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Research Article

The Modified Coupled Hirota Equation: Riemann-Hilbert Approach and N-Soliton Solutions

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The Cauchy initial value problem of the modified coupled Hirota equation is studied in the framework of Riemann-Hilbert approach. The N-soliton solutions are given in a compact form as a ratio of $(N+1)\times(N+1)$ determinant and $N\times N$ determinant, and the dynamical behaviors of the single-soliton solution are displayed graphically.

1. Introduction

The Hirota equation

$$iu_t + \frac{1}{2}u_{xx} + |u|^2 u - i\alpha u_{xxx} - 6i\alpha |u|^2 u_x = 0 \qquad (1)$$

is an important integrable model [1], where α is a real parameter. This equation was initially proposed by Hirota [1] as a model for describing the ultrashort pulses sufferred from higher-order dispersion and self-steepening effect [2]. This Hirota equation is an integrable generalization of the well-known nonlinear Schrödinger equation (NLSE). Subsequently, the multisolitons, breathers, rogue waves, and high-order rogue waves for the Hirota equation (1) were studied by many researchers via generalized Darboux transformation method and other methods [2–6]. For the integrability and other types of solutions of the Hirota model (1), we refer to [7–9].

The aim of this paper is to study the modified coupled Hirota equation in the following form [10]:

$$\begin{aligned} &iu_t + \frac{1}{2}u_{xx} + \left(|u|^2 + |v|^2\right)u \\ &\quad + i\epsilon\left[u_{xxx} + \left(6|u|^2 + 3|v|^2\right)u_x + 3uv^* v_x\right] = 0, \\ &iv_t + \frac{1}{2}v_{xx} + \left(|u|^2 + |v|^2\right)v \\ &\quad + i\epsilon\left[v_{xxx} + \left(6|v|^2 + 3|u|^2\right)v_x + 3vu^* u_x\right] = 0, \end{aligned} \qquad (2)$$

where $u = u(x,t), v = v(x,t)$ represent complex field envelope, ϵ is a small dimensionless real parameter, and "$*$" denotes complex conjugation. The coupled higher-order Hirota equations were firstly studied in [11], where the authors used them to describe electronmagnetic pulse propogation in coupled optical waveguides and obtained soliton solutions by the inverse scattering approach. For dark soliton solutions and composite rogue waves for the coupled Hirota equation, we refer to [10, 12–14]. In the present paper, we shall seek the solutions $u(x,t), v(x,t)$ at any later time t for prescribed initial conditions $u(x,0), v(x,0)$. That is, we shall solve an Cauchy problem for the modified coupled Hirota equation (2); actually we are going to construct the multisoliton solutions for this system with the aid of the Riemann-Hilbert approach [15–20]. We mention that there exist many other efficient methods to investigate the exact solition solutions for the nonlinear evolution equations; for instance, the first integral method is used to study the exact 1-soliton solutions for a variety of Boussinesq-like equations [21], the G'/G–expansion approach is utilized to investigate the dispersive dark optical soliton for the Schrödinger-Hirota equation [22], the extended trial equation method is used to study the dispersive optical solitons for the Schrödinger-Hirota equation [23], and the Bäcklund transformation method is adopted to study the optical solitons for the Schrödinger-Hirota equation with power law nonlinearity [24].

The structure of this paper is arranged as follows. In Section 2, we start with the spectral analysis of the Lax pair of

(2) and then we shall formulate the corresponding Riemann-Hilbert problem for this equations. In Section 3, we shall solve the Riemann-Hilbert problem and discuss the spatial and temporal evolutions of scattering data. In Section 4, N-soliton solutions of (2) will be constructed. In Section 5, we graphically show the behavior of single-soliton solutions for (2).

2. Riemann-Hilbert Formulation

In this section, we shall study the direct scattering problems for (2) and establish the corresponding Riemann-Hilbert problem.

2.1. Direct Scattering Process. The modified coupled Hirota equation (2) is Lax integrable with the linear spectral problem

$$Y_x = UY = \left(\frac{i}{12\epsilon}\lambda\sigma + \widetilde{U}\right)Y, \tag{3}$$

$$Y_t = VY = \left(\frac{i}{192\epsilon^2}i\left(\lambda^3 + 2\lambda^2\right)\sigma + \widetilde{V}\right)Y, \tag{4}$$

where λ is a spectral parameter and $Y(x,t,\lambda)$ is a matrix function. The matrices $\sigma, \widetilde{U}, \widetilde{V}$ are defined as follows:

$$\begin{gathered}\sigma = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ \widetilde{U} = \begin{pmatrix} 0 & -u & -v \\ u^* & 0 & 0 \\ v^* & 0 & 0 \end{pmatrix}, \\ \widetilde{V} = \frac{1}{16\epsilon}\lambda^2\widetilde{U} + \lambda V_1 + V_2,\end{gathered} \tag{5}$$

with

$$V_1 = \frac{1}{4}\begin{pmatrix} ie & -\dfrac{u}{2\epsilon} - iu_x & -\dfrac{v}{2\epsilon} - iv_x \\ \dfrac{u^*}{2\epsilon} - iu_x^* & -i|u|^2 & -ivu^* \\ \dfrac{v^*}{2\epsilon} - iv_x^* & -iuv^* & -i|v|^2 \end{pmatrix},$$

$$V_2 = \begin{pmatrix} \epsilon\left(e_1 + e_2\right) + \dfrac{i}{2}e & \epsilon e_3 - \dfrac{i}{2}u_x & \epsilon e_4 - \dfrac{i}{2}v_x \\ -\epsilon e_3^* - \dfrac{i}{2}u_x^* & -\epsilon e_1 - \dfrac{i}{2|u|^2} & \epsilon e_5 - \dfrac{i}{2}vu^* \\ -\epsilon e_4^* - \dfrac{i}{2}v_x^* & -\epsilon e_5^* - \dfrac{i}{2}uv^* & -\epsilon e_2 - \dfrac{i}{2|v|^2} \end{pmatrix}, \tag{6}$$

in which

$$\begin{aligned} e &= |u|^2 + |v|^2, \\ e_1 &= uu_x^* - u^*u_x, \\ e_2 &= vv_x^* - v^*v_x, \\ e_3 &= u_{xx} + 2eu, \\ e_4 &= v_{xx} + 2ev, \\ e_5 &= u^*v_x - vu_x^*. \end{aligned} \tag{7}$$

Moreover, we mention that a nonlocal two-wave interaction system associated with an easier 3×3 matrix spectral problem is investigated with the aid of the Riemann-Hilbert approach by [25].

Let us study (2) for the localized solutions; we simply suppose that the potentials u and v decay to zero sufficiently fast as $x \longrightarrow \pm\infty$, and ϵ is taken to be positive without loss of generality. It is convenient for us to introduce a new matrix spectral function

$$J(x,t,\lambda) = Y(x,t,\lambda)E_1^{-1}, \tag{8}$$

where

$$E_1 = e^{(i/12\epsilon)\lambda\sigma x + (i/192\epsilon^2)(\lambda^3 + 2\lambda^2)\sigma t} \tag{9}$$

E_1 is a solution of the above spectral equations (3) and (4) at $x \longrightarrow \pm\infty$. Hence, (3) and (4) can be rewritten as

$$J_x = \frac{i\lambda}{12\epsilon}[\sigma, J] + \widetilde{U}J, \tag{10}$$

$$J_t = \frac{i}{192\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)[\sigma, J] + \widetilde{V}J, \tag{11}$$

where $[\sigma, J] = \sigma J - J\sigma$. We point out that the matrix Q possesses the symmetry condition, i.e.,

$$\widetilde{U}^\dagger = -\widetilde{U}, \tag{12}$$

where the superscript † denotes the Hermitian of a matrix.

In the scattering process, the Jost solutions $J_\pm(x,\lambda)$ of (10) fulfill the following asymptotic condition:

$$\begin{aligned} J_+(x,\lambda) &\longrightarrow \mathrm{I}, \quad x \longrightarrow +\infty, \\ J_-(x,\lambda) &\longrightarrow \mathrm{I}, \quad x \longrightarrow -\infty, \end{aligned} \tag{13}$$

where I denotes the 3×3 unit matrix and the subscripts in $J_\pm$ represent to which end of the x axis the boundary conditions are set. With the aid of an identity from the matrix calculus, i.e.,

$$(\det J)_x = \det J \cdot \operatorname{tr}\left(J_x J^{-1}\right), \tag{14}$$

where $\det(J)$ denotes the determinant of matrix J and $\operatorname{tr}(\cdot)$ represents the trace of a matrix, combined with the fact that $\operatorname{tr}(Q) = 0$, it follows that

$$\det J_\pm = 1 \tag{15}$$

for all (x,λ). Besides, it is easy to check that the matrix Jost solutions $J_\pm$ solve the following Volterra integral equations:

$$J_+(x,\lambda) = I - \int_x^{+\infty} e^{(i/12\epsilon)\lambda\sigma(x-\zeta)}\widetilde{U}J_+(\zeta,\lambda)e^{(-i/12\epsilon)\lambda\sigma(x-\zeta)}d\zeta,$$

$$J_-(x,\lambda) = I + \int_{-\infty}^{x} e^{(i/12\epsilon)\lambda\sigma(x-\zeta)} \widetilde{U} J_-(\zeta,\lambda) e^{(-i/12\epsilon)\lambda\sigma(x-\zeta)} d\zeta. \tag{16}$$

Hence $J_\pm(x,\lambda)$ admit analytical continuations if the above Volterra integrals converge. Let us split $J_\pm$ into column vectors, i.e., $J_\pm = ([J_\pm]_1, [J_\pm]_2, [J_\pm]_3)$, and then the column vectors $[J_-]_1, [J_+]_2, [J_+]_3$ can be analytically continued to the upper half plane $\lambda \in \mathbb{C}_+$, while the column vectors $[J_+]_1, [J_-]_2, [J_-]_3$ can be analytically continued to the lower half plane $\lambda \in \mathbb{C}_-$.

Denoting $E = e^{(i/12\epsilon)\lambda\sigma x}$, it is easy to show that both J_+E and J_-E are fundamental matrix solutions of the spectral problem (3), which indicates that they must be linearly related by a matrix $S(\lambda)$– the so-called scattering matrix. That is,

$$J_-E = J_+ES(\lambda), \quad \lambda \in \mathbb{R}, \tag{17}$$

with $S(\lambda) = (s_{ij}(\lambda))_{3\times 3}$. In view of (15) and (17), clearly one obtains

$$\det S(\lambda) = 1. \tag{18}$$

From (17), one can easily deduce

$$S(\lambda) = E^{-1}J_+^{-1}J_-E, \quad \lambda \in \mathbb{R}, \tag{19}$$

which implies that one has to study the analytic properties of J_+^{-1} before deriving the analytic property about the entries of $S(\lambda)$. Let us begin with the adjoint spectral equation of (10):

$$K_x = \frac{i}{12\epsilon}\lambda[\sigma, K] - K\widetilde{U} \tag{20}$$

One can simply find that $J_\pm^{-1}$ satisfy the above equation (20), where $J_\pm^{-1}$ is partitioned into rows in the following form:

$$J_+^{-1} = \begin{pmatrix} [J_+^{-1}]^1 \\ [J_+^{-1}]^2 \\ [J_+^{-1}]^3 \end{pmatrix}, \qquad J_-^{-1} = \begin{pmatrix} [J_-^{-1}]^1 \\ [J_-^{-1}]^2 \\ [J_-^{-1}]^3 \end{pmatrix}. \tag{21}$$

By similar discussions, we know that the row vectors $[J_+^{-1}]^1, [J_-^{-1}]^2, [J_-^{-1}]^3$ are analytic in $\lambda \in \mathbb{C}_+$, while other row vectors $[J_-^{-1}]^1, [J_+^{-1}]^2, [J_+^{-1}]^3$ are analytic for $\lambda \in \mathbb{C}_-$. Thanks to the analytic property of J_+^{-1} and J_-, it follows that s_{11} admits analytic continuation to $\mathbb{C}_+$, and $s_{22}, s_{33}, s_{23}, s_{32}$ can be analytically continued to $\mathbb{C}_-$, while $s_{13}, s_{31}, s_{12}, s_{21}$ are only defined for$\lambda \in \mathbb{R}$. Similarly we have that r_{11} admits analytic continuation to $\mathbb{C}_-$, and $r_{22}, r_{33}, r_{23}, r_{32}$ can be analytically continued to $\mathbb{C}_+$, while $r_{13}, r_{31}, r_{12}, r_{21}$ are only defined for $\lambda \in \mathbb{R}$.

2.2. Riemann-Hilbert Problem. Next, we shall construct the Riemann-Hilbert problem. To this end, we introduce the following matrix function:

$$\begin{aligned} P_+ &= J_-H_1 + J_+H_2 + J_+H_3 \\ &= J_+\left(ES(\lambda)E^{-1}H_1 + H_2 + H_3\right) \\ &= J_+E(S(\lambda)H_1 + H_2 + H_3)E^{-1} \\ &= J_+E\begin{pmatrix} s_{11} & 0 & 0 \\ s_{21} & 1 & 0 \\ s_{31} & 0 & 1 \end{pmatrix}E^{-1} \end{aligned} \tag{22}$$

where

$$\begin{aligned} H_1 &= \operatorname{diag}(1,0,0), \\ H_2 &= \operatorname{diag}(0,1,0), \\ H_3 &= \operatorname{diag}(0,0,1). \end{aligned} \tag{23}$$

Therefore $\det(P_+) = s_{11}$. Moreover, in view of (16), it is easy to verify that the large-λ asymptotics of these analytical functions are

$$P_+(x,\lambda) \longrightarrow I, \quad \lambda \in \mathbb{C}_+ \longrightarrow \infty. \tag{24}$$

From the analytic property of $J_\pm$, we get that P_+ is analytic in $\lambda \in \mathbb{C}_+$. To obtain the analytic counterpart of P_+ in $\mathbb{C}_-$, we begin with the inverse matrix of $S(\lambda)$, that is,

$$R(\lambda) = S^{-1}(\lambda) = E^{-1}J_-^{-1}J_+E, \tag{25}$$

where $R(\lambda) = (r_{ij})_{3\times 3}$. Then we shall investigate the following matrix function:

$$\begin{aligned} P_- &= \begin{pmatrix} [J_-^{-1}]^1 \\ [J_+^{-1}]^2 \\ [J_+^{-1}]^3 \end{pmatrix} = H_1[J_-^{-1}] + H_2[J_+^{-1}] + H_3[J_+^{-1}] \\ &= \left(H_1ER(\lambda)E^{-1} + H_2 + H_3\right)J_+^{-1} \\ &= E\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}E^{-1}J_+^{-1}, \end{aligned} \tag{26}$$

which is analytic for $\lambda \in \mathbb{C}_-$. Similarly, one has the following asymptotics property:

$$P_-(x,\lambda) \longrightarrow \infty, \quad \lambda \in \mathbb{C}_- \longrightarrow \infty. \tag{27}$$

By now, we have obtained two analytic matrix functions P_+ and P_-, which are analytic in $\mathbb{C}_+$ and $\mathbb{C}_-$, respectively. On the axes, they are related by

$$P_-(x,\lambda)P_+(x,\lambda) = G(x,\lambda), \quad \lambda \in \mathbb{R}, \tag{28}$$

where

$$G(x,\lambda) = E\left(H_1 R(\lambda) + H_2 + H_3\right) \cdot E^{-1} J_+^{-1} J_+ E\left(S(\lambda) H_1 + H_2 + H_3\right) E^{-1} = E \begin{pmatrix} r_{11}s_{11} + r_{12}s_{21} + r_{13}s_{31} & r_{12} & r_{13} \\ s_{21} & 1 & 0 \\ s_{31} & 0 & 1 \end{pmatrix} E^{-1} = \begin{pmatrix} 1 & r_{12}e^{-(i/4\epsilon)\lambda x} & r_{13}e^{-(i/4\epsilon)\lambda x} \\ s_{21}e^{(i/4\epsilon)\lambda x} & 1 & 0 \\ s_{31}e^{(i/4\epsilon)\lambda x} & 0 & 1 \end{pmatrix}, \quad (29)$$

where $r_{11}s_{11} + r_{12}s_{21} + r_{13}s_{31} = 1$ is followed by the fact that $R(\lambda)S(\lambda) = \mathrm{I}$. Equation (28) is just the matrix Riemann-Hilbert problem we needed, and the associated canonical condition is as follows:

$$P_\pm(x,\lambda) \longrightarrow I, \quad \lambda \longrightarrow \infty. \quad (30)$$

2.3. Symmetric Properties. In the remaining subsection, we shall investigate the symmetric properties which will be used later. To this end, we firstly take the Hermitian transpose of the spectral equation (10), that is,

$$\left(J_\pm^\dagger(x,\lambda^*)\right)_x = -\frac{i\lambda}{12\epsilon}\left(J_\pm^\dagger(x,\lambda^*)\sigma - \sigma J_\pm^\dagger(x,\lambda^*)\right) + J_\pm^\dagger(x,\lambda^*)\widetilde{U}^\dagger = \frac{i\lambda}{12\epsilon}\left[\sigma, J_\pm^\dagger(x,\lambda^*)\right] - J_\pm^\dagger(x,\lambda^*)\widetilde{U}, \quad (31)$$

where the symmetric condition $\widetilde{U}^\dagger = -\widetilde{U}$ is used. It is easy to verify from (31) that $J_\pm^\dagger(x,\lambda^*))$ also fulfills the adjoint spectral equation (20). As discussed above, we know that $J_\pm^{-1}(x,\lambda)$ solves (20) and the boundary conditons (30); thus we have

$$J_\pm^\dagger(x,\lambda^*) = J_\pm^{-1}(x,\lambda). \quad (32)$$

Moreover, in view of (22), (26), and (32), one gets

$$(P_+)^\dagger(x,\lambda^*) = E\begin{pmatrix} s_{11}^* & s_{21}^* & s_{31}^* \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} E^{-1} J_+^\dagger(x,\lambda^*) = E\begin{pmatrix} s_{11}^* & s_{21}^* & s_{31}^* \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} E^{-1} J_+^{-1}(x,\lambda). \quad (33)$$

On the other hand, one simply finds

$$S^\dagger(\lambda^*) = E^{-1} J_-^\dagger \left(J_+^{-1}\right)^\dagger E = E^{-1} J_-^{-1} J_+ E = R(\lambda). \quad (34)$$

Using (33) and (34), one has the following symmetric property:

$$(P_+)^\dagger(x,\lambda^*) = E\begin{pmatrix} s_{11}^* & s_{21}^* & s_{31}^* \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} E^{-1} J_+^{-1}(x,\lambda) = E\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} E^{-1} J_+^{-1}(x,\lambda) = P_-(x,\lambda). \quad (35)$$

Next, we are going to investigate the property of s_{11}, which matters a lot in later analysis. From (34), one has the following relations:

$$s_{11}^*(\lambda^*) = r_{11}(\lambda), \quad s_{jj}^*(\lambda^*) = r_{jj}(\lambda), \quad j = 2,3, \quad s_{kl}^*(\lambda^*) = r_{lk}(\lambda), \quad k,l = 1,2,3, k \neq l. \quad (36)$$

By (36), we know that if $\lambda_1 \in \mathbb{C}_+$ is a zero of s_{11}, then $\lambda_1^* \in \mathbb{C}_-$ is also a zero of r_{11}.

3. Solutions to Riemann-Hilbert Problem

In this section, we shall solve the Riemann-Hilbert problem (28) in both regular and nonreguluar case. Before this, from (15) and (22), we simply see that

$$\det P_+(\lambda) = s_{11}(\lambda), \quad (37)$$

and, similarly, we also have

$$\det P_-(\lambda) = r_{11}(\lambda). \quad (38)$$

Now we firstly consider the regular Riemann-Hilbert problem (28), i.e., $\det P_+ = s_{11} \neq 0$ and $\det P_- = r_{11} \neq 0$ in their analytic domain. Equation (28) can be rewritten as

$$(P_+)^{-1}(\lambda) - P_-(\lambda) = \widehat{G}(\lambda)(P_+)^{-1}(\lambda), \quad \lambda \in \mathbb{R}, \quad (39)$$

where $\widehat{G} = I - G$. By Plemelj's formula [26], we simply get that the solution to the regular Riemann-Hilbert problem (28) under condition (30) takes the following form:

$$(P_+)^{-1}(\lambda) = I + \frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{\widehat{G}(\xi)(P_+)^{-1}(\xi)}{\xi - \lambda} d\xi, \quad \lambda \in \mathbb{C}_+. \quad (40)$$

Next, we turn to investigate the nonregular Riemann-Hilbert problem (28); that is, $\det P_+(\lambda) = s_{11}(\lambda)$ and $\det P_-(\lambda) = r_{11}(\lambda)$ possess simple zeros. From the discussion in the last section, we can assume that $\{\lambda_k \in \mathbb{C}_+, 1 \leq k \leq N\}$ are simple zeros of $s_{11}(\lambda)$; then we can simply denote the zeros of $r_{11}(\lambda)$ by $\{\lambda_k^* \in \mathbb{C}_-, 1 \leq k \leq N\}$. Under this circumstance, since all the zeros are simple ones, then the kernal of $P_+(\lambda_k)$ is one-dimensional and can be spanned by a single column vector v_k; therefore one has

$$P_+(\lambda_k)v_k = 0, \quad 1 \le k \le N, \tag{41}$$

By taking the Hermitian conjugate to (41), one simply deduces that

$$v_k^\dagger P_+^\dagger(\lambda_k) = 0, \quad 1 \le k \le N. \tag{42}$$

Taking account of (35), one arrives at

$$v_k^\dagger P_-(\lambda_k^*) = 0, \quad 1 \le k \le N. \tag{43}$$

For simplicity, we denote

$$\widehat{v}_k := v_k^\dagger, \quad 1 \le k \le N. \tag{44}$$

Moreover, we can derive the explicit expressions for v_k. To this end, let us start from the fact that J_+ satisfies the spectral equation (10), that is,

$$J_{+,x} = \frac{i\lambda}{12\epsilon}[\sigma, J_+] + \widetilde{U}J_+. \tag{45}$$

By taking the x-derivative to (22), one simply has

$$\begin{aligned}P_{+,x} &= J_{+,x}E\begin{pmatrix} s_{11} & 0 & 0 \\ s_{21} & 1 & 0 \\ s_{31} & 0 & 1\end{pmatrix}E^{-1} \\ &+ J_+\left(\frac{i\lambda}{12\epsilon}\right)\sigma E\begin{pmatrix} s_{11} & 0 & 0 \\ s_{21} & 1 & 0 \\ s_{31} & 0 & 1\end{pmatrix}E^{-1} \\ &+ J_+E\begin{pmatrix} s_{11} & 0 & 0 \\ s_{21} & 1 & 0 \\ s_{31} & 0 & 1\end{pmatrix}E^{-1}\left(-\frac{i\lambda}{12\epsilon}\right)\sigma,\end{aligned} \tag{46}$$

and, inserting (45) into (46), we have

$$P_{+,x} = \frac{i\lambda}{12\epsilon}[\sigma, P_+] + \widetilde{U}P_+. \tag{47}$$

Taking the x-derivative to (41), in view of (47), one has

$$P_+(\lambda_k)\left(v_{k,x} - \frac{i\lambda_k}{12\epsilon}\sigma v_k\right) = 0. \tag{48}$$

By similar procedure, we have

$$P_+(\lambda_k)\left(v_{k,t} - \frac{i\left(\lambda_k^3 + 2\lambda_k^2\right)}{192\epsilon^2}\sigma v_k\right) = 0. \tag{49}$$

Taking account of (48) and (49), it is easy to obtain that

$$v_k = e^{(i\lambda_k/12\epsilon)\sigma x + (i(\lambda_k^3+2\lambda_k^2)/192\epsilon^2)\sigma t}v_{k0}, \quad 1 \le k \le N. \tag{50}$$

where v_{k0} is a constant vector. By (44), we conclude that

$$\widehat{v}_k = v_{k0}^\dagger e^{-(i\lambda_k^*/12\epsilon)\sigma x - i(\lambda_k^{*3}+2\lambda_k^{*2})\sigma t/192\epsilon^2}, \quad 1 \le k \le N. \tag{51}$$

In the remaining subsection, we shall construct a matrix function $\Upsilon(x,t;\lambda)$ which could cancel all the zeros of $P_\pm$. Firstly, we define a matrix function

$$\Upsilon_1(\lambda) = I + \frac{\lambda_1^* - \lambda_1}{\lambda - \lambda_1^*}\frac{v_1\widehat{v}_1}{\widehat{v}_1 v_1}. \tag{52}$$

It turns out that $\Upsilon_1(\lambda)$ only admits a simple pole singularity at $\lambda = \lambda_1^* \in \mathbb{C}_-$. Moreover, it can be shown easily that

$$\begin{aligned}\Upsilon_1^{-1}(\lambda) &= I - \frac{\lambda_1^* - \lambda_1}{\lambda - \lambda_1}\frac{v_1\widehat{v}_1}{\widehat{v}_1 v_1}, \\ \Upsilon_1(\lambda_1)v_1 &= 0, \\ \widehat{v}_1\Upsilon_1^{-1}(\lambda_1^*) &= 0,\end{aligned} \tag{53}$$

and

$$\begin{aligned}\det\Upsilon_1(\lambda) &= \frac{\lambda - \lambda_1}{\lambda - \lambda_1^*}, \\ \det\left(P_+(\lambda)\Upsilon_1^{-1}(\lambda)\right)\big|_{\lambda=\lambda_1} &\ne 0, \\ \det\left(\Upsilon(\lambda)P_-(\lambda)\right)\big|_{\lambda=\lambda_1^*} &\ne 0.\end{aligned} \tag{54}$$

Hence, we can construct the following two matrix functions:

$$\begin{aligned}\Upsilon(\lambda) &= \Upsilon_N(\lambda)\Upsilon_{N-1}(\lambda)\cdots\Upsilon_1(\lambda), \\ \Upsilon^{-1}(\lambda) &= \Upsilon_1^{-1}(\lambda)\Upsilon_2^{-1}(\lambda)\cdots\Upsilon_N^{-1}(\lambda),\end{aligned} \tag{55}$$

in which

$$\begin{aligned}\Upsilon_j(\lambda) &= I + \frac{\lambda_j^* - \lambda_j}{\lambda - \lambda_j^*}\frac{w_j\widehat{w}_j}{\widehat{w}_j w_j}, \\ \Upsilon_j^{-1}(\lambda) &= I - \frac{\lambda_j^* - \lambda_j}{\lambda - \lambda_j}\frac{w_j\widehat{w}_j}{\widehat{w}_j w_j}, \\ &\qquad j = 2, \ldots, N,\end{aligned} \tag{56}$$

with

$$\begin{aligned}w_j &= \Upsilon_{j-1}(\lambda_j)\Upsilon_{j-2}(\lambda_j)\cdots\Upsilon_1(\lambda_j)v_j, \\ \widehat{w}_j &= \Upsilon_1^{-1}(\lambda_j^*)\Upsilon_2^{-1}(\lambda_j^*)\cdots\Upsilon_{j-1}^{-1}(\lambda_j^*)\widehat{v}_j, \\ &\qquad j = 2, \ldots, N.\end{aligned} \tag{57}$$

From the above discussions, we could define

$$\begin{aligned}P^+(\lambda) &= P_+(\lambda)\Upsilon^{-1}(\lambda), \\ P^-(\lambda) &= \Upsilon(\lambda)P_-(\lambda)\end{aligned} \tag{58}$$

It follows easily that the functions $P^\pm$ are analytic in $\mathbb{C}_\pm$, respectively; moreover, $\det P^\pm$ are nonzero in their corresponding analytic zones and $P^\pm(\lambda) \longrightarrow I$ as $\lambda \longrightarrow \infty$.

Therefore, the nonregular Riemann-Hilbert problem (28) turns into regular one

$$P^{-}(\lambda) P^{+}(\lambda) = \Upsilon(\lambda) G(\lambda) \Upsilon^{-1}(\lambda), \quad \lambda \in \mathbb{R}, \tag{59}$$

and the normalization condition

$$P^{\pm}(\lambda) \longrightarrow I, \quad \lambda \longrightarrow \infty. \tag{60}$$

4. The Inverse Problem

In this section, with the aid of large-λ expansion of the solutions to the Riemann-Hilbert problem (28), we shall reconstruct the potentials u and v. To this end, we firstly investigate the following asymptotic expansion:

$$P_{+} = P_{+,0} + \frac{1}{\lambda} P_{+,1} + \frac{1}{\lambda^2} P_{+,2} + O\left(\frac{1}{\lambda^3}\right). \tag{61}$$

Inserting (61) into (10) and comparing the coefficients of the powers of λ, we find

$$\begin{aligned} i\left[\sigma, P_{+,0}\right] &= 0, \\ i\left[\sigma, P_{+,1}\right] + \widetilde{U} P_{+,0} &= 0, \end{aligned} \tag{62}$$

from which we can deduce

$$\begin{aligned} P_{+,0} &= I, \\ \widetilde{U} &= -\frac{i}{12\epsilon}\left[\sigma, P_{+,1}\right]. \end{aligned} \tag{63}$$

Therefore, we have

$$\begin{aligned} u &= -\frac{i}{4\epsilon}\left(P_{+,1}\right)_{12}, \\ v &= -\frac{i}{4\epsilon}\left(P_{+,1}\right)_{13}, \end{aligned} \tag{64}$$

where $(P_{+,1})_{ij}$ denotes the (i,j)-th entry of the matrix $P_{+,1}$.

To present concise expressions for u and v, we need to find $P_{+,1}$. To do this, we begin to simplify the expressions of $\Upsilon(\lambda)$ and $\Upsilon^{-1}(\lambda)$. Since $\Upsilon(\lambda)$ has simple pole singularities at $\lambda_j^* \in \mathbb{C}_-$, $j = 1, 2, \ldots, N$, one can simply set

$$\begin{aligned} \Upsilon(\lambda) &= I + \sum_{j=1}^{N} \frac{z_j \widehat{v}_j}{\lambda - \lambda_j^*}, \\ \Upsilon^{-1}(\lambda) &= I - \sum_{j=1}^{N} \frac{v_j \widehat{z}_j}{\lambda - \lambda_j}, \end{aligned} \tag{65}$$

where $z_j, \widehat{z}_j$ are column vector and row vector related to $v_j, \widehat{v}_j$. In view of $\Upsilon(\lambda)\Upsilon^{-1}(\lambda) = I$, one has

$$\operatorname{Res}_{\lambda=\lambda_j} \Upsilon(\lambda) \Upsilon^{-1}(\lambda) = \Upsilon\left(\lambda_j\right) v_j = 0, \quad 1 \le j \le N. \tag{66}$$

Inserting (65) into (66), one simply gets

$$\left(z_1, z_2, \ldots, z_N\right) M = \left(v_1, v_2, \ldots, v_N\right), \tag{67}$$

where

$$M = \left(M_{ij}\right)_{N\times N} = \left(\frac{\widehat{v}_i v_j}{\lambda_i^* - \lambda_j}\right)_{N\times N}. \tag{68}$$

By solving the linear equations (67), $\Upsilon(\lambda)$ and $\Upsilon^{-1}(\lambda)$ can be rewritten as

$$\begin{aligned} \Upsilon(\lambda) &= I + \sum_{i,j=1}^{N} \frac{v_i \widehat{v}_j \left(M^{-1}\right)_{ij}}{\lambda - \lambda_j^*}, \\ \Upsilon^{-1}(\lambda) &= I - \sum_{i,j=1}^{N} \frac{v_i \widehat{v}_j \left(M^{-1}\right)_{ij}}{\lambda - \lambda_j}. \end{aligned} \tag{69}$$

Now we are in a position to reconstruct the potentials. By Plemelj's formula [26], the nonregular Riemann-Hilbert problem (59) can be solved as follows:

$$\begin{aligned} &\left(P^{+}\right)^{-1}(\lambda) \\ &\quad = I + \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{\Upsilon(\zeta) \widehat{G}(\zeta) \Upsilon^{-1}(\zeta) \left(P^{+}\right)^{-1}(\zeta)}{\zeta - \lambda} d\zeta, \end{aligned} \tag{70}$$

where $\widehat{G} = I - G$. As $\lambda \longrightarrow \infty$, one has

$$\begin{aligned} &\left(P^{+}\right)^{-1}(\lambda) \\ &\quad \longrightarrow I \\ &\qquad - \frac{1}{2\pi i \lambda} \int_{-\infty}^{+\infty} \Upsilon(\zeta) \widehat{G}(\zeta) \Upsilon^{-1}(\zeta) \left(P^{+}\right)^{-1}(\zeta)\, d\zeta, \end{aligned} \tag{71}$$

hence

$$\begin{aligned} &P^{+}(\lambda) \\ &\quad \longrightarrow I \\ &\qquad + \frac{1}{2\pi i \lambda} \int_{-\infty}^{+\infty} \Upsilon(\zeta) \widehat{G}(\zeta) \Upsilon^{-1}(\zeta) \left(P^{+}\right)^{-1}(\zeta)\, d\zeta. \end{aligned} \tag{72}$$

It follows from (69) that

$$\Upsilon(\lambda) \longrightarrow I + \frac{1}{\lambda} \sum_{i,j=1}^{N} v_i \widehat{v}_j \left(M^{-1}\right)_{ij}, \quad \lambda \longrightarrow \infty. \tag{73}$$

In view of (58), (72), and (73), one finds

$$\begin{aligned} &P_{+,1}(x,t) \\ &\quad = \sum_{i,j=1}^{N} v_i \widehat{v}_j \left(M^{-1}\right)_{ij} \\ &\qquad + \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \Upsilon(\zeta) \widehat{G}(\zeta) \Upsilon^{-1}(\zeta) \left(P^{+}\right)^{-1}(\zeta)\, d\zeta, \end{aligned} \tag{74}$$

where

$$\begin{aligned} &\widehat{G}(x,t;\lambda) = I - G \\ &\quad = -E \begin{pmatrix} 0 & r_{12}(t,\lambda) & r_{13}(t,\lambda) \\ s_{21}(t,\lambda) & 0 & 0 \\ s_{31}(t,\lambda) & 0 & 0 \end{pmatrix} E^{-1}. \end{aligned} \tag{75}$$

At the end of the section, we shall establish the temporal evolution of scattering data $s_{21}, s_{31}, r_{12}, r_{13}$. Let us firstly study the evolution of $s_{21}(\lambda), s_{31}(\lambda)$, since $J(x,\lambda)$ fulfills (11), i.e.,

$$J_{-,t} = \frac{i}{192\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)[\sigma, J_-] + \widetilde{U} J_-, \tag{76}$$

and then we have

$$(J_-E)_t = \frac{i}{192\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)[\sigma, J_-E] + \widetilde{U} J_-E \tag{77}$$

In view of (17), one simply has

$$J_-E = J_+ES \tag{78}$$

Then by (77) and (78), we have

$$(J_+ES)_t = \frac{i}{192\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)[\sigma, J_+ES] + \widetilde{U} J_+ES. \tag{79}$$

Since the potentials u and v decay to zero sufficiently fast as $x \longrightarrow \infty$, then we know that $\widetilde{U}$ tends to zero matrix as $x \longrightarrow \infty$. Thus by taking the limit $x \longrightarrow \infty$ to (79), one has

$$S_t = \frac{i}{192\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)[\sigma, S]. \tag{80}$$

From (80) we have

$$\begin{aligned} s_{21}(t;\lambda) &= s_{21}(0;\lambda)\exp\left(\frac{i}{64\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)t\right), \\ s_{31}(t;\lambda) &= s_{31}(0;\lambda)\exp\left(\frac{i}{64\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)t\right). \end{aligned} \tag{81}$$

Similarly, one can derive

$$\begin{aligned} r_{12}(t;\lambda) &= r_{12}(0;\lambda)\exp\left(\frac{-i}{64\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)t\right), \\ r_{13}(t;\lambda) &= r_{13}(0;\lambda)\exp\left(\frac{-i}{64\epsilon^2}\left(\lambda^3 + 2\lambda^2\right)t\right). \end{aligned} \tag{82}$$

Notice that the initial data $r_{12}(0,\lambda), r_{13}(0,\lambda)$ and $s_{21}(0,\lambda)$, $s_{31}(0,\lambda)$ are determined by the initial values $u(0,x), v(0,x)$.

5. Soliton Solutions

In this section, we shall construct a explicit formula of the N-soliton solutions for the modified coupled Hirota equation. As is known to all, the soliton solutions correspond to the vanishing scattering coefficients, i.e., $s_{21} = s_{31} = r_{12} = r_{13} = 0$. In this case, $\widehat{G} = 0$, thus

$$P_{+,1} = \sum_{i,j=1}^{N} v_i \widehat{v}_j \left(M^{-1}\right)_{ij}. \tag{83}$$

Taking account of (64), one has

$$\begin{aligned} u &= -\frac{i}{4\epsilon}\sum_{i,j=1}^{N}\left(v_i\widehat{v}_j\right)_{12}\left(M^{-1}\right)_{ij}, \\ v &= -\frac{i}{4\epsilon}\sum_{i,j=1}^{N}\left(v_i\widehat{v}_j\right)_{13}\left(M^{-1}\right)_{ij}. \end{aligned} \tag{84}$$

Denote $v_{k0} = (\alpha_k, \beta_k, \gamma_k)^T, \theta_k = (i\lambda_k/12\epsilon)x + (i/192\epsilon^2)(\lambda_k^3 + 2\lambda_k^2)t$, then we have

$$v_k = \left(\alpha_k e^{-2\theta_k}, \beta_k e^{\theta_k}, \gamma_k e^{\theta_k}\right)^T, \tag{85}$$

and, therefore, we have

$$\begin{aligned} u &= -\frac{i}{4\epsilon}\sum_{i,j=1}^{N}\alpha_i\beta_j^* e^{-2\theta_i+\theta_j^*}\left(M^{-1}\right)_{ij}, \\ v &= -\frac{i}{4\epsilon}\sum_{i,j=1}^{N}\alpha_i\gamma_j^* e^{-2\theta_i+\theta_j^*}\left(M^{-1}\right)_{ij}. \end{aligned} \tag{86}$$

5.1. Single-Soliton Solutions. When $N = 1$, the single-soliton solutions for the modified coupled Hirota equations (2) take the following form:

$$\begin{aligned} u &= \frac{-\lambda_{1I}\alpha_1\beta_1^*}{2\epsilon} \\ &\cdot \frac{\exp\left(-\lambda_{1I}x/4\epsilon + \left(\left(\lambda_{1I}^2 - 3\lambda_{1R}^2 - 4\lambda_{1R}\right)/64\epsilon^2\right)\lambda_{1I}t\right)\exp\left(-i\lambda_{1R}x/4\epsilon - it\left[\lambda_{1R}^3 - 3\lambda_{1R}\lambda_{1I}^2 + 2\lambda_{1R}^2 - 2\lambda_{1I}^2\right]/64\epsilon^2\right)}{|\alpha_1|^2 + \left(|\beta_1|^2 + |\gamma_1|^2\right)\exp\left(-\lambda_{1I}x/2\epsilon - \left(\left(3\lambda_{1R}^2 - \lambda_{1I}^2 + 4\lambda_{1R}\right)/32\epsilon^2\right)\lambda_{1I}t\right)} \\ v &= \frac{-\lambda_{1I}\alpha_1\gamma_1^*}{2\epsilon} \\ &\cdot \frac{\exp\left(-\lambda_{1I}x/4\epsilon + \left(\left(\lambda_{1I}^2 - 3\lambda_{1R}^2 - 4\lambda_{1R}\right)/64\epsilon^2\right)\lambda_{1I}t\right)\exp\left(-i\lambda_{1R}x/4\epsilon - it\left[\lambda_{1R}^3 - 3\lambda_{1R}\lambda_{1I}^2 + 2\lambda_{1R}^2 - 2\lambda_{1I}^2\right]/64\epsilon^2\right)}{|\alpha_1|^2 + \left(|\beta_1|^2 + |\gamma_1|^2\right)\exp\left(-\lambda_{1I}x/2\epsilon - \left(\left(3\lambda_{1R}^2 - \lambda_{1I}^2 + 4\lambda_{1R}\right)/32\epsilon^2\right)\lambda_{1I}t\right)} \end{aligned} \tag{87}$$

with $\lambda_1 = \lambda_{1R} + i\lambda_{1I}, (\lambda_{1I}) > 0$. Denote

$$x_0 = \frac{4\epsilon}{\lambda_{1I}} \ln \frac{|\alpha_1|}{\sqrt{|\beta_1|^2 + |\gamma_1|^2}}, \qquad \sigma_0 = \frac{\lambda_{1R}}{96\epsilon^2}\left(\lambda_{1R}^2 - 3\lambda_{1I}^2\right), \tag{88}$$

and then the above single-soliton solutions can be rewritten as follows:

$$\begin{aligned} u(x,t) &= -\frac{\lambda_{1I}}{4\epsilon} \frac{\alpha_1\beta_1}{|\alpha_1| \sqrt{|\beta_1|^2 + |\gamma_1|^2}} \\ &\cdot \operatorname{sech}\left[-\frac{\lambda_{1I}}{4\epsilon}\left(x + \frac{3\lambda_{1R}^2 + 4\lambda_{1R} - \lambda_{1I}^2}{16\epsilon} t + x_0\right)\right] \\ &\cdot \exp\left\{-\frac{i\lambda_{1R}}{6\epsilon} x - \frac{i\left(\lambda_{1R}^2 - \lambda_{1I}^2\right)t}{48\epsilon^2} - i\sigma_0\right\}, \\ v(x,t) &= -\frac{\lambda_{1I}}{4\epsilon} \frac{\alpha_1\gamma_1}{|\alpha_1| \sqrt{|\beta_1|^2 + |\gamma_1|^2}} \\ &\cdot \operatorname{sech}\left[-\frac{\lambda_{1I}}{4\epsilon}\left(x + \frac{3\lambda_{1R}^2 + 4\lambda_{1R} - \lambda_{1I}^2}{16\epsilon} t + x_0\right)\right] \\ &\cdot \exp\left\{-\frac{i\lambda_{1R}}{6\epsilon} x - \frac{i\left(\lambda_{1R}^2 - \lambda_{1I}^2\right)t}{48\epsilon^2} - i\sigma_0\right\}. \end{aligned} \tag{89}$$

The amplitude functions $|u|$ and $|v|$ both admit the shape of a hyperbolic secant with peak amplitude $\lambda_{1I}/4|\epsilon|$, and their velocities are $-(3\lambda_{1R}^2 + 4\lambda_{1R} - \lambda_{1I}^2)/16\epsilon$. The phases of the single-soliton solutions $u(x,t)$ and $v(x,t)$ depend linearly on both space x and time t, and parameters x_0 and σ_0 are the initial location and phase of the solitary waves.

Setting $\alpha_1 = \beta_1 = i, \gamma_1 = -i, \lambda_{1R} = \lambda_{1I} = 1, \epsilon = 1/12$, we plot the graphics of single-soliton solutions for the modified coupled Hirota equations (2) in Figures 1 and 2.

When the data are chosen as before, as displayed in Figures 1 and 2, both the peak amplitudes of $|u|$ and $|v|$ are $3\sqrt{2}/2$, the velocities are equal to -4.5, and the initial location and phase are determined, respectively, by $x_0 = -\ln 2/6$ and $\sigma_0 = -3$. Moreover, the solitary waves propagate from the left to the right.

5.2. N-Soliton Solutions. When $N \geq 2$, the N-soliton solutions for the modified coupled Hirota equations (2) can be rewritten as

$$u = \frac{i}{4\epsilon} \frac{\det F_1}{\det M}, \qquad v = \frac{i}{4\epsilon} \frac{\det F_2}{\det M}, \tag{90}$$

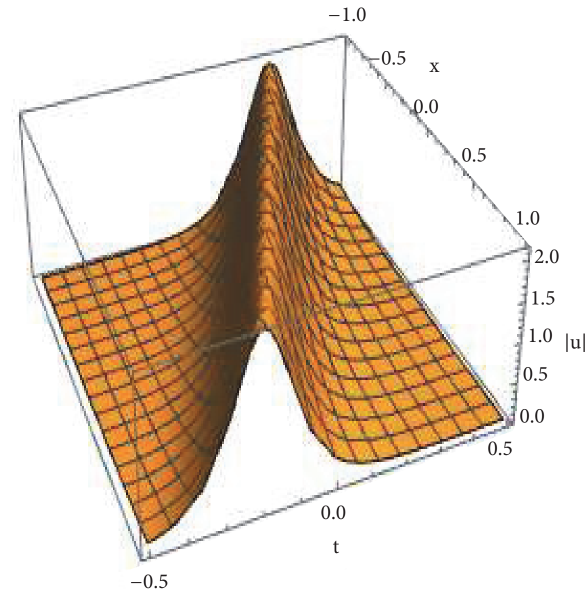

Figure 1: The modula of single-soliton, solution u in 3D plot.

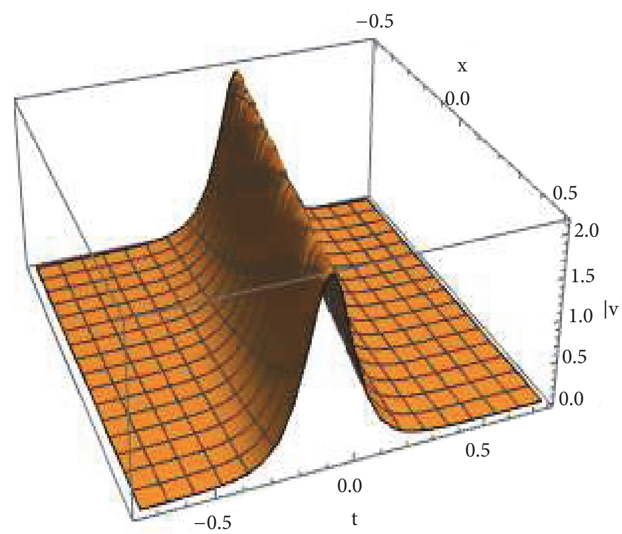

Figure 2: The modula of single-soliton, solution v in 3D plot.

where the two $(N+1)\times(N+1)$ matrices F_1, F_2 are defined as follows:

$$F_1 = \begin{pmatrix} 0 & \beta_1^* e^{\theta_1^*} & \beta_2^* e^{\theta_2^*} & \cdots & \beta_N^* e^{\theta_N^*} \\ \alpha_1 e^{-2\theta_1} & M_{11} & M_{12} & \cdots & M_{1N} \\ \alpha_2 e^{-2\theta_2} & M_{21} & M_{22} & \cdots & M_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_N e^{-2\theta_N} & M_{N1} & M_{N2} & \cdots & M_{NN} \end{pmatrix}, \tag{91}$$

$$F_2 = \begin{pmatrix} 0 & \gamma_1^* e^{\theta_1^*} & \gamma_2^* e^{\theta_2^*} & \cdots & \gamma_N^* e^{\theta_N^*} \\ \alpha_1 e^{-2\theta_1} & M_{11} & M_{12} & \cdots & M_{1N} \\ \alpha_2 e^{-2\theta_2} & M_{21} & M_{22} & \cdots & M_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_N e^{-2\theta_N} & M_{N1} & M_{N2} & \cdots & M_{NN} \end{pmatrix}. \quad (92)$$

6. Conclusions

Starting from the spectral analysis of the Lax pair of the modified coupled Hirota equation (2), we managed to construct the corresponding matrix Riemann-Hilbert problem. We mainly discussed the solutions to the general nonregular matrix Riemann-Hilbert problem; after a regularization procedure, we constructed two matrix functions $\Upsilon(\lambda)$ and $\Upsilon^{-1}(\lambda)$ to eliminate the zeros, which transformed the nonregular Riemann-Hilbert problem into regular one, which could be solved directly by applying the Plemeljs formula [26]. Subsequently, the N-soliton solutions to the modified coupled Hirota equation (2) were obtained by the reconstruction of potentials, which was displayed in a compact form as a ratio of $(N+1)\times(N+1)$ determinant and $N \times N$ determinant. In addition, the dynamical behaviors of the single-soliton solutions were shown graphically. We point out that in the present paper we only treat the case when the potentials fulfill the vanishing boundary conditions; for the general case when the potentials do not vanish at the infinity, more general solutions could be obtained, which may be studied in the future.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

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