_1, p_2, \omega) \times S\left(q_1', q_2', \omega^{-1}\right)$ into $S\left(r_1, r_2, \omega^{-1}\right)$, $(p_i \neq q_i)$ or $S\left(\infty, \infty, \omega^{-1}\right)$, $(p_i = q_i)$ by

$$b(f, g) = f^{\sim} * g \qquad f \in D(p_1, p_2, \omega), g \in S\left(q_1', q_2', \omega^{-1}\right)$$

where $f^{\sim}(x) = f(-x)$. It is easy to see $\|b\| \leq 1$. The b lifts to a linear map B from $D(p_1, p_2, \omega) \otimes_\gamma S\left(q_1', q_2', \omega^{-1}\right)$ into $S\left(r_1, r_2, \omega^{-1}\right)$ or $S\left(\infty, \infty, \omega^{-1}\right)$ and $\|B\| \leq 1$ by Theorem 6 in (Bonsall-Duncan, [1]).

Definition 2.2. The range of B,with the quotient norm, will be denoted by $\Lambda_S^D(G)$.

Thus $\Lambda_S^D(G)$ is a Banach space of functions on G which can be viewed as a subspace of $S\left(r_1, r_2, \omega^{-1}\right)$ or $S\left(\infty, \infty, \omega^{-1}\right)$ and every element h of $\Lambda_S^D(G)$ has at least one expansion of the form

$$h = \sum_{i=1}^{\infty} f_i^{\sim} * g_i,$$

where $f_i \in D(p_1, p_2, \omega)$, $g_i \in S\left(q_1', q_2', \omega^{-1}\right)$, and $\sum_{i=1}^{\infty} \|f_i\|_D \|g_i\|_S < \infty$,

with the expansion converging in the norm of $S\left(r_1, r_2, \omega^{-1}\right)$ or $S\left(\infty, \infty, \omega^{-1}\right)$. Furthermore the norm on $\Lambda_S^D(G)$ will be denoted by $\|.\|_{\Lambda_S^D}$.

Proposition 2.3. *$D(p_1, p_2, \omega)$ and $S(p_1, p_2, \omega)$ are an essential Banach $L_\omega^1(G)$-modules.*

Proposition 2.4. *$\Lambda_S^D(G)$ is an essential Banach $L_\omega^1(G)-$module.*

Proof. It is easy to prove that $\Lambda_S^D(G)$ is a Banach $L_\omega^1(G)-$module.Let $(e_\alpha)_{\alpha \in I}$ be an approximate identity bounded in $L_\omega^1(G)$ it is also an approximate identity in $D(p_1, p_2, \omega)$ from Proposition 2.3. Assume that $\|e_\alpha\|_{1,\omega} \leq K$ for all $\alpha \in I$. Let $h \in \Lambda_S^D(G)$ be given; we get

$$h = \sum_{i=1}^{\infty} f_i^{\sim} * g_i, f_i \in D(p_1, p_2, \omega), g_i \in S\left(q_1', q_2', \omega^{-1}\right)$$

where $\sum_{i=1}^{\infty} \|f_i\|_D \|g_i\|_S < \infty$. Hence we have

$$\|h - e_\alpha * h\|_{\Lambda_S^D(G)} = \left\| \sum_{i=1}^{\infty} (f_i^{\sim} - e_\alpha * f_i^{\sim}) * g_i \right\|_{\Lambda_S^D(G)} \leq \sum_{i=1}^{\infty} \|f_i^{\sim} - e_\alpha * f_i^{\sim}\|_D \|g_i\|_S$$

and also we obtain

$$\lim_{\alpha \in I} \|h - e_\alpha * h\|_{\Lambda_S^D(G)} = 0.$$

Consequently, by Corollary 15.3 in (Doran-Wichmann, [3]), we get

$$(\Lambda_S^D(G))_e = \Lambda_S^D(G).$$

3. Multipliers from $D(p_1, p_2, \omega)$ To $D(q_1, q_2, \omega)$

In this section, we will extend Theorem 2 in (Murthy-Unni, [11]) as a multipliers of from $D(p_1, p_2, \omega)$ to $D(q_1, q_2, \omega)$ by using the method in (Rieffel, [13]). Let us mention that we assume $\omega_1 = \omega_2$ to simplify our proof and let us recall that (Murthy-Unni, [11]) defines the space $\tau(p_1, \omega_1, p_2, \omega_2)$ to be the set of all functions u which can be written in the form

$$u = \sum_{j=1}^{\infty} f_j * g_j$$

where $f_j \in C_c(G) \subset D(p_1, \omega_1, p_2, \omega_2)$ and $g_j \in S(p_1', \omega_1^{-1}, p_2', \omega_2^{-1})$ with $\sum_{j=1}^{\infty} \|f_j\|_D \|g_j\|_S < \infty$ and they prove that the space of multipliers $M(D(p_1, \omega_1, p_2, \omega_2))$ is isometrically isomorphic to $\tau(p_1, \omega_1, p_2, \omega_2)^*$, the conjugate space of $\tau(p_1, \omega_1, p_2, \omega_2)$.

Since following (Rieffel, [13]) we get a general theorem. We start by recalling the following definition.

Definition 3.1. Let K be the closed linear subspace of $D(p_1, p_2, \omega) \otimes_{L^1_\omega} S\left(q_1', q_2', \omega^{-1}\right)$ which is spanned by all elements of the form

$$(\varphi * f) \otimes g - f \otimes (\varphi^{\sim} * g)$$

where $f \in D(p_1, p_2, \omega)$, $g \in S\left(q_1', q_2', \omega^{-1}\right)$ and $\varphi \in L^1_\omega(G)$. Then the Banach $L^1_\omega(G)-$module tensor product $D(p_1, p_2, \omega) \otimes_{L^1_\omega} S\left(q_1', q_2', \omega^{-1}\right)$ is defined to be the quotient Banach space

$$D(p_1, p_2, \omega) \otimes_{L^1_\omega} S\left(q_1', q_2', \omega^{-1}\right) = D(p_1, p_2, \omega) \otimes_\gamma S\left(q_1', q_2', \omega^{-1}\right) \diagup K$$

Lemma 3.2. *Let G be locally a compact abelian group and $1 < p_i,\, q_i^{'} < \infty$, $\frac{1}{p_i} + \frac{1}{q_i^{'}} \geq 1$, $(i = 1, 2)$. Given any $\varphi \in C_c(G)$ define T_φ by $T_\varphi(f) = f * \varphi$. Then $T_\varphi \in Hom_{L^1_\omega}(D(p_1, p_2, \omega),\, D(q_1, q_2, \omega))$ and the inequality*

$$\|T_\varphi\| \leq \|\varphi\|_{1,\omega}^{\frac{p_1}{q_1}} \|\varphi\|_{p_1^{'},\omega}^{1-\frac{p_1}{q_1}}$$

or the inequality

$$\|T_\varphi\| \leq \|\varphi\|_{1,\omega}^{\frac{p_2}{q_2}} \|\varphi\|_{p_2^{'},\omega}^{1-\frac{p_2}{q_2}}$$

is satisfied.

Proof. Since $C_c(G) \subset L^p_\omega(G)$ for all p and ω, using the Proposition 2.1, Proposition 2.2 and Riesz-Thorin's interpolation theorem, it is obtained.

Definition 3.3. Let G be a locally compact abelian group. If every element of $Hom_{L^1_\omega}(D(p_1, p_2, \omega), D(q_1, q_2, \omega))$ can be approximated in the ultraweak operator topology by operators of the form T_φ, $\varphi \in C_c(G)$ then G is called to satisfy property $P^{p_1,p_2,\omega}_{q_1,q_2,\omega}$.

Theorem 3.4. *Let G be a locally compact abelian group. If $1 < p_i^{'},\, q_i < \infty$, $\frac{1}{p_i} + \frac{1}{q_i^{'}} = \frac{1}{r_i} + 1$ and $\frac{1}{p_i} + \frac{1}{q_i^{'}} \geq 1$ $(i = 1, 2)$ then G satisfies property $P^{p_1,p_2,\omega}_{q_1,q_2,\omega}$ if and only if the kernel of B is K and the space $D(p_1, p_2, \omega) \otimes_{L^1_\omega} S\left(q_1', q_2', \omega^{-1}\right)$ is isometrically isomorphic to the space $\Lambda^D_S(G)$.*

Proof. Suppose that G satisfies property $P^{p_1,p_2,\omega}_{q_1,q_2,\omega}$. It is easy to see that $K \subset KerB$. To show that $KerB \subset K$ it is suffices to show $K^\perp \subset (KerB)^\perp$. Let $F \in K^\perp$ be given. From the isometric isomorphism

$$K^\perp \cong (D(p_1, p_2, \omega) \otimes_{L^1_\omega} S\left(q_1', q_2', \omega^{-1}\right))^* \cong Hom_{L^1_\omega}(D(p_1, p_2, \omega), D(q_1, q_2, \omega))$$

there is a multiplier $T \in Hom_{L^1_\omega}(D(p_1, p_2, \omega), D(q_1, q_2, \omega))$ corresponding F such that

$$< t, F >= \sum_{i=1}^{\infty} < g_i, Tf_i >, \tag{3.1}$$

where $t \in KerB$, $t = \sum\limits_{i=1}^{\infty} f_i \otimes g_i$ and $\sum\limits_{i=1}^{\infty} \|f_i\|_D \|g_i\|_S < \infty$. We wish to show that $\sum\limits_{i=1}^{\infty} < g_i,\, Tf_i >= 0$, since G satisfies property $P^{p_1,p_2,\omega}_{q_1,q_2,\omega}$ there is a net (φ_j), of elements $C_c(G)$ such that the operators T_{φ_j} defined in Lemma 3.2 converge T in the ultraweak operator topology.

$$\lim_j \sum_{i=1}^{\infty} < g_i, T_{\varphi_j} f_i >= \sum_{i=1}^{\infty} < g_i, T f_i > . \tag{3.2}$$

Thus to prove it suffices to show that

$$\sum_{i=1}^{\infty} < g_i, f_i * \varphi_j >= 0 \tag{3.3}$$

for each j. On the other hand, we have

$$\sum_{i=1}^{\infty} < g_i, f_i * \varphi_j >=< \sum_{i=1}^{\infty} f_i^{\sim} * g_i, \varphi_j >= 0 \tag{3.4}$$

Hence from (3.2) and (3.4) we get $F \in (KerB)^{\perp}$ and also using the following

B^- isomorphism such that $B^- \circ \Phi = B$

$$\begin{array}{ccc} (D(p_1,p_2,\omega)\otimes_{L^1_\omega} S(q_1',q_2',\omega^{-1})) \to^{B} \Lambda_S^D(G) \to^{i} S(r_1,r_2,\omega^{-1}) \\ \Phi \searrow \qquad\qquad\qquad\qquad \nearrow B^- \\ (D(p_1,p_2,\omega)\otimes_{\gamma} S(q_1',q_2',\omega^{-1}))/KerB \end{array}$$

we have $(D(p_1,p_2,\omega)\otimes_{L^1_\omega} S(q_1',q_2',\omega^{-1})) \cong \Lambda_S^D(G)$.

Suppose conversely that $KerB = K$. We will show that the set $N = \{T_\varphi \mid \varphi \in C_C(G)\}$ is everywhere dense in $Hom_{L^1_\omega}(D(p_1,p_2,\omega), D(q_1,q_2,\omega))$ in the ultraweak operator topology. It is sufficient to show that the set of the linear functionals which corresponds to the operators T_φ, denoted by M, is everywhere dense in $(D(p_1,p_2,\omega)\otimes_{L^1_\omega} S(q_1',q_2',\omega^{-1}))^*$ in the weak* topology.

But to show this it is sufficient to prove that $M^{\perp} = KerB$. Since $(D(p_1,p_2,\omega) \otimes_{L^1_\omega} S(q_1',q_2',\omega^{-1}))^* \cong (KerB)^{\perp}$ then $< t, F >= 0$ for all $t \in KerB$ and $F \in M$. Thus $T \in M^{\perp}$. That means $KerB \subset M^{\perp}$. Conversely for every $t \in M^{\perp}$ and $F \in M$ we have $< t, F >= 0$. Using (3.4) and Hann-Banach theorem we find that $\sum_{i=1}^{\infty} f_i^{\sim} * g_i = 0$. Therefore $M^{\perp} \subset KerB$. This completes the proof.

Corollary 3.5 Let G be a locally compact abelian group and $1 < p_i, q_i^{'} < \infty$, $\frac{1}{p_i} + \frac{1}{q_i^{'}} = \frac{1}{r_i} + 1$, $\frac{1}{p_i} + \frac{1}{q_i'} \geq 1$, $(i = 1, 2)$. If G satisfies property $P_{q_1,q_2,\omega}^{p_1,p_2,\omega}$ then we have the identification

$$Hom_{L^1_\omega}(D(p_1,p_2,\omega), D(q_1,q_2,\omega)) \cong \Lambda_S^D(G)^*.$$

4. Multipliers from $L^1_\omega(G)$ To $\Lambda^D_S(G)$

Proposition 4.1. *Let G be a locally compact abelian group. $Hom_{L^1_\omega}(L^1_\omega(G),$ $\Lambda^D_S(G))$ is an essential Banach module over $L^1_\omega(G)$.*

Proof. It is easy to see that $(Hom_{L^1_\omega}(L^1_\omega(G), \Lambda^D_S(G))$ is a $L^1_\omega(G)-$Banach module, defined by $(fT)(g) = T(f * g)$, for all $f \in L^1_\omega(G)$ and $T \in (Hom_{L^1_\omega}(L^1_\omega$ $(G),\ \Lambda^D_S(G))$. On the other hand take $(e_\alpha)_{\alpha\in I}$ bounded approximate identity in $L^1_\omega(G)$. For every $T \in (Hom_{L^1_\omega}(L^1_\omega(G), \Lambda^D_S(G))$ we obtain

$$\begin{aligned}\|e_\alpha T - T\| &= \sup_{\|f\|_{1.\omega}=1} \|(e_\alpha T - T)(f)\|_{\Lambda^D_S(G)} \\ &= \sup_{\|f\|_{1.\omega}=1} \|T(e_\alpha * f) - T(f)\|_{\Lambda^D_S(G)} \leq \sup_{\|f\|_{1.\omega}=1} \|T\| \, \|e_\alpha * f - f\|_{1,\omega} .\end{aligned}$$

This completes the proof by Corollary 15. 3 in (Doran-Wichmann, [3]).

Theorem 4.2. *Let G be a locally compact abelian group. The space $Hom_{L^1_\omega}$ $(L^1_\omega(G), \Lambda^D_S(G))$ is isometrically isomorphic to the space $\Lambda^D_S(G)$.*

Proof. It is the consequence of the Theorem 3.3. in (Datry-Muraz, [2]).

Remark 1. (1) If $p_1 = p_2 = p$ and $q_1 = q_2 = q$, we get $\frac{1}{p} + \frac{1}{q'} \geq 1$ and $\frac{1}{p} + \frac{1}{q'} = \frac{1}{r} + 1$, then it is obtained Theorem 1 in (Murthy-Unni, [11]):

$$Hom_{L^1_\omega}(L^p_\omega(G), L^q_\omega(G)) \cong (L^p_\omega(G) \otimes_{L^1_\omega} L^{q'}_{\omega^{-1}}(G))^* \cong (\Lambda^p_{q'}(G))^*$$

$$= \left\{ t = \sum_{i=1}^{\infty} f_i^{\sim} * g_i \mid \sum_{i=1}^{\infty} \|f_i\|_{p,\omega} \|g_i\|_{q',\omega^{-1}} < \infty, f_i \in L^p_\omega(G), g_i \in L^{q'}_{\omega^{-1}}(G) \right\}^*$$

(2) If $p_1 = p_2 = p$ and $q_1 \neq q_2$, we have a new multipliers space such that

$$Hom_{L^1_\omega}(L^p_\omega(G), L^{q_1}_\omega(G) \cap L^{q_2}_\omega(G)) \cong (L^p_\omega(G) \otimes_{L^1_\omega} S\left(q_1', q_2', \omega^{-1}\right))^* \cong (\Lambda^p_S(G))^*$$

$$= \left\{ t = \sum_{i=1}^{\infty} f_i^{\sim} * g_i \mid \sum_{i=1}^{\infty} \|f_i\|_{p,\omega} \|g_i\|_S < \infty,\ f_i \in L^p_\omega(G),\ g_i \in S\left(q_1', q_2', \omega^{-1}\right) \right\}^*$$

(3) If $p_1 \neq p_2$, $q_1 = q_2 = q$ we get the folowing new multipliers space such that

$$Hom_{L^1_\omega}(L^{p_1}_\omega(G) \cap L^{p_2}_\omega(G), L^q_\omega(G)) \cong (L^{p_1}_\omega(G) \cap L^{p_2}_\omega(G) \otimes_{L^1_\omega} L^{q'}_{\omega^{-1}}(G))^* \cong (\Lambda^D_{q'}(G))^*$$
$$= \left\{ t = \sum_{i=1}^{\infty} f_i^{\sim} * g_i \mid \sum_{i=1}^{\infty} \|f_i\|_D \|g_i\|_{q',\omega^{-1}} < \infty,\ f_i \in D(p_1, p_2, \omega), g_i \in L^{q'}_{\omega^{-1}}(G) \right\}^*$$

Note that in Remarks 1, 2 and 3, the norm of t is the infimum of the expression for all representations of t

(4) If $\omega = 1$, it is obtained the classical case of $L^p(G)-$spaces.

ACKNOWLEDGMENT

The author would like to thank to the referee for the significant suggestions.

REFERENCES

1. F. F. Bonsall and J. Duncan, *Complete Normed Algebras. Berlin*, Heidelberg, Newyork: Springer-Verlag, 1973.
2. C. Datry and G. Muraz, *Multiplier Problems, Proceedings of the third Asian Mathematical Conference*, Diliman-Philippines, 2000.
3. R. S. Doran and J. Wichmann, *Approximate Identities and Factorazation in Banach Modules*, Lecture notes in Mathematics, 768. Springer-Verlag, 1979.
4. N. Dunford and J. T. Schwartz, *Linear Operators, I*, Interscience Publisher, 1958.
5. Gaudry G. I. Multipliers of Weighted Lebesque and Measure Spaces. *Proc. London Math. Soc.*, **19(3)** (1969), 327-340.
6. A. Grothendieck, *Produits tensoriels topologiques et espaces nucleaires*, American Mathematical Society, Providence, Rhode Island, 1955.
7. E. Hewitt, K. A. Ross, *Abstract Harmonic Analysis I*, Springer-Verlag, 1963.
8. R. Larsen, *An Introduction to Theory of Multipliers*, New York, Springer-Verlag, 1970.
9. T. S. Liu and J. K. Wang, Sums and Intersections of Lebesque Spaces, *Math. Scand*, **23** (1968), 241-251.
10. T. S. Liu and A. V. Rooij, *Mathematiche Nachrichten Band,* **42** (1969), 1-3.
11. G. N. K. Murthy and K. R.Unni, *Multipliers on Weighted Spaces. Functional Analysis and its Applications*, International conference. Lecture Notes in Mathematics 399, Springer-Verlag, 1973.
12. H. Reiter, *Classical Harmonic Analysis and Locally Compact Groups+*, Oxford at the Clarendon Press. 1968.
13. M. A. Rieffel, Multipliers and Tensor Product of L^p-spaces of Locally Compact Groups, S*tudia Mathematica, T*, **XXXIII** (1969), 71-82.
14. R. Schatten, Theory of Cross-Spaces, *Annals of Mathematics studies*, **26** (1950).

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