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Research Article

Solving Signal Control Problems with Second-Order Sensitivity Information of Equilibrium Network Flows

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The equilibrium network signal control problem is represented as a Stackelberg game. Due to the characteristics of a Stackelberg game, solving the upper-level problem and lower-level problem iteratively cannot be expected to converge to the solution. The reaction function of the lower-level problem is the key information to solve a Stackelberg game. Usually, the reaction function is approximated by the network sensitivity information. This paper firstly presents the general form of the second-order sensitivity formula for equilibrium network flows. The second-order sensitivity information can be applied to the second-order reaction function to solve the network signal control problem efficiently. Finally, this paper also demonstrates two numerical examples that show the computation of second-order sensitivity and the speed of convergence of the nonlinear approximation algorithm.

1. Introduction

The network signal control problem (NSCP) is to find the optimal signal setting which improves the performance of existing facilities in a transportation network. Conventional methods for optimizing signal settings can be divided into two types: stage-based and group-based approaches [1–6]. The stage-based approach divided the signal cycle into separate stages and solved the optimal signal settings for each group of compatible traffic movements in stages. This approach is regarded as superior in the concern for safety and loss of capacity with phase switching [6]. The group-based approach considered each group of traffic streams having right-of-way in the time domain directly. Compared with the stage-based approach, the group-based approach has a higher degree of flexibility in signal timing arrangement [3, 7]. However, the most optimization models proposed so far usually converged to a local optimal solution and without taking traffic rerouting effects into account when solving NSCP [5]. The equilibrium network signal control problem (ENSCP) is used to find an optimal network signal design when the network flow pattern is constrained to be equilibrium. Friesz [8] points out that this is a problem of interest because of Braess' paradox [9]. This paradox shows that the congestion of the network may be severer when adding capacity to a congested network without taking the reaction of network users into consideration. Hence, in practice, the equilibrium network signal design problem must be solved by constraining the network flow pattern to meet user equilibrium. The user equilibrium network design with fixed transportation demand has been studied in both discrete [10] and continuous [11] versions. To help solve the signal control problem, Allsop [12] pointed out that the route choices of road users should be considered the impacts of signal settings changing. Gartner et al. [13] and Fisk [14] described the signal control problem as a Stackelberg or leader-follower game between road users and the administration. The Stackelberg game can be represented as a bilevel problem, where the upper-level problem aims to find the optimal signal setting or link capacity expansions which maximizes system performance, and the lower-level problem aims to solve the user equilibrium (UE) flows, respectively [15, 16].

Marcotte [17], Sheffi and Powell [18], Heydecker and Khoo [19], Smith and van Vuren [20], Tan et al. [21], Cantarella et al. [22], Gartner et al. [13], Smith et al. [23], van Vuren and van Vliet [24], and others proposed algorithms to solve the network problem. However, when calculating optimal settings in general road networks, there were no efficient solution algorithms that are combined with anticipating the reactions of road users. Moreover, the iterative optimization assignment (IOA) procedure which solves signal settings and equilibrium flows cannot be expected to converge to the true solution and might lead to a decline in network performance [14, 25]. The sensitivity analysis-based algorithm evaluates the influence factors as the derivatives of the reaction functions with respect to the upper-level decision variables. The derivative information is obtained by implementing sensitivity analysis for a given solution of the user equilibrium problem [26–30]. For the singularity, algorithms can only solve a small network problem. Cho [31] proposed a generalized inverse method, Cho et al. [32] proposed a row reduction method, Patriksson [33], Josefsson, and Patriksson [34] proposed directional derivative method, and Yang and Bell [35] proposed a column reduction method to overcome the singularity problem. In the sensitivity analysis-based linear approximation algorithm, the sensitivity information is used to create a linear approximation of the reaction function and is then inserted into the upper-level problem, iterated until the solutions converge [34, 36, 37]. Recently, Chiou has conducted several studies related to optimal design of area traffic control with equilibrium network flows and proposed a number of computational algorithms to solve the problem, such as the projected Quasi-Newton method and the bundle subgradient projection method [38–41]. Moreover, the ENSCP, combined with an explicit traffic model, TRANSYT, was proposed to evaluate the performance index of the system more precisely [4, 38, 42–44].

The remainder of this paper is organized as follows: Section 2 introduces the equilibrium network flow models and the first-order sensitivity formula obtained by row reduction method. Section 3 introduces the matrix calculus theory and the second-order sensitivity formula. The network signal control model and solution algorithm are presented in Section 4. Finally, a numerical example and conclusion are presented in Sections 5 and 6, respectively.

2. Sensitivity Analysis of Equilibrium Network Flows

2.1. Equilibrium Network Flow Models. Consider a transportation network consisting of a finite set of nodes $i \in N$ and a finite set of arcs $a \in A$ together with a nonempty set of origin-destination (OD) pairs $w \in W$. For each $w \in W$, there is a nonempty finite set of paths, $p \in P_w$. Let real numbers, nonnegative reals, and positive reals be denoted by R, R_+, and R_{++}, respectively. The path flow vector, h, arc flow vector, f, and travel demand vector, T, are denoted in the following equations:

$$h = \left(h_p : p \in P, P = \bigcup_{w \in W} P_w \right) \in R_+^{|P|},$$
$$f = \left(f_a : a \in A \right) \in R_+^{|A|}, \tag{1}$$
$$T = \left(T_w : w \in W \right) \in R_{++}^{|W|},$$

where $|P|$, $|A|$, and $|W|$ denote the cardinalities of P, A, and W, respectively. The relationship between arc flow, path flow, and travel demand is given by

$$\Delta h = f, \qquad \Lambda h = T, \tag{2}$$

where Δ is a $|A| \times |P|$ matrix, with $\Delta_{ap} = 1$, if arc a belongs to path p and $\Delta_{ap} = 0$ otherwise; Λ is a $|W| \times |P|$ matrix, with $\Lambda_{wp} = 1$, if OD pair w belongs to path p and $\Lambda_{wp} = 0$ otherwise. In general, Δ is called arc/path incidence matrix and Λ is called OD/path incidence matrix.

In sensitivity analysis, a vector of perturbation parameters with dimension $\rho, s \in R^\rho$, is introduced. The arc cost function and travel demand function are supposed to be influenced by s. Let $t(f, s)$ be the arc cost function vector and let $T(s)$ be the travel demand function vector. The path cost function vector is given by $c(h, s) = \Delta^T t(f, s)$. When the network equilibrium is reached, the following equations must be satisfied:

$$h_p > 0 \Longrightarrow c_p(h, s) = \mu_w, \quad p \in P_w, \; w \in W,$$
$$h_p = 0 \Longrightarrow c_p(h, s) \geq \mu_w, \quad p \in P_w, \; w \in W, \tag{3}$$

where μ_w is the equilibrium path cost of OD pair w. Equations (3) are recognized as Wardrop equilibrium conditions; say, there is no traveler can change path unilaterally to improve his travel time [45]. Generally, the equilibrium network flow problem can be written in the form of variational inequality (VI) problem as follows [46]. Find $f \in \Omega(s)$ such that

$$t(f(s), s)^T (u - f) \geq 0, \quad \forall u \in \Omega(s),$$
$$\Omega(s) = \{ f \mid \Delta h = f, \Lambda h = T(s), h \geq 0 \}, \tag{4}$$

where Ω is the feasible arc flow solution set of the network flow problems.

An equivalent VI can be written with the cost function in terms of path flow variable h rather than arc flow variable f as follows. Find $h \in \Gamma(s)$ such that

$$c(h, s)^T (u - h) \geq 0, \quad \forall u \in \Gamma(s),$$
$$\Gamma(s) = \{ h \mid \Lambda h = T(s), h \geq 0 \}, \tag{5}$$

where Γ is the feasible path flow solution set of the network flow problems.

2.2. First-Order Sensitivity Formula for Equilibrium Network Flows. The classical first-order sensitivity analysis for equilibrium network flows was proposed by Tobin and Friesz [47]. However, Tobin and Friesz method had a strong requirement on the topology of network which may not hold in practical networks [48]. Cho et al. proposed the row reduction method to overcome this issue [32]. In this paper, we only summarize the key results of the row reduction method, and the readers are encouraged to refer to the original paper [32] for more details.

In the row reduction method, a maximal set of rows from Δ, say Δ_1, is selected for which the combined matrix $[\Delta_1, \Lambda]^T$ is of full row rank. Hence, Δ can be partitioned as $[\Delta_1; \Delta_2]$. Assume that the number of independent arcs is $\alpha_1$and the number of dependent arcs is α_2, respectively. Therefore, $|A| = \alpha = \alpha_1 + \alpha_2$. For a differentiable function, $f : S \rightarrow R^m$, let the partial derivative of f with respect to s (the Jacobian matrix of f) be denoted by $D_s f$. Let the second-order partial derivative of f with respect to s be denoted by $H_s f$. The first-order sensitivity formula can be expressed as

$$\begin{bmatrix} D_s f(s) \\ D_s \mu(s) \end{bmatrix} = \begin{bmatrix} D_f t(f,s) & -M^T \\ M & 0 \end{bmatrix}^{-1} \begin{bmatrix} -D_s t(f,s) \\ \Delta_2 N_2 D_s T(s) \end{bmatrix}, \quad (6)$$

where

$$\begin{aligned} M &= [\Delta_2 N_1 \;\; -I], \\ N_1 &= \Delta_1^T M_{11} + \Lambda^T M_{21}, \\ N_2 &= \Delta_1^T M_{12} + \Lambda^T M_{22}, \\ \left(\begin{bmatrix} \Delta_1 \\ \Lambda \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Lambda \end{bmatrix}^T \right)^{-1} &= \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}. \end{aligned} \quad (7)$$

3. Second-Order Sensitivity Formula for Equilibrium Network Flows

The sensitivity analysis-based nonlinear approximation heuristic algorithm (NLAA) was firstly proposed by Cho and Lin [37]. With the first-order and the second-order sensitivity information, the reaction function of the lower-level problem can be approximated by a nonlinear function. However, Cho and Lin did not provide the general form of the second-order sensitivity information. In this section, we will introduce the preliminary of matrix calculus theory and derive the general form of the second-order sensitivity formula for equilibrium network flows.

3.1. Preliminary Definitions and Theorems. To derive the second-order sensitivity formula, we introduce some definitions and theorems of matrix calculus as follows [49, 50].

Definition 1 (Kronecker product). Let U be an $m \times n$ matrix and let V be a $p \times q$ matrix; then the Kronecker product of U and V, denoted by $U \otimes V$, is an $mp \times nq$ matrix defined by

$$U \otimes V = \begin{bmatrix} u_{11}V & \cdots & u_{1n}V \\ \vdots & \ddots & \vdots \\ u_{m1}V & \cdots & u_{mn}V \end{bmatrix}. \quad (8)$$

Definition 2 (Vector operator). Let U be an $m \times n$ matrix and U_j is the jth column of U; then $\operatorname{vec} U$ is the $mn \times 1$ vector:

$$\operatorname{vec} U = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}. \quad (9)$$

Definition 3. Let U be an $m \times n$ real matrix function of a $p \times q$ matrix of real variables s. The derivative of U with respect to s is the $mn \times pq$ matrix:

$$D_s U = \frac{\partial \operatorname{vec} U}{\partial (\operatorname{vec} s)^T}. \quad (10)$$

Theorem 4 (chain rule for matrix functions, Magnus and Neudecker, 1985 [49]). *Let S be a subset of $R^{m\times n}$ and assume that $U : S \to R^{p\times q}$ is differentiable at an interior point y of S. Let P be a subset of $R^{p\times q}$ such that $U(x) \in P$ for all $x \in S$ and assume that $V : P \to R^{r\times s}$ is differentiable at an interior point $z = U(y)$ of P. Then the composite function $F : S \to R^{r\times s}$ defined by $F(x) = V(U(x))$ is differentiable at y and*

$$D_y F = (D_z V)(D_y U). \quad (11)$$

Theorem 5 (Magnus and Neudecker, 1985 [49]). *Let $U : S \to R^{m\times r}$ and $V : S \to R^{r\times n}$ be two matrix functions defined and differentiable on an open set S in $R^{p\times q}$. Then the simple product UV is differentiable on S and the Jacobian matrix is the $mn\times pq$ matrix:*

$$D_s(UV) = \frac{\partial \operatorname{vec} UV}{\partial (\operatorname{vec} s)^T} = (V^T \otimes I_m) D_s U + (I_n \otimes U) D_s V, \quad (12)$$

where I_m and I_n are the identity matrices of size m and n, respectively.

Theorem 6 (Magnus and Neudecker, 1999 [50]). *Let $f : S \to R^m$ be a function defined on a set S in R^n. Let r be an interior point of S and let $B(s_0; r)$ be an n-ball lying in S. Let s be a point in R^n with $\|s\| < r$, so that $s_0 + s \in B(s_0; r)$. If f is twice differentiable at s_0, then the second-order Taylor expansion of function f at $s_0 + s$ is*

$$f(s_0 + s) = f(s_0) + df(s_0; s) + \frac{1}{2} d^2 f(s_0; s), \quad (13)$$

where $df(s_0; s)$ and $d^2 f(s_0; s)$ are the first differential and the second differential of f at s_0, respectively, and

$$\begin{aligned} df(s_0; s) &= D_s f(s_0) \cdot (s - s_0), \\ d^2 f(s_0; s) &= \left((s - s_0)^T \otimes I_m\right) \cdot H_s f(s_0) \cdot (s - s_0). \end{aligned} \quad (14)$$

3.2. Second-Order Sensitivity Formula for Equilibrium Network Flows. To derive the second-order sensitivity formula for equilibrium network flows, it is intuitive to take derivative of (6) with respect to s. For convenience, let

$$\begin{aligned} \begin{bmatrix} D_f t(f(s), s) & -M^T \\ M & 0 \end{bmatrix}^{-1} &= U(f(s), s), \\ \begin{bmatrix} -D_s t(f(s), s) \\ \Delta_2 N_2 D_s T(s) \end{bmatrix} &= V(f(s), s), \end{aligned} \quad (15)$$

where U is an $(\alpha+\alpha_2)\times(\alpha+\alpha_2)$ matrix and V is an $(\alpha+\alpha_2)\times\rho$ matrix, respectively.

Lemma 7. *The second-order sensitivity for equilibrium network flows is*

$$\begin{bmatrix} H_s f \\ H_s \mu \end{bmatrix} = \left(V^T \otimes I_{(\alpha+\alpha_2)}\right) D_s U + (I_k \otimes U) D_s V, \quad (16)$$

where

$$U(f(s),s)=\begin{bmatrix} D_f t(f(s),s) & -M^T \\ M & 0 \end{bmatrix}^{-1},$$
$$V(f(s),s)=\begin{bmatrix} -D_s t(f(s),s) \\ \Delta_2 N_2 D_s T(s) \end{bmatrix}. \quad (17)$$

Proof. Since the first-order sensitivity is the product of (17), the second-order sensitivity can be obtained by taking derivative of the product with respect to s directly. According to Theorem 5, the formula of the second-order sensitivity is expressed as (16) and the proof is complete. □

4. Network Signal Control Model and Solution Algorithm

4.1. Network Signal Control Model. Consider the signal optimization problem, where the aim of the regulating agency is to minimize a network performance function $Z(s)$ such as total travel time or gas consumption, with fixed OD travel demand, where travelers select routes on the network in an optimal user fashion. Notably, S denotes the set of feasible signal control variables. For any given $s \in S$, a user optimal arc flow solution $f(s) \in \Omega$ exists and the problem of the regulator is to solve

$$P_1: \quad \min_{s\in X_1} \quad Z(s)=P(f(s),s)$$
$$\text{s.t.} \quad \text{user equilibrium}. \quad (18)$$

In the general problem, the signal variable that can be set by the controlling agent is green time. By specifying the cost functions t_a for each network arc a in terms of these variables and assuming that the behavioral hypothesis for route choice follows the first principle of Wardrop [45], problem P_1 can be presented as

$$P_2: \quad \min_{s\in X_1} \quad Z(s)=\sum_{a\in A} t_a(f(s),s)\, f_a(s) \quad (19)$$
$$\text{s.t.} \quad t(f(s),s)\cdot(u-f)\geq 0, \quad \forall u\in\Omega. \quad (20)$$

If $t(s,f)$ is strictly monotone, then, for each $s\geq 0$, (20) has a unique solution and function $f(s)$ is (continuously) differentiable at every point $s\geq 0$. Thus, P_2 can be rewritten as P_3:

$$P_3: \quad \min_{s\in X_1} \quad Z(s)=\sum_{a} t_a(f(s),s)\, f_a(s)$$
$$\text{s.t.} \quad s\geq 0. \quad (21)$$

Also, given $R(s,f(s)) = \min t(f(s),s)\cdot(u-f)$, then P_2 is equivalent to P_4:

$$P_4: \quad \min_{s\in X_1} \quad Z(s)=\sum_{a} t_a(f(s),s)\, f_a(s)$$
$$\text{s.t.} \quad R(f(s),s)=0. \quad (22)$$

4.2. Solution Algorithms. The iterative optimization assignment (IOA) method described by Tan et al. [21] is proceeded as follows. First, fix s and solve (20) for f; then fix f and solve (19) for s, continuing this process until $s^{k+1}-s^k\to 0$ or $f^{k+1}-f^k\to 0$. The final solution (f^N,s^N) is termed the Nash solution. Notably, that the solution obtained using the IOA algorithm is not necessarily an optimal solution of the equilibrium network control problem [14]. The sensitivity analysis of equilibrium network flows [32, 47] was used to solve the equilibrium network signal design problem [26, 36, 51].

4.2.1. A Sensitivity Analysis-Based Linear Approximation Heuristic Algorithm. The challenge in solving problem P_2 is that, since the lower level of the problem cannot be represented in closed form, it is impossible to obtain an explicit reaction function that can be plugged into the upper level. In the sensitivity analysis-based linear approximation heuristic algorithm, the sensitivity information is used to create a linear approximation of the reaction function and is then inserted into the upper-level problem, iterated until the solutions converge (abbreviated as LAA) [36].

The heuristic is detailed as follows.

Algorithm A1.

Step 0. Determine a fixed small value $\delta>0$ and an initial value s^0. Set $k=0$.

Step 1. Solve (18) given s^k and yielding f^k.

Step 2. Calculate the sensitivity information $D_s f$ by (6).

Step 3. Using $D_s f$, Taylor expansion and Theorem 6 form the linear approximation f^{k+1}, $f^{k+1}=f^k+D_s f\cdot(s^{k+1}-s^k)$. Since f^k, s^k, and $D_s f$ are known, f^{k+1} can be replaced by a function of s^{k+1}. Thus, $f^{k+1}=A+Bs^{k+1}$.

Step 4. Reformulate (21) as

$$\min_{s} \quad \sum_{a} t_a\left(A+Bs^{k+1},s\right)\cdot\left(A+Bs^{k+1}\right)$$
$$\text{s.t.} \quad s\geq 0. \quad (23)$$

Step 5. Solve the problem in Step 4 using any software package which can solve the optimal solution for s^{k+1}. If $|s^{k+1}-s^k|\leq\delta$, then stop; otherwise set $k=k+1$ and go to Step 1.

4.2.2. A Sensitivity Analysis-Based Nonlinear Approximation Heuristic Algorithm. In the sensitivity analysis-based linear approximation heuristic algorithm, the reaction function of the lower level is based on approximation by a linear function. In this section, the reaction function of the lower-level problem is based on approximation by a nonlinear function and is plugged into the upper-level problem and is iterated until the solutions converge (abbreviated as NLAA) [37].

Algorithm A2.

Step 0. Determine a fixed small value $\delta>0$ and an initial value s^0. Set $k=0$.

Step 1. Solve (20) given s^k and yielding f^k.

Step 2. Calculate the sensitivity information $D_s f$ and $H_s f$ by (6) and (16).

Step 3. Using $D_s f$ and $H_s f$, Taylor expansion and Theorem 6 form the nonlinear approximation f^{k+1}:

$$f^{k+1} = f^k + D_s f \cdot \left(s^{k+1} - s^k\right) + \frac{1}{2} \cdot \left(\left(s^{k+1} - s^k\right)^T \otimes I_m\right) \cdot H_s f \cdot \left(s^{k+1} - s^k\right). \tag{24}$$

Since f^k, s^k, $D_s f$, and $H_s f$ are known, f^{k+1} can be replaced by a function of s^{k+1}. Thus, $f^{k+1} = A + Bs^{k+1} + C(s^{k+1})^2$.

Step 4. Reformulate (21) as

$$\begin{aligned} \min_s \quad & \sum_a t_a\left(A + Bs^{k+1} + C\left(s^{k+1}\right)^2, s\right) \cdot \left(A + Bs^{k+1} + C\left(s^{k+1}\right)^2\right) \\ \text{s.t.} \quad & s \geq 0. \end{aligned} \tag{25}$$

Step 5. Solve the problem in Step 4 using any software package which can solve the optimal solution for s^{k+1}. If $|s^{k+1} - s^k| \leq \delta$; then stop; otherwise set $k = k + 1$ and go to Step 1.

In addition to describing the algorithm in more detail, we will provide a proof that if this algorithm converges, it converges to an optimal solution of problem P_2.

Lemma 8. *If algorithm A2 converges, it converges to a critical point of* P_2.

Proof. If the sequence s^k converges to s^*, $s^k \rightarrow s^*$, then we know that

(1) if we set $s^0 = s^*$, $f^0 = f^*$, then $s^1 = s^*$, $f^1 = f^*$; and

(2) let

$$\widehat{Z}(s) = \sum_{a\in A} t_a\left(A + Bs + Cs^2, s\right) f_a(s). \tag{26}$$

Then, by the Karush-Kuhn-Tucker necessary conditions for optimality of vectors $s^* \geq 0$, we know that the following must be true:

(i)

$$\frac{\partial \widehat{Z}(s^*)}{\partial s^i} = 0 \quad \text{if } s^{*i} > 0,\ i = 1, \ldots, n, \tag{27}$$

(ii)

$$\frac{\partial \widehat{Z}(s^*)}{\partial s^i} \geq 0 \quad \text{if } s^{*i} = 0,\ i = 1, \ldots, n. \tag{28}$$

So, taking the derivative with respect to s, we get

$$\begin{aligned} & \frac{\partial}{\partial s^i}\left[\sum_a t_a\left(A + Bs + Cs^2, s\right)\left(A + Bs + Cs^2\right)\right]\Bigg|_{s=s^*} \\ & = \sum_a t_a\left(A + Bs + Cs^2, s\right) \cdot (B + 2Cs)\Bigg|_{s=s^*} \\ & + \sum_a\left[\frac{\partial t_a\left(A + Bs + Cs^2, s\right)}{\partial\left(A + Bs + Cs^2\right)} \cdot (B + 2Cs) + \frac{\partial t_a\left(A + Bs + Cs^2, s\right)}{\partial s}\right] \cdot \left(A + Bs + Cs^2\right)\Bigg|_{s=s^*}. \end{aligned} \tag{29}$$

Further, we know

$$\begin{aligned} B &= D_s f|_{s=s^*}, \\ C &= \frac{1}{2} \cdot H_s f|_{s=s^*}, \\ f(s) &= A + Bs + Cs^2. \end{aligned} \tag{30}$$

So, substituting (29), we know

$$\begin{aligned} & \frac{\partial}{\partial s^i}\left[\sum_a t_a\left(A + Bs + Cs^2, s\right)\left(A + Bs + Cs^2\right)\right]\Bigg|_{s=s^*} \\ & = \sum_a t_a(f(s), s) \cdot (D_s f + H_s f \cdot s)\Bigg|_{s=s^*} \\ & + \sum_a\left[\frac{\partial t_a(f(s), s)}{\partial f} \cdot (D_s f + H_s f \cdot s) + \frac{\partial t_a(f(s), s)}{\partial s}\right] \cdot f_a\Bigg|_{s=s^*, f=f^*} \\ & = \sum_a t_a(f(s), s) \cdot D_s f(s)\Bigg|_{s=s^*} \\ & + \sum_a\left[\frac{\partial t_a(f(s), s)}{\partial f} \cdot D_s f(s) + \frac{\partial t_a(f(s), s)}{\partial s}\right] \cdot f_a\Bigg|_{s=s^*, f=f^*} \\ & = \frac{\partial}{\partial s}\left[\sum_a t_a(f(s), s) \cdot f_a(s)\right]\Bigg|_{s=s^*, f=f^*}. \end{aligned} \tag{31}$$

So, we know if conditions (i) and (ii) are satisfied, then the following should also be satisfied:

(iii)

$$\frac{\partial Z(s^*)}{\partial s^i} = 0 \quad \text{if } s^{*i} > 0,\ i = 1, \ldots, n, \tag{32}$$

(iv)

$$\frac{\partial Z(s^*)}{\partial s^i} \geq 0 \quad \text{if } s^{*i} = 0,\ i = 1, \ldots, n. \tag{33}$$

□

5. Numerical Example

This section provides two numerical examples which illustrate the computation of second-order sensitivity and the performance of NLAA. The first example demonstrates the computation of second-order sensitivity in detail. The second example focuses on the speed of convergence between LAA and NLAA.

Example 1. The first example is chosen from Dickson [25] and Fisk [14]. The network topology is shown in Figure 1. The set of OD pairs is $\{(1,2),(3,4)\}$ and a signal exists at the intersection of arcs 1 and 3. The cost functions used are

$$t_1 = \frac{f_1}{s_1}, \qquad t_2 = 2f_2, \qquad t_3 = \frac{2f_3}{s_3}, \tag{34}$$

where s_a denotes the green time on arc a and the cycle time, $s_1 + s_3$, is equal to 20.

Additionally, the travel demand T_1 from node 1 to node 2 is 10 and the travel demand T_2 from node 3 to node 4 is 10. Table 1 lists the arc cost functions $t_a(f_a, s_a)$ and the system objective function $Z(s)$.

5.1. First-Order and Second-Order Sensitivity Formulas. In this example, s_3 can be replaced by 20-s_1, and s_1 that is the only perturbation parameter (control variable) should be considered. Therefore, ρ is equal to 1. Together with (6) and (15), the first-order sensitivity with respect to s_1 can be rewritten as

$$\begin{bmatrix} D_s f \\ D_s \mu \end{bmatrix} = U \cdot V = \begin{bmatrix} \frac{s_1}{1+2s_1} & \frac{-s_1}{1+2s_1} & 0 & \frac{-2s_1}{1+2s_1} & 0 \\ \frac{-s_1}{1+2s_1} & \frac{s_1}{1+2s_1} & 0 & \frac{-s_1}{1+2s_1} & 0 \\ 0 & 0 & 0 & 0 & -1 \\ \frac{2s_1}{1+2s_1} & \frac{1}{1+2s_1} & 0 & \frac{2}{1+2s_1} & 0 \\ 0 & 0 & 1 & 0 & \frac{-1}{-20+s_1} \end{bmatrix} \times \begin{bmatrix} \frac{f_1}{s_1^2} \\ 0 \\ \frac{-2f_3}{(20-s_1)^2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{f_1}{s_1(1+2s_1)} \\ \frac{-f_1}{s_1(1+2s_1)} \\ 0 \\ \frac{2f_1}{s_1(1+2s_1)} \\ \frac{-2f_3}{(20-s_1)^2} \end{bmatrix}, \tag{35}$$

Table 1: Arc cost functions and the system objective function in Example 1.

$t_a(f_a, s_a) = P_a + Q_a(f_a/s_a)$		
$Z(s) = \sum_a (t_a(f_a, s_a) \cdot f_a)$		
Arc number	P_a	Q_a
1	2	1
2	0	2
3	0	2

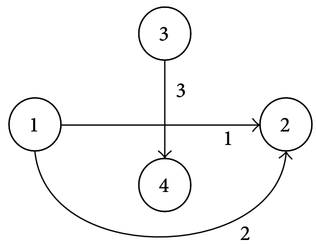

Figure 1: The network topology in Example 1.

where

$$D_s f = \begin{bmatrix} \frac{f_1}{s_1(1+2s_1)} \\ \frac{-f_1}{s_1(1+2s_1)} \\ 0 \end{bmatrix}, \qquad D_s \mu = \begin{bmatrix} \frac{2f_1}{s_1(1+2s_1)} \\ \frac{-2f_3}{(20-s_1)^2} \end{bmatrix}. \tag{36}$$

In (36), the sensitivity information of arc flow represents the change of arc flow on arc a, respectively, when the control variable s_1 increases one unit. Since $f_1 \geq 0$ and $s_1 > 0$, the equilibrium flow on arc 1 will increase when s_1 increases one unit. In the meanwhile, the equilibrium flow on arc 2 will decrease. Because OD pair (3, 4) has only one path (arc 3), s_1 will not affect the equilibrium flow on arc 3.

From Lemma 7, the second-order sensitivity with respect to control variable is

$$\begin{bmatrix} H_s f \\ H_s \mu \end{bmatrix} = \left(V^T \otimes I_{(\alpha+\alpha_2)}\right) D_s U + \left(I_k \otimes U\right) D_s V. \tag{37}$$

By Theorem 4, $D_s U$ can be derived by the chain rule for matrix functions as follows:

$$\begin{aligned} D_s U &= \left(D_{f,\mu} U\right) \begin{bmatrix} D_s f \\ D_s \mu \end{bmatrix} + \left(D_s U\right) \\ &= \begin{bmatrix} \frac{\partial \operatorname{vec} U}{\partial \operatorname{vec} f} & \frac{\partial \operatorname{vec} U}{\partial \operatorname{vec} \mu} \end{bmatrix} \begin{bmatrix} D_s f \\ D_s \mu \end{bmatrix} + \begin{bmatrix} \frac{\partial \operatorname{vec} U}{\partial \operatorname{vec} s_1} \end{bmatrix}. \end{aligned} \tag{38}$$

In this example, the matrix U is only dependent on s_1. Hence, $D_{f,\mu}U = 0$ and (38) can be rewritten as

$$D_sU = \left[\frac{\partial\ \mathrm{vec}\,U}{\partial\ \mathrm{vec}\,s_1}\right] = \begin{bmatrix} \dfrac{\partial U_1}{\partial s_1} \\ \dfrac{\partial U_2}{\partial s_1} \\ \dfrac{\partial U_3}{\partial s_1} \\ \dfrac{\partial U_4}{\partial s_1} \\ \dfrac{\partial U_5}{\partial s_1} \end{bmatrix}, \tag{39}$$

where U_j represents the jth column of matrix U, and

$$\frac{\partial U_1}{\partial s_1} = \begin{bmatrix} \dfrac{1}{(1+2s_1)^2} \\ \dfrac{-1}{(1+2s_1)^2} \\ 0 \\ \dfrac{2}{(1+2s_1)^2} \\ 0 \end{bmatrix}, \qquad \frac{\partial U_2}{\partial s_1} = \begin{bmatrix} \dfrac{-1}{(1+2s_1)^2} \\ \dfrac{1}{(1+2s_1)^2} \\ 0 \\ \dfrac{-2}{(1+2s_1)^2} \\ 0 \end{bmatrix},$$

$$\frac{\partial U_3}{\partial s_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad \frac{\partial U_4}{\partial s_1} = \begin{bmatrix} \dfrac{-2}{(1+2s_1)^2} \\ \dfrac{2}{(1+2s_1)^2} \\ 0 \\ \dfrac{-4}{(1+2s_1)^2} \\ 0 \end{bmatrix},$$

$$\frac{\partial U_5}{\partial s_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{2}{(-20+s_1)^2} \end{bmatrix}. \tag{40}$$

Similarly, D_sV can be derived as

$$\begin{aligned} D_sV &= \left(D_{f,\mu}V\right)\begin{bmatrix} D_sf \\ D_s\mu \end{bmatrix} + \left(D_sV\right) \\ &= \left[\frac{\partial\ \mathrm{vec}\,V}{\partial\ \mathrm{vec}\,f}\ \ \frac{\partial\ \mathrm{vec}\,V}{\partial\ \mathrm{vec}\,\mu}\right]\begin{bmatrix} D_sf \\ D_s\mu \end{bmatrix} + \left[\frac{\partial\ \mathrm{vec}\,V}{\partial\ \mathrm{vec}\,s_1}\right] \\ &= \begin{bmatrix} \frac{1}{s_1^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-2}{(20-s_1)^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dfrac{f_1}{s_1(1+2s_1)} \\ \dfrac{-f_1}{s_1(1+2s_1)} \\ 0 \\ \dfrac{2f_1}{s_1(1+2s_1)} \\ \dfrac{-2f_3}{(20-s_1)^2} \end{bmatrix} \\ &\quad + \begin{bmatrix} \dfrac{-2f_1}{s_1^3} \\ 0 \\ \dfrac{-4f_3}{(20-s_1)^3} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{f_1}{s_1^3(1+2s_1)} - \dfrac{2f_1}{s_1^3} \\ 0 \\ \dfrac{-4f_3}{(20-s_1)^3} \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \tag{41}$$

According to Definition 1, (39), (40), and (38), (37) can be rewritten as

$$\begin{aligned} \begin{bmatrix} H_sf \\ H_s\mu \end{bmatrix} &= \left(V^T \otimes I_5\right) D_sU + \left(I_1 \otimes U\right) D_sV \\ &= \begin{bmatrix} \frac{f_1}{s_1^2}\cdot I_5 \\ 0\cdot I_5 \\ \frac{-2f_3}{(20-s_1)^2}\cdot I_5 \\ 0\cdot I_5 \\ 0\cdot I_5 \end{bmatrix}^T \begin{bmatrix} \dfrac{\partial U_1}{\partial s_1} \\ \dfrac{\partial U_2}{\partial s_1} \\ \dfrac{\partial U_3}{\partial s_1} \\ \dfrac{\partial U_4}{\partial s_1} \\ \dfrac{\partial U_5}{\partial s_1} \end{bmatrix} \\ &\quad + (1\cdot U)\begin{bmatrix} \dfrac{f_1}{s_1^3(1+2s_1)} - \dfrac{2f_1}{s_1^3} \\ 0 \\ \dfrac{-4f_3}{(20-s_1)^3} \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \dfrac{-4f_1}{s_1(1+2s_1)^2} \\ \dfrac{4f_1}{s_1(1+2s_1)^2} \\ 0 \\ \dfrac{-8f_1}{s_1(1+2s_1)^2} \\ \dfrac{4f_3}{(-20+s_1)^3} \end{bmatrix}. \end{aligned} \tag{42}$$

Therefore,

$$H_s f = \begin{bmatrix} \dfrac{-4f_1}{s_1(1+2s_1)^2} \\ \dfrac{4f_1}{s_1(1+2s_1)^2} \\ 0 \end{bmatrix}, \qquad H_s\mu = \begin{bmatrix} \dfrac{-8f_1}{s_1(1+2s_1)^2} \\ \dfrac{4f_3}{(-20+s_1)^3} \end{bmatrix}. \quad (43)$$

5.2. Computational Results of LAA and NLAA. Based on (36) and (43), the first differential and the second differential of equilibrium arc flow f_a can be obtained by (14). At iteration k,

$$\begin{aligned} Bs^{k+1} &= df(s_0; s) = D_s f \big|_{f=f^k, s_1=s_1^k} \cdot \left(s_1^{k+1} - s_1^k\right) \\ &= \begin{bmatrix} \dfrac{f_1^k}{s_1^k(1+2s_1^k)} \\ \dfrac{-f_1^k}{s_1^k(1+2s_1^k)} \\ 0 \end{bmatrix} \cdot \left(s_1^{k+1} - s_1^k\right) \\ &= \begin{bmatrix} \dfrac{f_1^k\left(s_1^{k+1}-s_1^k\right)}{s_1^k(1+2s_1^k)} \\ \dfrac{-f_1^k\left(s_1^{k+1}-s_1^k\right)}{s_1^k(1+2s_1^k)} \\ 0 \end{bmatrix}, \\ C\left(s^{k+1}\right)^2 &= \frac{1}{2} d^2 f(s_0; s) \\ &= \frac{1}{2}\left(\left(s_1^{k+1}-s_1^k\right)^T \otimes I_3\right) \cdot H_s f \big|_{f=f^k, s_1=s_1^k} \\ &\quad \cdot \left(s_1^{k+1}-s_1^k\right) \\ &= \frac{1}{2}\left(\left(s_1^{k+1}-s_1^k\right) \otimes \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) \cdot \begin{bmatrix} \dfrac{-4f_1^k}{s_1^k(1+2s_1^k)^2} \\ \dfrac{4f_1^k}{s_1^k(1+2s_1^k)^2} \\ 0 \end{bmatrix} \\ &\quad \cdot \left(s_1^{k+1}-s_1^k\right) \\ &= \begin{bmatrix} \dfrac{-4f_1^k\left(s_1^{k+1}-s_1^k\right)^2}{2s_1^k(1+2s_1^k)^2} \\ \dfrac{4f_1^k\left(s_1^{k+1}-s_1^k\right)^2}{2s_1^k(1+2s_1^k)^2} \\ 0 \end{bmatrix}. \end{aligned} \quad (44)$$

In this example, both LAA and NLAA are implemented in the MATLAB environment. Set $\delta = 0.001$ and the initial $s_1 = 10$; Table 2 lists the computational results of LAA and NLAA approaches, and it shows that NLAA is more efficient than LAA.

Table 2: Computational results of LAA and NLAA in Example 1.

Iteration	LAA			NLAA		
	f_1	s_1	Z	f_1	s_1	Z
1	8.44693	7.63647	47.23792	8.45125	7.70052	47.23576
2	8.45366	7.73667	47.23553	8.45326	7.73055	47.23552
3	8.45323	7.73019	47.23552	8.45326	7.73056	47.23552
4	8.45325	7.73040	47.23552			

Example 2. This example is a simplified real network which represents the afternoon rush hour traffic between the working area Hsinchu Science-Based Industrial Park (HSIP) and the residential area Jhubei city. The network topology follows Figure 2. In this period, there is a large amount of travel demand from HSIP (node 1) to Jhubei city (node 16). There are two parallel paths from HSIP to Jhubei city. One is a freeway (arc 2-arc 4-arc 16) and the other is a highway with 5 signal-controlled intersections (arc 1-arc 6-arc 8-arc 10-arc 12-arc 14). The objective of this problem is to find the optimal signal settings which minimize the system cost. The arc cost functions $t_a(f_a, s_a)$ and the system objective function $Z(s)$ are listed in Table 3. Table 4 lists the origin-destination demand. For the signal-controlled intersections, the arcs entering the same intersection share the same cycle time and the minimum green time for each approach is 10 sec.

In this example, we set $\delta = 0.1$ and the initial $s_a = Cyc_a/2$ for each signalized arc. Table 5 lists the computational results of LAA and NLAA, respectively. Two parallel paths from node 1 to node 16 (2-4-16 and 1-6-8-10-12-14) have the same equilibrium travel time 13.4069 min. Table 5 shows that NLAA only takes 6% iterations, compared with LAA, to attain the same level of precision. Figure 3 shows the convergence curves of LAA and NLAA, respectively. The convergence rate of LAA is slower than NLAA due to the zigzag effect. Compared with Example 1, NLAA has more improvement in the speed of convergence than in Example 2. It may imply that NLAA is more efficient to deal with more nonlinear problems.

6. Conclusions

The key information to solve the equilibrium network signal control problem (ENSCP) is the reaction function of the lower-level problem. Because the reaction function cannot be obtained explicitly, the sensitivity information of equilibrium network flows is used to approximate it. Based on the first-order sensitivity formula and the matrix calculus, this paper first presents the general form of the second-order sensitivity formula for equilibrium network flows. With the second-order sensitivity formula, the reaction function can be approximated more accurately by a nonlinear function. From HSIP to Jhubei city, a simplified real network example demonstrates the speed of convergence between LAA and NLAA. The NLAA has significant improvement in solving the ENSCP with complicated arc cost functions; in this example, the NLAA only takes 6% iterations to attain the same level of precision.

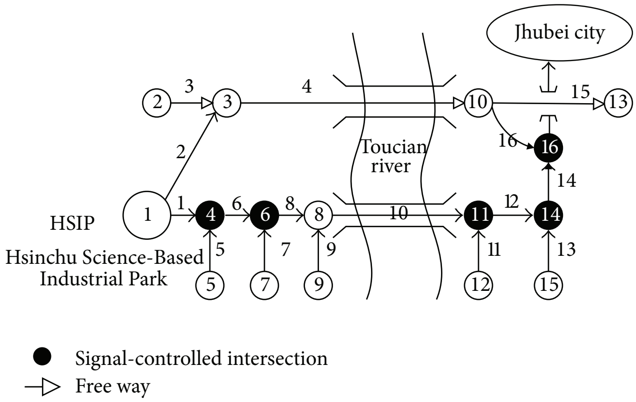

Figure 2: The network topology in Example 2.

Table 3: Arc cost functions and the system objective function in Example 2.

Signalized arc cost function	$t_a(f_a, s_a) = t_{0a}\left(1 + \alpha_a\left(\dfrac{f_a}{C_a(s_a/\mathrm{Cyc}_a)}\right)^{\beta_a}\right)$				
Nonsignalized arc cost function	$t_a(f_a, s_a) = t_{0a}\left(1 + \alpha_a\left(\dfrac{f_a}{C_a}\right)^{\beta_a}\right)$				
System objective function	$Z(s) = \sum_a (t_a(f_a, s_a) \cdot f_a)$				
Arc number	t_{0a} (min)	α_a	β_a	C_a (veh/min)	Cyc_a (sec)
1	1.8545	0.9200	3.5800	56.6667	300
2	0.8667	0.9200	3.5800	40.0000	—
3	1.8000	1.2700	3.9600	115.0000	—
4	2.2364	1.2700	3.9600	115.0000	—
5	0.2945	1.2100	2.3900	28.3333	300
6	0.1964	1.4200	2.3200	85.0000	300
7	0.3818	0.8600	4.3400	85.0000	300
8	1.0154	1.2700	3.9600	68.3333	—
9	1.0000	1.2100	2.3900	20.0000	—
10	1.0154	1.2700	3.9600	68.3333	180
11	0.3273	0.9200	3.5800	56.6667	180
12	0.9818	1.4200	2.3200	85.0000	150
13	0.6545	0.8600	4.3400	113.3333	150
14	1.2000	1.5000	2.4400	113.3333	150
15	3.8727	1.2700	3.9600	115.0000	—
16	0.4909	0.9200	3.5800	40.0000	150

Table 4: Origin-destination demand table in Example 2 (unit: veh/hr).

		Destination						
		4	6	8	11	13	14	16
Origin	1	50	275	475	400	1250	275	2250
	2	0	0	0	0	2550	0	1400
	5	0	150	250	200	0	150	250
	7	0	0	500	400	0	300	450
	9	0	0	0	325	0	225	350
	12	0	0	0	0	0	125	175
	15	0	0	0	0	0	0	900

This study focuses on the NLAA and a simplified delay formula is adopted to reflect the influence of traffic congestion. Practically, a traffic propagation model, such as TRANSYT model, should be included when solving the ENSCP. Since the derivatives of TRANSYT model have been obtained explicitly [4, 42], it can be extended to second-order derivatives and applied to NLAA in the future research.

Compared with LAA, the number of multiplications for matrix multiplication is greatly increasing in NLAA due to the Kronecker-product operation. NLAA has polynomial complexity with the network size and the number of perturbation parameters because of the property of the Kronecker product. There is still opportunity to improve the

Table 5: Computational results of LAA and NLAA in Example 2.

Arc number	LAA			NLAA		
	s_a (sec)	f_a (veh/min)	t_a (min)	s_a (sec)	f_a (veh/min)	t_a (min)
1	195.9103	31.4891	2.8118	195.8881	31.4901	2.8123
2	—	51.4275	2.8272	—	51.4266	2.8271
3	—	65.8333	2.0510	—	65.8333	2.0510
4	—	117.2609	5.3043	—	117.2599	5.3042
5	104.0897	16.6667	1.5529	104.1119	16.6667	1.5522
6	181.5056	47.3225	0.4263	181.4711	47.3234	0.4265
7	118.4944	27.5000	0.5199	118.5289	27.5000	0.5197
8	—	67.7391	2.2611	—	67.7401	2.2612
9	—	15.0000	1.6084	—	15.0000	1.6084
10	167.0451	62.3225	2.2192	167.0853	62.3234	2.2181
11	12.9549	5.0000	0.9517	12.9147	5.0000	0.9587
12	123.8799	45.2391	1.4849	123.8808	45.2401	1.4849
13	26.1201	15.0000	0.8256	26.1192	15.0000	0.8256
14	45.4083	42.3225	4.2042	45.4092	42.3234	4.2042
15	—	63.3333	4.3361	—	63.3333	4.3361
16	104.5917	53.9275	5.2760	104.5908	53.9266	5.2759
Iteration number		101			6	
Objective value (Z)		2188.2886			2188.2404	

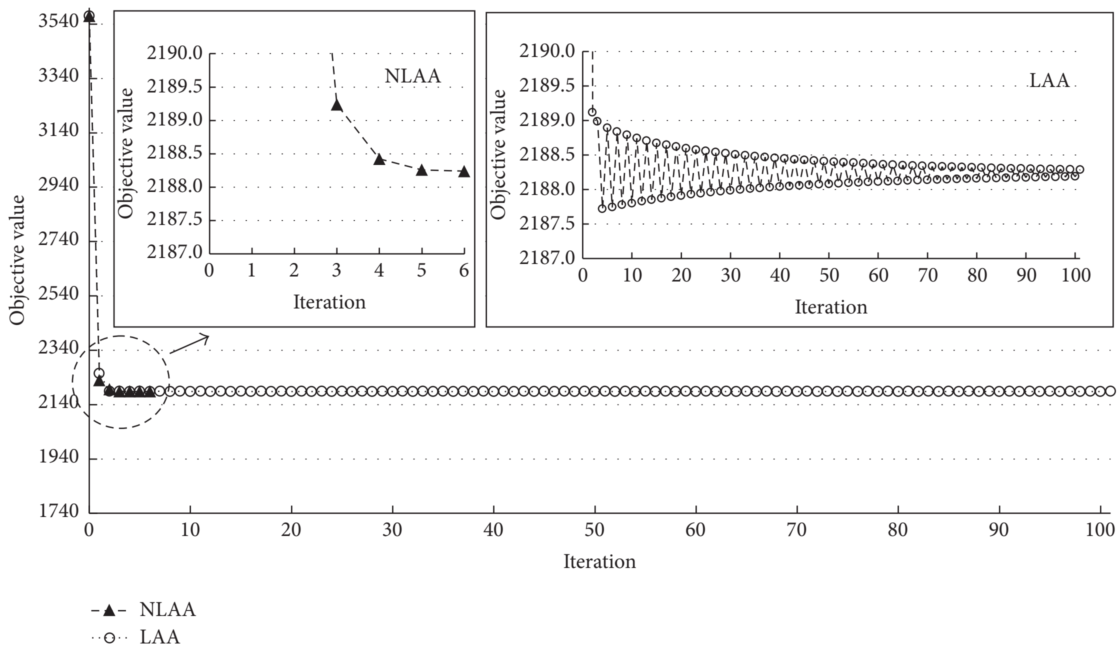

Figure 3: The convergence curves of LAA and NLAA in Example 2.

computing efficiency through adopting effective Kronecker-product algorithms.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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