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ON VECTOR EQUILIBRIUM PROBLEM WITH MULTIFUNCTIONS

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Abstract. In this paper, a vector equilibrium problem (VEP) with multifunctions is considered. Using the asymptotic cone of the solution set of (VEP), we give conditions under which the solution set is nonempty and compact, and then extend it to a random vector equilibrium problem with multifunctions.

1. INTRODUCTION

Let X be a nonempty convex and closed subset of $\mathbb{R}^n$ and $Y = \mathbb{R}^m$. Let $F : X \times X \to 2^Y$ be a multifunction and $C : X \to 2^Y$ a multifucnction such that $C(x)$ is a convex cone in Y with $intC(x) \neq \emptyset$ and $C(x) \neq Y$ for all $x \in X$. In this paper, we consider the following vector equilibrium problem:

(VEP) Find $\bar{x} \in X$ such that

$$F(\bar{x}, x) \subset Y \setminus (-intC(\bar{x})) \text{ for any } x \in X.$$

We denote the solution set of (VEP) by E_w. When $C(x)$ is a constant convex cone for any $x \in X$, the above problem (VEP) is reduced to the one studied in [1, 3, 8, 14]. If the above F is single-valued, then the problem (VEP) becomes the one studied in [7, 12].

Recently, using asymptotic analysis, F. Flores-Bazán and F. Flores-Bazán [7] obtained characterizations of nonemptiness and compactness of the solution set for (VEP) when F is single-valued. Ansari and Flores-Bazán [1] extended the results in [7] to (VEP) when $C(x)$ is a constant cone.

In this paper, following the ideas in [7] and using asymptotic cones, we give conditions under which the solution set of (VEP) is nonempty and compact. Also, following the ideas in Kalmoun [9, 10], we extend the conditions to a random vector equilibrium problem.

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2. Nonemptiness and Compactness of Solution Set

In this section, we extend Lemma 3.2, Theorem 3.7 and Theorem 3.13 in [7] to the vector equilibrium problem (VEP) with multifunctions so that we can obtain conditions under which the solution set of (VEP) is nonempty and compact. Our approach follows ideas of [7] in which vector equilibrium problems with single-valued functions were considered.

We now recall some known definitions:

Definition 2.1. Let X be a nonempty convex subset of $\mathbb{R}^n$ and $Y = \mathbb{R}^m$. Let $G : X \to 2^Y$ be a multifunction and C a convex cone in Y with $C \neq Y$.

(1) ([15]) G is called upper (lower) $C-$convex on X if for any $x_1, x_2 \in X,\ t \in [0,1]$,

$$tG(x_1) + (1-t)G(x_2) \subset G(tx_1 + (1-t)x_2) + C$$
$$(G(tx_1 + (1-t)x_2) \subset tG(x_1) + (1-t)G(x_2) - C, \text{ respectively})$$

holds. If G is both upper C-convex and lower C-convex, we say that G is C-convex.

(2) ([13]) G is said to be upper (lower) C-lower semicontinuous at $\bar{x} \in X$ if for any open set V in $\mathbb{R}^m$ with $G(\bar{x}) \cap V \neq \emptyset$, there exists a neighborhood U of $\bar{x}$ such that for any $x \in U \cap X$

$$G(x) \cap (V + C) \neq \emptyset$$
$$(G(x) \cap (V - C) \neq \emptyset, \text{ respectively}).$$

If G is upper (lower) C-lower semicontinuous at every $x \in X$, then G is called upper (resp. lower) C-lower semicontinuous on X.

If G is both upper C-lower semicontinuous on X and lower C-lower semicontinuous on X, then G is said to be C-lower semicontinuous on X.

Example 2.1. Define a multifunction $G : \mathbb{R} \to 2^{\mathbb{R}}$ by for any $x \in \mathbb{R}$, $G(x) = [x^2, \infty)$. Then G is $\mathbb{R}_+$-convex.

Definition 2.2 [6]. Let K be a closed set in $\mathbb{R}^n$. Then we define the asymptotic cone of K as the closed set

$$K^\infty = \{x \in \mathbb{R}^n \mid \exists t_n \downarrow 0,\ \ x_n \in K,\ t_n x_n \to x\}.$$

It is known that K is bounded if and only if $K^\infty = \{0\}$.

We give assumptions which will be used for next theorems.

Let X be a nonempty convex and closed subset of $\mathbb{R}^n$ and $Y = \mathbb{R}^m$.

(H_0) Let $C : X \to 2^Y$ be a multifunction such that $C(x)$ is a nonempty convex cone in Y with $intC(x) \neq \emptyset$ and $C(x) \neq Y$ for all $x \in X$.

(H_1) $F : X \times X \to 2^Y$ be a multifunction satisfying the following conditions:

(A_1) for all $x \in X,\ F(x,x) \subset [C(x) \cap (-C(x))]$.

(A_2) for all $x, y \in X, F(x,y) \subset Y \setminus (-intC(x))$ implies $F(y,x) \subset Y \setminus intC(y)$.

(A_3) for all $x \in X,\ y \mapsto F(x,y)$ is $C(x)$-convex and upper $C(x)$-lower semi-continuous on X.

(A_4) for all $x, y \in X$, the set $\{\xi \in [x,y] : F(\xi, y) \subset Y \setminus (-intC(\xi))\}$ is closed. Here $[x,y]$ stands for the closed line segment joining x and y.

We give an existence theorem for the problem (VEP) in compact setting: its proof can be done by using methods in [2, 4, 11] and so we omit its proof.

Theorem 2.1. *Let X be a nonempty convex and compact subset of $\mathbb{R}^n$ and $Y = \mathbb{R}^m$. Let $C : X \to 2^Y$ be a multifunction satisfying (H_0) and let $F : X \times X \to 2^Y$ be a multifunction satisfying (H_1). Then E_w is nonempty and closed.*

In order to give conditions for the soution set E_w of the problem (VEP) in noncompact setting to be nonempty and compact, we consider the asymptotic cone ${E_w}^\infty$ of E_w and its related set R_0 defined as follows:

$$R_0 := \bigcap_{y \in X} \{v \in X^\infty : F(y, z + \lambda v) \subset Y \setminus intC(y) \\ \forall \lambda > 0,\ \forall z \in X \text{ with } F(y,z) \subset -C(y)\}.$$

First we give relationships between ${E_w}^\infty$ and R_0.

Proposition 2.1. *Let X be a nonempty convex and closed subset of $\mathbb{R}^n$ and $Y = \mathbb{R}^m$. Let $C : X \to 2^Y$ be a multifunction satisfying (H_0) and $F : X \times X \to 2^Y$ a multifunction satisfying (A_2) and (A_3). Then ${E_w}^\infty \subset R_0$. If, in addition, (A_4) holds and there exists $x^* \in X$ such that*

$$F(y, x^*) \subset -C(y) \textit{ for all } y \in X,$$

then ${E_w}^\infty = R_0$.

Proof. Let $v \in (E_w)^\infty$. Then there exists $t_n \downarrow 0,\ \ v_n \in E_w$ such that $t_n v_n \to v$. Let y be any fixed in X. Since $v_n \in E_w$, we have

$$F(v_n, y) \subset Y \setminus (-intC(v_n)) \text{ for all } n \in \mathbb{N}.$$

By (A_2), we have

$$F(y, v_n) \subset Y \setminus intC(y) \text{ for all } n \in \mathbb{N}.$$

Take any $z \in X$ satisfying $F(y, z) \subset -C(y)$. Let $\lambda > 0$ be any fixed. For n sufficiently large, by the lower $C(y)$-convexity of $F(y, \cdot)$,

$$\begin{aligned} F(y, (1-\lambda t_n)z + \lambda t_n v_n)) &\subset (1-\lambda t_n)F(y,z) + \lambda t_n F(y, v_n) - C(y) \\ &\subset -C(y) + [Y \setminus intC(y)] - C(y) \\ &\subset Y \setminus intC(y). \end{aligned}$$

Since $(1-\lambda t_n)z+\lambda t_n v_n \to z+\lambda v$ and $F(y, \cdot)$ is upper $C(y)$-lower semicontinuous, $F(y, z+\lambda v) \subset Y \setminus intC(y)$. Thus $v \in R_0$. Hence $(E_w)^\infty \subset R_0$

Assume that there exists $x^* \in X$ such that $F(y, x^*) \subset -C(y)$ for all $y \in X$. Let $v \in R_0$. Then $v \in X^\infty$ for all $y \in X$ and $F(y, x^*+\lambda v) \subset Y \setminus intC(y)$ for all $\lambda > 0$. Let $y \in X$ and $\lambda > 0$ be any fixed. Consider $y_t := ty+(1-t)(x^*+\lambda v),\ t \in (0,1)$. Since $x^* \in X$ and $v \in X^\infty$, $y_t \in X$. Since $F(y_t, \cdot)$ is upper $C(y_t)$-convex,

$$tF(x_t, y) + (1-t)F(y_t, x^* + \lambda v) \subset F(y_t, y_t) + C(y_t)$$

Since $F(y_t, y_t) \subset C(y_t)$, $F(y_t, x^*+\lambda v) \subset Y \setminus intC(y)$ and $C(y_t)+Y\setminus(-intC(y_t)) \subset Y \setminus (-intC(y_t))$, we have

$$F(y_t, y) \subset Y \setminus (-intC(y_t)).$$

Since y_t converges to $x^* + \lambda v$ as $t \to 0+$, it follows from (A4) that

$$F(x^* + \lambda v, y) \subset Y \setminus (-intC(x^* + \lambda v)).$$

Thus for all $\lambda > 0$, $x^*+\lambda v \in E_w$. Hence $v \in (E_w)^\infty$. Consequently, ${E_w}^\infty = R_0$. ■

Now we give conditions assuring the nonemptiness and compactness of the solution set E_w of the problem (VEP).

Theorem 2.2. *Let X be a nonempty convex and closed subset of $\mathbb{R}^n$ and $Y = \mathbb{R}^m$. Let $C : X \to 2^Y$ be a multifunction satisfying (H_0) and $F : X \times X \to 2^Y$ a multifunction satisfying (H_1). If $R_0 = \{0\}$, E_w is nonempty and compact. Moreover, if there exists $x^* \in X$ such that*

$$F(y, x^*) \subset -C(y) \textit{ for all } y \in X,$$

then E_w is nonempty and compact if and only if $R_0 = \{0\}$.

Proof. For every $n \in \mathbb{N}$, such that $X_n := \{x \in X : \|x\| \leq n\}$. Then we may suppose that $X_n \neq \emptyset$ for all $n \in \mathbb{N}$. Also, X_n is a nonempty convex and compact subset of $\mathbb{R}^n$. By Theorem 2.1, for all $n \in \mathbb{N}$, there exists $x_n \in X_n$ such that

$$(2.1) \qquad F(x_n, y) \subset Y \setminus (-intC(x_n)) \text{ for any } y \in X_n.$$

Suppose to the contrary that $\{x_n\}$ is not bounded. Then, up to a subsequence, $\|x_n\| \to \infty$ and $\frac{x_n}{\|x_n\|} \to v$ for some $v \in X$. Then $v \in X^\infty$ and $v \neq 0$. Let $y \in X$ be any fixed. Then it is clear that $F(x_n, y) \subset Y \setminus (-intC(x_n))$ for n sufficiently large. By assumption (A_2), we have

$$F(y, x_n) \subset Y \setminus intC(y)$$

for n sufficiently large. Take $z \in X$ such that

$$F(y, z) \subset -C(y).$$

By the lower $C(y)$-convexity of $F(y, \cdot)$, for any $\lambda > 0$ and n sufficiently large,

$$\begin{aligned} F\left(y, (1 - \tfrac{\lambda}{\|x_n\|})z + \tfrac{\lambda}{\|x_n\|}x_n\right) &\subset (1 - \tfrac{\lambda}{\|x_n\|})F(y, z) + \tfrac{\lambda}{\|x_n\|}F(y, x_n) - C(y) \\ &\subset -C(y) + [Y \setminus intC(y)] - C(y) \\ &\subset Y \setminus intC(y). \end{aligned}$$

Since $(1 - \frac{\lambda}{\|x_n\|})z + \frac{\lambda}{\|x_n\|}x_n \to z + \lambda v$ and $F(y, \cdot)$ is upper $C(x)$-lower semicontinuous, $F(y, z + \lambda v) \subset Y \setminus intC(y)$. Hence $v \in R_0$. However, it contradicts the assumption that $R_0 = \{0\}$. Thus $\{x_n\}$ is bounded. Therefore, up to a subsequence, $x_n \to \bar{x}$ for some $\bar{x} \in X$. Let x be any fixed in X. Then it follows from (2.1) that for n sufficiently large, $F(x_n, x) \subset Y \setminus (-intC(x_n))$, and hence, by (A_2), $F(x, x_n) \subset Y \setminus intC(x)$. Since $F(x, \cdot)$ is upper $C(x)$-lower semicontinuous, $F(x, \bar{x}) \subset Y \setminus intC(x)$. We argue exactly as in proof of Proposition 2.1 to obtain that $F(\bar{x}, x) \subset Y \setminus (-intC(\bar{x}))$ for any $x \in X$. Hence $\bar{x} \in E_w$. Thus E_w is nonempty. By Proposition 2.1, $(E_w)^\infty \subset R_0$. So, by assumption, $(E_w)^\infty = \{0\}$ and hence E_w is bounded. Now we will prove that E_w is closed. Let $\{z_n\}$ be a sequence in E_w converging to some $z \in X$. Then for all $n \in \mathbb{N}$ and all $y \in X$, $F(z_n, y) \subset Y \setminus (-intC(x_n))$. Thus, by (A_2), for all $n \in \mathbb{N}$ and all $y \in X$, $F(y, z_n) \subset Y \setminus intC(y)$. Since F is upper $C(y)$-lower semicontinuous, $F(y, z) \subset Y \setminus intC(y)$ for all $y \in X$. By same argument as in proof of Proposition 2.1, $F(z, y) \subset Y \setminus (-intC(z))$. Thus $z \in E_w$ and hence E_w is closed. Consequently, E_w is nonempty and compact.

Conversely, assume that E_w is nonempty and compact, and that there exists $x^* \in X$ such that $F(y, x^*) \subset -C(y)$ for all $y \in X$. Let $v \in R_0$. Then for all

$y \in X$ and all $\lambda > 0$, $F(y, x^* + \lambda v) \subset Y \setminus intC(y)$. By same argument as in proof of Proposition 2.1, we have, for all $y \in X$ and all $\lambda > 0$,

$$F(x^* + \lambda v, y) \subset Y \setminus (-intC(x^* + \lambda v)).$$

Thus for all $\lambda > 0$, $x^* + \lambda v \in E_w$. Since E_w is bounded, $v = 0$. Hence $R_0 = \{0\}$. ∎

3. Random Vector Equilibrium Problem with Multifunctions

In this section, we will extend the first part of Theorem 2.2 to a random vector equilibrium problem with multifunctions. Our approach follows ideas of [9, 10] in which random vector equilibrium problems with single-valued functions were considered.

Let $(\Omega, \mathcal{A})$ be a measurable space where $\mathcal{A}$ is a σ-algebra of subsets of Ω. Let E be a topological space and let $\mathcal{B}(\mathcal{E})$ be the σ-algebra of all Borel sets of E. Let $\mathcal{A} \otimes \mathcal{B}(\mathcal{E})$ be σ-algebra generated by all subsets of the form of $A \times B$, where $A \in \mathcal{A}$ and $B \in \mathcal{B}(\mathcal{E})$.

Definition 3.1 [5]. *Let $(\Omega, \mathcal{A})$ be a measurable space and Y is a complete separable metric space. Consider a multifunction $F : \Omega \to 2^Y$.*

(1) *If there exists a measurable function $f : \Omega \to Y$ such that $f(\omega) \in F(\omega)$ for all $\omega \in \Omega$, then F is said to have a measurable selection f.*

(2) *If there is a countable family of measurable selections (f_i) such that $(f_i(\omega))$ is dense in $F(\omega)$, i.e., $F(\omega) = \overline{\cup_{i \geq 1} f_i(\omega)}$, for each $\omega \in \Omega$, then F is said to have a Castaing representation.*

Lemma 3.1 [5]. *Assume that $(\Omega, \mathcal{A})$ is a complete measurable space and Y is a complete separable metric space. If $F : \Omega \to 2^Y$ is a multifunction such that $GrF \in \mathcal{A} \otimes \mathcal{B}(\mathcal{Y})$, where GrF is the graph of F, then F has a Castaing representation.*

Let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measurable space and X a convex and closed subset of $\mathbb{R}^n$. Let $Y = \mathbb{R}^m$. Let $C : \Omega \times X \to 2^Y$ be a multifunction such that for any $(\omega, x) \in \Omega \times X$, $C(\omega, x)$ is a convex cone in Y with $intC(\omega, x) \neq \emptyset$ and $C(\omega, x) \neq Y$. Let $F : \Omega \times X \times X \to 2^Y$ be a multifunction.

Now we consider the following random vector equilibrium problem (RVEP):

(RVEP) Find a function $\gamma : \Omega \to X$ such that

$$F(\omega, \gamma(\omega), y) \subset Y \setminus -intC(\omega, \gamma(\omega)) \text{ for any } (\omega, y) \in \Omega \times X.$$

As in [10], for each $\omega \in \Omega$, $\gamma(\omega)$ is called a deterministic solution of (RVEP) and the function γ is said to be a random solution of (RVEP) when it is measurable.

We obtain the random version of the first part of Theorem 2.2 as follows:

Theorem 3.1. *Let $(\Omega, \mathcal{A}, \mu)$ be a complete σ-finite measurable space and X a nonempty convex and closed subset of $\mathbb{R}^n$. Let $Y = \mathbb{R}^m$. Let $C : \Omega \times X \to 2^Y$ be a multifunction such that for any $(\omega, x) \in \Omega \times X$, $C(\omega, x)$ is a convex cone in Y with $intC(\omega, x) \neq \emptyset$ and $C(\omega, x) \neq Y$. Suppose that the multifunction $F : \Omega \times X \times X \to 2^Y$ satisfies the following conditions:*

(RF_1) for any $y \in X$,
$\{(\omega, x) \in \Omega \times X : F(\omega, x, y) \subset Y \setminus (-intC(\omega, x))\} \in \mathcal{A} \otimes \mathcal{B}(\mathcal{X})$.

(RF_2) for all $\omega \in \Omega, x \in X$,
$F(\omega, x, x) \subset [C(\omega, x) \cap (-C(\omega, x))]$.

(RF_3) for all $\omega \in \Omega, x, y \in X$,
$F(\omega, x, y) \subset Y \setminus (-intC(\omega, x))$ implies $F(\omega, y, x) \subset Y \setminus intC(\omega, y)$.

(RF_4) for all $\omega \in \Omega, x \in X,\ y \mapsto F(\omega, x, y)$ is
$C(\omega, x)$-convex and $C(\omega, x)$-lower semicontinuous on X.

(RF_5) for all $\omega \in \Omega, x, y \in X,$ the set
$\{\xi \in [x, y] : F(\omega, \xi, y) \subset Y \setminus (-intC(\omega, \xi))\}$ is closed.

(RF_6) for each $\omega \in \Omega$,
$R_0^\omega := \cap_{y \in X}\{v \in X^\infty : F(\omega, y, z + \lambda v) \subset Y \setminus intC(\omega, y)$
$\forall \lambda > 0,\ \forall z \in X$ with $F(\omega, y, z) \subset -C(y)\} = \{0\}$.

Then there exists a countable family of measurable functions $\{\gamma_i\}_{i \geq 1} : \Omega \to X$ such that

(1) $F(\omega, \gamma_i(\omega), y) \subset Y \setminus (-intC(\omega, \gamma_i(\omega)))$ for any $(\omega, y) \in \Omega \times X$;
(2) $\overline{\cup_{i \geq 1}\gamma_i(\omega)} = \{x \in X : F(\omega, x, y) \subset Y \setminus (-intC(\omega, x))$ for all $y \in X\}$ for any $\omega \in \Omega$; and
(3) $\overline{\cup_{i \geq 1}\gamma_i(\omega)}$ is compact for any $\omega \in \Omega$.

Proof. By Theorem 2.2, for each $\omega \in \Omega$, there exists $x_\omega \in X$ such that

$$F(\omega, x_\omega, y) \subset Y \setminus (-intC(\omega, x_\omega)) \text{ for any } y \in X.$$

Since X is separable, there exists a sequence $\{y_n\}$ in X such that

$$\overline{\{y_1, y_2, \cdots\}} = X.$$

Define a multifunction $S : \Omega \to X$ by for any $\omega \in \Omega$,

$$S(\omega) = \cap_{y \in X}\{x \in X : F(\omega, x, y) \subset Y \setminus (-intC(\omega, x))\}.$$

Then it follows from Theorem 2.2 that for each $\omega \in \Omega$, $S(\omega)$ is nonempty and compact. Now we will prove that

$$\bigcap_{n=1}^{\infty} \{x \in X : F(\omega, x, y_n) \subset Y \setminus (-intC(\omega, x))\} \subset S(\omega).$$

Indeed, suppose to the contrary that

$$x \in \bigcap_{n=1}^{\infty} \{x \in X : F(\omega, x, y_n) \subset Y \setminus (-intC(\omega, x))\} \tag{3.1}$$

but $x \notin S(\omega)$. Since $x \notin S(\omega)$, there exists $y \in X$ such that

$$F(\omega, x, y) \cap (-intC(\omega, x)) \neq \emptyset.$$

Moreover, there exists a subsequence $\{y_{n_k}\}$ of $\{y_n\}$ such that $y_{n_k} \to y$. Since $F(\omega, x, \cdot)$ is lower $C(\omega, x)$-lower semicontinuous, $F(\omega, x, y_{n_k}) \cap (-intC(\omega, x)) \neq \emptyset$ for k sufficiently large. This contradicts (3.1). Thus $GrS = \bigcap_{n=1}^{\infty} \{(\omega, x) \in \Omega \times X : F(\omega, x, y_n) \subset Y \setminus (-intC(\omega, x))\}$. By assumption (RF$_1$), $GrS \in \mathcal{A} \otimes \mathcal{B}(\mathcal{X})$, GrS is the graph of the multifunction S. By Lemma 3.1, the multifunction S has a Castaing representation, i.e., there exists a countable family of measurable selections $\{\gamma_i\}_{i \geq 1}$ of S such that for any $\omega \in \Omega$, $S(\omega) = \overline{\cup_{i \geq 1} \gamma_i(\omega)}$. Hence the conclusions of Theorem 3.1 hold. ■

Additional Note:

During the revision of this paper, the authors became aware that Ansari and Flores-Bazán [1] also considered the problem (VEP) in Banach space setting where $C(x)$ is a constant cone. They obtained several necessary and/or sufficient conditions for the solution set to be nonempty and compact.

References

1. Q. H. Ansari and F. Flores-Bazán, Recession methods for generalized vector equilibrium problems, preprint.

2. Q. H. Ansari, I. V. Konnov and J. C. Yao, On generalized vector equilibrium problems, *Nonlinear Analysis* **47** (2001), 543-554.

3. Q. H. Ansari, W. Oettli and D. Schläger, A generalization of vectorial equilibrium, *Mathematical Methods of Operations Research* **46** (1997), 147-152.

4. Q. H. Ansari and J. C. Yao, An existence result for the generalized vector equilibrium problem, *Applied Mathematics Letters* **12** (1999), 53-56.

5. J. P. Aubin and H. Frankowska, *Set-Valued Analysis*, Birkhäser Boston, 1990.

6. A. Auslender and M. Teboulle, *Asymptotic Cones and Functions in Optimization and Variational Inequalities*, Springer-Verlag New York, Inc., 2003.

7. Fabián Flores-Bazán and Fernando Flores-Bazán, Vector equilibrium problems under asymptotic analysis, *Journal of Global Optimization* **26** (2003), 141-166.

8. P. G. Georgiev and T. Tanaka, Vector-valued set-valued variants of Ky Fan's inequality, *Journal of Nonlinear and Convex Analysis* **1** (2000), 245-254.

9. E. M. Kalmoun, Some deterministic and random vector equilibrium problems, *Journal of Mathematical Analysis and Applications* **267**(2002), 62-75.

10. E. M. Kalmoun, From deterministic to random vector equilibrium problems, *Journal of Nonlinear and Convex Analysis* **4** (2003), 77-85.

11. I. V. Konnov and J. C. Yao, Existence of solutions for generalized vector equilibrium problems, *Journal of Mathematical Analysis and Applications* **233** (1999), 328-335.

12. G. M. Lee and S. Kum, *Vector variational inequalities in a Hausdorff topological vector space* in "Vector Variational Inequalities and Vector Equilibria", edited by F. Giannessi, Kluwer Academic Publishers, 2000, pp. 307-320.

13. D. T. Luc, *Theory of Vector Optimization*, Lecture Notes in Economics and Mathematical Systems, 319, Springer-Verlag, Berlin, 1989.

14. W. Song, *Vector equilibrium problems with set-valued mappings* in "Vector Variational Inequalities and Vector Equilibria", Ed. F. Giannessi, Kluwer Academic Publishers, 2000, pp. 403-421.

15. N. X. Tan, On the existence of solutions to systems of vector quasi-optimization problems, *Mathematical Methods of Operations Research* **60** (2004), 53-71.

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