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Research Article

Krein Space-Based H_∞ Fault Estimation for Discrete Time-Delay Systems

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This paper investigates the finite-time H_∞ fault estimation problem for linear time-delay systems, where the delay appears in both state and measurement equations. Firstly, the design of finite horizon H_∞ fault estimation is converted into a minimum problem of certain quadratic form. Then we introduce a stochastic system in Krein space, and a sufficient and necessary condition for the minimum is derived by applying innovation analysis approach and projection theory. Finally, a solution to the H_∞ fault estimation is obtained by recursively computing a partial difference Riccati equation, which has the same dimension as the original system. Compared with the conventional augmented approach, the solving of a high dimension Riccati equation is avoided.

1. Introduction

Krein-space theory has proven to be an effective tool in dealing with the indefinite quadratic control/filtering problems. Some recent researches on H_∞ filtering have led to an interesting connection with Kalman filtering in Krein space [1, 2]. Comparing with the linear estimation approaches in Hilbert space, the Krein-space theory can lead to not only less conservative results but also computationally attractive algorithms. It has been shown in [1] that a finite horizon linear estimation problem can be cast into a problem of calculating the minimum point of a certain quadratic form. By applying linear estimation in Krein space, one can calculate recursively the minimum point via Riccati equation. In [2] the authors consider the H_∞ prediction problem for time-varying continuous-time systems with delayed measurements in the finite horizon case. The necessary and sufficient condition for the existence of an H_∞ predictor is obtained by applying a reorganized innovation approach in Krein space.

On the other hand, fault estimation is one of the most important issues. The paper [3] designs a fuzzy fault detection filter for T-S fuzzy systems with intermittent measurements, and all the results are formulated in the form of linear matrix inequalities. In [4], a sufficient condition for the existence of a fault filter is exploited in terms of certain linear matrix inequality. Reference [5] is concerned with the robust fault detection problem for a class of discrete-time networked systems with distributed sensors. The existence of the desired fault detection filter can be determined from the feasibility of a set of linear matrix inequalities. The paper [6] addresses the fault detection problem for discrete-time Markovian jump systems. The characterization of the gains of the desired fault detection filters is derived in terms of the solution to a convex optimization problem that can be easily solved by using the semidefinite program method. As for the fault estimation problem, the Krein-space approach has received much attention so far [7–12]. A Krein-space approach is presented in [7] to H_∞ fault estimation for LDTV system, where the augmented approach [13] is also used. Different from [7], a more further result is obtained by a Krein-space approach and nonaugmented approach for the same problem in [8]. Recently, by applying Krein-space approach and reorganized innovation approach, [9] considers the finite-horizon H_∞ fault estimation for linear discrete time-varying systems with delayed measurements [9]. Finite-horizon H_∞ fault estimation for uncertain linear discrete time-varying systems with known inputs is considered in [10].

Recently, we note that time-delay systems have received much attention [14–19]. For optimal estimation problem, when the delay appears in state, the reorganized innovation approach is not suitable for estimation problem. Motivated by this point, we consider the finite-horizon problem H_∞ fault estimation for linear discrete systems with time delay, where the delay appears in both state and measurement, which contain [9] as a special case. To the best of our knowledge, this problem has not yet been investigated, and this constitutes the primary motivation for our research. On the other hand, we desire to obtain the necessary and sufficient condition for the existence of an H_∞ fault estimator. A natural idea is to use Krein space to deal with the finite-horizon H_∞ fault estimation for linear time-delay systems, and this gives rise to another motivation of our work. The main contributions of the paper are highlighted as follows. (i) The necessary and sufficient condition will be derived for fault estimation problem with time delay. (ii) Compared with the augmentation approach [13], our result on estimation is given based on a partial difference Riccati equation, and hence the solving of an high dimension Riccati equation is avoided.

The organization of this paper is as follows. The problem statement is given in Section 2. Section 3 presents the fault estimator design in terms of a partial difference Riccati equation. A numerical example is given to demonstrate the effectiveness of the approach in Section 4, and the paper is concluded in Section 5.

Notation. Throughout this paper, a real symmetric matrix $P > 0$ (≥ 0) denotes P being a positive definite (or positive semidefinite) matrix. I denotes an identity matrix of appropriate dimension. The superscripts "-1" and "T" represent the inverse and transpose of a matrix. $\mathscr{R}^n$ denotes the n-dimensional Euclidean space. $\mathscr{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ii} = 1$. For stochastic vectors α and β, inner product $\langle\alpha, \beta\rangle$ equals the covariance matrix of α and β. $\theta(k) \in l_2[0, N]$ means $\sum_{k=0}^{N}\theta^T(k)\theta(k) < \infty$. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problems Statement

Consider the following linear systems with time delay:

$$x(k+1) = \sum_{i=0}^{d} A_i x(k-h_i) + B_d d(k) + B_f f(k), \quad (1)$$

$$y(k) = \sum_{i=0}^{d} C_i x(k-l_i) + D_f f(k) + v(k), \quad (2)$$

where $x(k) \in \mathscr{R}^n$, $d(k) \in \mathscr{R}^p$, and $f(k) \in \mathscr{R}^r$ are the state, the driving disturbance, and the fault to be estimated, respectively. Also, $y(k) \in \mathscr{R}^m$ and $v(k) \in \mathscr{R}^m$ are measurements and noises, respectively. Without loss of generality, the delays are assumed to be of an increasing order: $0 = h_0 < h_1 < \cdots < h_d$, $0 = l_0 < l_1 < \cdots < l_d$. Moreover, it is assumed that $d(k)$, $f(k)$, and $v(k)$ belong to $l_2[0, N]$. For simplicity of presentation, we assume that A_i, B_d, B_f, C_i, and D_f are constant matrices even though the later development and results can be easily adapted to the time-varying case.

Problem. Given the observation $\{y(0), \ldots, y(N)\}$, seek a fault estimator $r(k)$ $(k = 0, \ldots, N)$ such that

$$\sup_{(x_0,w)\neq 0} \frac{\sum_{k=0}^{N}\|r(k)-f(k)\|^2}{(x_0-\hat{x}_0)^T\Pi_0^{-1}(x_0-\hat{x}_0)+\sum_{k=0}^{N}\|w(k)\|^2} < \gamma^2, \quad (3)$$

where γ is a given positive scalar, Π_0 is a positive definite matrix, and

$$w(k) = \begin{bmatrix} d^T(k) & f^T(k) & v^T(k)\end{bmatrix}^T. \quad (4)$$

Without loss of generality, the initial state estimator $\hat{x}_0$ is assumed to be zero. The value of $x(-k)$ is assumed to be zero, where $1 \leq k \leq \tau$, $\tau = \max(h_d, l_d)$, $E\{x(-i)x^T(-j)\} = 0$.

Define the fault estimation error between $r(k)$ and $f(k)$ as

$$v_f(k) = r(k) - f(k), \quad (5)$$

and introduce the following quadratic form:

$$J_N = x_0^T\Pi_0^{-1}x_0 + \sum_{k=0}^{N}\|w(k)\|^2 - \gamma^{-2}\sum_{k=0}^{N}\left\|v_f(k)\right\|^2. \quad (6)$$

Obviously, the H_∞ performance (3) is satisfied if and only if $J_N > 0$ for all $(x_0, w(k)) \neq 0$.

3. Main Result

We consider constructing an equivalent Krein-space problem to the minimum for J_N. To do so we need to introduce the following stochastic systems in a Krein space:

$$\mathbf{x}(k+1) = \sum_{i=0}^{d} A_i\mathbf{x}(k-h_i) + B_d\mathbf{d}(k) + B_f\mathbf{f}(k), \quad (7)$$

$$\mathbf{y}(k) = \sum_{i=0}^{d} C_i\mathbf{x}(k-l_i) + D_f\mathbf{f}(k) + \mathbf{v}(k), \quad (8)$$

$$\mathbf{r}(k) = \mathbf{f}(k) + \mathbf{v}_f(k), \quad (9)$$

with

$$\left\langle \begin{bmatrix}\mathbf{x}(0)\\ \mathbf{d}(k)\\ \mathbf{f}(k)\\ \mathbf{v}(k)\\ \mathbf{v}_f(k)\end{bmatrix}, \begin{bmatrix}\mathbf{x}(0)\\ \mathbf{d}(j)\\ \mathbf{f}(j)\\ \mathbf{v}(j)\\ \mathbf{v}_f(j)\end{bmatrix}\right\rangle = \operatorname{diag}\left(\Pi_0, I\delta_{kj}, I\delta_{kj}, I\delta_{kj}, -\gamma^2 I\delta_{kj}\right). \quad (10)$$

Let $\mathbf{y}_r(k) = \begin{bmatrix}\mathbf{y}^T(k) & \mathbf{r}^T(k)\end{bmatrix}^T$; then the linear space generated by the measurements in the Krein space up to time N can be written as

$$\mathscr{L}\{\mathbf{y}_r(k), 0 \leq k \leq N\}. \quad (11)$$

It is readily known that $\mathbf{y}_r(k)$ satisfies

$$\mathbf{y}_r(k) = \sum_{i=0}^{d} C_{ri}\mathbf{x}(k-l_i) + D_{fr}\mathbf{f}(k) + \mathbf{v}_r(k), \quad (12)$$

$$C_{ri} = \begin{bmatrix} C_i \\ 0 \end{bmatrix}, \qquad D_{fr} = \begin{bmatrix} D_f \\ I \end{bmatrix}, \qquad \mathbf{v}_r(k) = \begin{bmatrix} \mathbf{v}(k) \\ \mathbf{v}_f(k) \end{bmatrix}. \quad (13)$$

In the sequel, we denote the Krein-space projection of $\mathbf{m}(k)$ onto $\mathscr{L}\{\{\mathbf{y}_r(j)\}_{j=0}^{s}\}$ by $\widehat{\mathbf{m}}(k \mid s)$. Construct the innovations

$$\widetilde{\mathbf{y}}_r(k) = \mathbf{y}_r(k) - \widehat{\mathbf{y}}_r(k \mid k-1). \quad (14)$$

Defining $\widetilde{\mathbf{x}}(k \mid j) = \mathbf{x}(k) - \widehat{\mathbf{x}}(k \mid j)$, we further have

$$\widetilde{\mathbf{y}}_r(k) = \sum_{i=0}^{d} C_{ri}\widetilde{\mathbf{x}}(k-l_i \mid k-1) + D_{fr}\mathbf{f}(k) + \mathbf{v}_r(k). \quad (15)$$

Furthermore, we define the cross-covariance matrices of the state estimation error:

$$P(i,j,k) = E\left\{(\mathbf{x}(i) - \widehat{\mathbf{x}}(i \mid k))(\mathbf{x}(j) - \widehat{\mathbf{x}}(j \mid k))^T\right\}. \quad (16)$$

By employing the Krein-space theory, a necessary and sufficient condition for the existence of the desired H_∞ fault estimator is given in the following.

Lemma 1. *Consider the stochastic systems* (7)–(9). *For a given* $\gamma > 0$, *a fault estimator* $\mathbf{r}(k)$ *achieving the performance* (3) *exists if and only if*

$$\Theta(k) = \sum_{i=0}^{d}\sum_{j=0}^{d} C_i P(k-l_i, k-l_j, k-1) C_j^T + I + D_f D_f^T > 0,$$

$$\Xi(k) = (1-\gamma^2) I - D_f^T \Theta^{-1}(k) D_f < 0. \quad (17)$$

Furthermore, if the above conditions are satisfied, the desired H_∞ *fault estimator is given by*

$$\mathbf{r}(k) = D_f^T \Theta^{-1}(k)\widetilde{\mathbf{y}}(k), \quad (18)$$

where $\widetilde{\mathbf{y}}(k) = \mathbf{y}(k) - \widehat{\mathbf{y}}(k \mid k-1)$, *and the minimum of the quadratic form* J_N *is*

$$\min J_N = \sum_{k=0}^{N} \widetilde{\mathbf{y}}^T(k)\Theta^{-1}(k)\widetilde{\mathbf{y}}(k). \quad (19)$$

Proof. It can be seen from (12) and (15) that

$$\begin{aligned} R_{\widetilde{\mathbf{y}}_r(k)} &= \langle \widetilde{\mathbf{y}}_r(k), \widetilde{\mathbf{y}}_r(k) \rangle \\ &= \sum_{i=0}^{d}\sum_{j=0}^{d} C_{ri} P(k-l_i, k-l_j, k-1) C_{ri}^T + D_{fr} D_{fr}^T - \gamma^2 I \\ &= \begin{bmatrix} \Theta(k) & D_f \\ D_f^T & (1-\gamma^2) I \end{bmatrix}. \end{aligned} \quad (20)$$

Equation (20) can be further written as

$$M(k) R_{\widetilde{\mathbf{y}}_r(k)} M^T(k) = \Lambda(k), \quad (21)$$

where

$$M(k) = \begin{bmatrix} I & 0 \\ -D_f^T \Theta^{-1}(k) & I \end{bmatrix}, \qquad \Lambda(k) = \begin{bmatrix} \Theta(k) & 0 \\ 0 & \Xi(k) \end{bmatrix}. \quad (22)$$

We can draw the conclusion from (17) and (21) that $R_{\widetilde{\mathbf{y}}_r(k)}$ and $R_{\mathbf{v}_r(k)}$ have the same inertia. Therefore following the same line as in [1], the minimum value of J_N can be obtained as follows:

$$\begin{aligned} \min J_N &= \sum_{k=0}^{N} \widetilde{\mathbf{y}}_r^T(k) R_{\widetilde{\mathbf{y}}_r(k)}^{-1} \widetilde{\mathbf{y}}_r(k) \\ &= \sum_{k=0}^{N} \left[\widetilde{\mathbf{y}}^T(k) \quad \mathbf{r}^T(k) - \widetilde{\mathbf{y}}^T(k)\Theta^{-1}(k) D_f\right] \\ &\quad \times \begin{bmatrix} \Theta^{-1}(k) & 0 \\ 0 & \Xi^{-1}(k) \end{bmatrix} \begin{bmatrix} \widetilde{\mathbf{y}}(k) \\ \mathbf{r}(k) - D_f^T \Theta^{-1}(k)\widetilde{\mathbf{y}}(k) \end{bmatrix}. \end{aligned} \quad (23)$$

Thus the rest of the proof is clear. □

In the following, we are devoted to the estimator design in terms of the solution to a partial difference Riccati equation.

Lemma 2. *The state estimate is recursively calculated as*

$$\begin{aligned} \widehat{\mathbf{x}}(k+1 \mid k) &= \sum_{i=0}^{d} A_i \widehat{\mathbf{x}}(k-h_i \mid k), \\ \widehat{\mathbf{x}}(k-j \mid k) &= \widehat{\mathbf{x}}(k-j \mid k-1) + K_j(k)\widetilde{\mathbf{y}}_r(k) \\ &= \widehat{\mathbf{x}}(k-j \mid k-1) + \sum_{i=0}^{d} P(k-j, k-h_i, k-1) C_i^T \\ &\quad \Theta^{-1}(k)\widetilde{\mathbf{y}}(k), \quad j = 0, \ldots, l, \end{aligned} \quad (24)$$

where the initial values are $\widehat{\mathbf{x}}(-j \mid -1) = 0$ *and* $K_j(k)$ *can be calculated by*

$$\begin{aligned} K_j(k) &= \sum_{i=0}^{d} P(k-j, k-h_i, k-1) C_{ri}^T R_{\widetilde{\mathbf{y}}_r(k)}^{-1}, \\ R_{\widetilde{\mathbf{y}}_r(k)} &= \begin{bmatrix} \Theta(k) & D_f \\ D_f^T & (1-\gamma^2) I \end{bmatrix}, \end{aligned} \quad (25)$$

while $P(.,.,.)$ is calculated by the partial difference Riccati equation as

$$\begin{aligned} P(k-i,k-j,k) &= P(k-i,k-j,k-1) \\ &\quad - K_i(k) R_{\tilde{\mathbf{y}}_r(k)} K_j^T(k), \quad 0 \le i \le j \le \tau, \end{aligned} \tag{26}$$

$$P(k+1,k-j,k) = \sum_{i=0}^{d} A_i P(k-h_i,k-j,k), \quad 0 \le j \le \tau, \tag{27}$$

$$\begin{aligned} P(k+1,k+1,k) &= \sum_{i=0}^{d}\sum_{j=0}^{d} A_i P(k-h_i,k-h_j,k) A_j^T \\ &\quad + B_d B_d^T + B_f B_f^T, \end{aligned} \tag{28}$$

$$P(k-i,k-j,k) = P^T(k-j,k-i,k), \tag{29}$$

$$P(-i,-j,-1) = P_0(-i,-j), \quad 0 \le i \le j, \ 0 \le j \le \tau. \tag{30}$$

Proof. Applying projection theory, we have

$$\hat{\mathbf{x}}(k+1 \mid k) = \sum_{i=0}^{d} A_i \hat{\mathbf{x}}(k-h_i \mid k), \tag{31}$$

$$\hat{\mathbf{x}}(k-j \mid k) = \hat{\mathbf{x}}(k-j \mid k-1) + K_j(k)\tilde{\mathbf{y}}_r(k), \tag{32}$$

where $K_j(k)$ is given as

$$K_j(k) = E\left\{\mathbf{x}(k-j)\tilde{\mathbf{y}}_r^T(k)\right\} R_{\tilde{\mathbf{y}}_r(k)}^{-1}. \tag{33}$$

Noting that $\mathbf{x}(k-j) = \hat{\mathbf{x}}(k-j \mid k-1) + \tilde{\mathbf{x}}(k-j \mid k-1)$, then based on (15) one has

$$\begin{aligned} K_j(k) &= E\left\{\tilde{\mathbf{x}}(k-j \mid k-1)\tilde{\mathbf{y}}_r^T(k)\right\} R_{\tilde{\mathbf{y}}_r(k)}^{-1} \\ &= E\left\{\tilde{\mathbf{x}}(k-j \mid k-1)\left[\sum_{i=0}^{d} C_{ri}\tilde{\mathbf{x}}(k-l_i \mid k-1) \right.\right. \\ &\qquad \left.\left. + D_{fr}\mathbf{f}(k) + \mathbf{v}_r(k)\right]^T\right\} R_{\tilde{\mathbf{y}}_r(k)}^{-1} \\ &= \sum_{i=0}^{d} P(k-j,k-l_i,k-1) C_{ri}^T R_{\tilde{\mathbf{y}}_r(k)}^{-1}. \end{aligned} \tag{34}$$

Then one has

$$\begin{aligned} \hat{\mathbf{x}}(k-j \mid k) &= \hat{\mathbf{x}}(k-j \mid k-1) + K_j(k)\tilde{\mathbf{y}}_r(k) \\ &= \hat{\mathbf{x}}(k-j \mid k-1) \\ &\quad + \sum_{i=0}^{d} P(k-j,k-l_i,k-1) C_{ri}^T R_{\tilde{\mathbf{y}}_r(k)}^{-1}\tilde{\mathbf{y}}_r(k) \\ &= \hat{\mathbf{x}}(k-j \mid k-1) + \sum_{i=0}^{d} P(k-j,k-l_i,k-1) \\ &\quad \times \left[C_i^T \ \ 0\right] M^T(k)\Lambda^{-1}(k) M(k)\tilde{\mathbf{y}}_r(k) \\ &= \hat{\mathbf{x}}(k-j \mid k-1) \\ &\quad + \sum_{i=0}^{d} P(k-j,k-l_i,k-1) C_i^T \Theta^{-1}(k)\tilde{\mathbf{y}}(k). \end{aligned} \tag{35}$$

Next it follows from (7) and (32) that

$$\begin{aligned} \tilde{\mathbf{x}}(k-j \mid k) &= \tilde{\mathbf{x}}(k-j \mid k-1) - K_j(k)\tilde{\mathbf{y}}_r(k), \\ \tilde{\mathbf{x}}(k-i \mid k) &= \tilde{\mathbf{x}}(k-i \mid k-1) - K_i(k)\tilde{\mathbf{y}}_r(k). \end{aligned} \tag{36}$$

Based on (36), we have the estimation error covariance matrices

$$\begin{aligned} P(k-i,k-j,k) &= E\left\{\tilde{\mathbf{x}}(k-i \mid k)\tilde{\mathbf{x}}^T(k-j \mid k)\right\} \\ &= E\left\{\tilde{\mathbf{x}}(k-i \mid k-1)\tilde{\mathbf{x}}^T(k-j \mid k-1)\right\} \\ &\quad + E\left\{K_i(k)\tilde{\mathbf{y}}_r(k)\tilde{\mathbf{y}}_r^T(k)K_j^T(k)\right\} \\ &\quad - E\left\{\tilde{\mathbf{x}}(k-i \mid k-1)\tilde{\mathbf{y}}_r^T(k)\right\}K_j^T(k) \\ &\quad - K_i(k)E\left\{\tilde{\mathbf{y}}_r(k)\tilde{\mathbf{x}}^T(k-j \mid k-1)\right\} \\ &= P(k-i,k-j,k-1) \\ &\quad - K_i(k)R_{\tilde{\mathbf{y}}_r(k)}K_j^T(k), \end{aligned} \tag{37}$$

which is (26). Combining (7) and (31), we obtain

$$\tilde{\mathbf{x}}(k+1 \mid k) = \sum_{i=0}^{d} A_i \tilde{\mathbf{x}}(k-h_i \mid k) + B_d\mathbf{d}(k) + B_f\mathbf{f}(k). \tag{38}$$

Furthermore, one has

$$\begin{aligned} &P(k+1,k-j,k) \\ &\quad = E\{\tilde{\mathbf{x}}(k+1\mid k)\tilde{\mathbf{x}}^T(k-j\mid k)\} \\ &\quad = E\left\{\left[\sum_{i=0}^{d} A_i\tilde{\mathbf{x}}(k-h_i\mid k) + B_d\mathbf{d}(k) + B_f\mathbf{f}(k)\right]\tilde{\mathbf{x}}^T(k-j\mid k)\right\} \\ &\quad = \sum_{i=0}^{d} A_i P(k-h_i,k-j,k), \\ &P(k+1,k+1,k) \\ &\quad = E\{\tilde{\mathbf{x}}(k+1\mid k)\tilde{\mathbf{x}}^T(k+1\mid k)\} \\ &\quad = E\left\{\left[\sum_{i=0}^{d} A_i\tilde{\mathbf{x}}(k-h_i\mid k) + B_d\mathbf{d}(k) + B_f\mathbf{f}(k)\right] \times \left[\sum_{i=0}^{d} A_i\tilde{\mathbf{x}}(k-h_i\mid k) + B_d\mathbf{d}(k) + B_f\mathbf{f}(k)\right]^T\right\} \\ &\quad = \sum_{i=0}^{d}\sum_{j=0}^{d} A_i P(k-h_i,k-h_j,k)A_j^T + B_dB_d^T + B_fB_f^T. \end{aligned} \tag{39}$$

Finally (29) is straightforward by virtue of the definition of (16). Thus the proof is completed here. □

Theorem 3. *Consider the system* (1)-(2). *For a given* $\gamma > 0$, *a fault estimator* $r(k)$ *that achieves the performance index* (3) *exists if and only if* $\Theta(k) > 0$ *and* $\Xi(k) < 0$, *where* $\Theta(k)$ *and* $\Xi(k)$ *are defined in Lemma 1. In this case, one possible finite-time* H_∞ *fault estimator is given by*

$$\begin{aligned} r(k) &= D_f^T\Theta^{-1}(k)\tilde{y}(k) \\ &= D_f^T\Theta^{-1}(k)\left[y(k) - \sum_{i=0}^{d} C_i\hat{x}(k-l_i\mid k-1)\right], \end{aligned} \tag{40}$$

where

$$\begin{aligned} &\hat{x}(k+1\mid k) = \sum_{i=0}^{d} A_i\hat{x}(k-h_i\mid k), \\ &\hat{x}(k-j\mid k) \\ &\quad = \hat{x}(k-j\mid k-1) \\ &\qquad + \sum_{i=0}^{d} P(k-j,k-l_i,k-1)C_i^T\Theta^{-1}(k)\tilde{y}(k), \\ &\qquad\qquad j = 0,\ldots,l. \end{aligned} \tag{41}$$

Proof. According to [1], we can see that the fault estimation problem addressed for the deterministic systems (1) with (2) and (5) is partially equivalent to that for the stochastic systems (7) with (8) and (9) in a Krein space, and therefore the proof is readly and we omitted here. □

Remark 4. In fact, the problem mentioned in this paper can be converted into the problem in [7, 8] by applying augmented approach. However, due to the existence of time delay, we need to solve a high dimension Riccati equation. Here the solutions to the fault estimator can be obtained by solving partial difference Riccati equations (26)–(28), which have the same dimension as the original system (1). Therefore solving a high dimension Riccati equation is avoided. Here we present a simple explanation. Because the multiplications and divisions cost much more in computation than additions, hence we only use the number of multiplications and divisions as the operation count. Denote C_{aug} and C_{new} as the numbers of multiplications and divisions for augmentation method and our proposed approach in one step, respectively. According to [18], one can see that the order of h_d in C_{aug} is 3, while the order of h_d in C_{new} is 2. Therefore if h_d is large enough, $C_{\text{aug}} \gg C_{\text{new}}$.

Remark 5. Recently, by applying Krein-space approach and reorganized innovation approach, [9] has considered the finite-horizon H_∞ fault estimation for linear discrete time-varying systems with two-channel single measurement delay. In this paper, we have investigated the finite-horizon problem H_∞ fault estimation for linear discrete systems with time delay, where the delay appears in both state and measurement, which contain [9] as a special case.

4. Numerical Example

Consider the linear discrete-time system:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.3 & 0.5 \\ 0 & 0.4 \end{bmatrix} x(k) + \begin{bmatrix} 0.2 & 0.1 \\ -0.05 & 0.2 \end{bmatrix} x(k-1) \\ &\quad + \begin{bmatrix} 0.4 & 0.1 \\ -0.5 & 0.3 \end{bmatrix} x(k-2) \\ &\quad + \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix} d(k) + \begin{bmatrix} 1.2 \\ 1.8 \end{bmatrix} f(k), \\ y(k) &= \begin{bmatrix} -0.5 & 0.5 \end{bmatrix} x(k) \\ &\quad + \begin{bmatrix} 0.5 & 0 \end{bmatrix} x(k-1) + \begin{bmatrix} 0.7 & -0.3 \end{bmatrix} x(k-2) \\ &\quad + 2.5 f(k) + v(k). \end{aligned} \tag{42}$$

The finite time horizon concerned here is $[0, 100]$. The driving disturbance and measurement noise are selected as $d(k) = 0.4\cos(k)$, $v(k) = 0.6\sin(k)$. The fault to be estimated is assumed to be

$$f(k) = \begin{cases} 1, & 10 \le k \ge 25,\ 50 \le k \ge 70, \\ 0, & \text{others}. \end{cases} \tag{43}$$

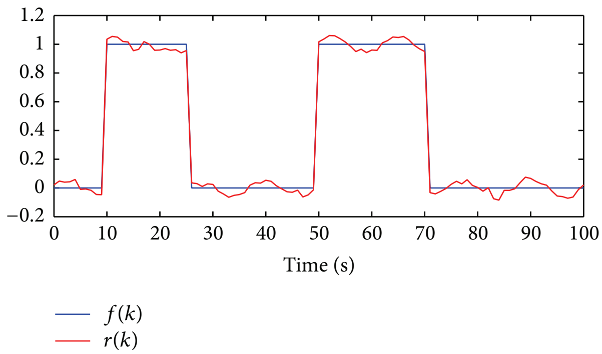

Figure 1: Fault and its estimation.

Set initial values as $x_0 = [1 \quad -0.5]^T$, $P(i, j, -1) = 0$, $-2 \le i \le -1$, $P(i, j, -1) = 0$, $-2 \le j \le -1$, $P(0, 0, -1) = I$. By using the result given in Theorem 3, the desired fault estimator is designed with $\gamma = 0.85$. Figure 1 shows the fault and its estimate, which confirm that the designed estimator performs very well.

5. Conclusion

The finite-time H_∞ fault estimation problem for linear time-delay systems has been investigated. The design of finite horizon H_∞ fault estimation has been converted into a minimum problem of certain quadratic form. Then an stochastic system in Krein space has been proposed, and a sufficient and necessary condition for the minimum has been derived by applying innovation analysis approach and projection theory. Finally a solution to the H_∞ fault estimation has been obtained by recursively computing a partial difference Riccati equation. Compared with the conventional augmented approach, the presented approach lessens the computational demand when the delay is large. In the further study, we will consider the H_∞ fault estimation problem for linear time-delay systems with multiplicative noise.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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