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SOME PROPERTIES OF SEMI-G-PREINVEX FUNCTIONS

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Abstract. In this paper, a new class of functions called semi-G-preinvex functions, which is generalization of semipreinvex functions and G-preinvex functions, is introduced. Some examples are given to show that there exist functions which are semi-G-preinvex functions, but are not G-invex functions, semipreinvex functions and G-preinvex functions. Furthermore, some interesting properties and characterizations of semi-G-preinvexity are established. Our results extend and improve the corresponding ones in the literature.

1. Introduction

It is well known that the research on convexity and generalized convexity is one of the most important aspects in mathematical programming and optimization theory and application (see [1, 2, 3, 4]). In fact, there are a number of nonlinear programming problems whose objective and constraints functions are nonconvex. Therefore, in the recent years attempts are made by several authors to define various nonconvex classes of functions and to study their optimality criteria in solving such types of problems ([5-17] and the references therein).

Over the years, many generalizations of this concept have been given in the literature. Avriel [10] introduced the definition of r-convex functions, which is a generalization of convex functions. Later, Ben-Israel and Mond [11] considered a class of nondifferentiable functions that were called preinvex in [12] as a generalization of convexity. Antczak [13] considered a class of r-preinvex functions, which is a generalization of r-convex functions, and obtained some optimality results for constrained optimization problems. Yang and Chen [14] present a wider class of generalized convex functions, called semipreinvex functions, which includes the classes of preinvex

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functions and arc-connected convex functions. On the other hand, Avriel et al. [4] introduced the definition of G-convex functions, which is another generalization of convex functions, and obtained some characterizations of G-convexity (where G is a continuous real-valued increasing function). As a generalization of G-convexity and invexity, Antczak [16] introduced the concept of G-invexity and obtained some optimality conditions for constrained optimization problems under assumptions of G-invexity. Antczak [17] obtained a class of noncovex functions–G-preinvex functions, which is a generalization of G-invex functions [16], preinvex functions [5, 12] and r-preinvex functions[13] ($r > 0$) with respect to the same η. By using the G-preinvexity, some optimality results for constrained optimization problems were derived.

Motivated and inspired by works of references above and the references therein, in this paper, we propose a new class of noncovex functions-semi–G-preinvex functions and discuss some important characterizations. The concept of semi-G-preinvexity unifies the concepts of semipreinvexity, r-semipreinvexity(with $r > 0$) and G-preinvexity. Some examples are given for the illustration of our results. Our paper extends and generalizes the corresponding ones in the literature ([6, 14, 15, 16, 17, 18, 19, 24]).

2. Preliminaries

Throughout this paper, if not otherwise specified, let X be a nonempty subset of R^n. Let $f : X \to R$ be a real-valued function and $\eta : X \times X \to R^n$ be a vector-valued function. And let $I_f(X)$ be the range of f, i.e., the image of X under f.

Now we recall some definitions.

Definition 2.1. A set X is said to be invex if there exists a vector-valued function $\eta : X \times X \to R^n$ such that for each $x, y \in X$ such that $\eta(x, y) \neq \mathbf{0}$ if $x \neq y$, and

$$y + \lambda\eta(x, y) \in X,\ \forall\lambda \in [0, 1]$$

Definition 2.2. Let $X \subseteq R^n$ be an invex set with respect to $\eta : X \times X \to R^n$ and let $f : X \to R$ be a mapping. We say that f is preinvex iff

$$f(y + \lambda\eta(x, y)) \leq \lambda f(x) + (1 - \lambda)f(y), \forall x, y \in X, \lambda \in [0, 1].$$

Definition 2.3. Let X be a nonempty invex (with respect to η) subset of R^n, $f : X \to R$ is said to be (strictly) G-preinvex at y with respect to η iff there exists a continuous real-valued increasing function $G : I_f(X) \to R$ and a vector-valued function $\eta : X \times X \to R^n$ such that, for all $x \in X (x \neq y)$ and any $\lambda \in [0, 1] (\lambda \in (0, 1))$,

$$f(y + \lambda\eta(x, y)) \leq G^{-1}(\lambda G(f(x)) + (1 - \lambda)G(f(y)))\ (<).$$

If the inequality above is satisfied for any $y \in X$ then f is (strictly) G-preinvex on X with respect to η.

Example 2.1. This example illustrates that a G-preinvex function is not necessarily a preinvex function with respect to the same η.

Let $X = [-1, 1]$, and let

$$f(x) = \ln(3 - 2|x|),$$

$$\eta(x, y) = \begin{cases} -y + 2x - x^2, & \text{if } 0 \le x \le 1, 0 \le y \le 1, \\ x - y, & \text{if } -1 \le x < 0, -1 \le y < 0, \\ -y - x^2 - 2x, & \text{if } -1 \le x < 0, 0 \le y \le 1, \\ -x - y, & \text{if } 0 \le x \le 1, -1 \le y < 0. \end{cases}$$

Then, by Definition 2.3, f is G-preinvex function on X with respect to η, where $G(t) = e^t$. But let $x = 1, y = 0, \lambda = \frac{1}{2}$, we have

$$f(y + \lambda\eta(x, y)) = f(\frac{1}{2}) = \ln 2 > \lambda f(x) + (1 - \lambda)f(y) = \frac{1}{2}\ln 3.$$

Thus, f is not preinvex function on X with respect to the same η. And we also can show f is not 1-convex (see r-convex in [10]) function on X, for

$$f(y + \lambda(x - y)) > \ln(\lambda e^{f(x)} + (1 - \lambda)e^{f(y)})$$

holds when $x = \frac{1}{2}, y = -\frac{1}{2}, \lambda = \frac{1}{2}$.

Definition 2.4. [20]. $X \subseteq R^n$ is said to be semi-connected set if there exists $\eta : X \times X \times [0, 1] \to R^n$ such that, for any $x, y \in X, \lambda \in [0, 1],\ y + \lambda\eta(x, y, \lambda) \in X$.

Definition 2.5. [14]. Let $X \subseteq R^n$ be semi-connected set with respect to $\eta : X \times X \times [0, 1] \to R^n$, $f : X \to R$ is said to be semipreinvex with respect to η iff for any $x, y \in X, \lambda \in [0, 1], \lim_{\lambda\to 0} \lambda\eta(x, y, \lambda) = 0$ and $f(y + \lambda\eta(x, y, \eta)) \le \lambda f(x) + (1 - \lambda)f(y)$.

Definition 2.6. Let $X \subseteq R^n$ be semi-connected set with respect to $\eta : X \times X \times [0, 1] \to R^n$, $f : X \to R$ is said to be (strictly) semi-G-preinvex at y with respect to η iff there exists a continuous real-valued increasing function $G : I_f(X) \to R$ and a vector-valued function $\eta : X \times X \times [0, 1] \to R^n$ such that, for all $x \in X(x \ne y), \lambda \in [0, 1](\lambda \in (0, 1)), \lim_{\lambda\to 0} \lambda\eta(x, y, \lambda) = 0$ and

$$f(y + \lambda\eta(x, y, \lambda)) \le G^{-1}(\lambda G(f(x)) + (1 - \lambda)G(f(y)))\ (<).$$

If the inequality above is satisfied for any $y \in X$ then f is (strictly) semi-G-preinvex on X with respect to η.

Remark 2.1. Every r-semipreinvex function [15] ($r > 0$) with respect to η is semi-G-preinvex function with respect to the same function η, where $G(t) = e^{rt}$.

Remark 2.2. Every G-preinvex function [17] with respect to η is semi-G-preinvex function with respect to the same function G, where $\eta(x, y, \lambda) = \eta(x, y)$. However, the converse is not true.

Example 2.2. This example illustrates that there exist functions, which are semi-G-preinvex functions, but are not G-preinvex functions with respect to the same function G.

Let $X = (-\frac{\pi}{2}, \frac{\pi}{2})$, it is easy to check that X is a semi-connected set with respect to $\eta(x, y, \lambda)$ and $\lim_{\lambda\to 0} \lambda\eta(x, y, \lambda) = 0$, where

$$\eta(x,y,\lambda) = \begin{cases} 0, & |x| \geq |y|; \\ -x - \frac{y}{\sqrt{\lambda}}, & |x| < |y|, 0 < \lambda \leq 1. \end{cases}$$

Let $f : X \to R$ be defined by $f(x) = \tan|x|$.

From definition 2.6, we can verify that f is a semi-G-preinvex function with respect to η, where $G(t) = \arctan t$. However, f is not G-preinvex with respect to $\eta^*(x, y) = 0$ for the same G.

Remark 2.3. Every G-convex function [4] is semi-G-preinvex function with respect to $\eta(x, y, \lambda) = x - y$, and every G-invex function [16] is semi-G-preinvex function with respect to the same G. However, the converse is not true.

Example 2.3. This example illustrates that there exist some functions, which are semi-G-preinvex functions, but are neither G-convex functions nor G-invex functions with respect to the same function G.

Let $X = [-1, 1]$, it is easy to check that X is a semi-connected set with respect to $\eta(x, y, \lambda)$ and $\lim_{\lambda\to 0} \lambda\eta(x, y, \lambda) = 0$, where

$$\eta(x,y,\lambda) = \begin{cases} \dfrac{x-y}{\sqrt{\lambda}}, & 0 \leq x \leq 1, 0 \leq y \leq 1, x \geq y, 0 < \lambda \leq 1; \\ \dfrac{x-y}{\sqrt{\lambda}}, & -1 \leq x < 0, -1 \leq y < 0, x \leq y, 0 < \lambda \leq 1; \\ -y - x^2 + 2x, & 0 \leq x \leq 1, 0 \leq y \leq 1, x < y; \\ \lambda^2(x-y), & -1 \leq x < 0, -1 \leq y < 0, x > y; \\ -y - x^2 - 2x, & -1 \leq x < 0, 0 \leq y \leq 1, x \leq -y; \\ -x - y, & 0 \leq x \leq 1, -1 \leq y < 0, x \geq -y; \\ -x - y, & -1 \leq x < 0, 0 \leq y \leq 1, x > -y; \\ 0, & 0 \leq x \leq 1, -1 \leq y < 0, x < -y. \end{cases}$$

Let $f : X \to R$ be defined by $f(x) = \ln(3 - |x|)$. Then, we can verify that f is a semi-G-preinvex function with respect to η, where $G(t) = e^t$. However, by letting $x = 1, y = -\frac{1}{2}, \lambda = \frac{1}{2}$, we have

$$f(y+\lambda(x-y)) = \ln\frac{11}{4} > G^{-1}(\lambda G(f(x)) + (1-\lambda)G(f(y))) = \ln\frac{9}{4}.$$

Thus, f is not G-convex. Furthermore, we observe that $f(x)$ is not differentiable, so f is also not G-invex on X.

Remark 2.4. Every semipreinvex function [14] with respect to η is semi-G-preinvex function with respect to the same η, where $G(t) = t$. However, the converse is not true.

Example 2.4. This example illustrates that a semi-G-preinvex function is not necessarily a semipreinvex function with respect to the same η.

Let $X = R$, it is easy to check that X is a semi-connected set with respect to $\eta(x, y, \lambda)$ and $\lim_{\lambda\to 0} \lambda\eta(x, y, \lambda) = 0$, where

$$\eta(x,y,\lambda) = \begin{cases} \dfrac{x-y}{\sqrt{\lambda}}, & x<0, y<0, x>y, 0<\lambda\le 1; \\ \lambda(x-y), & x\ge 0, y\ge 0, x\ge y; \\ \lambda(x-y), & x<0, y<0, x\le y; \\ x-y, & x\ge 0, y\ge 0, x<y; \\ -x-y, & x\ge 0, y<0, x<-y; \\ -\frac{1}{2}x-y, & x<0, y\ge 0, x>-y; \\ -y, & x\ge 0, y<0, x\ge -y; \\ 0, & x<0, y\ge 0, x\le -y. \end{cases}$$

Let $f : X \to R$ be defined by $f(x) = \ln(|x| + 2)$. Then, we can verify that f is a semi-G-preinvex function with respect to η, where $G(t) = e^t$. However, by letting $x = 1, y = -2, \lambda = \frac{1}{2}$, we have

$$f(y+\lambda\eta(x,y,\lambda)) = \ln\frac{7}{2} > \lambda f(x) + (1-\lambda)f(y) = \ln\sqrt{12}.$$

Thus, f is not semipreinvex function with respect to the same η.

We give some lemmas, which be useful in the sequel.

Lemma 2.1. *G^{-1} is increasing if and only if G is increasing.*

Lemma 2.2. [18]. *Let I be an index set. $(S_i)_{i\in I}$ is a family of semi-connected subsets in R^{n+1} with respect to the same function $\eta : R^{n+1} \times R^{n+1} \times [0, 1] \to R^{n+1}$, then their intersection $\bigcap_{i\in I} S_i$ is a semi-connected set with respect to the same η.*

3. Some Properties of Semi-G-preinvexity

In this section, we discuss some interesting properties and characterizations of semi-G-preinvexity, which extend and generalize the corresponding ones in the literature.

Firstly, we introduce the definition of semi-G-invex set with respect to η, which will enable us to search geometric properties (characterizations) of semi-G-preinvexity.

Definition 3.1. Let T be a given subset of $R^n \times R$. Then T is said to be semi-G-invex set with respect to η iff there exists a vector-valued function $\eta : R^n \times R^n \times [0,1] \to R^n$ and a continuous real-valued increasing function $G : R \to R$ such that

$$(x_2 + \lambda\eta(x_1, x_2, \lambda), G^{-1}(\lambda G(y_1) + (1-\lambda)G(y_2))) \in T$$

holds for any $(x_1, y_1) \in T, (x_2, y_2) \in T$ and any $\lambda \in [0,1]$.

From the definition of semi-G-invex set and semi-G-preinvexity, we can characterize semi-G-preinvexity by using epigraph E_f as follows:

Theorem 3.1. *Let $X \subseteq R^n$ be a semi-connected set with respect to η and f be a real-valued function defined on X. Then f is a semi-G-preinvex function on X with respect to η if and only if its epigraph $E_f = \{(x, \alpha) : x \in X, \alpha \in R, f(x) \le \alpha\}$ is a semi-G-invex set with respect to η.*

Then, we give some properties and characterizations of semi-G-preinvexity.

Theorem 3.2. *Let $X \subseteq R^n$ be a semi-connected set with respect to η and f be a real-valued function defined on X. Then f is a semi-G-preinvex function on X with respect to η if and only if for each pair of points $x, y \in X$, the following relation*

$$f(y + \lambda\eta(x, y, \lambda)) \le G^{-1}(\lambda G(\alpha) + (1-\lambda)G(\beta))$$

is fulfilled for any $\lambda \in [0,1]$, whenever $f(x) \le \alpha$ and $f(y) \le \beta$.

Proof. Let $x, y \in X$ such that

$$f(x) \le \alpha \text{ and } f(y) \le \beta.$$

By hypothesis $y + \lambda\eta(x, y, \lambda) \in X$ for any $\lambda \in [0,1]$. Since f is a semi-G-preinvex function on X with respect to η then for any $\lambda \in [0,1]$,

$$f(y + \lambda\eta(x, y, \lambda)) \le G^{-1}(\lambda G(f(x)) + (1-\lambda)G(f(y))).$$

So, we have

$$f(y + \lambda\eta(x, y, \lambda)) \le G^{-1}(\lambda G(\alpha) + (1-\lambda)G(\beta)).$$

Conversely, let $(x, \alpha) \in E_f$ and $(y, \beta) \in E_f$, by assumption, for $\forall\lambda \in [0,1]$ we have

$$f(y + \lambda\eta(x, y, \lambda)) \le G^{-1}(\lambda G(\alpha) + (1-\lambda)G(\beta)).$$

By the definition of epigraph of f, it follows that

$$(y + \lambda\eta(x, y, \lambda), G^{-1}(\lambda G(\alpha) + (1-\lambda)G(\beta))) \in E_f$$

i.e., E_f is a semi-G-invex set. Hence, from Theorem 3.1, we can get that f is a semi-G-preinvex function on X with respect to η. ■

Theorem 3.3. *Let $F \subset R^{n+1}$ defined by $F := \{(x, \alpha) : x \in R^n, \alpha \in R\}$ be any set which is semi-G-invex with respect to η and let $f(x) = \inf\{\alpha : (x, \alpha) \in F\}$. Then f is a semi-G-preinvex function on X with respect to η.*

Proof. Let $\alpha_1, \beta_1 \in R$ and $x, y \in R^n$ such that

$$f(x) < \alpha_1 \text{ and } f(y) < \beta_1.$$

Then there exist α_2 and β_2 such that $(x, \alpha_2) \in F$ and $(y, \beta_2) \in F$ and, moreover,

$$f(x) \leq \alpha_2 < \alpha_1 \text{ and } f(y) \leq \beta_2 < \beta_1.$$

By assumption, F is a semi-G-invex set with respect to η. Thus, by Definition 3.1, we have

$$(y + \lambda\eta(x, y, \lambda), G^{-1}(\lambda G(\alpha_2) + (1 - \lambda)G(\beta_2))) \in F, \ \forall \lambda \in [0, 1].$$

By definition of f we get, for any $\lambda \in [0, 1]$,

$$f(y + \lambda\eta(x, y, \lambda)) \leq G^{-1}(\lambda G(\alpha_2) + (1 - \lambda)G(\beta_2)).$$

By using Theorem 3.2, we get the conclusion of this theorem. ■

Theorem 3.4. *Let $f : X \to R$ be semi-G-preinvex function with respect to η. Let $\mu = \inf_{u \in X} f(u)$, then set $E = \{u \in X : f(u) = \mu\}$ is a semi-connected set with respect to the same η. If f is strictly semi-G-preinvex function with respect to η, then E is singleton.*

Proof. (i) Let $x, y \in E$ and $\lambda \in [0, 1]$. From the semi-G-preinvexity with respect to η of f, we have

$$\begin{aligned} f(y + \lambda\eta(x, y, \lambda)) &\leq G^{-1}(\lambda G(f(x)) + (1 - \lambda)G(f(y))) \\ &\leq G^{-1}(\lambda G(\mu) + (1 - \lambda)G(\mu)) = \mu. \end{aligned}$$

Thus

$$y + \lambda\eta(x, y, \lambda) \in E.$$

Hence, E is semi-connected with respect to η.

(ii) Let $x, y \in E (x \neq y)$ s.t. $f(x) = f(y) = \mu$. Since E is semi-connected with respect to η, for all $\lambda \in [0, 1]$, we have

$$y + \lambda\eta(x, y, \lambda) \in E \subset X.$$

Since f is strictly semi-G-preinvex function with respect to η, thus

$$f(y+\lambda\eta(x,y,\lambda)) < G^{-1}(\lambda G(f(x)) + (1-\lambda)G(f(y))) = \mu,$$

which contradicts the fact that $\mu = \inf_{u\in X} f(u)$. Therefore, E is singleton. ■

Theorem 3.5. *Let $X \subseteq R^n$ be semi-connected set with respect to $\eta(x,y,\lambda)$, $f : X \to R$ is semi-G-preinvex functions with respect to the same η if and only if its epigraph $E_f = \{(x,u) : x \in X, u \in R, f(x) \le u\}$ is semi-connected with respect to $\eta^* : E_f \times E_f \times [0,1] \to R^{n+1}$, where*

$$\eta^*((y,v),(x,u),\lambda) = \begin{cases} \eta(y,x,0), & \lambda = 0; \\ (\eta(y,x,\lambda), \frac{G^{-1}(\lambda G(u)+(1-\lambda)G(v))-u}{\lambda}), & 0 < \lambda \le 1. \end{cases}$$

for all $(x,u),(y,v) \in E_f$.

Proof. (i) When $\lambda = 0$, the Theorem is obvious.

(ii) When $0 < \lambda \le 1$. Let $(x,u) \in E_f$, $(y,v) \in E_f$, i.e., $f(x) \le u$ and $f(y) \le v$. By assumption, f is semi-G-preinvex with respect to η, we have

$$\begin{aligned} f(y+\lambda\eta(x,y,\lambda)) &\le G^{-1}(\lambda G(f(x)) + (1-\lambda)G(f(y))) \\ &\le G^{-1}(\lambda G(u) + (1-\lambda)G(v)),\ \lambda \in (0,1]. \end{aligned}$$

Thus

$$(x+\lambda\eta(y,x,\lambda), G^{-1}(\lambda G(u) + (1-\lambda)G(v))) \in E_f.$$

That is,

$$(x,u) + \lambda(\eta(y,x,\lambda), \frac{G^{-1}(\lambda G(u) + (1-\lambda)G(v)) - u}{\lambda}) \in E_f,\ \lambda \in (0,1].$$

Hence, E_f is a semi-connected set with respect to

$$\eta^*((y,v),(x,u),\lambda) = (\eta(y,x,\lambda), \frac{G^{-1}(\lambda G(u) + (1-\lambda)G(v)) - u}{\lambda}).$$

Conversely, assume that E_f is a semi-connected set with respect to η^*. Let $x, y \in X$ and $u, v \in R$ such that $f(x) \le u$, $f(y) \le v$. Then,

$$(x,u) \in E_f \text{ and } (y,v) \in E_f.$$

From the semi-connectedness of the set E_f with respect to η^*, we have

$$(x,u) + \lambda\eta^*((y,v),(x,u),\lambda) \in E_f,\ \lambda \in (0,1].$$

It follows that

$$(x,u)+\lambda(\eta(y,x,\lambda),\frac{G^{-1}(\lambda G(u)+(1-\lambda)G(v))-u}{\lambda})\in E_f,\ \ \lambda\in(0,1].$$

Then

$$(x+\lambda\eta(y,x,\lambda),G^{-1}(\lambda G(u)+(1-\lambda)G(v)))\in E_f,\ \ \lambda\in(0,1].$$

That is

$$f(y+\lambda\eta(x,y,\lambda))\leq G^{-1}(\lambda G(u)+(1-\lambda)G(v)).$$

Thus, by Theorem 3.2, f is semi-G-preinvex function with respect to η on X. ■

Theorem 3.6. *Let I be an index set. $X\subseteq R^n$ be semi-connected set with respect to $\eta:R^n\times R^n\times[0,1]\to R^n$, and a family of real-valued functions $(f_i)_{i\in I}$ be semi-G-preinvex with respect to the same η. Then, the function $f(x)=\sup_{i\in I}f_i(x)$ is a semi-G-preinvex function with respect to the same η on X.*

Proof. (i) When $\lambda=0$, the Theorem is obvious.

(ii) When $0<\lambda\leq 1$, from the semi-G-preinvexity with respect to the same η of every f_i and by Theorem 3.5, we can obtain that epigraph

$$E_{f_i}=\{(x,u)|x\in X,u\in R,f_i(x)\leq u\}$$

is semi-connected set in $R^n\times R$ with respect to

$$\eta^*((y,v),(x,u),\lambda)=(\eta(y,x,\lambda),\frac{G^{-1}(\lambda G(u)+(1-\lambda)G(v))-u}{\lambda}).$$

Therefore, their intersection

$$\begin{aligned}\bigcap_{i\in I}E_{f_i}&=\{(x,u)|x\in X,u\in R,f_i(x)\leq u,i\in I\}\\&=\{(x,u)|x\in X,u\in R,f(x)\leq u\}.\end{aligned}$$

Obviously, the intersection $\bigcap_{i\in I}E_{f_i}$ is the epigraph of f. Then, by Lemma 2.2, we can get the intersection is a semi-connected set in $R^n\times R$ with respect to η^*. From Theorem 3.5, f is semi-G-preinvex with respect to η. ■

Theorem 3.7. *Let $X\subseteq R^n$ be a semi-connected set with respect to η. If $f:X\to R$ is a semi-G-preinvex function on X with respect to η, then the level set $S_\alpha=\{x\in X:f(x)\leq\alpha\}$ is a semi-connected set with respect to η for each $\alpha\in R$.*

Proof. From the definitions of semi-G-preinvexity and the level set, we can get the result. ■

Remark 3.1. Since semi-G-preinvexity is a generalization of G-(r/pre)invexity and (semi-)preinvexity, our results extend and generalize the corresponding results in literatures.

4. Conclusions

In this paper, we have introduced a new kind of generalized convex function called a semi-G-preinvex function. Examples are given to show that the class of function is a generalization of G-preinvex functions, semipreinvex functions, G-invex functions and G-convex functions. some interesting properties and characterizations of semi-G-preinvexity are established. Our results extend and improve the corresponding ones in the literature.

An interesting topic for our future research is to investigate semi-G-preinvex functions and semicontinuity. Another interesting research topic is to explore some characterizations of these semi-G-preinvex functions and their applications in nonlinear programming problems.

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