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COVER-INCOMPARABILITY GRAPHS AND 2-COLORED DIAGRAMS OF POSETS*

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Abstract. As a continuation of the study of cover-incomparability graphs of posets (C-I graphs), the notion of 2-colored diagrams is introduced and used in characterizations of posets whose C-I graphs belong to certain natural classes of graphs. As a particular instance, posets whose C-I graphs are chordal are characterized using a single 2-colored diagram. Some other instances are characterized in a similar way.

1. INTRODUCTION

Cover-incomparability graphs of posets, or shortly C-I graphs, were introduced in [4] in relation with transit functions, where these graphs appear precisely as the underlying graphs of the standard transit function on posets. On the other hand, C-I graphs can be defined as the edge union of the covering and incomparability graph of a poset. In the paper that followed [8], the authors proved that the complexity of recognizing whether a given graph is the C-I graph of some poset is in general NP-complete. Then the graphs that are C-I graphs among the class of block graphs and

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split graphs [5], as well as distance-hereditary graphs [9] and cographs [6], respectively, were characterized, yielding linear time recognition algorithms of these classes. (See also a recent paper on k-trees that are C-I graphs [7].)

As shown in [4], for the C-I graphs that belong to a class of graphs enjoying a forbidden induced subgraph characterization, there exists a characterization in terms of a list of forbidden isometric subposets. As special instances of this meta theorem, the posets whose C-I graphs are chordal, distance-hereditary, and Ptolemaic, respectively, were characterized using forbidden isometric subposets [4]. Note that the list of forbidden isometric subposets of a poset P may be very large compared to the list of forbidden induced subgraphs of the corresponding C-I graph of P. In this paper, we continue the study of cover-incomparability graphs, by introducing the notion of a *2-colored diagram*, which enables a more compact presentation of the list of forbidden subposets for posets whose C-I graphs belong to some class of graphs.

The 2-colored diagram is a variant of the Hasse diagram, in which two types of edges appear – the so-called *normal* and *bold* edges. The normal edge in a 2-colored diagram presents the covering relation between its elements, while the bold edge can either present the covering relation or the incomparability relation between its elements. Hence, a 2-colored diagram with bold edges presents a family of posets. In this paper, we use 2-colored diagrams mainly for the presentation of forbidden subposets for a family of C-I graphs that belong to some special classes. In particular, the poset whose C-I graphs are chordal are characterized in terms of just one forbidden 2-colored diagram (see Section 3). (These posets were characterized in [4] in terms of three forbidden isometric subposets.) In Section 4 we demonstrate the use of this new concept by characterizing three more families of posets, each of which has the property that its C-I graphs belong to a class, defined by forbidding one graph as an induced subgraph. Each of the three graphs belongs to the list of nine forbidden induced subgraphs in the characterization of line graphs [2], thus the resulting 2-colored diagrams present a non-complete list of forbidden substructures for the characterization of posets with C-I line graphs. Two of the other six graphs from the list of forbidden induced subgraphs for line graphs cannot appear as subgraphs of C-I graphs. Nevertheless, for the rest of the graphs from the list, the analysis of forbidden 2-colored diagrams becomes very tedious, so the complete characterization of the posets with C-I line graphs is left for an eventual future study. In the following section we present all the basic concepts, used in the paper, and give the formal definition of 2-colored diagrams and subposets.

2. Basic and Main Concepts

Let $P = (V, \leq)$ be a poset. If $u \leq v$ but $u \neq v$, then we write $u < v$. If u and v are in V, then v *covers* u in P if $u < v$ and there is no w in V with $u < w < v$. If $u \leq v$ we will sometimes say that u is *below* v, and that v is *above* u. Also, we will write $u \lhd v$ if v covers u; and $u \lhd \lhd v$ if u is below v but not covered by v. By

$u \parallel v$ we denote that u and v are incomparable. Let V' be a nonempty subset of V. Then there is a natural poset $Q = (V', \leq')$, where $u \leq' v$ if and only if $u \leq v$ for any $u, v \in V'$. The poset Q is called a *subposet* of P and its notation is simplified to $Q = (V', \leq)$. If, in addition, together with any two comparable elements u and v of Q, a chain of shortest length between u and v of P is also in Q, we say that Q is an *isometric subposet*. The *C-I graph* of P is denoted by G_P and defined as the graph with vertex set V and uv is an edge of G_P if $u \lhd v$ or $v \lhd u$ or $u \parallel v$. Finally recall that a poset P is *dual* to a poset Q if for any $x, y \in P$ the following holds: $x \leq y$ in P if and only if $y \leq x$ in Q.

Let us list a few preliminary observations about C-I graphs of posets (most of the proofs are trivial, for others cf. [4]).

Lemma 2.1. *If P is a poset, then*

(i) *the C-I graph of P is connected;*

(ii) *points of P, which are independent (pairwise non-adjacent) in the C-I graph of P lie on a common chain;*

(iii) *an antichain of P corresponds to a complete subgraph in the C-I graph of P;*

(iv) *the C-I graph of P contains no induced cycles of length greater than 4;*

(v) *if Q is the dual poset of P, then G_P isomorphic to G_Q.*

By abuse of language, and because of the bijective correspondence between the points of P and vertices of G_P, we will sometimes call the elements of P also vertices.

Note that if two points u, v in a poset P are not comparable, then in G_P they are adjacent. In addition, u and v are adjacent also in the case when $u \lhd v$ or $v \lhd u$. Can we somehow join these two cases? Let P be a poset, and let u be a minimal element and v be a maximal element of P, and let u and v be incomparable. Then in the relation $\leq$ of poset P we add the pair $u \leq v$, and denote the resulting poset by P'. Indeed, P' is well defined (i.e. we get a reflexive, anti-symmetric and transitive relation), and clearly $u \lhd v$ in P'. Hence the covering graph of P' is isomorphic to the graph obtained from the covering graph of P by adding edge uv. However the resulting cover-incomparability graphs are isomorphic, i.e. $G_P = G_{P'}$.

We can generalize the above idea as follows. Let P be a poset and k, m integers. For any $i \in \{1, \ldots, k\}$, let u_i be a minimal element of P, and let v_j for any $j \in \{1, \ldots, m\}$ be a maximal element of P. Set $I_n = \{1, \ldots, n\}$. Let I be the set of all ordered pairs $(i, j) \in I_k \times I_m$ such that u_i and v_j are incomparable. Let A be an arbitrary subset (possibly empty) of I. Then, for all $(i, j) \in A$ we add $u_i \leq v_j$ to the relation of P, and obtain relation that is clearly again a partial order which we denote by $P(A)$. The covering graphs of P and $P(A)$ differ only by edges $u_i v_j$, where $(i, j) \in A$. Yet, for any $A \subseteq I$ the corresponding poset $P(A)$ has the same cover-incomparability graph as P, i.e. $G_P = G_{P(A)}$. Now, by *2-colored diagram $\mathcal{P}$*

we denote the family of $2^{|I|}$ posets obtained by the above construction. We describe the family $\mathcal{P}$ by the Hasse diagram of initial poset P using normal edges, added by bold edges between u_i and v_j for all $(i, j) \in I$. Any subset of the set of bold edges can thus be chosen and removed arbitrarily to obtain one of the Hasse diagrams of a poset from the family $\mathcal{P}$. Hence one drawing, using normal and bold edges, suffices to describe all posets of $\mathcal{P}$. Note that the notion of *dual 2-colored diagram* makes sense (as the family of duals of the posets from the corresponding family), and can also be described by such a drawing.

In a similar way, but with an additional condition, we define 2-colored diagrams as subposets, the concept which become interesting when we wish to forbid a family of subposets. Namely, we say that a poset Q *does not contain a 2-colored diagram* $\mathcal{P}$ (with respect to the set of bold edges I) if there exists no subposet $P(A)$ in Q, for $A \subset I$, such that for any bold edge xy in $P(A)$, we have $x \lhd_Q y$ (where x is minimal and y maximal element). The latter condition is quite strong, stronger than the usual subposet concept. Notably, a bold edge means that either the two elements are incomparable (when the corresponding pair of indices is not in A), or they are in a covering relation already in Q (not just in $P(A)$). In other words, if Q does not contain a 2-colored diagram $\mathcal{P}$ (with respect to the set of bold edges I), and if Q contains a subposet $P(A)$ for some $A \subset I$, then it must be the case that there is a bold edge xy in $P(A)$ such that $x <_Q y$ but x is not covered by y in Q (i.e., there exists an element z such that $x < z < y$).

3. C-I Graphs of Posets and Chordal Graphs

Recall that a graph is chordal, if it contains no induced cycles of length greater than 3. Since C-I graphs do not contain induced cycles of length greater than 4 (by Lemma 2.1 (iv)), the following result describes all forbidden isometric subposets for a poset P whose C-I graph is chordal.

Lemma 3.1. ([4]). *If P is a poset, then G_P contains an induced C_4 if and only if P contains one of the posets R_1, R_2 and R_3 from Fig. 1 as an isometric subposet.*

The notion of isometric subposet that appears in the above lemma was invented because the notion of the usual subposet was not restrictive enough to prove the mentioned characterizations. Now, we present a characterization of posets with chordal C-I graphs by forbidding only one 2-colored diagram $\mathcal{P}_c$, see Fig. 2.

Theorem 3.2. *If P is a poset, then G_P is chordal if and only if P does not contain a 2-colored diagram $\mathcal{P}_c$.*

Proof. Let P be a poset such that G_P is not a chordal graph. By Lemma 2.1*(iv)* we infer that G_P contains an induced C_4. Denote its vertices by u, x, v, y, as shown on the right-hand side of Figure 2. Set $S = \{u, x, v, y\}$.

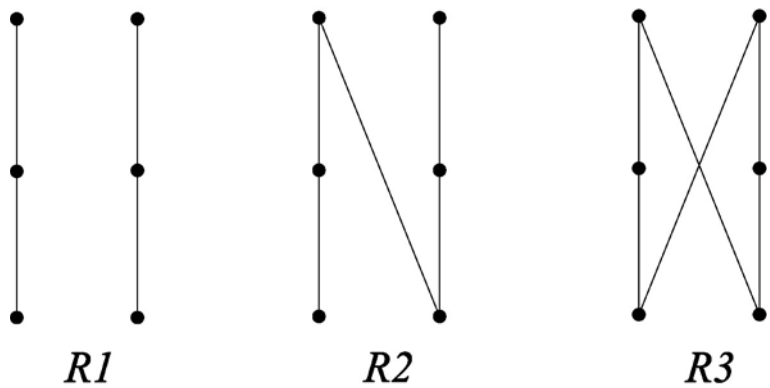

Fig. 1. Forbidden isometric subposets for C_4.

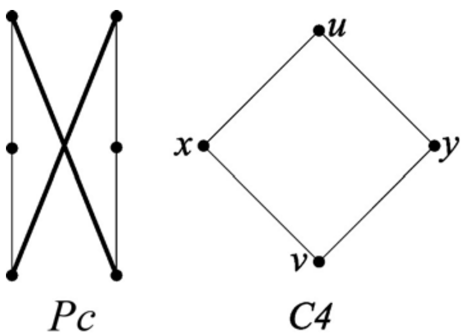

Fig. 2. Forbidden 2-colored diagram for C_4.

Since u and v are not adjacent in G_P, they must lie on the same chain in P and there exists an element w that lies between u and v. Without loss of generality we may assume that $u < w < v$, and let us denote by S_1 a chain $u \lhd \ldots \lhd w \ldots \lhd v$. In the same way we infer that x and y lie on the same chain, and there is an element z between them. Again we may assume that $x < z < y$, and denote by S_2 a chain $x \lhd \ldots \lhd z \ldots \lhd y$. Clearly u, x, v, y do not lie on the same chain in P (because the lowest point on a chain is in G_P adjacent to only one point on that chain).

Now we claim that S_1 and S_2 have no common elements. Suppose first that one of the four points, say u, is below the other three points. Then, as u is adjacent to x and y in G_P, we infer that u would be covered in P by both x and y. Hence x and y would be incomparable in P, and so adjacent in G_P, which is a contradiction. By duality we derive that no point is above all other points from S, and so none of the elements from S can be in both chains S_1 and S_2. If there is an element $t \notin S$ that lies on S_1 and S_2 then we get $u \lhd \lhd y$ (and $x \lhd \lhd v$) which is again not possible, because u and y are adjacent (and also x and v are adjacent) in G_P. Thus the chains S_1 and

S_2 have no common elements.

Consider the subposet of P induced by the elements from $S' = S \cup \{w, z\}$. By the same argument as in the previous paragraph, w is incomparable with x and y, and z is incomparable with u and v. It is possible, but not necessary, that $x < v$, resp. $u < y$, but only when $x \lhd v$ and $u \lhd y$. Altogether this implies that $(S', \leq)$ is a subposet in P that belongs to the 2-colored diagram $\mathcal{P}_c$.

For the converse note that the existence of a 2-colored diagram $\mathcal{P}_c$ in a poset P yields an induced C_4 in G_P, induced by the endpoints of the bold edges. Hence G_P is not chordal and this direction is also proved. ■

4. 2-Colored Diagrams of Some Subposets

In this section we consider three graph classes denoted $\mathcal{F}(G)$, where G is a fixed forbidden induced subgraph for the class $\mathcal{F}(G)$. The forbidden induced subgraphs that we consider are the following: the so-called true-twin graph that we denote by G_1, the graph $K_5 - e$, and the claw $K_{1,3}$, depicted in Figures 3, 5 and 7, respectively. Thus the three graph classes are the G_1-free graphs, ($K_5 - e$)-free graphs and claw-free graphs, respectively. Each of these graph classes is characterized by using forbidden 2-colored diagrams. We conclude the paper by characterizing the posets whose C-I graphs are Ptolemaic, which is obtained by deriving forbidden 2-colored diagrams from the forbidden isometric subposets characterization of Ptolemaic graphs from [4].

Theorem 4.1. *If P is a poset, then G_P belongs to $\mathcal{F}(G_1)$ if and only if P does not contain 2-colored diagrams P_1, P_2 and P_3 from Figure 4 and their duals.*

Proof. Suppose P contains one of the 2-colored diagrams $P_i, i = 1, 2, 3$. Then clearly G_P contains the graph from Figure 3 as an induced subgraph.

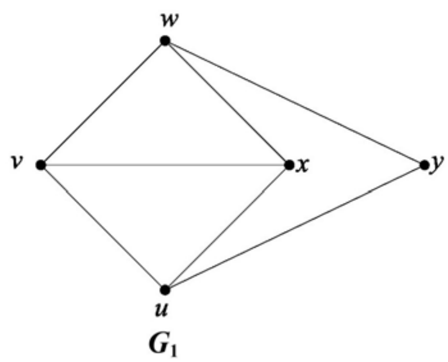

Fig. 3. The forbidden induced subgraph for family $\mathcal{F}(G_1)$.

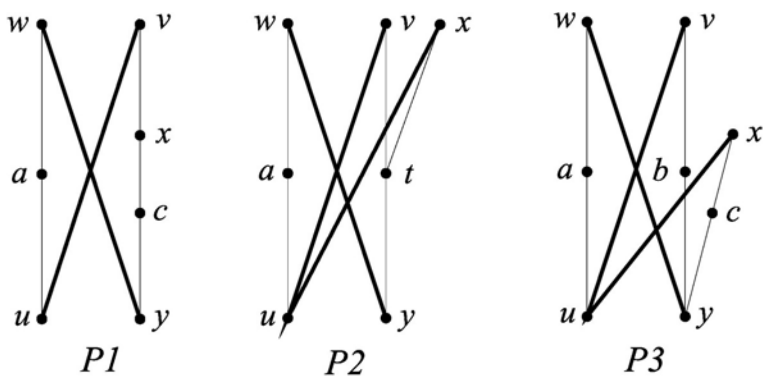

Fig. 4. Forbidden 2-colored diagrams for posets whose C-I graphs do not contain G_1, depicted in Figure 3, as an induced subgraph.

Conversely, suppose $G_P \notin \mathcal{F}(G_1)$. Then G_P contains an induced subgraph G_1 shown in Figure 3, with vertices labeled by u, v, w, x and y. Let $X = \{u, v, w, x, y\}$. Graph G_P has (at least) two induced 4-cycles. First consider the induced 4-cycle with vertices u, y, w and v. From Lemma 3.1 and from the analogous arguments, as used in the proof of Theorem 3.2, it follows that there exist two disjoint chains $S_1 = u \lhd \ldots \lhd a \lhd \ldots \lhd w$ and $S_2 = y \lhd \ldots \lhd b \lhd \ldots \lhd v$ in P, where a and b are two arbitrary elements from those chains. (The case when $w \lhd\lhd u$ or $v \lhd\lhd y$, can be dealt in a similar way, and as a result yield duals of 2-colored diagrams from Figure 4.)

The same arguments used on the induced 4-cycle with vertices u, y, w and x lead to two disjoint chains S_1 and $S_3 = y \lhd \ldots \lhd c \lhd \ldots \lhd x$, where c is an arbitrary element between y and x in P. Note that in S_3, $x \lhd\lhd y$ is not possible, since in this case $x \lhd\lhd y \lhd\lhd v$, which implies that x and v are not adjacent in G_P, a contradiction.

Since x and v are adjacent in G_P we have three possibilities.

Case (i). $x \lhd v$ in P. In this case consider the subposet P' of P induced by the elements of $S = X \cup \{a, c\}$. In this subposet $u \lhd a \lhd w$ and $y \lhd c \lhd x \lhd v$. Since G_1 is an induced subgraph of G_P beside the mentioned covering relations in P' the only other possible covering relations are $u \lhd v$ and $y \lhd w$. Thus P' is a subposet of P that belongs to the 2-colored diagram P_1.

Case (ii). $v \lhd x$ in P. In this case consider the subposet P' of P induced by the elements of $S = X \cup \{a, b\}$. In this subposet $u \lhd a \lhd w$ and $y \lhd b \lhd v \lhd x$. Since G_1 is an induced subgraph of G_P the only other possible covering relations in P' (beside already mentioned ones) are $u \lhd x$ and $y \lhd w$. Thus P' is a subposet of P that belongs to the 2-colored diagram P_1.

Case (iii). $x \parallel v$ in P. In this case we distinguish two subcases with respect to the number of elements in $S_2 \cap S_3$.

Subcase (a). There exists $t \in S_2 \cap S_3$ different from y. In this case consider the subposet P' of P induced by the elements of $S = X \cup \{a, t\}$. In this subposet $u \lhd a \lhd w$, $y \lhd t \lhd v$ and $t \lhd x$. Since G_1 is an induced subgraph of G_P, the only other possible covering relations in P' (besides already mentioned ones) are $u \lhd v$, $u \lhd x$ and $y \lhd w$. Thus P' is a subposet of P that belongs to the 2-colored diagram P_2.

Subcase (b). $S_2 \cap S_3 = \{y\}$. Consider the subposet P' of P induced by the elements of $S = X \cup \{a, b, c\}$. In this subposet $u \lhd a \lhd w$, $y \lhd b \lhd v$ and $y \lhd c \lhd x$. Since G_1 is an induced subgraph of G_P the only other possible covering relations in P' (besides already mentioned ones) are $u \lhd v$, $u \lhd x$ and $y \lhd w$. Thus P' is a subposet of P that belongs to the 2-colored diagram P_3. ∎

Theorem 4.2. *If P is a poset, then G_P belongs to $\mathcal{F}(K_5 - e)$ if and only if P does not contain 2-colored diagrams P_4 and P_5 from Figure 6 and their duals.*

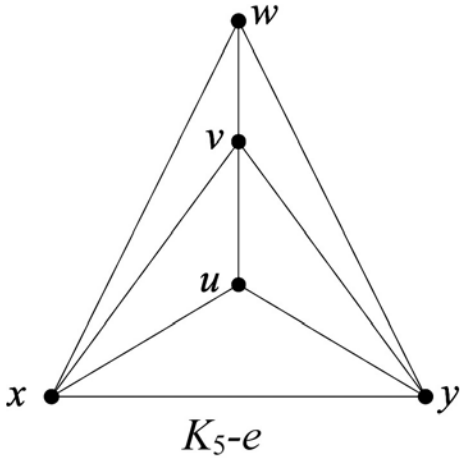

Fig. 5. The forbidden induced subgraph for family $\mathcal{F}(K_5 - e)$.

Proof. Suppose P contains one of the 2-colored diagrams P_4 and P_5. Then clearly G_P contains the graph from Figure 5 as an induced subgraph.

Suppose that G_P contains an induced subgraph shown in Figure 5, with vertices labeled as u, v, w, x and y. Note that x, y and v appear symmetrically, and so do vertices u and w.

Since u and w are not adjacent in G_P they lie on a common chain in P but are not in a covering relation. Without loss of generality we may assume that $u \lhd\!\lhd w$ in P. We have two possibilities:

Case (i). There exists vertex from $\{x, y, v\}$, say v (we can choose one, since this three vertices appear symmetrically), such that $u \lhd v \lhd w$ in P. Since x and y are adjacent to w and u in G_P, v is not in a covering relation with neither of $\{x, y\}$. Thus v is incomparable with x and y in P, as it is adjacent to both vertices in G_P. Note that

x and y are adjacent in G_P and hence they are either incomparable or in a covering relation in P. If they are incomparable then P_4 is an isometric subposet of P. In the other case, when x and y are in a covering relation in P, P contains P_5 as isometric subposet, since the only other possible covering relations in P are $u \lhd z_2$ and $z_1 \lhd w$, where $z_1, z_2 \in \{x, y\}$ and $z_1 \lhd z_2$.

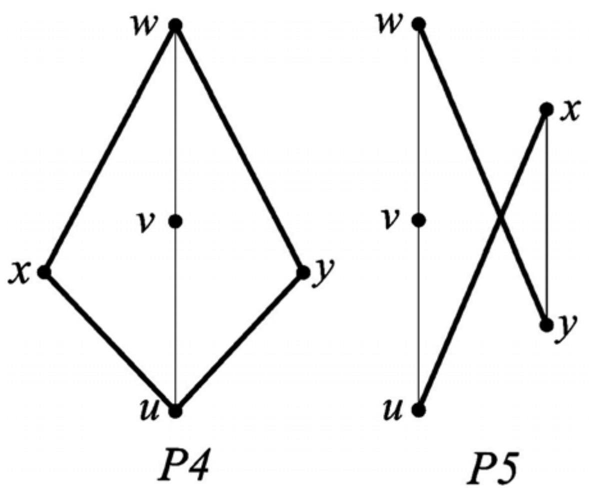

Fig. 6. Forbidden 2-colored diagrams for posets whose C-I graphs do not contain $K_5 - e$ as an induced subgraph.

Case (ii). There is no vertex $z \in \{x, y, v\}$ such that $u \lhd z \lhd w$ in P. Since $u \lhd \lhd w$ in P there exists $a \neq x, y, v$ in P such that $u < a < w$ in P. Let $u = s_0, s_1, \ldots, s_{n-1}, s_n = w$ be a chain from u to w in P of the shortest length. As x, y, v are all adjacent to both u and w and since the condition from Case (i) is not satisfied, it is clear that $s_i \notin \{x, y, v\}$ for every $i = 1, \ldots, n-1$. Since x, y and v induce a clique in G_P, these three vertices form a bipartite subposet (i.e. each maximal chain in this subposet is of length 1) in P. Thus at least two of these three vertices, say x and y, are incomparable in P. Since x and y are adjacent to w and u in G_P, x and y are incomparable with s_i, $i = 1, \ldots, n-1$. Thus the subposet of P induced by the elements $\{u, s_1, s_2, x, y\}$ belongs to the 2-colored diagram P_4. ∎

In the definition of 2-colored diagram, the condition on the existence of a bold edge xy was that x is a minimal and y is a maximal element of the poset. Note that without this condition, the ambiguity could arise, because the elements below x (or the elements above y, respectively), might or might not be in relation with y (x, respectively), depending on whether the bold edge xy is included or not. However, no ambiguity can arise, if we have a minimal element x, a maximal element y, and an element z which is in a 2-colored diagram connected only with a bold edge to y

and to x. This is the case with the 2-colored diagram in Figure 8, which appears in a characterization of the posets whose C-I graphs are claw-free. The following result is a straighforward exercise and is left to the reader.

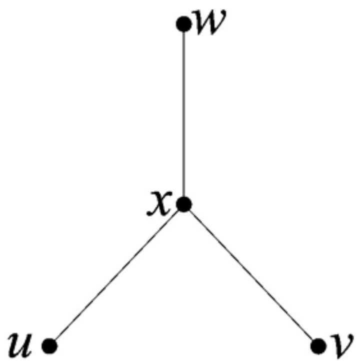

Fig. 7. The forbidden induced subgraph (claw) for family $\mathcal{F}(K_{1,3})$

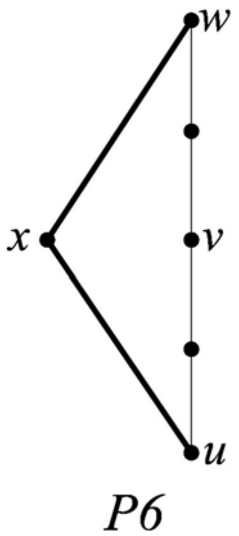

Fig. 8. Forbidden 2-colored diagram for posets whose C-I graphs are claw-free.

Theorem 4.3. *If P is a poset, then G_P contains an induced claw if and only if P contains a 2-colored diagram P_6 from Figure 8.*

We conclude the paper with the forbidden 2-colored diagrams characterization of posets whose C-I graphs are Ptolemaic, which uses a result from [4]. Recall that Ptolemaic graphs are the graphs, which are both chordal and distance-hereditary. A distance-hereditary graph is a graph in which every non-shortest path has at least one *chord* (i.e. an edge connecting two non-consecutive vertices on a path). In [1], distance-hereditary graphs G were characterized as the graphs, which have no house, long cycle (cycle of length at least five), domino and 3-fan as an induced subgraph. As a straightforward consequence of the following result

Theorem 4.4. ([4, Corollary 4.2]). *If P is a poset, then G_P is Ptolemaic if and only if P is does not contain any R_1, R_2, R_3 (from Figure 1) and any Q_1, Q_2, Q_3, Q_4 and Q_5 (from Figure 9) as an isometric subposet.*

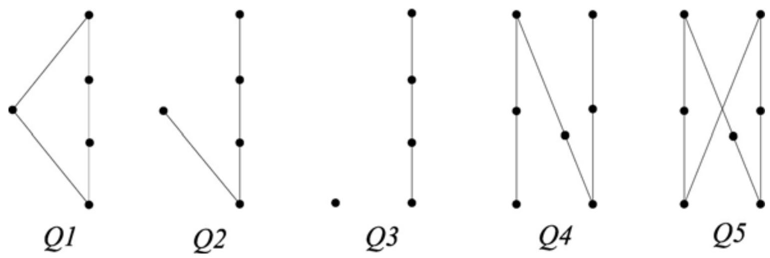

Fig. 9. Forbidden isometric subposets for 3-fan.

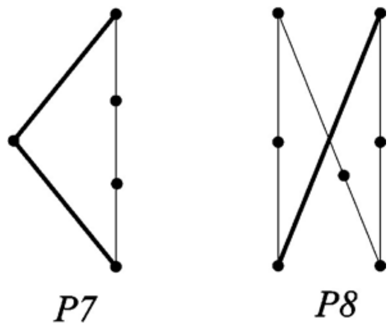

Fig. 10. Forbidden 2-colored diagram for 3-fan.

we derive the following characterization, using 2-colored diagrams.

Theorem 4.5. *If P is a poset, then G_P is Ptolemaic if and only if P has no 2-colored diagram P_c (from Figure 2) and no P_7 and P_8 (from Figure 10).*

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