Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2015, Article ID 826020, 8 pages
http://dx.doi.org/10.1155/2015/826020

Research Article

Effect of Leakage Delay on Stability of Neutral-Type Genetic Regulatory Networks

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Received 16 June 2014; Accepted 6 August 2014

Academic Editor: Yonghui Xia

The stability of neutral-type genetic regulatory networks with leakage delays is considered. Firstly, we describe the model of genetic regulatory network with neutral delays and leakage delays. Then some sufficient conditions are derived to ensure the asymptotic stability of the genetic regulatory network by the Lyapunov functional method. Further, the effect of leakage delay on stability is discussed. Finally, a numerical example is given to show the effectiveness of the results.

1. Introduction

Biology networks are important research tools of complex biological phenomena, which have been attracting attention of scientists and engineers [1–4]. There are many kinds of biological networks, such as metabolic network, signal network, genetic regulatory network (GRN), and protein interaction network. GRN, which describes the complex interactions between genes (mRNA) and its products (proteins), is often modelled by a kind of dynamical system. The study of GRNs can help scientists understand many important and complex phenomena of living cells. Up to now, several models of GRNs have been established, for example, Boolean network models [5], Bayesian network models [6], Petri network models [7, 8], and the differential equation models. It is more convenient to analyze the dynamical behaviors by using differential equation models and have been widely studied by many experts [9–12].

As we all know, stability is one of the most important issues of longtime quality behavior of dynamical systems. Some important biological secrets have been revealed and many significant results have been reported [13–15]. On the other hand, time delay is a common phenomenon that describes the dynamical behaviors. The existence of time delay can make the dynamical behaviors more complicated and may cause destabilization, oscillation, bifurcation, and chaos. So time delay should be taken into consideration when modelling the genetic networks. There are many types of time delays such as discrete delays [16], continuous delays [17, 18], random delays [19], and mixed delays [20]. In recent years, the neutral delay has attracted attention because it can embody some important information about the derivation of the past state [21, 22]. In [23, 24], the authors considered asymptotic stability of BAM neural networks of neutral type. Lakshmanan and Balasubramaniam [25] obtained some new results of robust stability analysis for neutral-type neural networks with time-varying delays and Markovian jumping parameters. However, the stability analysis for genetic regulatory networks with neutral delay has been rarely investigated. In [26], Jung et al. discussed the stability for genetic regulatory network with neutral delay. Further, Jiao et al. [27] investigated the asymptotic stability of genetic regulatory network with neutral delay, which extended and improved the main results of Jung et al. [26]. On the other hand, leakage delay is another type of delay, which exists in the negative feedback term of system [28, 29]. So far, few studies have been paid to the stability analysis for GRNs with leakage delay. In [30], the authors discussed the global asymptotic stability for genetic regulatory networks with leakage delay. In [31], stability was investigated for a class of uncertain impulsive stochastic genetic regulatory networks with time-varying leakage delay. As far as we know, there is no result on the stability of genetic regulatory networks with neutral delays and leakage delays. Motivated by the discussion above, in this paper, we consider the stability of genetic regulatory networks of neutral type with leakage delay.

The paper is organized as follows. In Section 2, the problem is formulated and some preliminaries and assumption are given. In Section 3, the asymptotic stability of neutral-type genetic regulatory networks with leakage delay is investigated. In Section 4, an illustrative example is given to show the effectiveness of the theoretical results. Some conclusions are proposed in Section 5.

Notations. Throughout this paper, R^n and $R^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively. A^T denotes the transposition of matrix A. Diag $(\cdot)$ denotes the diagonal matrix. Asterisk $(*)$ represents the symmetric block of the symmetric matrix.

2. Preliminaries

GRNs with n mRNA and n proteins can be modelled by the following differential equations:

$$\begin{aligned} \dot{m}_i(t) &= -a_i m_i(t) + \sum_{j=1}^{n} \omega_{ij} g_j\left(p_j(t-\sigma(t))\right) + I_i, \\ \dot{p}_i(t) &= -c_i p_i(t) + d_i m_i\left(t-\tau_1(t)\right), \end{aligned} \tag{1}$$

where $m_i(t)$ and $p_i(t)$ denote the concentrations of mRNA and protein of the ith node at time t, respectively. a_i and c_i are the degradation rates of the mRNA and protein, respectively. d_i is the translation rate, $\sigma(t)$ is the feedback regulation delays, and $\tau(t)$ is the translation delays satisfying $0 \le \sigma(t) \le \sigma$, $0 \le \tau_1(t) \le \tau_1$, where σ and τ are constants. Consider $g_j(p_j(s)) = (p_j(s)/\beta_j)^{H_j}/(1+p_j(s)/\beta_j)^{H_j}$, where H_j is the Hill coefficient. β_j is a positive constant which denotes the feedback regulation of the protein on the transcription. The coupling matrix $W = (\omega_{ij})_{n\times n} \in R^{n\times n}$ of the GRNs is defined as follows:

$$\omega_{ij} = \begin{cases} \alpha_{ij}, & \text{if transcription factor } j \text{ is an activator of gene } i, \\ 0, & \text{if there is no link from } j \text{ to } i, \\ -\alpha_{ij}, & \text{if transcription factor } j \text{ is a repressor of gene } i. \end{cases} \tag{2}$$

Let $(m^*, p^*)^T$ be an equilibrium point of (1). The equilibrium point can be shifted to the origin by transformation: $x_i(t) = m_i(t) - m_i^*$, $y_i(t) = p_i(t) - p_i^*$. System (1) can be changed into the following compact matrix form:

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + Wf\left(y(t-\sigma(t))\right), \\ \dot{y}(t) &= -Cy(t) + D_1 x\left(t-\tau_1(t)\right), \end{aligned} \tag{3}$$

where $A = \text{diag}[a_1, a_2, \ldots, a_n], C = \text{diag}[c_1, c_2, \ldots, c_n], D_1 = \text{diag}[d_1, d_2, \ldots, d_n]$, and $f(y(t)) = g(y(t) + p^*) - g(p^*)$ with $f(0) = 0$.

In this paper, we will consider the GRNs with neutral delays and leakage delays:

$$\begin{aligned} \dot{x}(t) &= -Ax\left(t-\rho_1\right) + Wf\left(y(t-\sigma(t))\right), \\ \dot{y}(t) &= -Cy\left(t-\rho_2\right) + D_1 x\left(t-\tau_1(t)\right) + D_2 \dot{x}\left(t-\tau_2(t)\right), \end{aligned} \tag{4}$$

where $\rho_1 > 0$ and $\rho_2 > 0$ denote the leakage delays. $\tau_2(t)$ is the neutral delay and satisfies $0 \le \tau_2(t) \le \tau_2$, $\dot{\tau}_2(t) \le \tau^*$.

The system (4) has the following equivalent form:

$$\begin{aligned} &\frac{d}{dt}\left[x(t) - A\int_{t-\rho_1}^{t} x(\mu)\, d\mu\right] = -Ax(t) + Wf\left(y(t-\sigma(t))\right), \\ &\frac{d}{dt}\left[y(t) - C\int_{t-\rho_2}^{t} y(\mu)\, d\mu\right] \\ &\quad = -Cy(t) + D_1 x\left(t-\tau_1(t)\right) + D_2 \dot{x}\left(t-\tau_2(t)\right). \end{aligned} \tag{5}$$

The initial conditions are given as follows:

$$\begin{aligned} x_i(s) &= \phi_i(s), \quad s \in (-\tau, 0), \\ y_i(s) &= \psi_i(s), \quad s \in (-\sigma, 0), \end{aligned} \tag{6}$$

where $\tau = \max\{\rho_1, \sigma\}$ and $\sigma = \max\{\rho_2, \tau_1, \tau_2\}$.

In this paper, we introduce the following assumption and lemmas.

Assumption 1. There exist constants k_i^-, k_i^+ such that the regulatory function $f_i(\cdot)$ satisfies

$$k_i^- \le \frac{f_i(x) - f_i(y)}{x - y} \le k_i^+, \tag{7}$$

for all $x, y \in \mathbb{R}$, $x \ne y$ and $i = 1, 2, \ldots, n$.

Remark 2. In the above assumption, k_i^-, k_i^+ are some real constants, which may be positive, zero, or negative. Compared with the existing results in [32], this is less conservative and less restrictive.

The following lemmas will be used to derive our main results.

Lemma 3 (see [33]). *Given any real matrix of appropriate dimension $M = M^T > 0$ and a vector function $\omega(\cdot) : [a, b] \to \mathbb{R}^n$, such that the integrations concerned are well defined, then*

$$\begin{aligned} &\left[\int_a^b \omega(s)\, ds\right]^T M \left[\int_a^b \omega(s)\, ds\right] \\ &\quad \le (b-a)\int_a^b \omega^T(s) M \omega(s)\, ds. \end{aligned} \tag{8}$$

Lemma 4 (see [34, 35]). *For any diagonal matrices $\Lambda > 0$, if Assumption 1 holds, then*

$$\begin{bmatrix} y(t) \\ f(y(t)) \end{bmatrix}^T \begin{bmatrix} -\Lambda L & \Lambda K \\ * & -\Lambda \end{bmatrix} \begin{bmatrix} y(t) \\ f(y(t)) \end{bmatrix} \geq 0, \quad (9)$$

$$\begin{bmatrix} y(t-\sigma(t)) \\ f(y(t-\sigma(t))) \end{bmatrix}^T \begin{bmatrix} -\Lambda L & \Lambda K \\ * & -\Lambda \end{bmatrix} \begin{bmatrix} y(t-\sigma(t)) \\ f(y(t-\sigma(t))) \end{bmatrix} \geq 0, \quad (10)$$

where $K = \operatorname{diag}(k_1^- k_1^+, \ldots, k_n^- k_n^+)$ *and* $L = \operatorname{diag}((k_1^- + k_1^+)/2, \ldots, (k_n^- + k_n^+)/2)$.

3. Main Results

In this section, the asymptotic stability of system (5) is investigated. Some sufficient conditions are obtained to ensure the stability of system (5) based on the Lyapunov functional approach.

Theorem 5. *Let Assumption 1 hold. Then the trivial solution of system (5) is asymptotically stable if there exist symmetric positive definite matrices P, Q, symmetric matrices P_i, Q_i ($i = 1, 2$), diagonal positive definite matrix Q_3, and positive definite matrices Q_4, Q_5, Q_6 such that the following LMIs hold:*

$$\prod = \left(\Pi_{i,j}\right)_{16\times 16} < 0, \quad (11)$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ * & S_{22} & S_{23} \\ * & * & S_{33} \end{bmatrix} > 0, \quad (12)$$

$$\begin{bmatrix} T_{11} & T_{12} \\ * & S_{22} \end{bmatrix} > 0, \quad (13)$$

$$\begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} > 0, \quad (14)$$

where

$$\begin{gathered}
\Pi_{1,1} = -PA - AP + P_1 + \rho_1^2 P_2 + R_1 + \tau_2 R_3, \\
\Pi_{1,4} = T_{12}^T, \qquad \Pi_{1,6} = APA, \\
\Pi_{1,12} = PW, \qquad \Pi_{1,14} = -R_{12}, \\
\Pi_{2,2} = \tau_1 T_{22} + Q_3 - Q_4 - Q_4^T + R_1 + \tau_2 R_{22}, \\
\Pi_{2,3} = -Q_4 A, \qquad \Pi_{2,5} = R_{12}^T, \qquad \Pi_{2,12} = Q_4 W, \\
\Pi_{3,3} = -P_1, \qquad \Pi_{4,4} = -T_{12} - T_{12}^T + \tau_1 T_{11}, \\
\Pi_{4,5} = -D_1^T Q_6^T D_2, \qquad \Pi_{4,7} = D_1 Q^T, \\
\Pi_{4,8} = D_1^T Q_5^T, \qquad \Pi_{4,13} = -D_1^T QC, \\
\Pi_{5,5} = (\tau^* - 1) Q_3 - D_2^T Q_6 D_2 - D_2^T Q_6^T D_2 \\
+ \tau_2 R_{11} - R_{12} - R_{12}^T, \\
\Pi_{5,7} = D_2^T Q^T, \qquad \Pi_{5,8} = D_2^T Q_5^T + D_2^T Q_6, \\
\Pi_{5,9} = D_2^T Q_6 C, \qquad \Pi_{5,13} = -D_2^T QC, \\
\Pi_{6,6} = -P_2, \qquad \Pi_{6,12} = -APW, \\
\Pi_{7,7} = -QC - CQ + Q_1 + \rho_2^2 Q_2 - \Lambda_1 L, \\
\Pi_{7,10} = S_{13}^T, \qquad \Pi_{7,11} = \Lambda_1 K, \qquad \Pi_{7,12} = S_{23}^T, \\
\Pi_{7,13} = CQC, \qquad \Pi_{8,8} = \sigma S_{33} - Q_5 - Q_5^T, \\
\Pi_{8,9} = -Q_5 C, \qquad \Pi_{9,9} = -Q_1, \\
\Pi_{10,10} = \sigma S_{11} - S_{13} - \Lambda_2 L, \\
\Pi_{10,12} = \sigma S_{12} - S_{23}^T + \Lambda_2 K, \\
\Pi_{11,11} = -\Lambda_1, \qquad \Pi_{12,12} = \sigma S_{22} - \Lambda_2, \\
\Pi_{13,13} = -Q_2, \qquad \Pi_{14,14} = -R_1, \\
\Pi_{15,15} = -R_2, \qquad \Pi_{16,16} = -\frac{R_3}{\tau_2}.
\end{gathered} \quad (15)$$

Proof. Consider the following Lyapunov-Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \quad (16)$$

where

$$\begin{aligned}
V_1(t) = & \left[x(t) - A\int_{t-\rho_1}^{t} x(\mu)\, d\mu\right]^T \\
& \times P\left[x(t) - A\int_{t-\rho_1}^{t} x(\mu)\, d\mu\right] \\
& + \left[y(t) - C\int_{t-\rho_2}^{t} y(\mu)\, d\mu\right]^T \\
& \times Q\left[y(t) - C\int_{t-\rho_2}^{t} y(\mu)\, d\mu\right], \\
V_2(t) = & \int_{t-\rho_1}^{t} x^T(s) P_1 x(s)\, ds \\
& + \rho_1 \int_{t-\rho_1}^{t} \int_{s}^{t} x^T(\mu) P_2 x(\mu)\, d\mu\, ds \\
& + \int_{t-\rho_2}^{t} y^T(s) Q_1 y(s)\, ds \\
& + \rho_2 \int_{t-\rho_2}^{t} \int_{s}^{t} y^T(\mu) Q_2 y(\mu)\, d\mu\, ds, \\
V_3(t) = & \int_0^t \int_{\mu-\sigma(\mu)}^{\mu} \begin{bmatrix} y(\mu-\sigma(\mu)) \\ f(y(\mu-\sigma(\mu))) \\ \dot{y}(s) \end{bmatrix}^T \\
& \times \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ * & S_{22} & S_{23} \\ * & * & S_{33} \end{bmatrix}
\end{aligned}$$

$$
\begin{aligned}
&\times \begin{bmatrix} y(\mu-\sigma(\mu)) \\ f(y(\mu-\sigma(\mu))) \\ \dot{y}(s) \end{bmatrix} ds\, d\mu \\
&+\int_0^t \int_{\mu-\tau_1(\mu)}^{\mu} \begin{bmatrix} x(\mu-\tau_1(\mu)) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} T_{11} & T_{12} \\ * & S_{22} \end{bmatrix} \\
&\times \begin{bmatrix} x(\mu-\tau_1(\mu)) \\ \dot{x}(s) \end{bmatrix} ds\, d\mu \\
&+\int_0^t \int_{\mu-\tau_2(\mu)}^{\mu} \begin{bmatrix} \dot{x}(\mu-\tau_2(\mu)) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} \\
&\times \begin{bmatrix} \dot{x}(\mu-\tau_2(\mu)) \\ \dot{x}(s) \end{bmatrix} ds\, d\mu, \\
V_4(t) &= \int_{-\sigma}^{0} \int_{t+\mu}^{t} \dot{y}^T(s) S_{33} \dot{y}(s)\, ds\, d\mu \\
&+ \int_{-\tau_1}^{0} \int_{t+\mu}^{t} \dot{x}^T(s) T_{22} \dot{x}(s)\, ds\, d\mu, \\
V_5(t) &= \int_{t-\tau_2}^{t} \left[x^T(s) R_1 x(s) + \dot{x}^T(s) R_2 \dot{x}(s) \right] ds \\
&+ \int_{-\tau_2}^{0} \int_{t+\mu}^{t} \left[x^T(s) R_3 x(s) \right. \\
&\left. + \dot{x}^T(s) R_{22} \dot{x}(s) \right] ds\, d\mu \\
&+ \int_{t-\tau_2(t)}^{t} \dot{x}^T(s) Q_3 \dot{x}(s)\, ds.
\end{aligned}
\tag{17}
$$

Calculating the derivative of $V(t)$ along with the solution of the system (5), we have

$$
\begin{aligned}
\dot{V}_1(t) &= 2\left[x(t) - A\int_{t-\rho_1}^{t} x(\mu)\, d\mu \right]^T \\
&\times P\left[-Ax(t) + Wf(y(t-\sigma(t))) \right] \\
&+ 2\left[y(t) - C\int_{t-\rho_2}^{t} y(\mu)\, d\mu \right]^T \\
&\times \left[-Cy(t) + D_1 x(t-\tau_1(t)) + D_2 \dot{x}(t-\tau_2(t)) \right] \\
&= -2x^T(t) PAx(t) + 2x^T(t) PWf(y(t-\sigma(t))) \\
&+ 2x^T(t) APA\left(\int_{t-\rho_1}^{t} x(\mu)\, d\mu \right) \\
&- 2\left(\int_{t-\rho_1}^{t} x(\mu)\, d\mu \right)^T APWf(y(t-\sigma(t))) \\
&- 2y^T(t) QCy(t) + 2x^T(t-\tau_1(t)) D_1 Q^T y(t) \\
&+ 2\dot{x}(t-\tau_2(t)) D_2^T Q^T y(t) \\
&+ 2y^T(t) CQC\left(\int_{t-\rho_2}^{t} y(\mu)\, d\mu \right) \\
&- 2\left(\int_{t-\rho_2}^{t} y(\mu)\, d\mu \right)^T CQD_1 x(t-\tau_1(t)) \\
&- 2\dot{x}(t-\tau_2(t)) D_2^T QC\left(\int_{t-\rho_2}^{t} y(\mu)\, d\mu \right), \\
\dot{V}_2(t) &= x^T(t) P_1 x(t) - x^T(t-\rho_1) P_1 x(t-\rho_1) \\
&- \rho_1 \int_{t-\rho_1}^{t} x^T(\mu) P_2 x(\mu)\, d\mu + \rho_1^2 x^T(t) P_2 x(t) \\
&+ y^T(t) Q_1 y(t) - y^T(t-\rho_2) Q_1 y(t-\rho_2) \\
&- \rho_2 \int_{t-\rho_2}^{t} y^T(\mu) Q_2 y(\mu)\, d\mu + \rho_2^2 y^T(t) Q_2 y(t) \\
&\le x^T(t) P_1 x(t) - x^T(t-\rho_1) P_1 x(t-\rho_1) \\
&- \left[\int_{t-\rho_1}^{t} x(\mu)\, d\mu \right]^T P_2 \left[\int_{t-\rho_1}^{t} x(\mu)\, d\mu \right] \\
&+ \rho_1^2 x^T(t) P_2 x(t) + y^T(t) Q_1 y(t) \\
&- y^T(t-\rho_2) Q_1 y(t-\rho_2) \\
&- \left[\int_{t-\rho_2}^{t} y(\mu)\, d\mu \right]^T Q_2 \left[\int_{t-\rho_2}^{t} y(\mu)\, d\mu \right] \\
&+ \rho_2^2 y^T(t) Q_2 y(t), \\
\dot{V}_3(t) &= \int_{\mu-\sigma(t)}^{t} \begin{bmatrix} y(t-\sigma(t)) \\ f(y(t-\sigma(t))) \\ \dot{y}(s) \end{bmatrix}^T \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ * & S_{22} & S_{23} \\ * & * & S_{33} \end{bmatrix} \\
&\times \begin{bmatrix} y(t-\sigma(t)) \\ f(y(t-\sigma(t))) \\ \dot{y}(s) \end{bmatrix} ds \\
&+ \int_{t-\tau_1(t)}^{t} \begin{bmatrix} x(t-\tau_1(t)) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} T_{11} & T_{12} \\ * & S_{22} \end{bmatrix} \\
&\times \begin{bmatrix} x(t-\tau_1(t)) \\ \dot{x}(s) \end{bmatrix} ds \\
&+ \int_{t-\tau_2(t)}^{t} \begin{bmatrix} \dot{x}(t-\tau_2(t)) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ * & R_{22} \end{bmatrix} \\
&\times \begin{bmatrix} \dot{x}(t-\tau_2(t)) \\ \dot{x}(s) \end{bmatrix} ds \\
&\le \tau_1 x^T(t-\tau_1(t)) T_{11} x(t-\tau_1(t)) \\
&+ 2x^T(t) T_{12}^T x(t-\tau_1(t)) \\
&- 2x^T(t-\tau_1(t)) T_{12}^T x(t-\tau_1(t))
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{t-\tau_1}^{t} \dot{x}^T(s) T_{22}\dot{x}(s)\,ds \\
&+\tau_2\dot{x}^T(t-\tau_2(t)) R_{11}\dot{x}(t-\tau_2(t)) \\
&+2\dot{x}^T(t) R_{12}^T\dot{x}(t-\tau_2(t)) \\
&-2\dot{x}^T(t-\tau_2(t)) R_{12}^T\dot{x}(t-\tau_2(t)) \\
&+\int_{t-\tau_2}^{t} \dot{x}^T(s) R_{22}\dot{x}(s)\,ds \\
&+\sigma y^T(t-\sigma(t)) S_{11} y(t-\sigma(t)) \\
&+\sigma f^T(y(t-\sigma(t))) S_{22} f(y(t-\sigma(t))) \\
&+\int_{t-\sigma}^{t} \dot{y}^T(s) S_{33}\dot{y}(s)\,ds \\
&+\sigma y^T(t-\sigma(t)) S_{12} f(y(t-\sigma(t))) \\
&+y^T(t-\sigma(t)) S_{13}[y(t)-y(t-\sigma(t))] \\
&+f^T(y(t-\sigma(t))) S_{23}[y(t)-y(t-\sigma(t))],
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_4(t) &= \sigma\dot{y}^T(t) S_{33}\dot{y}(t) - \int_{-\sigma}^{0} \dot{y}^T(t+t) S_{33}\dot{y}(t+t)\,dt \\
&\quad+\tau_1\dot{x}^T(t) T_{22}\dot{x}(t) - \int_{-\tau_1}^{0} \dot{x}^T(t+t) T_{22}\dot{x}(t+t)\,dt \\
&= \sigma\dot{y}^T(t) S_{33}\dot{y}(t) - \int_{t-\sigma}^{t} \dot{y}^T(s) S_{33}\dot{y}(s)\,ds \\
&\quad+\tau_1\dot{x}^T(t) T_{22}\dot{x}(t) - \int_{t-\tau_1}^{t} \dot{x}^T(s) T_{22}\dot{x}(s)\,ds,
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_5(t) &= x^T(t) R_1 x(t) + \dot{x}^T(t) R_1\dot{x}(t) \\
&\quad- x^T(t-\tau_2) R_1 x(t-\tau_2) \\
&\quad- \dot{x}^T(t-\tau_2) R_2\dot{x}(t-\tau_2) \\
&\quad+ \tau_2 x^T(t) R_3 x(t) + \tau_2\dot{x}^T(t) R_{22}\dot{x}(t) \\
&\quad- \int_{t-\tau_2}^{t} x^T(s) R_3 x(s)\,ds \\
&\quad- \int_{t-\tau_2}^{t} \dot{x}^T(s) R_{22}\dot{x}(s)\,ds \\
&\quad+ \dot{x}^T(t) Q_3\dot{x}(t) - \dot{x}^T(t-\tau_2(t)) \\
&\quad\times Q_3\dot{x}(t-\tau_2(t))[1-\dot{\tau}_2(t)] \\
&\le x^T(t) R_1 x(t) + \dot{x}^T(t) R_1\dot{x}(t) \\
&\quad- x^T(t-\tau_2) R_1 x(t-\tau_2) \\
&\quad- \dot{x}^T(t-\tau_2) R_2\dot{x}(t-\tau_2) \\
&\quad+ \tau_2 x^T(t) R_3 x(t) + \tau_2\dot{x}^T(t) R_{22} \\
&\quad- \frac{1}{\tau_2}\left(\int_{t-\tau_2}^{t} x(s)\,ds\right)^T R_3\left(\int_{t-\tau_2}^{t} x(s)\,ds\right) \\
&\quad- \int_{t-\tau_2}^{t} \dot{x}^T(s) R_{22}\dot{x}(s)\,ds + \dot{x}^T(t) Q_3\dot{x}(t) \\
&\quad+ \dot{x}^T(t-\tau_2(t)) Q_3\dot{x}(t-\tau_2(t))(\tau^*-1).
\end{aligned} \tag{18}
$$

In addition, note that

$$
\begin{aligned}
0 &= 2\dot{x}^T(t) Q_4[-\dot{x}(t)+\dot{x}(t)] \\
&= 2\dot{x}^T(t) Q_4[-\dot{x}(t) - Ax(t-\rho_1) + Wf(y(t-\sigma(t)))] \\
&= -2\dot{x}^T(t) Q_4\dot{x}(t) - 2\dot{x}^T(t) Q_4 Ax(t-\rho_1) \\
&\quad+ 2\dot{x}^T(t) Q_4 Wf(y(t-\sigma(t))),
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
0 &= 2\dot{y}^T(t) Q_5[-\dot{y}(t)+\dot{y}(t)] \\
&= 2\dot{y}^T(t) Q_5[-\dot{y}(t) - Cy(t-\rho_2) \\
&\qquad + D_1 x(t-\tau_1(t)) + D_2\dot{x}(t-\tau_2(t))] \\
&= -2\dot{y}^T(t) Q_5\dot{y}(t) - 2\dot{y}^T(t) Q_5 Cy(t-\rho_2) \\
&\quad+ 2x^T(t-\tau_1(t)) D_1 Q_5^T\dot{y}(t) \\
&\quad+ 2\dot{x}^T(t-\tau_2(t)) D_2^T Q_5^T\dot{y}(t),
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
0 &= 2\dot{x}^T(t-\tau_2(t)) D_2^T Q_6 \\
&\quad\times[-D_2\dot{x}(t-\tau_2(t)) + D_2\dot{x}(t-\tau_2(t))] \\
&= 2\dot{x}^T(t-\tau_2(t)) D_2^T Q_6 \\
&\quad\times[-D_2\dot{x}(t-\tau_2(t)) + \dot{y}(t) + Cy(t-\rho_2) \\
&\qquad -D_1 x(t-\tau_1(t))] \\
&= -2\dot{x}^T(t-\tau_2(t)) D_2^T Q_6 D_2\dot{x}(t-\tau_2(t)) \\
&\quad+ 2\dot{x}^T(t-\tau_2(t)) D_2^T Q_6\dot{y}(t) \\
&\quad+ 2\dot{x}^T(t-\tau_2(t)) D_2^T Q_6 Cy(t-\rho_2) \\
&\quad- 2x^T(t-\tau_1(t)) D_1^T Q_6^T D_2\dot{x}^T(t-\tau_2(t)).
\end{aligned} \tag{21}
$$

From (9) to (21), we have

$$
\dot{V}(t) \le \xi^T(t)\prod\xi(t), \tag{22}
$$

where $\prod$ is defined in (11) and

$$\begin{aligned}\xi^T(t) = \Big[& x^T(t), \dot{x}^T(t), x^T(t-\rho_1), x^T(t-\tau_1(t)), \\ & x^T(t-\tau_2(t)), \left(\int_{t-\rho_1}^{t} x(\mu)\, d\mu\right)^T, \\ & y^T(t), \dot{y}^T(t), y^T(t-\rho_2), y^T(t-\sigma(t)), \\ & f^T(y(t)), f^T(y(t-\sigma(t))), \\ & \left(\int_{t-\rho_2}^{t} y(\mu)\, d\mu\right)^T, x^T(t-\tau_2), \dot{x}^T(t-\tau_2), \\ & \left(\int_{t-\tau_2}^{t} x(\mu)\, d\mu\right)^T \Big]\end{aligned} \tag{23}$$

which implies $\dot{V}(t) \leq 0$. It is easy to prove $\dot{V}(t) \leq \lambda_{\min}(\prod)\xi^T(t)\xi(t) \leq \lambda_{\min}(\prod)[x^T(t)x(t) + y^T(t)y(t)]$, where $\lambda_{\min}(\prod) < 0$. According to the Lyapunov stability theory, the GRNs (5) are asymptotically stable. The proof is complete. □

Remark 6. If the leakage delay $\rho_1 = \rho_2 = 0$, the model is the same as that in [27]. We can also deduce the sufficient conditions for guaranteeing system stability. So the model in [27] is a special case of the model in this paper. In fact, the leakage delay has a large effect on the dynamic behaviors of the model. It often leads to the instability. This phenomenon can be observed in Figure 1. Considering the impact of leakage delay, we analyze GRNs with neutral delays and leakage delays simultaneously.

4. Simulation Results

In this section, an example is given to demonstrate the feasibility and efficiency of the theoretical results above. The simulation is conducted on MATLAB, and the ordinary differential equation is solved by Runge-Kutta method.

Example 7. Consider the following neutral-type genetic regulatory networks with leakage delays:

$$\begin{aligned}\dot{x}(t) &= -Ax(t-\rho_1) + Wf(y(t-\sigma(t))), \\ \dot{y}(t) &= -Cy(t-\rho_2) + D_1 x(t-\tau_1(t)) + D_2 \dot{x}(t-\tau_2(t)),\end{aligned} \tag{24}$$

where

$$A = \begin{bmatrix} 6 & 0 \\ 0 & 7 \end{bmatrix}, \qquad W = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 8 & 0 \\ 0 & 0.8 \end{bmatrix}, \qquad f(y) = \frac{y^2}{1+y^2}. \tag{25}$$

Then we have

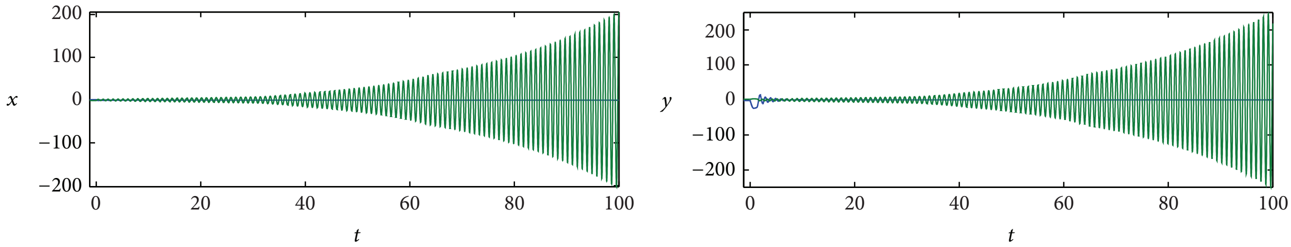
$$\begin{gathered} k_i^+ = 0.65, \qquad k_i^- = 0, \\ K = \operatorname{diag}[0.325, 0.325], \qquad L = 0. \end{gathered} \tag{26}$$

Time-varying delays and the leakage delays are chosen as

$$\tau_1 = 0.995, \qquad \tau_2 = 1, \qquad \rho_1 = \rho_2 = 0.01, \qquad \sigma = 0.01. \tag{27}$$

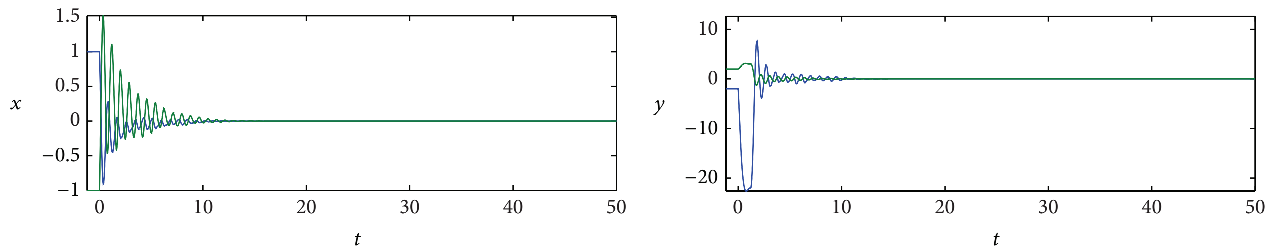
By using the MATLAB LMI Toolbox, we obtain

$$\begin{gathered}
P = \begin{bmatrix} 0.3039 & 0.0035 \\ 0.0035 & 0.1728 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.0011 & 0.0000 \\ 0.0000 & 0.0544 \end{bmatrix}, \\
P_1 = \begin{bmatrix} 1.4529 & 0.0114 \\ 0.0114 & 0.9774 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 64.7658 & 0.5258 \\ 0.5258 & 59.4535 \end{bmatrix}, \\
Q_1 = \begin{bmatrix} 0.0007 & 0.0001 \\ 0.0001 & 0.0627 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 4.2036 & 0.0059 \\ 0.0059 & 17.3389 \end{bmatrix}, \\
Q_3 = \begin{bmatrix} 0.0100 & 0 \\ 0 & 0.0100 \end{bmatrix}, \qquad Q_4 = \begin{bmatrix} 0.0397 & 0.0007 \\ 0.0011 & 0.0178 \end{bmatrix}, \\
Q_5 = \begin{bmatrix} 0.0001 & 0.0000 \\ 0.0000 & 0.0100 \end{bmatrix}, \qquad Q_6 = \begin{bmatrix} 0.0001 & 0.0000 \\ 0.0000 & 0.0101 \end{bmatrix}, \\
T_{11} = \begin{bmatrix} 0.1108 & 0.0002 \\ 0.0002 & 0.0870 \end{bmatrix}, \qquad T_{12} = \begin{bmatrix} 0.2207 & 0.0005 \\ 0.0005 & 0.1803 \end{bmatrix}, \\
T_{22} = \begin{bmatrix} 0.0022 & 0.0001 \\ 0.0001 & 0.0011 \end{bmatrix}, \qquad S_{11} = \begin{bmatrix} 0.0084 & 0.0005 \\ 0.0005 & 0.7474 \end{bmatrix}, \\
S_{12} = \begin{bmatrix} -25.6827 & -0.0276 \\ -1.7215 & -21.2464 \end{bmatrix}, \\
S_{13} = \begin{bmatrix} 0.0003 & -0.0000 \\ -0.0000 & 0.0344 \end{bmatrix}, \\
S_{22} = \begin{bmatrix} 10.4912 & -1.0149 \\ -1.0149 & 9.3772 \end{bmatrix}, \\
S_{23} = \begin{bmatrix} 0.0097 & -0.0195 \\ -0.0004 & -0.1500 \end{bmatrix}, \\
S_{33} = \begin{bmatrix} 0.0018 & 0.0002 \\ 0.0002 & 0.1414 \end{bmatrix}, \qquad R_{11} = \begin{bmatrix} 0.1961 & 0.0029 \\ 0.0029 & 0.1490 \end{bmatrix}, \\
R_{12} = \begin{bmatrix} 0.0188 & 0.0002 \\ 0.0004 & 0.0048 \end{bmatrix}, \qquad R_{22} = \begin{bmatrix} 0.2200 & 0.0077 \\ 0.0077 & 0.1084 \end{bmatrix}, \\
R_1 = \begin{bmatrix} 0.0044 & 0.0001 \\ 0.0001 & 0.0014 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 12.3518 & 0 \\ 0 & 12.3518 \end{bmatrix}, \\
R_3 = \begin{bmatrix} 3.4620 & 0.0037 \\ 0.0037 & 3.1125 \end{bmatrix}, \qquad \Lambda_1 = \begin{bmatrix} 0.0084 & 0 \\ 0 & 0.0084 \end{bmatrix}, \\
\Lambda_2 = \begin{bmatrix} 0.7930 & 0 \\ 0 & 0.7930 \end{bmatrix}.
\end{gathered} \tag{28}$$



(a) (b)

Figure 1: State trajectory of the system with $\rho_1 = \rho_2 = 0.24$ for Example 7 with two nodes.



(a) (b)

Figure 2: State trajectory of the system with $\rho_1 = \rho_2 = 0.13$ for Example 7 with two nodes.

Table 1: Effect of leakage delay on the stability.

ρ (leakage delay)	Condition	Stability
$0 < \rho \leq 0.229$	Satisfied	Stable
$0.229 < \rho \leq 0.239$	Dissatisfied	Stable
$\rho \geq 0.240$	Dissatisfied	Unstable

This means that all conditions in Theorem 5 are satisfied. Hence, the GRN model (24) is asymptotically stable. On the other hand, we have the following simulation results shown in Figure 2, which also show the correctness of the theoretic analysis.

Remark 8. In order to reflect the effect of the leakage delays on the stability of GRNs, we have given the simulation results by using different leakage delays. Table 1 shows the conclusions when the leakage delays are changed. When leakage delays specially $\rho_1 = \rho_2 = 0.23$, the conditions in Theorem 5 are not satisfied. However, the system is still stable. Thus, there still exist rooms for improving the sufficient conditions in Theorem 5.

5. Conclusion

In this paper, we have investigated the stability of neural-type GRNs with leakage delays. Based on Lyapunov stability theory and linear matrix inequalities, some sufficient conditions have been obtained to guarantee stability for GRNs. It is noted that the leakage delay and neutral delay are considered together in the paper, which extended and improved the previous results in [27, 30]. Finally, a numerical simulation has been given to show the effectiveness of our theoretic results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was jointly supported by the National Natural Science Foundation of China under Grant 11226116, the Foundation of Key Laboratory of Advanced Process Control for Light Industry (Jiangnan University), Ministry of Education, China, and the Fundamental Research Funds for the Central Universities (JUSRP51317B, JUDCF13042).

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