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Research Article

Fixed Point Theorems for Ćirić-Berinde Type Contractive Multivalued Mappings

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We give a Ćirić-Berinde type contractive condition for multivalued mappings and analyze the existence of fixed point for these mappings.

1. Introduction and Preliminaries

In 2012, Samet et al. [1] introduced the notions of α-ψ-contractive mapping and α-admissible mappings in metric spaces and obtained corresponding fixed point results, which are generalizations of ordered fixed point results (see [1]). Since then, by using their idea, some authors investigated fixed point results in the field. Asl et al. [2] extended some of results in [1] to multivalued mappings by introducing the notions of α_*-ψ-contractive mapping and α_*-admissible mapping.

Recently, Salimi et al. [3] modified the notions of α-ψ-contractive mapping and α-admissible mappings by introducing another function η. And then, they gave generalizations of the results of Samet et al. [1] and Karapınar and Samet [4]. Hussain et al. [5] extended these modified notions to multivalued mappings. That is, they introduced the notion of α-η-contractive multifunctions and gave fixed point results for these multifunctions.

Very recently, Ali et al. [6] generalized and extended the notion of α-ψ-contractive mapping by introducing the notion of (α, ψ, ξ)-contractive multivalued mappings and obtained fixed point theorems for these mappings in complete metric spaces.

The purpose of this paper is to introduce the notion of Ćirić-Berinde type contractive multivalued mappings and to generalize and extend the notion of α-η-contractive multifunctions and to establish fixed point theorems for Ćirić-Berinde type contractive multivalued mappings.

Let (X, d) be a metric space. We denote by $CB(X)$ the class of nonempty closed and bounded subsets of X and by $CL(X)$ the class of nonempty closed subsets of X. Let $H(\cdot,\cdot)$ be the generalized Hausdorff distance on $CL(X)$; that is, for all $A, B \in CL(X)$,

$$H(A,B) = \begin{cases} \max\left\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(b,A)\right\}, & \text{if the maximum exists,} \\ \infty, & \text{otherwise,} \end{cases} \tag{1}$$

where $d(a, B) = \inf\{d(a, b) : b \in B\}$ is the distance from point a to subset B.

For $A, B \in CL(X)$, let $D(A, B) = \sup_{x\in A}\inf_{y\in B} d(x, y)$.

Then, we have $D(A, B) \le H(A, B)$ for all $A, B \in CL(X)$.

From now on, we denote by

$$M(x,y) = \max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{1}{2}\{d(x,Ty) + d(y,Tx)\}\right\} \tag{2}$$

for a multivalued map $T : X \to CL(X)$ and $x, y \in X$.

We denote by Ξ the class of all functions $\xi : [0,\infty) \to [0,\infty)$ such that

(1) ξ is continuous;

(2) ξ is nondecreasing on $[0,\infty)$;

(3) $\xi(t) = 0$ if and only if $t = 0$;

(4) ξ is subadditive.

Also, we denote by Ψ the family of all nondecreasing functions $\psi : [0,\infty) \to [0,\infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$, where ψ^n is the nth iterate of ψ.

Note that if $\psi \in \Psi$, then $\psi(0) = 0$ and $0 < \psi(t) < t$ for all $t > 0$.

Let (X, d) be a metric space, and let $\alpha : X \times X \to [0,\infty)$ be a function.

We consider the following conditions:

(1) for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} x_n = x$, we have

$$\alpha(x_n, x) \geq 1 \quad \forall n \in \mathbb{N}; \tag{3}$$

(2) for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and a cluster point x of $\{x_n\}$, we have

$$\lim_{n\to\infty} \inf \alpha(x_n, x) \geq 1; \tag{4}$$

(3) for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and a cluster point x of $\{x_n\}$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that

$$\alpha(x_{n(k)}, x) \geq 1 \quad \forall k \in \mathbb{N}. \tag{5}$$

Remark 1. (1) implies (2) and (2) implies (3).

Note that if (X, d) is a metric space and $\xi \in \Xi$, then $(X, \xi \circ d)$ is a metric space.

Let (X, d) be a metric space, and let $T : X \to CL(X)$ be a multivalued mapping. Then, we say that

(1) T is called *α_*-admissible* [2] if

$$\alpha(x, y) \geq 1 \quad \text{implies } \alpha_*(Tx, Ty) \geq 1, \tag{6}$$

where $\alpha_*(Tx, Ty) = \inf\{\alpha(a, b) : a \in Tx, b \in Ty\}$;

(2) T is called *α-admissible* [7] if, for each $x \in X$ and $y \in Tx$ with $\alpha(x, y) \geq 1$, we have $\alpha(y, z) \geq 1$ for all $z \in Ty$.

Lemma 2. *Let (X, d) be a metric space, and let $T : X \to CL(X)$ be a multivalued mapping. If T is α_*-admissible, then it is α-admissible.*

Proof. Suppose that T is an α_*-admissible mapping.

Let $x \in X$ and $y \in Tx$ be such that $\alpha(x, y) \geq 1$.

Let $z \in Ty$ be given.

Since T is α_*-admissible, $\alpha(y, z) \geq \alpha_*(Tx, Ty) \geq 1$. □

Lemma 3. *Let (X, d) be a metric space, and let $\xi \in \Xi$ and $B \in CL(X)$.*

If $a \in X$ and $\xi(d(a, B)) < c$, then there exists $b \in B$ such that $\xi(d(a, b)) < c$.

Proof. Let $\epsilon = c - \xi(d(a, B))$.

Since $\xi(d(a, B)) < c$ and $\xi \circ d$ is metric on X, there exists $b \in B$ such that $\xi(d(a, b)) < \xi(d(a, B)) + \epsilon$ by definition of infimum. Hence, $\xi(d(a, b)) < c$. □

Let (X, d) be a metric space.

A function $f : X \to [0,\infty)$ is called *upper semicontinuous* if, for each $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x$, we have $\lim_{n\to\infty} f(x_n) \leq f(x)$.

A function $f : X \to [0,\infty)$ is called *lower semicontinuous* if, for each $x \in X$ and $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x$, we have $f(x) \leq \lim_{n\to\infty} f(x_n)$.

For a multivalued map $T : X \to CL(X)$, let $f_T : X \to [0,\infty)$ be a function defined by $f_T(x) = d(x, Tx)$.

2. Fixed Point Theorems

In this section, we establish fixed point theorems for Ćirić-Berinde type contractive multivalued mappings.

Theorem 4. *Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0,\infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible.*

Assume that, for all $x, y \in X, \alpha(x, y) \geq 1$ implies

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)), \tag{7}$$

where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing.

Also, suppose that the following are satisfied:

(1) *there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;*

(2) *either T is continuous or f_T is lower semicontinuous.*

Then T has a fixed point in X.

Proof. Let $x_0 \in X$ and $x_1 \in Tx_0$ be such that $\alpha(x_0, x_1) \geq 1$. Let c be a real number with $\xi(d(x_0, x_1)) < \xi(c)$.

If $x_0 = x_1$, then x_1 is a fixed point.

Let $x_0 \neq x_1$.

If $x_1 \in Tx_1$, then x_1 is a fixed point. Let $x_1 \notin Tx_1$. Then $d(x_1, Tx_1) > 0$.

From (7) we obtain

$$\begin{aligned}
0 &< \xi(d(x_1, Tx_1)) \\
&\leq \xi(H(Tx_0, Tx_1)) \\
&\leq \psi\Big(\xi\Big(\max\Big\{d(x_0, x_1), d(x_0, Tx_0), d(x_1, Tx_1), \\
&\qquad \frac{1}{2}\{d(x_0, Tx_1) + d(x_1, Tx_0)\}\Big\}\Big)\Big) \\
&\quad + L\xi(d(x_1, Tx_0)) \\
&\leq \psi\Big(\xi\Big(\max\Big\{d(x_0, x_1), d(x_0, x_1), d(x_1, Tx_1), \\
&\qquad \frac{1}{2}\{d(x_0, Tx_1) + d(x_1, x_1)\}\Big\}\Big)\Big) \\
&\quad + L\xi(d(x_1, x_1))
\end{aligned}$$

$$\le \psi\left(\xi\left(\max\left\{d\left(x_0,x_1\right),d\left(x_0,x_1\right),d\left(x_1,Tx_1\right),\frac{1}{2}\left\{d\left(x_0,x_1\right)+d\left(x_1,Tx_1\right)\right\}\right\}\right)\right)$$
$$=\psi\left(\xi\left(\max\left\{d\left(x_0,x_1\right),d\left(x_1,Tx_1\right)\right\}\right)\right). \tag{8}$$

If $\max\{d(x_0,x_1),d(x_1,Tx_1)\}=d(x_1,Tx_1)$, then we have $0<\xi(d(x_1,Tx_1))\le\psi(\xi(d(x_1,Tx_1)))<\xi(d(x_1,Tx_1))$, which is a contradiction.

Thus, $\max\{d(x_0,x_1),d(x_1,Tx_1)\}=d(x_0,x_1)$, and hence we have

$$0<\xi\left(d\left(x_1,Tx_1\right)\right)\le\psi\left(\xi\left(d\left(x_0,x_1\right)\right)\right)<\psi\left(\xi\left(c\right)\right). \tag{9}$$

Hence, there exists $x_2\in Tx_1$ such that

$$\xi\left(d\left(x_1,x_2\right)\right)<\psi\left(\xi\left(c\right)\right). \tag{10}$$

Since T is α-admissible, from condition (1) and $x_2\in Tx_1$, we have

$$\alpha\left(x_1,x_2\right)\ge 1. \tag{11}$$

If $x_2\in Tx_2$, then x_2 is a fixed point. Let $x_2\notin Tx_2$.
Then $d(x_2,Tx_2)>0$, and so $\xi(d(x_2,Tx_2))>0$.
From (7) we obtain

$$\begin{aligned}
0&<\xi\left(d\left(x_2,Tx_2\right)\right)\\
&\le\xi\left(H\left(Tx_1,Tx_2\right)\right)\\
&\le\psi\left(\xi\left(\max\left\{d\left(x_1,x_2\right),d\left(x_1,Tx_1\right),d\left(x_2,Tx_2\right),\frac{1}{2}\left\{d\left(x_1,Tx_2\right)+d\left(x_2,Tx_1\right)\right\}\right\}\right)\right)\\
&\quad+L\xi\left(d\left(x_2,Tx_1\right)\right)\\
&\le\psi\left(\xi\left(\max\left\{d\left(x_1,x_2\right),d\left(x_1,x_2\right),d\left(x_2,Tx_2\right),\frac{1}{2}\left\{d\left(x_1,Tx_2\right)+d\left(x_2,x_2\right)\right\}\right\}\right)\right)\\
&\quad+L\xi\left(d\left(x_2,x_2\right)\right)\\
&\le\psi\left(\xi\left(\max\left\{d\left(x_1,x_2\right),d\left(x_1,x_2\right),d\left(x_2,Tx_2\right),\frac{1}{2}\left\{d\left(x_1,x_2\right)+d\left(x_2,Tx_2\right)\right\}\right\}\right)\right)\\
&=\psi\left(\xi\left(\max\left\{d\left(x_1,x_2\right),d\left(x_2,Tx_2\right)\right\}\right)\right).
\end{aligned} \tag{12}$$

If $\max\{d(x_1,x_2),d(x_2,Tx_2)\}=d(x_2,Tx_2)$, then we have $\xi(d(x_2,Tx_2))\le\psi(\xi(d(x_2,Tx_2)))<\xi(d(x_2,Tx_2))$, which is a contradiction.

Thus, $\max\{d(x_1,x_2),d(x_2,Tx_2)\}=d(x_1,x_2)$, and hence we have

$$\xi\left(d\left(x_2,Tx_2\right)\right)\le\psi\left(\xi\left(d\left(x_1,x_2\right)\right)\right)<\psi^2\left(\xi\left(c\right)\right). \tag{13}$$

Hence, there exists $x_3\in Tx_2$ such that

$$\xi\left(d\left(x_2,x_3\right)\right)<\psi^2\left(\xi\left(c\right)\right). \tag{14}$$

Since T is α-admissible, from $x_2\in Tx_1$ and $\alpha(x_1,x_2)\ge 1$, we have

$$\alpha\left(x_2,x_3\right)\ge 1. \tag{15}$$

By induction, we obtain a sequence $\{x_n\}\subset X$ such that, for all $n\in\mathbb{N}\cup\{0\}$,

$$\begin{gathered}\alpha\left(x_n,x_{n+1}\right)\ge 1,\\ x_{n+1}\in Tx_n,\quad x_n\ne x_{n+1},\quad \xi\left(d\left(x_n,x_{n+1}\right)\right)<\psi^n\left(\xi\left(c\right)\right).\end{gathered} \tag{16}$$

Let $\epsilon>0$ be given.
Since $\sum_{n=0}^{\infty}\psi^n(\xi(c))<\infty$, there exists $N\in\mathbb{N}$ such that

$$\sum_{n\ge N}\psi^n\left(\xi\left(c\right)\right)<\xi\left(\epsilon\right). \tag{17}$$

For all $m>n\ge N$, we have

$$\begin{aligned}\xi\left(d\left(x_n,x_m\right)\right)&\le\sum_{k=n}^{m-1}\psi^k\left(\xi\left(c\right)\right)\\ &<\sum_{n\ge N}\psi^n\left(\xi\left(c\right)\right)<\xi\left(\epsilon\right)\end{aligned} \tag{18}$$

which implies $d(x_n,x_m)<\epsilon$ for all $m>n\ge N$. Hence, $\{x_n\}$ is a Cauchy sequence in X.

It follows from the completeness of X that there exists

$$x_*=\lim_{n\to\infty}x_n\in X. \tag{19}$$

Suppose that T is continuous.
We have

$$\begin{aligned}d\left(x_*,Tx_*\right)&\le d\left(x_*,x_{n+1}\right)+d\left(x_{n+1},Tx_*\right)\\ &\le d\left(x_*,x_{n+1}\right)+H\left(x_n,Tx_*\right).\end{aligned} \tag{20}$$

By letting $n\to\infty$ in the above inequality, we obtain $d(x_*,Tx_*)=0$, and so $x_*\in Tx_*$.

Assume that f_T is lower semicontinuous.

Then, $f_T(x_*)\le\lim_{n\to\infty}f_T(x_n)$. Hence, $d(x_*,Tx_*)\le\lim_{n\to\infty}d(x_n,Tx_n)\le\lim_{n\to\infty}d(x_n,x_{n+1})=0$. Thus, $x_*\in Tx_*$. □

Corollary 5. *Let (X,d) be a complete metric space, and let $\alpha:X\times X\to[0,\infty)$ be a function. Suppose that $T:X\to CL(X)$ is an α-admissible mapping.*

Assume that, for all $x,y\in X$,

$$\xi\left(\alpha\left(x,y\right)H\left(Tx,Ty\right)\right)\le\psi\left(\xi\left(M\left(x,y\right)\right)\right)+L\xi\left(d\left(y,Tx\right)\right), \tag{21}$$

where $L\ge 0$, $\xi\in\Xi$, and $\psi\in\Psi$ is strictly increasing.

Also, suppose that conditions (1) and (2) of Theorem 4 are satisfied.

Then T has a fixed point in X.

Remark 6. If we have $\xi(t) = t$ for all $t \geq 0$, $L = 0$, and T is continuous, then Corollary 5 reduces to Theorem 3.4 of [7].

Let $(X, \preceq)$ be an ordered set and $A, B \subset X$. We say that $A \preceq B$ whenever, for each $a \in A$, there exists $b \in B$ such that $a \preceq b$.

Corollary 7. *Let $(X, \preceq, d)$ be a complete ordered metric space. Suppose that a multivalued mapping $T : X \to CL(X)$ satisfies*

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)) \quad (22)$$

for all $x, y \in X$ with $Tx \preceq Ty$ (resp., $Ty \preceq Tx$), where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing.

Assume that, for each $x \in X$ and $y \in Tx$ with $Tx \preceq Ty$ (resp., $Ty \preceq Tx$), we have $Ty \preceq Tz$ (resp., $Tz \preceq Ty$) for all $z \in Ty$.

Also, suppose that the following are satisfied:

(1) *there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $Tx_0 \preceq Tx_1$ (resp., $Tx_1 \preceq Tx_0$);*

(2) *either T is continuous or f_T is lower semicontinuous.*

Then T has a fixed point in X.

Remark 8. If we have $\xi(t) = t$ for all $t \geq 0$, $L = 0$, and T is continuous, then Corollary 7 reduces to Corollary 3.6 of [7].

From Theorem 4 we obtain the following result.

Corollary 9. *Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that $T : X \to CL(X)$ is an α_*-admissible mapping.*

Assume that, for all $x, y \in X, \alpha(x, y) \geq 1$ implies

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)), \quad (23)$$

where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing.

Also, suppose that conditions (1) and (2) of Theorem 4 are satisfied.

Then T has a fixed point in X.

Remark 10. If we have $L = 0$ in Corollary 9, then Corollary 9 reduces to Theorem 2.5 of [6].

Corollary 11. *Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that $T : X \to CL(X)$ is an α_*-admissible mapping.*

Assume that, for all $x, y \in X$,

$$\xi(\alpha(x, y) H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)), \quad (24)$$

where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing.

Also, suppose that conditions (1) and (2) of Theorem 4 are satisfied.

Then T has a fixed point in X.

Remark 12. In Corollary 11, let $\xi(t) = t$ for all $t \geq 0$ and $\alpha(x, y) = 1$ for all $x, y \in X$ and $\psi(t) = kt$ for all $t \geq 0$, where $k \in [0, 1)$. If T is single valued map, then Corollary 11 reduces to Theorem 2.2 of [8].

Theorem 13. *Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α-admissible.*

Assume that, for all $x, y \in X, \alpha(x, y) \geq 1$ implies

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)), \quad (25)$$

where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing and upper semicontinuous function. Also, suppose that the following are satisfied:

(1) *there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;*

(2) *for a sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$ and a cluster point x of $\{x_n\}$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that, for all $k \in \mathbb{N} \cup \{0\}$,*

$$\alpha(x_{n(k)}, x) \geq 1. \quad (26)$$

Then T has a fixed point in X.

Proof. Following the proof of Theorem 4, we obtain a sequence $\{x_n\} \subset X$ with $\lim_{n\to\infty} x_n = x_* \in X$ such that, for all $n \in \mathbb{N} \cup \{0\}$,

$$x_{n+1} \in Tx_n, \quad x_n \neq x_{n+1}, \quad \alpha(x_n, x_{n+1}) \geq 1. \quad (27)$$

From (2) there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that

$$\alpha(x_{n(k)}, x_*) \geq 1. \quad (28)$$

Thus, we have

$$\begin{aligned} \xi(d(x_{n(k)+1}, Tx_*)) &= \xi(H(Tx_{n(k)}, Tx_*)) \\ &\leq \psi(\xi(M(x_{n(k)}, x_*))) \\ &\quad + L\xi(d(x_*, x_{n(k)+1})), \end{aligned} \quad (29)$$

where

$$\begin{aligned} &M(x_{n(k)}, x_*) \\ &= \max\Big\{d(x_{n(k)}, x_*), d(x_{n(k)}, x_{n(k)+1}), d(x_*, Tx_*), \\ &\qquad \frac{1}{2}\{d(x_{n(k)}, Tx_*) + d(x_*, x_{n(k)+1})\}\Big\}. \end{aligned} \quad (30)$$

We have

$$\lim_{k\to\infty} M(x_{n(k)}, x_*) = d(x_*, Tx_*), \quad (31)$$

and so

$$\lim_{k\to\infty} \xi(M(x_{n(k)}, x_*)) = \xi(d(x_*, Tx_*)). \quad (32)$$

Suppose that $d(x_*, Tx_*) \neq 0$.

Since ψ is upper semicontinuous,

$$\lim_{k\to\infty} \psi(\xi(M(x_{n(k)}, x_*))) \leq \psi(\xi(d(x_*, Tx_*))). \quad (33)$$

Letting $k \to \infty$ in inequality (29) and using continuity of ξ, we obtain

$$\begin{aligned} 0 &< \xi(d(x_*, Tx_*)) \\ &\leq \lim_{k\to\infty} \psi(\xi(M(x_{n(k)}, x_*))) + \lim_{k\to\infty} L\xi(d(x_*, x_{n(k)+1})) \\ &\leq \psi(\xi(d(x_*, Tx_*))) \\ &< \xi(d(x_*, Tx_*)) \end{aligned} \tag{34}$$

which is a contradiction. Hence, $d(x_*, Tx_*) = 0$, and hence x_* is a fixed point of T. □

The following example shows that upper semicontinuity of ψ cannot be dropped in Theorem 13.

Example 14. Let $X = [0, \infty)$, and let $d(x, y) = |x - y|$ for all $x, y \geq 0$.

Define a mapping $T : X \to CL(X)$ by

$$Tx = \begin{cases} \left\{\frac{1}{2}, 1\right\} & (x = 0), \\ \left\{\frac{3}{4}x\right\} & (0 < x \leq 1), \\ \{2x\} & (x > 1). \end{cases} \tag{35}$$

Let $\xi(t) = t$ for all $t \geq 0$, and let

$$\psi(t) = \begin{cases} \frac{4}{5}t & (t \geq 1), \\ \frac{3}{4}t & (0 \leq t < 1). \end{cases} \tag{36}$$

Then, $\xi \in \Xi$, and $\psi \in \Psi$ and ψ is a strictly increasing function.

Let $\alpha, \eta : X \times X \to [0, \infty)$ be defined by

$$\alpha(x, y) = \begin{cases} 4 & (0 \leq x, y \leq 1), \\ 0 & \text{otherwise}. \end{cases} \tag{37}$$

Obviously, condition (2) of Theorem 13 is satisfied. Condition (1) of Theorem 13 is satisfied with $x_0 = 1/4$.

We show that (7) is satisfied.

Let $x, y \in X$ be such that $\alpha(x, y) \geq 1$.

Then, $0 \leq x, y \leq 1$.

If $x = y$, then obviously (7) is satisfied.

Let $x \neq y$.

If $x = 0$ and $0 < y \leq 1$, then we obtain

$$\begin{aligned} \xi(H(Tx, Ty)) &= H\left(\left\{\frac{1}{2}, 1\right\}, \frac{3}{4}y\right) \\ &\leq \frac{1}{4} \leq \psi(d(x, Tx)) \leq \psi(\xi(M(x, y))). \end{aligned} \tag{38}$$

Let $0 < x \leq 1$ and $0 < y \leq 1$.

Then, we have

$$\begin{aligned} \xi(H(Tx, Ty)) = d(Tx, Ty) &= d\left(\frac{3}{4}x, \frac{3}{4}y\right) \\ &= \frac{3}{4}|x - y| = \psi(d(x, y)) \\ &\leq \psi(\xi(M(x, y))). \end{aligned} \tag{39}$$

Thus, (7) is satisfied.

We now show that T is α-admissible.

Let $x \in X$ be given, and let $y \in Tx$ be such that $\alpha(x, y) \geq 1$.

Then, $0 \leq x, y \leq 1$.

Obviously, $\alpha(y, z) \geq 1$ for all $z \in Ty$ whenever $0 < y \leq 1$.

If $y = 0$, then $Ty = \{1/2, 1\}$. Hence, for all $z \in Ty$, $\alpha(y, z) \geq 1$.

Hence, T is α-admissible. Thus, all hypotheses of Theorem 13 are satisfied. However, T has no fixed points.

Note that ψ is not upper semicontinuous.

Corollary 15. *Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that $T : X \to CL(X)$ is an α-admissible mapping.*

Assume that, for all $x, y \in X$,

$$\xi(\alpha(x, y) H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)), \tag{40}$$

where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing and upper semicontinuous function. Also, suppose that conditions (1) and (2) of Theorem 13 are satisfied.

Then T has a fixed point in X.

Corollary 16. *Let $(X, \preceq, d)$ be a complete ordered metric space. Suppose that a multivalued mapping $T : X \to CL(X)$ satisfies*

$$\xi(H(Tx, Ty)) \leq \psi(\xi(M(x, y))) + L\xi(d(y, Tx)) \tag{41}$$

for all $x, y \in X$ with $Tx \preceq Ty$ (resp., $Ty \preceq Tx$), where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing and upper semicontinuous function.

Assume that, for each $x \in X$ and $y \in Tx$ with $Tx \preceq Ty$ (resp., $Ty \preceq Tx$), we have $Ty \preceq Tz$ (resp., $Tz \preceq Ty$) for all $z \in Ty$.

Also, suppose that the following are satisfied:

(1) *there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $Tx_0 \preceq Tx_1$ (resp., $Tx_1 \preceq Tx_0$);*

(2) *for a sequence $\{x_n\}$ in X with $x_n \preceq x_{n+1}$ (resp., $x_{n+1} \preceq x_n$) for all $n \in \mathbb{N} \cup \{0\}$ and a cluster point x of $\{x_n\}$, there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that, for all $k \in \mathbb{N} \cup \{0\}$,*

$$x_{n(k)} \preceq x \quad (\text{resp., } x \preceq x_{n(k)}). \tag{42}$$

Then T has a fixed point in X.

Remark 17. Corollary 16 is a generalization and extension of the result of [9] to multivalued mappings.

Corollary 18. *Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that a multivalued mapping $T : X \to CL(X)$ is α_*-admissible.*

Assume that, for all $x, y \in X$, $\alpha(x, y) \geq 1$ implies

$$\xi\left(H\left(Tx, Ty\right)\right) \leq \psi\left(\xi\left(M\left(x, y\right)\right)\right) + L\xi\left(d\left(y, Tx\right)\right), \quad (43)$$

where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing and upper semicontinuous function.

Also, suppose that conditions (1) and (2) of Theorem 13 are satisfied.

Then T has a fixed point in X.

Remark 19. By taking $L = 0$ in Corollary 18 and by applying Remark 1, Corollary 18 reduces to Theorem 2.6 of [6].

Corollary 20. *Let (X, d) be a complete metric space, and let $\alpha : X \times X \to [0, \infty)$ be a function. Suppose that $T : X \to CL(X)$ is an α_*-admissible mapping.*

Assume that, for all $x, y \in X$,

$$\xi\left(\alpha\left(x, y\right) H\left(Tx, Ty\right)\right) \leq \psi\left(\xi\left(M\left(x, y\right)\right)\right) + L\xi\left(d\left(y, Tx\right)\right), \quad (44)$$

where $L \geq 0$, $\xi \in \Xi$, and $\psi \in \Psi$ is strictly increasing and upper semicontinuous function.

Also, suppose that conditions (1) and (2) of Theorem 13 are satisfied.

Then T has a fixed point in X.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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References

[1] B. Samet, C. Vetro, and P. Vetro, "Fixed point theorems for $\alpha - \psi$-contractive type mappings," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 75, no. 4, pp. 2154–2165, 2012.

[2] J. H. Asl, S. Rezapour, and N. Shahzad, "On fixed points of α-ψ-contractive multifunctions," *Fixed Point Theory and Applications*, vol. 2012, article 212, 2012.

[3] P. Salimi, A. Latif, and N. Hussain, "Modified α-ψ-contractive mappings with applications," *Fixed Point Theory and Applications*, vol. 2013, article 151, 2013.

[4] E. Karapınar and B. Samet, "Generalized $\alpha - \psi$ contractive type mappings and related fixed point theorems with applications," *Abstract and Applied Analysis*, vol. 2012, Article ID 793486, 17 pages, 2012.

[5] N. Hussain, P. Salimi, and A. Latif, "Fixed point results for single and set-valued α-η-ψ-contractive mappings," *Fixed Point Theory and Applications*, vol. 2013, article 212, 2013.

[6] M. U. Ali, T. Kamran, and E. Karapınar, "(α, ψ, ξ)-contractive multivalued mappings," *Fixed Point Theory and Applications*, vol. 2014, article 7, 2014.

[7] B. Mohammadi, S. Rezapour, and N. Shahzad, "Some results on fixed points of α-ψ-Ciric generalized multifunctions," *Fixed Point Theory and Applications*, vol. 2013, article 24, 2013.

[8] V. Berinde, "Some remarks on a fixed point theorem for Ciric-type almost contractions," *Carpathian Journal of Mathematics*, vol. 25, no. 2, pp. 157–162, 2009.

[9] R. P. Agarwal, M. A. El-Gebeily, and D. O'Regan, "Generalized contractions in partially ordered metric spaces," *Applicable Analysis*, vol. 87, no. 1, pp. 109–116, 2008.