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Research Article

Stochastic Multicriteria Acceptability Analysis Based on Choquet Integral

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To reflect the interactions among criteria, Choquet integral is employed to stochastic multicriteria acceptability analysis. Models are first given to roughly identify the best and worst ranking orders of each alternative, based on which the weight information spaces are explored to support some alternative for ranking at some position and calculate the acceptability indices of alternatives. Models are then given to analyze the characters of information spaces, which can describe what kind of information supports alternatives for ranking at some position and can give an analysis about the effect of characters on the decision result. The proposed method considers not only the interactions between two criteria, but also the interactions among three, four, and more criteria. The proposed method can be considered as an extension of the existing ones.

1. Introduction

Multicriteria decision-making has been applied in many areas [1, 2]. Most of the existing multicriteria decision-making is to find the rankings of alternatives from the known information, while stochastic multicriteria acceptability analysis (SMAA [3]) is to find the information space that supports each alternative for the best ranking. Lahdelma and Salminen [4] introduced the SMAA-2 method, which extends the original SMAA by considering all the rankings in the analysis. SMAA and SMAA-2 methods assume the utility function is linear and the criteria are independent corresponding to decision maker's constant marginal value or risk-neutral behaviour. By using one real-life problem and a large number of artificial test problems, Lahdelma and Salminen [5] showed that, in most cases, slight nonlinearity does not significantly affect the SMAA results. Sometimes, there exist interactions among criteria [6–8]. For example, supplier selection is an important issue in supply chain management. Product quality, offering price, delivery time, and service quality are key criteria for supplier evaluation. From one side, delivery time and service quality are redundant criteria, because, in general, the supplier who has good service will deliver on time. Therefore, even if these two criteria can be very important, their comprehensive importance is smaller than the sum of the importance of the two criteria. From the other side, the two criteria, product quality and offering price, lead to a positive interaction because a supplier who supplies high quality and offers a low price is very well appreciated. Therefore, the comprehensive importance of quality and price should be greater than the sum of the importance of them.

By considering the interactions of the criteria, Angilella et al. [7, 8] applied Choquet integral [6] to SMAA-2 method, but they only consider the interactions of two criteria and neglect the interactions among three, four, or more criteria. For example, in a manufacturing enterprise, there are three kinds of equally important and necessary materials that make one product. If the number of any kind of material is zero, the product can not be produced. In such cases, these three kinds of materials can be considered to be three criteria, which have positive interactions (a numerical illustration is given in Example 2). In addition, SMAA method, SMAA-2 method, and Angilella et al.'s method [7, 8] only focus on exploring the spaces of the weight information but do not give an analysis.

By taking into account the decision maker's attitudinal character (orness), Ahn [9] presented a reverse decision-aiding method for analyzing the effect of orness on the multicriteria decision-making. Ma et al. [10] extended it to the situation when a few best or worst alternatives need to be identified. But Ahn's and Ma et al.'s models are all based on ordered weighted averaging (OWA) operator [11]. Moreover, they only analyze the impact of orness on the multicriteria decision-making. Actually, the properties of the aggregation operator can be expressed more specifically through different concepts except orness [12].

In this paper, Choquet integral and SMAA-2 are combined to deal with the multicriteria decision-making with interactions among criteria. Models are firstly given to roughly estimate the best and worst ranking orders of each alternative, based on which the information space that supports each alternative at some position is explored, and the acceptability indices of alternatives are calculated. Then the characters of Choquet integral are used to describe the information spaces to try to analyze the effect of these characters on the decision results. Several examples are also given to compare the proposed methods with the existing ones.

2. Basic Concepts of Choquet Integral

A fuzzy measure μ on X is a function μ: $P(X) \rightarrow [0,1]$, satisfying the axioms [13] (i) $\mu(\phi) = 0$ and (ii) $A \subset B \subset X$ implies $\mu(A) \le \mu(B)$. It is assumed that $\mu(X) = 1$ as usual.

The Möbius transform of μ is a set function on X defined as [14] $w_A = \sum_{B\subset A}(-1)^{|A\setminus B|}\mu(B), \forall A \subset X$. In terms of Möbius representation, (i) and (ii) can be represented by (iii) $w_\phi = 0$; (iv) $\forall i \in X$ and $\forall S \subseteq X\setminus\{i\}, \sum_{T\subseteq S} w_{T\cup\{i\}} \ge 0$. $\mu(X) = 1$ can be expressed as $\sum_{T\subseteq S} w_T = 1$.

The Choquet integral [6] is firstly defined on fuzzy measure [13], and then other transformations are defined. Suppose the performance of an alternative under criteria $(G_1, G_2, \ldots, G_n)$ is expressed as $x = (x_1, x_2, \ldots, x_n) \in R^n$. The Choquet integral with respect to the Möbius representation can be given as [15]

$$C(x) = \sum_{T\subseteq N} w_T \bigwedge_{i\in T} x_i, \tag{1}$$

where $w_\phi = 0$, $\sum_{T\subseteq N} w_T = 1$, $\forall i \in N$, $\forall S \subseteq N\setminus\{i\}$, $\sum_{T\subseteq S} w_{T\cup\{i\}} \ge 0$, and $N = \{1, 2, \ldots, n\}$.

In expression (1), w_T measures the interactions of the criteria that belong to T [12]. If $w_T > 0$, then the set of criteria c_j, $j \in T$, has positive interactions. Choquet integral uses the minimum value of the criteria evaluations in the coalition T as the value of T. Some authors [16–19] have tried to substitute the minimum operation with other ones. Marichal [20] denoted that other operations are not stable for the admissible positive linear transformation.

Choquet integral is continuous, nondecreasing, and stable under the same transformations of interval scales in the sense of the theory of measurement, and it coincides with the weighted arithmetic (WA) operator [21] and the ordered weighted averaging (OWA) operator [11]. Choquet integral has some characters [20], which can be described by the following.

The importance of criteria j is expressed by the Shapley value [20] as follows:

$$\varphi_j = \sum_{j\in T}\frac{1}{t} w_T, \tag{2}$$

where t is the cardinality of the coalition T; that is, $t = |T|$. The Shapley value is a fundamental concept in game theory expressing a power index. It can be interpreted as a weighted average value of the marginal contribution of criterion j alone in all combinations.

The interaction index expresses the sign and the magnitude of the interactions of the criteria in the coalition T [20] as follows:

$$\varphi_T = w_T. \tag{3}$$

The degree of orness is defined by [20]

$$\text{orn} = \frac{1}{n-1}\sum_{T\subseteq N}\frac{n-t}{t+1} w_T \tag{4}$$

which represents the degree to which the overall value is close to that of "min." In some sense, it also reflects the extent to which the overall value behaves like a minimum or has a conjunctive behavior.

An interesting phenomenon in aggregation is the veto effect and its counterpart, the favor effect. It seems reasonable to define indices that measure the degree of veto or favor of a given criterion. If the Choquet integral is considered, a natural definition of a degree of veto (resp., favor) consists in considering the probability [20] as follows:

$$\begin{aligned} v_j &= 1 - \frac{1}{n-1}\sum_{T\subseteq N\setminus j}\frac{1}{t+1} w_T, \quad j \in N, \\ o_j &= \frac{n}{n-1}\sum_{T\subseteq N\setminus j}\frac{1}{t+1}\left(w_{T\cup j} + w_T\right) - \frac{1}{n-1}, \quad j \in N. \end{aligned} \tag{5}$$

Here v_j measures the degree to which the decision maker demands that criterion G_j is satisfied. v_j is different from the weight of criterion G_j: we might have a high degree of veto on a not very important criterion. o_j is the degree to which the decision maker considers that a good score along criterion G_j is sufficient to be satisfied.

The dispersion is to measure how much of the information in the arguments is used. In a certain sense, the more disperse the weight vector is, the more the information about the individual criteria is being used in the aggregation process. The dispersion of Choquet integral can be defined by [20]

$$\begin{aligned} \text{dis} = &-\frac{1}{\ln n} \\ &\cdot \sum_{j=1}^{n}\sum_{T\subseteq N\setminus j}\frac{(n-t-1)!t!}{n!}\left[\sum_{K\subseteq T} w_{K\cup j}\ln\sum_{K\subseteq T} w_{K\cup j}\right]. \end{aligned} \tag{6}$$

3. Stochastic Multicriteria Acceptability Analysis Based on Choquet Integral

In a decision matrix, assume that $A = (A_1, A_2, \ldots, A_m)$ represents the set of alternatives and $G = (G_1, G_2, \ldots, G_n)$ represents the set of relevant criteria. Usually, it is difficult to obtain the exact information about the criteria evaluations and interactions between criteria, because the decision maker may not be willing or able to provide exact estimations of decision parameters under time pressure, lack of knowledge or data, and fear of commitment [22] or because the decision maker has limited attention and information processing capabilities to exact value judgements [23].

Stochastic multicriteria acceptability analysis (SMAA) has been developed in particular for situations where neither the criteria evaluations nor the criteria weight vectors are precisely known. The evaluation value of alternative A_i under criterion G_j is represented by the stochastic variable ξ_{ij} with a probability distribution f_X over the space $X \subseteq R_{m\times n}$. Similarly, the decision makers' unknown or partially known preferences are represented by a weight distribution with density function f_W over the space of all compatible weights W.

Considering the interactions of the criteria [6], the weight information can be defined as

$$W = \left\{ \{w_T, T \subseteq N\} : w_\phi = 0,\ \sum_{T\subseteq N} w_T = 1, \right.$$
$$\left. \sum_{T\subseteq S} w_{T\cup\{l\}} \geq 0,\ \forall S \subseteq N \setminus \{l\},\ \forall l \in N \right\}, \tag{7}$$

where $N = \{1, 2, \ldots, n\}$.

Based on Choquet integral [6], the overall evaluation of alternative A_i can be given as

$$u(i, \xi, w) = \sum_{T\subseteq N} w_T \bigwedge_{j\in T} \xi_{ij}, \tag{8}$$

where $\xi \in X$ and $w \in W$.

If $w_T = 0$, $t \geq 2$, then W reduces to the classical weight information set as follows:

$$W' = \left\{ \{w'_j,\ j = 1, 2, \ldots, n\} : \sum_{j=1}^{n} w'_j = 1, \right.$$
$$\left. w'_j \geq 0,\ j = 1, 2, \ldots, n \right\} \tag{9}$$

and (8) reduces to the WA mean [21] as follows:

$$u(i, \xi, w') = \sum_{j=1}^{n} w_j \xi_{ij}. \tag{10}$$

For each ξ in X and w in W, $u(i, \xi, w)$ provides a complete ranking of alternatives, and then the position of alternative A_i is denoted by

$$\operatorname{rank}(i, \xi, w) = 1 + \sum_{k\neq i} \rho(u(\xi_k, w) > u(\xi_i, w)), \tag{11}$$

where $\rho(\text{true}) = 1$ and $\rho(\text{false}) = 0$.

Here, we can give a rough estimation about the best and worst ranking orders of alternative A_i, which can be obtained by solving the following model:

$$\begin{aligned} &\text{Min / Max} \quad \operatorname{rank}(i, \xi, w) \\ &\text{s.t.} \quad w \in W,\ \xi \in X. \end{aligned} \tag{MOD 1}$$

When Choquet integral reduces to the OWA operator [11], (MOD 1) reduces to the one given by Ahn [9] and Ma et al. [10].

Suppose the optimal solutions of (MOD 1) are denoted by $\operatorname{rank}_*(i, \xi, w)$ and $\operatorname{rank}^*(i, \xi, w)$, respectively; then we have the following theorem.

Theorem 1. *Let $\xi \in X$, $w \in W$, and $w' \in W'$; then we have*

$$\begin{aligned} \operatorname{rank}^*(i, \xi, w) &\geq \operatorname{rank}^*(i, \xi, w'), \\ \operatorname{rank}_*(i, \xi, w) &\leq \operatorname{rank}_*(i, \xi, w'). \end{aligned} \tag{12}$$

Proof. Since $W' \subseteq W$, we have

$$\begin{aligned} \operatorname{rank}^*(i, \xi, w) &= \max_{\xi\in X, w\in W} \operatorname{rank}(i, \xi, w) \\ &= 1 + \max_{\xi\in X, w\in W} \sum_{k\neq i} \rho(u(\xi_k, w) > u(\xi_i, w)) \\ &\geq 1 + \max_{\xi\in X, w\in W'} \sum_{k\neq i} \rho(u(\xi_k, w') > u(\xi_i, w')) \\ &= \max_{\xi\in X, w\in W'} \operatorname{rank}(i, \xi, w') = \operatorname{rank}^*(i, \xi, w'), \\ \operatorname{rank}_*(i, \xi, w) &= \min_{\xi\in X, w\in W} \operatorname{rank}(i, \xi, w) \\ &= 1 + \min_{\xi\in X, w\in W} \sum_{k\neq i} \rho(u(\xi_k, w) > u(\xi_i, w)) \\ &\leq 1 + \min_{\xi\in X, w\in W'} \sum_{k\neq i} \rho(u(\xi_k, w') > u(\xi_i, w')) \\ &= \min_{\xi\in X, w\in W'} \operatorname{rank}(i, \xi, w') = \operatorname{rank}^*(i, \xi, w') \end{aligned} \tag{13}$$

which completes the proof. □

Theorem 1 means the best ranking order of alternative A_i obtained by considering the interactions of the criteria is not worse than that obtained without considering the interactions of the criteria, while the worst ranking order of alternative A_i obtained by considering the interactions of the criteria is not better than that obtained without considering the interactions of the criteria. That is because the information space is enlarged by considering the interactions of the criteria.

For each $\xi \in X$, suppose alternative A_i ranks rth, where $r \in [\operatorname{rank}_*(i, \xi, w), \operatorname{rank}^*(i, \xi, w)]$; we can compute the set of possible weights based on SMAA [3] as follows:

$$W_i^r(\xi) = \{w \in W, \operatorname{rank}(i, \xi, w) = r\} \tag{14}$$

which is called the favorable ranking weights of alternative A_i at position r.

On the basis of the favorable ranking weights, the ranking acceptability index that alternative A_i is at position r is given as

$$b_i^r = \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, dw\, d\xi \tag{15}$$

which describes the share of parameters supporting alternative A_i at position r in the obtained final ranking; in particular, b_i^1 measures the variety of parameters making alternative A_i the most preferred one.

Next, the characters of Choquet integral can be used to analyze the information space $W_i^r(\xi)$, such as the interactions of criteria in coalition T as follows:

$$\begin{aligned} &\text{Max / Min} \quad w_T, \quad T \subseteq N \\ &\text{s.t.} \qquad w \in W_i^r(\xi). \end{aligned} \tag{MOD 2}$$

The Shapley value of criterion j is as follows:

$$\begin{aligned} &\text{Max / Min} \quad \varphi_j(w) = \sum_{j \in T} \frac{1}{t} w_T, \quad T \subseteq N \\ &\text{s.t.} \qquad w \in W_i^r(\xi). \end{aligned} \tag{MOD 3}$$

The degree of veto or favor of a given criterion [12] is as follows:

$$\begin{aligned} &\text{Max / Min} \quad v_j(w) = 1 - \frac{1}{n-1} \sum_{T \subseteq N \setminus j} \frac{1}{t+1} w_T \\ &\text{s.t.} \qquad w \in W_i^r(\xi), \end{aligned} \tag{MOD 4}$$

$$\begin{aligned} &\text{Max / Min} \quad o_j(w) = \frac{n}{n-1} \sum_{T \subseteq N \setminus j} \frac{1}{t+1} \left(w_{T \cup j} + w_T\right) - \frac{1}{n-1} \\ &\text{s.t.} \qquad w \in W_i^r(\xi). \end{aligned} \tag{MOD 5}$$

The degree of orness [12] is as follows:

$$\begin{aligned} &\text{Max / Min} \quad \operatorname{orn}(w) = \frac{1}{n-1} \sum_{T \subseteq N} \frac{n-t}{t+1} w_T \\ &\text{s.t.} \qquad w \in W_i^r(\xi). \end{aligned} \tag{MOD 6}$$

The dispersion to measure is as follows:

$$\begin{aligned} &\text{Max / Min} \quad \operatorname{dis}(w) = -\sum_{j=1}^{n} \sum_{T \subseteq N \setminus j} \frac{(n-t-1)!t!}{n!} \sum_{K \subseteq T} w_{K \cup j} \\ &\text{s.t.} \qquad w \in W_i^r(\xi). \end{aligned} \tag{MOD 7}$$

By solving ((MOD 2)–(MOD 7)), we can roughly describe what kind of information $W_i^r(\xi)$ supports alternative A_i for ranking at position r. We can find that some alternatives might be identified for a lower range of characters and others for a higher range. The range of characters of two alternatives may be nonoverlapping, overlapping, or inclusion, or equivalent depending upon the end points of the ranges. An alternative with wider range of character is more probable to be selected than the one with a narrower range of character.

In real decision-making, the information about attribute weights is incompletely known because of time pressure, lack of knowledge or data, and the expert's limited expertise about the problem domain [22–25]. The proposed method can help the decision makers identify the corresponding alternatives in the case when the decision makers have difficulty in specifying the precise information about the criteria weight vector. Based on the known information about the criteria weight vector, the ranges of the above characters can be calculated, and the corresponding alternatives can be identified according to the results obtained by the proposed method; meanwhile the redundant alternatives can be removed.

((MOD 2)–(MOD 7)) analyze the ranges of the characters of the weight information space. We can calculate the central values of them to give a clear description of these characters as follows:

$$\begin{aligned} \overline{w}^i &= \frac{1}{b_i^1} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, w\, dw\, d\xi, \\ \overline{\varphi}^i &= \frac{1}{b_i^1} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, \varphi(w)\, dw\, d\xi \\ &= \varphi(w_i^c), \\ \overline{v}^i &= \frac{1}{b_i^1} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, v(w)\, dw\, d\xi \\ &= v(w_i^c), \\ \overline{o}^i &= \frac{1}{b_i^1} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, f(w)\, dw\, d\xi \\ &= o(w_i^c), \\ \overline{\operatorname{dis}}^i &= \frac{1}{b_i^1} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, \operatorname{dis}(w)\, dw\, d\xi \\ &= \operatorname{dis}(w_i^c), \\ \overline{\operatorname{orn}}^i &= \frac{1}{b_i^1} \int_{\xi \in X} f_X(\xi) \int_{w \in W_i^r(\xi)} f_W(w)\, \operatorname{orn}(w)\, dw\, d\xi \\ &= \operatorname{orn}(w_i^c). \end{aligned} \tag{16}$$

Table 1: A decision matrix with four alternatives evaluated by three criteria.

	G_1	G_2	G_3
A_1	0.3	0.9	0.6
A_2	0.7	0.8	0.5
A_3	0.5	1.0	0.4
A_4	0.4	0.85	0.6

Table 2: Best and worst ranking orders for each alternative.

	Rank$_*$			Rank*		
	(MOD 1)	By Ma et al.'s model	By Ahn's model	(MOD 1)	By Ma et al.'s model	By Ahn's model
A_1	1	1	1	4	4	4
A_2	1	1	1	4	4	4
A_3	1	1	1	4	4	4
A_4	1	2	>1	4	4	4

Table 3: The ranking acceptability indices obtained by OWA-SMAA-2.

	b_i^1	b_i^2	b_i^3	b_i^4
A_1	1	25	25	50
A_2	67	17	6	11
A_3	32	25	18	23
A_4	0	34	51	16

Table 4: The ranking acceptability indices obtained by Choquet-SMAA-2.

	b_i^1	b_i^2	b_i^3	b_i^4
A_1	3	5	19	73
A_2	88	5	5	2
A_3	3	44	29	24
A_4	6	46	47	1

The central weight vector describes the preference of a typical decision maker that makes alternative A_i the most preferred one, which can be presented to the decision makers in order to help them understand how different weights correspond to different choices.

For convenience, if the OWA operator and Choquet integral are used instead of WA mean in SMAA-2 [4], then we denote the methods as OWA-SMAA-2 and Choquet-SMAA-2, respectively. Angilella et al. [7, 8] proposed a method by integrating the SMAA-2 with the Choquet integral, but their method only considers the positive and negative interactions of two criteria, neglecting possible interactions among three, four, or more criteria. In this paper, Angilella et al.'s method [7, 8] is denoted as 2-Choquet-SMAA-2. In particular, if Choquet integral reduces to the WA mean [21], then Choquet-SMAA-2 reduces to SMAA-2 [4]; if the interactions between two criteria are only considered, that is, $w_T = 0, t > 2$, then Choquet-SMAA-2 reduces to 2-Choquet-SMAA-2 [7, 8].

4. Illustrative Examples

Example 1 (see [9]). Assume that an artificial decision problem characterized by four alternatives (i.e., A_1, A_2, A_3, and A_4) and three criteria (i.e., G_1, G_2, and G_3) is shown in Table 1.

By considering the interactions of the criteria, (MOD 1) is firstly used to estimate the best and worst ranking orders of each alternative, which are listed in Table 2. It is noted that A_4 is not one of the potential best alternatives in both Ahn's model [9] and Ma et al.'s model [10]. The best ranking order of A_4 is the second in Ma et al.'s model [10], while the best ranking order of A_4 is first in the proposed model. The ranking intervals of alternatives obtained by the proposed model are wider than those obtained by Ahn's model [9] and Ma et al.'s model [10], which is also consistent with the findings in Theorem 1.

Table 5: Determination of the ranges of orness under $r = 1$.

	A_1	A_2	A_3	A_4
Ahn's and Ma et al.'s models	[0.25, 0.25]	[0.25, 1.00]	[0.00, 0.67]	ϕ
The proposed model	[0.17, 0.92]	[0.00, 0.92]	[0.17, 1.00]	[0.08, 0.70]

Next, the ranking acceptability index b_i^r of alternative A_i at position r can be calculated. Tables 3 and 4 show the acceptability indices of alternatives obtained by using OWA-SMAA-2 and Chqouet-SMAA-2, respectively. It is noted that $b_4^1 = 0$ in OWA-SMAA-2, and $b_4^1 = 6$ in Choquet-SMAA-2, which shows that although the possibility that A_4 ranks first is low in Choquet-SMAA-2, it is possible for A_4 to rank first. But it is impossible for A_4 to rank first in OWA-SMAA-2.

But what kind of information supports alternative A_i for ranking at position r? The decision makers may be very interested about this question, which can help the decision maker analyze the decision result. Take orness as an example; by assuming that each alternative is the preferred one, we can estimate the ranges of orness, which are listed in Table 5.

It is noted that the range of orness of alternative A_2 is even wider than that of A_1 and thus has a greater chance of being chosen. Except for estimating the ranges of orness, we can also estimate the ranges of other characters, such as the Shapley values of the criteria, the interactions of the criteria, the veto and favor degree of each criterion, and the degree of the use of date. By using ((MOD 2)–(MOD 7)), the results are given in Table 6.

By analyzing Table 6, the lowest condition that alternative A_i ranks at position r can be obtained. For example, if A_1 is at the first ranking, then the Shapley value of criterion c_1 should not be bigger than 0.58, c_2 not bigger than 0.83, and c_3 not smaller than 0.08; w_2 should not be bigger than 0.67; the interaction effect w_{12} of criteria c_1 and c_2 should not be smaller than −0.67 and should not be bigger than 0.50; the interaction effect w_{23} of criteria c_1 and c_2 should not be smaller than −0.67; the interaction effect w_{123} of criteria c_1, c_2, and c_3 should not be smaller than −1.50 and should not be bigger than 0.67; v_1 should not be smaller than 0.08 and should not be bigger than 0.50; v_3 should not be smaller

Table 6: Determination of the ranges of characters under $r = 1$.

	A_1	A_2	A_3	A_4
φ_1	[0.00, 0.58]	[0.08, 1.00]	[0.00, 0.75]	[0.00, 0.47]
φ_2	[0.00, 0.83]	[0.00, 0.83]	[0.11, 1.00]	[0.00, 0.70]
φ_3	[0.08, 1.00]	[0.00, 0.83]	[0.00, 0.67]	[0.20, 1.00]
w_1	[0.00, 1.00]	[0.00, 1.00]	[0.00, 1.00]	[0.00, 0.60]
w_2	[0.00, 0.67]	[0.00, 0.50]	[0.33, 1.00]	[0.00, 0.44]
w_3	[0.00, 1.00]	[0.00, 1.00]	[0.00, 1.00]	[0.00, 1.00]
w_{12}	[−0.67, 0.50]	[−0.50, 1.00]	[−1.00, 0.50]	[−0.44, 0.50]
w_{13}	[−1.00, 1.00]	[−1.00, 1.00]	[−1.00, 1.00]	[−0.60, 1.00]
w_{23}	[−0.67, 1.00]	[−0.50, 1.00]	[−1.00, 0.50]	[−0.44, 1.00]
w_{123}	[−1.50, 0.67]	[−2.00, 1.00]	[−1.50, 1.00]	[−1.50, 0.75]
v_1	[0.08, 0.50]	[0.13, 1.00]	[0.00, 0.75]	[0.25, 0.75]
v_2	[0.00, 1.00]	[0.00, 1.00]	[0.00, 1.00]	[0.13, 1.00]
v_3	[0.13, 1.00]	[0.13, 1.00]	[0.00, 0.75]	[0.45, 1.00]
o_1	[0.00, 1.00]	[0.00, 1.00]	[0.08, 1.00]	[0.00, 0.70]
o_2	[0.25, 0.75]	[0.00, 0.75]	[0.33, 1.00]	[0.13, 0.55]
o_3	[0.19, 1.00]	[0.00, 1.00]	[0.08, 1.00]	[0.13, 1.00]
orn	[0.17, 0.92]	[0.00, 0.92]	[0.17, 1.00]	[0.08, 0.70]
dis	[0.00, 0.97]	[0.21, 1.00]	[0.39, 0.99]	[0.00, 0.97]

Table 7: The impact of orness on the decision-making by the proposed model.

Ranges of attitudinal character	Best alternatives
[0.00, 0.08)	A_2
[0.08, 0.17)	A_2 and A_4
[0.17, 0.70]	A_1, A_2, A_3, and A_4
(0.70, 0.92]	A_1, A_2, and A_3
(0.92, 1.00]	A_3

Table 8: The impact of orness on the decision-making by Ahn's and Ma et al.'s models.

Ranges of attitudinal character	Best alternatives
[0.00, 0.25)	A_3
[0.25, 0.67]	A_1, A_2, and A_3
(0.67, 1.00]	A_2

than 0.13; o_2 should not be smaller than 0.25 and should not be bigger than 0.75; o_3 should not be smaller than 0.19; orn should not be smaller than 0.17 and should not be bigger than 0.92; dis should not be bigger than 0.97.

From the reverse view, we can analyze the effect of these characters on the decision results. Take orness as an example; the corresponding potential optimal alternatives can be identified as the value of orness increases from zero to one. In Table 7, if orness takes the value between 0.00 and 0.08, then the potential optimal alternative is A_2; if orness takes the value between 0.08 and 0.17, then the potential optimal alternatives are A_2 and A_4; if orness takes the value between 0.92 and 1.00, then the potential optimal alternative is A_3. By comparing the proposed method with the ones given by Ahn and Ma et al., it is noted that the range of orness that contains the most potential optimal alternatives in Ahn's and Ma et al.'s methods is $[0.25, 0.67]$; the corresponding alternatives are A_1, A_2, and A_3 in Table 8. But Table 7 shows the range of orness that contains most potential optimal alternatives in the proposed method is $[0.17, 0.70]$, and the corresponding alternatives are A_1, A_2, A_3, and A_4. Similarly, we can analyze the impact of other characters on the decision results. Here we will not repeat them.

The central values of the characters can be further calculated; please see Table 9. These values show us how different weights correspond to different choices.

Example 2. An artificial decision problem characterized by four alternatives and three criteria is shown in Table 10.

By using SMAA-2, 2-Choquet-SMAA-2, and Choquet-SMAA-2, respectively, we can obtain the ranking acceptability indices of alternatives as listed in Tables 11–13.

It is noted that the results obtained by SMAA-2, 2-Choquet-SMAA-2, and Choquet-SMAA-2 are similar. If SMAA-2 and 2-Choquet-SMAA-2 are used, we can obtain $b_4^1 = 0$, which means it is impossible for A_4 to rank first; if Choquet-SMAA-2 is used, we can obtain $b_4^1 = 1$, which shows it is possible for A_4 to rank first.

Suppose alternatives A_1, A_2, A_3, and A_4 denote four manufacturers, who produce the same production, and criteria G_1, G_2, and G_3 denote three kinds of equally important and necessary raw materials which produce such production. The production is made if and only if these three kinds of materials are all obtained. In such cases, we have $A_4 \succ A_1 \sim$

Table 9: Determination of the central values of the characters.

	A_1	A_2	A_3	A_4
$\overline{\varphi}_1$	0.188	0.349	0.289	0.206
$\overline{\varphi}_2$	0.400	0.328	0.415	0.333
$\overline{\varphi}_3$	0.412	0.323	0.296	0.461
$\overline{w}_1$	0.241	0.205	0.262	0.164
$\overline{w}_2$	0.350	0.189	0.482	0.231
$\overline{w}_3$	0.298	0.197	0.264	0.262
$\overline{w}_{12}$	−0.063	0.145	−0.080	−0.063
$\overline{w}_{13}$	0.067	0.119	0.119	0.130
$\overline{w}_{23}$	0.271	0.111	−0.069	0.250
$\overline{w}_{123}$	−0.163	0.035	0.021	0.027
$\overline{v}_1$	0.379	0.656	0.475	0.505
$\overline{v}_2$	0.563	0.639	0.546	0.616
$\overline{v}_3$	0.589	0.632	0.482	0.736
$\overline{o}_1$	0.403	0.368	0.459	0.304
$\overline{o}_2$	0.537	0.353	0.576	0.384
$\overline{o}_3$	0.530	0.353	0.462	0.456
$\overline{\text{dis}}$	0.824	0.847	0.846	0.823
$\overline{\text{orn}}$	0.490	0.358	0.499	0.381

Table 10: A decision matrix with four alternatives evaluated by four criteria.

	G_1	G_2	G_3
A_1	0.7	0.7	0.0
A_2	0.0	0.7	0.7
A_3	0.7	0.0	0.7
A_4	0.2	0.2	0.2

Table 11: The ranking acceptability indices obtained by SMAA-2.

	b_i^1	b_i^2	b_i^3	b_i^4
A_1	33	33	25	8
A_2	33	34	25	8
A_3	33	33	25	8
A_4	0	0	25	76

$A_2 \sim A_3$. If SMAA-2 or 2-Choquet-SMAA-2 is used, the following equations have no solution:

$$\begin{matrix} A_1 \sim A_2 \\ A_2 \sim A_3 \\ A_4 \succ A_1 \end{matrix} \Longleftrightarrow \begin{cases} \sum_{T\subseteq N} w_T \left(\bigwedge_{j\in T} p_{1j} - \bigwedge_{j\in T} p_{2j} \right) = 0 \\ \sum_{T\subseteq N} w_T \left(\bigwedge_{j\in T} p_{2j} - \bigwedge_{j\in T} p_{3j} \right) = 0 \\ \sum_{T\subseteq N} w_T \left(\bigwedge_{j\in T} p_{4j} - \bigwedge_{j\in T} p_{3j} \right) > 0 \\ w_\phi = 0, \quad \sum_{T\subseteq S} w_T = 1, \\ \forall i \in N, \ \forall S \subseteq N \setminus \{i\}, \ \sum_{T\subseteq S} w_{T\cup\{i\}} \geq 0 \\ w_T = 0, \ t \geq 2. \end{cases} \tag{17}$$

Table 12: The rank acceptability indices obtained by 2-Choquet-SMAA-2.

	b_i^1	b_i^2	b_i^3	b_i^4
A_1	33	33	29	4
A_2	34	33	29	4
A_3	33	33	29	4
A_4	0	0	13	87

Table 13: The ranking acceptability indices obtained by Choquet-SMAA-2.

	b_i^1	b_i^2	b_i^3	b_i^4
A_1	33	33	27	7
A_2	33	33	27	7
A_3	33	33	27	7
A_4	1	1	19	80

Table 14: The ranking acceptability obtained by Choquet-SMAA-2 with constrains.

	b^1	b^2	b^3	b^4
A_1	0	32	34	33
A_2	0	34	35	32
A_3	0	34	32	35
A_4	100	0	0	0

That is because these three kinds of materials have positive interactions, which is not considered in SMAA-2 and 2-Choquet-SMAA-2. While Choquet-SMAA-2 is used, we can obtain the results as listed in Table 14.

From the analysis shown in Table 14, we can find that the proposed method not only can deal with the situation in which there exist interactions of two criteria but also can deal with the situation in which there exist interactions of three or more interactions. The proposed method can be considered to be a generalization of the existing ones.

5. Conclusions

This paper has investigated stochastic multicriteria acceptability analysis based on Choquet integral, which not only considers the interactions of two criteria, but also considers the interactions of three or more criteria. We have given models to roughly estimate the best and worst ranking orders of alternatives based on Choquet integral. We also have explored the weight information spaces that support some alternative for ranking at some position. The acceptability indices of alternatives have been calculated to describe the share of the information for alternatives. To describe these information spaces, models have been established to estimate the ranges of the characters. The impact of the characters on decision results has been analyzed. Several examples have been given to compare the proposed methods with the existing ones.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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