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Research Article

Dynamics of a Stochastic Cooperative Predator-Prey System with Beddington-DeAngelis Functional Response

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Stochastic cooperative predator-prey system with Beddington-DeAngelis functional response is studied. It presents an investigation of dynamic properties of the system. Our results show that there exists a unique positive solution to the system for any positive initial value, and the positive solution is stochastically bounded. Moreover, under some conditions, we analyze global asymptotic stability of the positive solutions. With small environmental noises, the stochastic system is getting more similar to the corresponding deterministic system. Neither of the species in the system will die out. Finally, simulations are carried out to conform to our result.

1. Introduction

As we all know, in mathematical biology, predator-prey system, competitive system, and cooperative system are the three rudimentary and important ecological systems. The dynamic relationship between species has long been and will continue to be a dominant theme in ecology due to its universal existence and importance. It is well-known that predator-prey systems are very important and extensive in the nature fields. One significant component of the predator-prey relationship is predator's functional response, that is, the rate of prey consumption by an average predator. There are many significant functional responses in order to model various different situations. In fact, most of the functional responses are prey-dependent; however, some biologists have argued that in many cases, especially when predators have to search for food and therefore have to share or compete for food, the traditional predator-prey systems with prey-dependent functional response fail to model the interference among predators, the functional response should be predator-dependent. In [1, 2], Beddington and DeAngelis proposed the following predator-prey model with Beddington-DeAngelis functional response:

$$\begin{aligned}\frac{dx}{dt} &= x(r-\theta x) - \frac{Exy}{a+bx+cy},\\ \frac{dy}{dt} &= -dy + \frac{\beta xy}{a+bx+cy}.\end{aligned}\tag{1}$$

Skalski and Gilliam [3] compared statistical evidence from 19 predator-prey systems, and then they claimed that three predator-dependent functional responses (Hassell-Varley, Beddington-DeAngelis, and Growley-Martin) can provide better description of predator feeding over a range of predator-prey abundances. And the Beddington-DeAngelis type functional response was even suitable in some cases.

But most of this work is restricted to predator-prey systems, little has been done for cooperative systems [4, 5]. May [6] suggested the following set of equations:

$$\begin{aligned}\frac{dx}{dt} &= r_1 x\left(1-\frac{x}{a_1+b_1 y}-c_1 x\right),\\ \frac{dy}{dt} &= r_2 y\left(1-\frac{y}{a_2+b_2 x}-c_2 y\right),\end{aligned}\tag{2}$$

to describe a pair of mutualists.

However, there is often the interaction among multiple species in nature, whose relationship is more complex than those in two species. Therefore, it is more realistic to consider the multiple-species predator-prey systems. In order to continue studying such models, in this paper, we consider a cooperative predator-prey system with Beddington-DeAngelis functional responses at first:

$$\begin{aligned} dx &= x\left(a_1 - b_1 x - \frac{c_2 y}{1+\alpha_2 x+\beta_2 y} - \frac{c_3 z}{1+\alpha_3 x+\beta_3 z}\right)dt,\\ dy &= y\left(a_2 - b_2 y - \frac{h_2 y}{f_2+g_2 z} + \frac{d_2 x}{1+\alpha_2 x+\beta_2 y}\right)dt,\\ dz &= z\left(a_3 - b_3 z - \frac{h_3 z}{f_3+g_3 y} + \frac{d_3 x}{1+\alpha_3 x+\beta_3 z}\right)dt, \end{aligned} \tag{3}$$

where species x is the prey of y and z, y, and z are cooperative species. All the parameters in system (3) are positive constants.

In fact, population dynamics is inevitably affected by environmental white noise which is an important component in an ecosystem. But the model (3) is deterministic and does not incorporate the effect of environmental noise. May [6] also pointed out the fact that due to environmental fluctuation, the birth rates, carrying capacity, competition coefficients, and other parameters involved in system exhibit random fluctuation to a greater or a lesser extent. Therefore many scholars rewrote the deterministic models as stochastic ones subjected to stochastic noises, for studying the effect of environmental variability on the population dynamics [7–9].

The parameters in the real ecosystems are often subject to lots of environmental noises, since they relate to climate, geographical distribution, geological features, human disaster, human intervention, and other environmental factors. Therefore, the logistics and energy flow, in which they are determined by groups, are fluctuating. The oscillation in population biomass is directly manifested as birth and death rates of random perturbation. Currently, one of the main ways considered in the literature to model the effect of the environmental fluctuations in population dynamics is to assume that the most sensitive parameter is the intrinsic growth rate. Thus, in this paper we introduce some stochastic perturbation into the intrinsic growth rate. Therefore, the intrinsic growth rate can be written as an average growth rate adding some small random perturbed terms. In general, by the well-known central limit theorem, the small terms follow some normal distributions, so we can use standard Brownian motions to represent the environmental fluctuations.

In this paper, taking into account the effect of randomly fluctuating environment, we introduce stochastic perturbation into growth rates a_1, a_2, and a_3 to become $a_1+\sigma_1\dot{B}_1(t)$, $a_2+\sigma_2\dot{B}_2(t)$, and $a_3+\sigma_3\dot{B}_3(t)$ in system, where σ_i^2 represents the intensity of the noise and $\dot{B}_i(t)$ is a standard white noise, namely, is $B_i(t)$ a standard Brownian motion defined on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$. Then the stochastic system takes the following form:

$$\begin{aligned} dx &= x\left(a_1 - b_1 x - \frac{c_2 y}{1+\alpha_2 x+\beta_2 y} - \frac{c_3 z}{1+\alpha_3 x+\beta_3 z}\right)dt\\ &\quad + \sigma_1 x dB_1(t),\\ dy &= y\left(a_2 - b_2 y - \frac{h_2 y}{f_2+g_2 z} + \frac{d_2 x}{1+\alpha_2 x+\beta_2 y}\right)dt\\ &\quad + \sigma_2 y dB_2(t),\\ dz &= z\left(a_3 - b_3 z - \frac{h_3 z}{f_3+g_3 y} + \frac{d_3 x}{1+\alpha_3 x+\beta_3 z}\right)dt\\ &\quad + \sigma_3 z dB_3(t). \end{aligned} \tag{4}$$

Considering system (4), the initial conditions $x(0) > 0$, $y(0) > 0$, and $z(0) > 0$ will be referred to.

2. Global Positive Solutions

Lemma 1. *For any initial value $(x_0, y_0, z_0) \in R_+^3$, where $R_+^3 = \{(x,y,z) \mid x > 0, y > 0, z > 0\}$, system (4) has a unique positive local solution $(x(t), y(t), z(t))$ for $t \in [0,\tau_e]$ almost surely, where τ_e is the explosion time.*

Proof. Consider the following equations:

$$\begin{aligned} du &= \left(a_1 - \frac{1}{2}\sigma_1^2 - b_1 e^u - \frac{c_2 e^v}{1+\alpha_2 e^v+\beta_2 e^u}\right.\\ &\quad \left. - \frac{c_3 e^w}{1+\alpha_3 e^v+\beta_3 e^w}\right)dt + \sigma_1 dB_1(t),\\ dv &= \left(a_2 - \frac{1}{2}\sigma_2^2 - b_2 e^v - \frac{h_2 e^v}{f_2+g_2 e^w}\right.\\ &\quad \left. + \frac{d_2 e^u}{1+\alpha_2 e^v+\beta_2 e^u}\right)dt + \sigma_2 dB_2(t),\\ dw &= \left(a_3 - \frac{1}{2}\sigma_3^2 - b_3 e^w - \frac{h_3 e^v}{f_3+g_3 e^w}\right.\\ &\quad \left. + \frac{d_3 e^u}{1+\alpha_3 e^w+\beta_3 e^u}\right)dt + \sigma_3 dB_3(t), \end{aligned} \tag{5}$$

on $t > 0$ with initial value $u(0) = \ln x(0)$, $v(0) = \ln y(0)$, and $w(0) = \ln z(0)$. The coefficients of (5) satisfy the local Lipschitz condition, thus there is a unique local solution $(u(t), v(t), w(t))$ on $[0,\tau_e)$. Then $x(t) = e^{u(t)}$, $y(t) = e^{v(t)}$, and $z(t) = e^{w(t)}$ are the unique positive local solutions with initial value $x_0 > 0$, $y_0 > 0$, and $z_0 > 0$ by Itô's formula. □

Theorem 2. *For any initial value $(x_0, y_0, z_0) \in R_+^3$, there is a unique solution $(x(t), y(t), z(t))$ of system (4) on $t \geqslant 0$, and the solution will remain in R_+^3 with probability 1.*

Proof. According to Lemma 1, we only need to show that $\tau_e = \infty$. Let $n_0 > 0$ be sufficiently large for x_0, y_0, and z_0

lying within the interval $[1/n_0, n_0]$. For each integer $n > n_0$, we define the stopping times

$$\tau_n = \inf\left\{t \in [0, \tau_e) : x(t) \notin \left(\frac{1}{n}, n\right), \quad y(t) \notin \left(\frac{1}{n}, n\right) \text{ or } z(t) \notin \left(\frac{1}{n}, n\right)\right\}. \tag{6}$$

Obviously, τ_n is increasing as $n \to \infty$. Set $\tau_\infty = \lim_{n\to\infty}\tau_n$; hence, $\tau_\infty \leqslant \tau_e$ almost surely. Now, we only need to show that $\tau_\infty = \infty$. If this statement is false, there is a pair of constants $T > 0$ and $\epsilon \in (0,1)$ such that $\mathscr{P}\{\tau_\infty \leqslant T\} > \epsilon$. Thus there exists an integer $n_1 > n_0$ such that

$$\mathscr{P}\{\tau_n \leqslant T\} \geqslant \epsilon, \quad n \geqslant n_1. \tag{7}$$

Define a C^3 function $V: R_+^3 \to R_+$ by

$$V(x, y, z) = (x - 1 - \ln x) + (y - 1 - \ln y) + (z - 1 - \ln z). \tag{8}$$

The nonnegativity of this function can be seen from $u - 1 - \ln u \geqslant 0$ and $\forall u > 0$. If $(x_0, y_0, z_0) \in R_+^3$, we have

$$\begin{aligned} dV = {} & (x-1)\left(a_1 - b_1 x - \frac{c_2 y}{1+\alpha_2 x + \beta_2 y} - \frac{c_3 z}{1+\alpha_3 x+\beta_3 z}\right)dt + \frac{\sigma_1^2}{2}dt \\ & + (y-1)\left(a_2 - b_2 y - \frac{h_2 y}{f_2 + g_2 z} + \frac{d_2 x}{1+\alpha_2 x+\beta_2 y}\right)dt + \frac{\sigma_2^2}{2}dt \\ & + (z-1)\left(a_3 - b_3 z - \frac{h_3 z}{f_3 + g_3 y} + \frac{d_3 x}{1+\alpha_3 x+\beta_3 z}\right)dt + \frac{\sigma_3^2}{2}dt \\ & + (x-1)\sigma_1 dB_1(t) + (y-1)\sigma_2 dB_2(t) \\ & + (z-1)\sigma_3 dB_3(t), \end{aligned} \tag{9}$$

then

$$\begin{aligned} LV = {} & (x-1)\left(a_1 - b_1 x - \frac{c_2 y}{1+\alpha_2 x+\beta_2 y} - \frac{c_3 z}{1+\alpha_3 x+\beta_3 z}\right) \\ & + \frac{\sigma_1^2}{2} \\ & + (y-1)\left(a_2 - b_2 y - \frac{h_2 y}{f_2+g_2 z} + \frac{d_2 x}{1+\alpha_2 x+\beta_2 y}\right) \\ & + \frac{\sigma_2^2}{2} \\ & + (z-1)\left(a_3 - b_3 z - \frac{h_3 z}{f_3+g_3 y} + \frac{d_3 x}{1+\alpha_3 x+\beta_3 z}\right) \\ & + \frac{\sigma_3^2}{2} \\ \leqslant {} & (x-1)(a_1 - b_1 x) + (y-1)\left(a_2 - b_2 y + \frac{d_2}{\alpha_2}\right) \\ & + (z-1)\left(a_3 - b_3 z + \frac{d_3}{\alpha_3}\right) + \frac{\sigma_1^2+\sigma_2^2+\sigma_3^2}{2} \\ \leqslant {} & x(a_1 - b_1 x) + y\left(a_2 - b_2 y + \frac{d_2}{\alpha_2}\right) \\ & + z\left(a_3 - b_3 z + \frac{d_3}{\alpha_3}\right) + \frac{\sigma_1^2+\sigma_2^2+\sigma_3^2}{2} \\ \leqslant {} & K, \end{aligned} \tag{10}$$

where K is a positive number. Substituting this inequality into (9), we see that

$$dV(x,y,z) \leqslant Kdt + (x-1)\sigma_1 dB_1(t) + (y-1)\sigma_2 dB_2(t) + (z-1)\sigma_3 dB_3(t). \tag{11}$$

Integrating both sides of the above inequality from 0 to $\tau_k \wedge T$ and then taking the expectations leads to

$$EV(x(\tau_n \wedge T), y(\tau_n \wedge T), z(\tau_n \wedge T)) \leqslant V(x_0, y_0, z_0) + KT. \tag{12}$$

Set $\Omega_n = \{\tau_n \leqslant T\}$, then we get $\mathscr{P}(\Omega_n) \geqslant \epsilon$ by inequality (7). Obviously, for every $\omega \in \Omega_n$, there are at least $x(\tau_n, \omega)$, $y(\tau_n, \omega)$, and $z(\tau_n, \omega)$ which equal either k or $1/k$, then $V(x(\tau_n,\omega), y(\tau_n,\omega), z(\tau_n,\omega))$ is no less than

$$\min\left\{(n - 1 - \ln n), \left(\frac{1}{n} - 1 - \ln\frac{1}{n}\right)\right\}. \tag{13}$$

It then follows from (9) that

$$\begin{aligned} & V(x_0, y_0, z_0) + KT \\ & \geqslant E\left[1_{\Omega_n(\omega)} V(x(\tau_n,\omega), y(\tau_n,\omega), z(\tau_n,\omega))\right] \\ & \geqslant \epsilon \min\left\{(n-1-\ln n), \left(\frac{1}{n} - 1 - \ln\frac{1}{n}\right)\right\}, \end{aligned} \tag{14}$$

where $1_{\Omega_n(\omega)}$ is the indicator function of Ω_n, letting $n \to \infty$, we have that

$$\infty > V(x_0, y_0, z_0) + KT = \infty \tag{15}$$

This completes the proof. □

3. Stochastic Boundedness

Definition 3. The solution $(x(t), y(t), z(t))$ of system (4) is said to be stochastically ultimately bounded, if for any $\epsilon \in$

(0, 1), there is a positive constant $\delta = \delta(\epsilon)$, such that for any initial value $(x_0, y_0, z_0) \in R_+^3$, the solution $(x(t), y(t), z(t))$ of system (4) has the property that

$$\limsup_{n\to\infty} \mathscr{P}\left\{ \left| x(t), y(t), z(t) \right| = \sqrt{x^2(t) + y^2(t) + z^2(t)} > \delta \right\} < \epsilon. \tag{16}$$

Assumption A. For any initial value $(x_0, y_0, z_0) \in R_+^3$, there exists $p \geqslant 1$ such that

$$x(0) < \frac{a_1 + (1/2)(p-1)\sigma_1^2}{b_1},$$
$$y(0) < \frac{a_2 + (d_2/\alpha_2) + (1/2)(p-1)\sigma_2^2}{b_2}, \tag{17}$$
$$z(0) < \frac{a_3 + (d_3/\alpha_3) + (1/2)(p-1)\sigma_3^2}{b_3}.$$

Lemma 4. *Assume that Assumption A holds. Let $(x(t), y(t), z(t))$ be a positive solution of (4) with any initial value $(x_0, y_0, z_0) \in R_+^3$, for all $p > 1$, then*

$$E[x^p(t)] \leqslant K_1(p),$$
$$E[y^p(t)] \leqslant K_2(p), \tag{18}$$
$$E[z^p(t)] \leqslant K_3(p),$$

where

$$K_1(p) := \left(\frac{a_1 + (1/2)(p-1)\sigma_1^2}{b_1}\right)^p,$$
$$K_2(p) := \left(\frac{a_2 + (d_2/\alpha_2) + (1/2)(p-1)\sigma_2^2}{b_2}\right)^p, \tag{19}$$
$$K_3(p) := \left(\frac{a_3 + (d_3/\alpha_3) + (1/2)(p-1)\sigma_3^2}{b_3}\right)^p.$$

Proof. Define the function $V_1 = x^p$, For $(x(t), y(t), z(t)) \in R_+^3$ and $p > 0$. By Itô's formula we get

$$\begin{aligned} dV_1 &= px_1^{p-1}dx + \frac{1}{2}p(p-1)x_1^{p-2}(dx)^2 \\ &= px^p\left[\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z}\right)dt + \sigma_1 dB_1(t)\right] \\ &\quad + \frac{1}{2}p(p-1)\sigma_1^2 dt \\ &= px^p\left[\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z} + \frac{1}{2}p(p-1)\sigma_1^2\right)dt + \sigma_1 dB_1(t)\right]. \end{aligned} \tag{20}$$

Integrating from 0 to t and taking expectations yields

$$\begin{aligned} &E[x^p(t)] - E[x^p(0)] \\ &= \int_0^t pE\left[x^p\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z} + \frac{1}{2}p(p-1)\sigma_1^2\right)\right]ds. \end{aligned} \tag{21}$$

So,

$$\begin{aligned} &\frac{dE[x^p(t)]}{dt} \\ &= pE\left[x^p\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z} + \frac{1}{2}p(p-1)\sigma_1^2\right)\right] \\ &\leqslant p\left\{\left(a_1 + \frac{1}{2}(p-1)\sigma_1^2\right)E[x^p(t)] - b_1E\left[x^{p+1}(t)\right]\right\} \\ &\leqslant p\left\{\left(a_1 + \frac{1}{2}(p-1)\sigma_1^2\right)E[x^p(t)] - b_1E[x^p(t)]^{(p+1)/p}\right\} \\ &= pE[x^p(t)]\left\{\left[\left(a_1 + \frac{1}{2}(p-1)\sigma_1^2\right)\right] - b_1E[x^p(t)]^{1/p}\right\}. \end{aligned} \tag{22}$$

Let $X(t) = E[x^p(t)]$, then we have

$$\frac{dX(t)}{dt} \leqslant pX(t)\left[\left(a_1 + \frac{1}{2}(p-1)\sigma_1^2\right) - b_1X^{1/p}(t)\right]. \tag{23}$$

From (17), we know

$$0 < b_1X^{1/p}(0) = b_1x(0) < a_1 + \frac{1}{2}(p-1)\sigma_1^2, \tag{24}$$

which by the standard comparison argument shows that

$$(E[x^p(t)])^{1/p} = X^{1/p} \leqslant \frac{a_1 + (1/2)(p-1)\sigma_1^2}{b_1}, \tag{25}$$

that is,

$$E[x^p(t)] \leqslant \left(\frac{a_1 + (1/2)(p-1)\sigma_1^2}{b_1}\right)^p. \tag{26}$$

Define the function $V_2 = y^p$, for $(x(t), y(t), z(t)) \in R^3_+$ and $p > 0$. By Itô's formula we get

$$\begin{aligned} dV_2 &= py^{p-1}dy + \frac{1}{2}p(p-1)y^{p-2}(dy)^2 \\ &= py^p\left[\left(a_2 - b_2 y - \frac{h_2 y}{f_2 + g_2 z} + \frac{d_2 x}{1+\alpha_2 x + \beta_2 y}\right)dt \right. \\ &\qquad \left. + \sigma_2 dB_2(t) + \frac{1}{2}(p-1)\sigma_2^2 dt\right] \\ &= py^p\left[\left(a_2 - b_2 y - \frac{h_2 y}{f_2 + g_2 z} + \frac{d_2 x}{1+\alpha_2 x + \beta_2 y}\right.\right. \\ &\qquad \left.\left. + \frac{1}{2}(p-1)\sigma_2^2\right)dt + \sigma_2 dB_2(t)\right]. \end{aligned} \tag{27}$$

Integrating from 0 to t and taking expectations yields

$$\begin{aligned} &E\left[y^p(t)\right] - E\left[y^p(0)\right] \\ &\quad = \int_0^t pE\left[y^p\left(a_2 - b_2 y - \frac{h_2 y}{f_2 + g_2 z} + \frac{d_2 x}{1+\alpha_2 x + \beta_2 y}\right)\right. \\ &\qquad \left. + \frac{1}{2}(p-1)\sigma_2^2\right]ds. \end{aligned} \tag{28}$$

So,

$$\begin{aligned} &\frac{dE\left[y^p(t)\right]}{dt} \\ &\quad = pE\left[y^p\left(a_2 - b_2 y - \frac{h_2 y}{f_2 + g_2 z} + \frac{d_2 x}{1+\alpha_2 x + \beta_2 y}\right)\right. \\ &\qquad \left. + \frac{1}{2}(p-1)\sigma_2^2\right] \\ &\quad \leqslant p\left\{\left(a_2 + \frac{d_2}{\alpha_2} + \frac{1}{2}(p-1)\sigma_2^2\right)E\left[y^p(t)\right]\right. \\ &\qquad \left. - b_2 E\left[y^{p+1}(t)\right]\right\} \\ &\quad \leqslant p\left\{\left(a_2 + \frac{d_2}{\alpha_2} + \frac{1}{2}(p-1)\sigma_2^2\right)E\left[y^p(t)\right]\right. \\ &\qquad \left. - b_2 E\left[y^p(t)\right]^{(p+1)/p}\right\} \\ &\quad = pE\left[y^p(t)\right]\left\{\left(a_2 + \frac{d_2}{\alpha_2} + \frac{1}{2}(p-1)\sigma_2^2\right)\right. \\ &\qquad \left. - b_2 E\left[y^p(t)\right]^{1/p}\right\}. \end{aligned} \tag{29}$$

Let $Y(t) = E[y^p(t)]$, then we have

$$\begin{aligned} &\frac{dY(t)}{dt} \\ &\quad \leqslant pY(t)\left[a_2 + \frac{d_2}{\alpha_2} + \frac{1}{2}(p-1)\sigma_2^2 - b_2 Y^{1/p}(t)\right]. \end{aligned} \tag{30}$$

From (17), we know that

$$0 < b_2 Y^{1/p}(0) = b_2 y(0) < a_2 + \frac{d_2}{\alpha_2} + \frac{1}{2}(p-1)\sigma_2^2, \tag{31}$$

which by the standard comparison argument shows that

$$\left(E\left[y^p(t)\right]\right)^{1/p} = Y^{1/p}(t) \leqslant \frac{a_2 + (d_2/\alpha_2) + (1/2)(p-1)\sigma_2^2}{b_2}, \tag{32}$$

that is,

$$E\left[y^p(t)\right] \leqslant \left(\frac{a_2 + (d_2/\alpha_2) + (1/2)(p-1)\sigma_2^2}{b_2}\right)^p. \tag{33}$$

Similarly, we can show that

$$E\left[z^p(t)\right] \leqslant \left(\frac{a_3 + (d_3/\alpha_3) + (1/2)(p-1)\sigma_3^2}{b_3}\right)^p. \tag{34}$$

This completes the proof. □

Theorem 5. *Assume that Assumption A holds, the solutions of system* (4) *with initial value* $(x_0, y_0, z_0) \in R^3_+$ *are stochastically ultimately bounded.*

Proof. If $(x(t), y(t), z(t)) \in R^3$, its norm here is denoted by $|x(t), y(t), z(t)| = (x^2(t) + y^2(t) + z^2(t))^{1/2}$, then

$$\left|(x(t), y(t), z(t))\right|^p \leqslant 3^{p/2}\left(|x(t)|^p + |y(t)|^p + |z(t)|^p\right) \tag{35}$$

by Lemma 4, $E[|x(t), y(t), z(t)|^p] \leqslant K(p)$, $t \in (0, +\infty)$. $K(p)$ is dependent on (x_0, y_0, z_0) and is defined by $K(p) = 3^{p/2}(K_1(p) + K_2(p) + K_3(p))$. By virtue of Chebyshev inequality, we can easily obtain that the solution $(x(t), y(t), z(t))$ of system (4) is stochastically ultimately bounded. □

4. Stochastic Permanence

Definition 6 (see [10]). The solution $(x(t), y(t), z(t))$ of system (4) is said to be stochastically permanent, if for any $\epsilon \in (0,1)$, there exist a pair of positive constants $\delta = \delta(\epsilon)$ and $\chi = \chi(\epsilon)$ such that for any initial value $(x_0, y_0, z_0) \in R^3_+$, the solution $(x(t), y(t), z(t))$ of system (4) has the properties that

$$\begin{aligned} &\liminf_{t\to\infty} \mathscr{P}\left\{\left|(x(t), y(t), z(t))\right| \geqslant \delta\right\} \geqslant 1 - \epsilon, \\ &\liminf_{t\to\infty} \mathscr{P}\left\{\left|(x(t), y(t), z(t))\right| \leqslant \chi\right\} \geqslant 1 - \epsilon. \end{aligned} \tag{36}$$

Assumption B. One has $1/2(\max\{\sigma_1,\sigma_2,\sigma_3\})^2 < \min\{a_1, a_2-(c_2/\alpha_2), a_3-(c_3/\alpha_3)\}$.

Theorem 7. *Under Assumption B, for any initial value $(x_0, y_0, z_0) \in R^3_+$, the solution $(x(t), y(t), z(t))$ of system (4) satisfies that*

$$\limsup_{t\to\infty} E\left[\frac{1}{\left|(x(t), y(t), z(t))\right|^{\theta}}\right] \leqslant H, \tag{37}$$

where θ is an arbitrary positive constant satisfying

$$\frac{\theta+1}{2}(\max\{\sigma_1,\sigma_2,\sigma_3\})^2 < \min\left\{a_1, a_2-\frac{c_2}{\alpha_2}, a_3-\frac{c_3}{\alpha_3}\right\}, \tag{38}$$

and k is an arbitrary positive constant satisfying

$$\theta\min\left\{a_1, a_2-\frac{c_2}{\alpha_2}, a_3-\frac{c_3}{\alpha_3}\right\} - \frac{\theta(\theta+1)}{2}(\max\{\sigma_1,\sigma_2,\sigma_3\})^2 - k > 0. \tag{39}$$

Proof. Define $V(x,y,z) = x+y+z$ for $(x,y,z)\in R^3_+$, then

$$\begin{aligned} dV(x,y,z) &= x\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z}\right)dt \\ &\quad + y\left(a_2 - b_2y - \frac{h_2y}{f_2+g_2z} + \frac{d_2x}{1+\alpha_2x+\beta_2y}\right)dt \\ &\quad + z\left(a_3 - b_3z - \frac{h_3z}{f_3+g_3y} + \frac{d_3x}{1+\alpha_3x+\beta_3z}\right)dt \\ &\quad + \sigma_1xdB_1(t) + \sigma_2ydB_2(t) + \sigma_3zdB_3(t). \end{aligned} \tag{40}$$

Define $U(x,y,z) = (1/(V(x,y,z)))$ for $(x,y,z)\in R^3_+$, by Itô's formula, we get

$$\begin{aligned} dU &= -U^2\left[x\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z}\right)\right. \\ &\quad + y\left(a_2 - b_2y - \frac{h_2y}{f_2+g_2z} + \frac{d_2x}{1+\alpha_2x+\beta_2y}\right) \\ &\quad \left.+z\left(a_3 - b_3z - \frac{h_3z}{f_3+g_3y} + \frac{d_3x}{1+\alpha_3x+\beta_3z}\right)\right]dt \\ &\quad + U^3\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right)dt \\ &\quad - U^2\left(\sigma_1xdB_1(t) + \sigma_2ydB_2(t) + \sigma_3zdB_3(t)\right) \\ &= LUdt - U^2\left(\sigma_1xdB_1(t) + \sigma_2ydB_2(t) + \sigma_3zdB_3(t)\right), \end{aligned} \tag{41}$$

where

$$\begin{aligned} LU &= -U^2\left[x\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z}\right)\right. \\ &\quad + y\left(a_2 - b_2y - \frac{h_2y}{f_2+g_2z} + \frac{d_2x}{1+\alpha_2x+\beta_2y}\right) \\ &\quad \left.+z\left(a_3 - b_3z - \frac{h_3z}{f_3+g_3y} + \frac{d_3x}{1+\alpha_3x+\beta_3z}\right)\right] \\ &\quad + U^3\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right). \end{aligned} \tag{42}$$

Under Assumption B, choosing a positive constant θ such that it satisfies (38). By Itô's formula, we get

$$\begin{aligned} d(1+U)^{\theta} &= \left[\theta(1+U)^{\theta-1}LU + \frac{\theta(\theta-1)}{2}U^4(1+U)^{\theta-2}\right. \\ &\quad \left.\times\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right)\right]dt \\ &\quad - \theta U^2(1+U)^{\theta-1}\left(\sigma_1xdB_1(t)+\sigma_2ydB_2(t)+\sigma_3zdB_3(t)\right), \end{aligned} \tag{43}$$

where

$$\begin{aligned} L(1+U)^{\theta} &= \theta(1+U)^{\theta-1}LU + \frac{\theta(\theta-1)}{2}U^4 \\ &\quad \times(1+U)^{\theta-2}\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right), \end{aligned} \tag{44}$$

then choosing a positive constant k such that it obeys (39), by Itô's formula,

$$de^{kt}(1+U)^{\theta} = e^{kt}d(1+U)^{\theta} + ke^{kt}(1+U)^{\theta}dt, \tag{45}$$

where

$$\begin{aligned} Le^{kt}(1+U)^{\theta} &= e^{kt}L(1+U)^{\theta} + ke^{kt}(1+U)^{\theta} \\ &= e^{kt}(1+U)^{\theta-2} \\ &\quad \times\left\{k(1+U)^2 + \frac{\theta(\theta+1)}{2}U^4\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right)\right. \\ &\quad + \theta U^3\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right) \\ &\quad - \theta(1+U)U^2 \\ &\quad \times\left[x\left(a_1 - b_1x - \frac{c_2y}{1+\alpha_2x+\beta_2y} - \frac{c_3z}{1+\alpha_3x+\beta_3z}\right)\right. \\ &\quad + y\left(a_2 - b_2y - \frac{h_2y}{f_2+g_2z} + \frac{d_2x}{1+\alpha_2x+\beta_2y}\right) \\ &\quad \left.\left.+z\left(a_3 - b_3z - \frac{h_3z}{f_3+g_3y} + \frac{d_3x}{1+\alpha_3x+\beta_3z}\right)\right]\right\} \end{aligned}$$

$$\begin{aligned}
&\leqslant e^{kt}(1+U)^{\theta-2} \\
&\quad \times \left\{ k(1+U)^2 + \frac{\theta(\theta+1)}{2}U^4\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right) \right. \\
&\qquad + \theta U^3\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right) \\
&\qquad - \theta U^2\left[\left(a_1x + \left(a_2 - \frac{c_2}{\alpha_2}\right)y + \left(a_3 - \frac{c_3}{\alpha_3}\right)z\right)\right. \\
&\qquad\qquad \left. - \left(b_1x^2 + \left(b_2 + \frac{h_2}{f_2}\right)y + \left(b_3 + \frac{h_3}{f_3}\right)z\right)\right] \\
&\qquad - \theta U^3\left[\left(a_1x + \left(a_2 - \frac{c_2}{\alpha_2}\right)y + \left(a_3 - \frac{c_3}{\alpha_3}\right)z\right)\right. \\
&\qquad\qquad \left.\left. - \left(b_1x^2 + \left(b_2 + \frac{h_2}{f_2}\right)y + \left(b_3 + \frac{h_3}{f_3}\right)z\right)\right]\right\} \\
&\leqslant e^{kt}(1+U)^{\theta-2} \\
&\quad \times \left[\left(k + \theta\max\left\{b_1, b_2 + \frac{h_2}{f_2}, b_3 + \frac{h_3}{f_3}\right\}\right)\right. \\
&\qquad + \left(2k - \theta\min\left\{a_1, a_2 - \frac{c_2}{\alpha_2}, a_3 - \frac{c_3}{\alpha_3}\right\}\right. \\
&\qquad\quad + \theta\max\left\{b_1, b_2 + \frac{h_2}{f_2}, b_3 + \frac{h_3}{f_3}\right\} \\
&\qquad\quad \left. + \theta\left(\max\left\{\sigma_1,\sigma_2,\sigma_3\right\}\right)^2\right)U \\
&\qquad + \left(k - \theta\min\left\{a_1, a_2 - \frac{c_2}{\alpha_2}, a_3 - \frac{c_3}{\alpha_3}\right\}\right. \\
&\qquad\quad \left.\left. + \frac{\theta(\theta+1)}{2}\left(\max\left\{\sigma_1,\sigma_2,\sigma_3\right\}\right)^2\right)U^2\right],
\end{aligned} \tag{46}$$

where

$$\begin{aligned}
&U^3\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right) \leqslant \left(\max\left\{\sigma_1,\sigma_2,\sigma_3\right\}\right)^2U, \\
&\frac{\theta(\theta+1)}{2}U^4\left(\sigma_1^2x^2+\sigma_2^2y^2+\sigma_3^2z^2\right) \\
&\quad \leqslant \frac{\theta(\theta+1)}{2}\left(\max\left\{\sigma_1,\sigma_2,\sigma_3\right\}\right)^2U^2.
\end{aligned} \tag{47}$$

Hence, it implies that there exists a positive constant K such that

$$Le^{kt}(1+U)^\theta \leqslant Ke^{kt}. \tag{48}$$

Then we have

$$\begin{aligned}
de^{kt}(1+U)^\theta &\leqslant Ke^{kt}dt - e^{kt}\theta U^2(1+U)^{\theta-1} \\
&\quad \times\left(\sigma_1 x dB_1(t) + \sigma_2 y dB_2(t) + \sigma_3 z dB_3(t)\right).
\end{aligned} \tag{49}$$

Integrating both sides of the above inequality from 0 to t and then taking the expectations leads to

$$\begin{aligned}
E\left[e^{kt}(1+U(t))^\theta\right] &\leqslant (1+U(0))^\theta \\
&\quad + \frac{K}{k}e^{kt} = (1+U(0))^\theta + H_1e^{kt},
\end{aligned} \tag{50}$$

where $H_1 = K/k$. So

$$\limsup_{t\to\infty} E\left[U(t)^\theta\right] \leqslant \limsup_{t\to\infty} E\left[(1+U(t))^\theta\right] \leqslant H_1. \tag{51}$$

Since that $(x+y+z)^\theta \leqslant 3^\theta(x^2+y^2+z^2)^{\theta/2} = 3^\theta|x,y,z|^\theta$, where $(x,y,z)\in R_+^3$, obviously

$$\begin{aligned}
&\limsup_{t\to\infty} E\left[\frac{1}{\left|(x(t),y(t),z(t))\right|^\theta}\right] \\
&\quad \leqslant 3^\theta \limsup_{t\to\infty} E\left[U(t)^\theta\right] \leqslant 3^\theta H_1 =: H,
\end{aligned} \tag{52}$$

as required. □

Theorem 8. *Under Assumption B, system* (4) *is stochastically permanent.*

Proof. By Theorem 5, we know that

$$\limsup_{t\to\infty} E\left|(x(t),y(t),z(t))\right|^p \leqslant K(p). \tag{53}$$

Now, for any $\epsilon > 0$, let $\chi = (K(p)/\epsilon)^{1/p}$. Then by Chebyshev's inequality, we can obtain the conclusion easily. □

5. Global Asymptotic Stability

Definition 9. Let $(x_1(t), y_1(t), z_1(t))$ be a positive solution of system (4). If we say that $(x_1(t), y_1(t), z_1(t))$ is globally asymptotically stable in expectation, it means that any other solution $(x_2(t), y_2(t), z_2(t))$ of system (4) has $t \geqslant 0$ and that we have initial value $(x_0, y_0, z_0) \in R_+^3$. That is

$$\begin{aligned}
\mathscr{P}\left\{\lim_{t\to\infty} E\left[\left|(x_1(t), y_1(t), z_1(t))\right.\right.\right. \\
\left.\left.\left. - (x_2(t), y_2(t), z_2(t))\right|\right] = 0\right\} = 1.
\end{aligned} \tag{54}$$

Lemma 10 (see [11]). *Suppose that an n-dimensional stochastic process $X(t)$ on $t \geqslant 0$ satisfies the condition*

$$E|X(t) - X(s)|^\alpha \geqslant c|t-s|^{1+\beta}, \quad 0 \leqslant s,\ t < \infty, \tag{55}$$

for some positive constants α, β, and c. Then there exists a continuous modification $\widetilde{X}(t)$ of $X(t)$ which has the property that for every $\vartheta \in (0, \beta/\alpha)$ there is a positive random variable $h(\omega)$ such that

$$\begin{aligned}
&\mathscr{P}\left\{\omega : \sup_{0<|t-s|<h(\omega),\,0\leqslant s,\,t<\infty} \frac{\left|\widetilde{X}(t,\omega) - \widetilde{X}(s,\omega)\right|}{|t-s|^\vartheta}\right. \\
&\quad \left. \leqslant \frac{2}{1-2^{-\vartheta}}\right\} = 1.
\end{aligned} \tag{56}$$

In other words, almost every sample path of $\widetilde{X}(t)$ is locally but uniformly Hölder continuous with exponent ϑ.

Lemma 11. *Let $(x(t), y(t), z(t))$ be a solution of system (4) on $t \geqslant 0$ with initial value $(x_0, y_0, z_0) \in R_+^3$, then almost every sample path of $(x(t), y(t), z(t))$ is uniformly continuous on $t \geqslant 0$.*

Proof. From system (4), we have the following stochastic integral equation

$$x(t) = x(0) + \int_0^t x(s) \times \left(a_1 - b_1 x(s) - \frac{c_2 y(s)}{1+\alpha_2 x(s) + \beta_2 y(s)} - \frac{c_3 z(s)}{1+\alpha_3 x(s) + \beta_3 z(s)}\right) ds + \int_0^t \sigma_1 x(s)\, dB_1(s). \tag{57}$$

Let $f(s) = x(s)(a_1 - b_1 x(s) - (c_2 y(s)/(1+\alpha_2 x(s) + \beta_2 y(s))) - (c_3 z(s)/(1+\alpha_3 x(s) + \beta_3 z(s))))$, $g(s) = \sigma_1 x(s)$, notice that

$$\begin{aligned} E|f(t)|^p &= E\left[\left|x\left(a_1 - b_1 x - \frac{c_2 y}{1+\alpha_2 x+\beta_2 y} - \frac{c_3 z}{1+\alpha_3 x + \beta_3 z}\right)\right|^p\right] \\ &\leqslant \frac{1}{2}E|x|^{2p} + \frac{1}{2}E \times \left[\left|a_1 - b_1 x - \frac{c_2 y}{1+\alpha_2 x+\beta_2 y} - \frac{c_3 z}{1+\alpha_3 x + \beta_3 z}\right|^{2p}\right] \\ &\leqslant \frac{1}{2}E|x|^{2p} + \frac{1}{2}E\left[\left|a_1 + b_1 x + c_2 y + c_3 z\right|^{2p}\right] \\ &\leqslant \frac{1}{2}E|x|^{2p} + \frac{4^{2p-1}}{2} \times \left(a_1^{2p} + b_1^{2p}E|x|^{2p} + c_2 E|y|^{2p} + c_3 E|z|^{2p}\right) \\ &\leqslant \frac{1}{2}K_1(2p) + \frac{4^{2p-1}}{2} \times \left(a_1^{2p} + b_1^{2p}K_1(2p) + c_2 K_2(2p) + c_3 K_3(2p)\right) \\ &=: K_4(p). \end{aligned} \tag{58}$$

On the other hand, by the moment inequality (see [12]) for stochastic integrals, we have that for $0 \leqslant t_1 < t_2 < \infty$ and $p > 2$,

$$\begin{aligned} E\left|\int_{t_1}^{t_2} g(s)\, dB_1(s)\right|^p &\leqslant \left[\frac{p(p-1)}{2}\right]^{p/2} (t_2 - t_1)^{(p-2)/2} \int_{t_1}^{t_2} E|g(s)|^p ds \\ &\leqslant \left[\frac{p(p-1)}{2}\right]^{p/2} (t_2 - t_1)^{(p-2)/2} \int_{t_1}^{t_2} \sigma_1^p k_1(p)\, ds \\ &= \left[\frac{p(p-1)}{2}\right]^{p/2} (t_2 - t_1)^{p/2} G_1(p), \end{aligned} \tag{59}$$

where $G_1(p) = \sigma_1^p k_1(p)$. Let $0 \leqslant t_1 < t_2 < \infty$, $t_2 - t_1 \leqslant 1$, and $(1/p) + (1/q) = 1$, we obtain

$$\begin{aligned} E|x(t_2) - x(t_1)|^p &= E\left|\int_{t_1}^{t_2} f(s)\, ds + \int_{t_1}^{t_2} g(s)\, dB_1(s)\right|^p \\ &\leqslant 2^{p-1}E\left|\int_{t_1}^{t_2} f(s)\, ds\right|^p + 2^{p-1}E\left|\int_{t_1}^{t_2} g(s)\, dB_1(s)\right|^p \\ &\leqslant 2^{p-1}\left(\int_{t_1}^{t_2} 1^q ds\right)^{p/q} E\left(\int_{t_1}^{t_2} |f(s)|^p ds\right) + 2^{p-1}\left[\frac{p(p-1)}{2}\right]^{p/2} (t_2 - t_1)^{p/2} G_1(p) \\ &\leqslant 2^{p-1}(t_2 - t_1)^p K_4(p) + 2^{p-1}\left[\frac{p(p-1)}{2}\right]^{p/2} (t_2 - t_1)^{p/2} G_1(p) \\ &\leqslant 2^{p-1}(t_2 - t_1)^{p/2}\left\{(t_2 - t_1)^{p/2} + \left[\frac{p(p-1)}{2}\right]^{p/2}\right\} K_5(p) \\ &\leqslant 2^{p-1}\left\{1 + \left[\frac{p(p-1)}{2}\right]^{p/2}\right\} K_5(p)(t_2 - t_1)^{p/2}, \end{aligned} \tag{60}$$

where $K_5(p) := \max\{K_4(p), G_1(p)\}$. Then, we have that almost every sample path of $x(t)$ is locally but uniformly Hölder continuous with exponent ϑ for every $\vartheta \in (0, (p-2)/2p\,)$ and therefore almost every sample path of $x(t)$ is uniformly continuous on $t \geqslant 0$. Similarly, we can show that almost every sample path of $y(t)$ and $z(t)$ is uniformly continuous on $t \geqslant 0$. □

Lemma 12 (see [13]). *Let $f(t)$ be a nonnegative function defined on $[0, \infty)$ such that $f(t)$ is integrable on $[0, \infty)$ and is uniformly continuous on $t \geqslant 0$. Then $\lim_{t\to\infty} f(t) = 0$.*

Theorem 13. *If*

$$A := b_1 - \frac{c_2\alpha_2}{\beta_2} - \frac{c_3\alpha_3}{\beta_3} - d_2 - d_3 > 0,$$
$$B := b_2 - c_2 - \frac{2h_2}{f_2} - \frac{d_2\beta_2}{\alpha_2} - \frac{h_3 g_3 \left(a_2 + d_2/\alpha_2 + \sigma_2^2\right)}{b_2 f_3^2} > 0,$$
$$C := b_3 - c_3 - \frac{2h_3}{f_3} - \frac{d_3\beta_3}{\alpha_3} - \frac{h_2 g_2 \left(a_3 + d_3/\alpha_3 + \sigma_3^2\right)}{b_3 f_2^2} > 0, \quad (61)$$

then system (4) *is globally asymptotically stable.*

Proof. Define

$$V(t) = \left|\ln x_1(t) - \ln x_2(t)\right| + \left|\ln y_1(t) - \ln y_2(t)\right| + \left|\ln z_1(t) - \ln z_2(t)\right|, \quad (62)$$

then $V(t)$ is a continuous positive function on $t \geqslant 0$. A direct calculation of the right differential $d^+V(t)$ of $V(t)$, and then applying Itô's formula, we have

$$\begin{aligned}
d^+V(t) &= \operatorname{sgn}(x_1(t) - x_2(t)) \times \left\{\left[\frac{dx_1(t)}{x_1(t)} - \frac{(dx_1(t))^2}{2x_1^2(t)}\right] - \left[\frac{dx_2(t)}{x_2(t)} - \frac{(dx_2(t))^2}{2x_2^2(t)}\right]\right\} \\
&+ \operatorname{sgn}(y_1(t) - y_2(t)) \times \left\{\left[\frac{dy_1(t)}{y_1(t)} - \frac{(dy_1(t))^2}{2y_1^2(t)}\right] - \left[\frac{dy_2(t)}{y_2(t)} - \frac{(dy_2(t))^2}{2y_2^2(t)}\right]\right\} \\
&+ \operatorname{sgn}(z_1(t) - z_2(t)) \times \left\{\left[\frac{dz_1(t)}{z_1(t)} - \frac{(dz_1(t))^2}{2z_1^2(t)}\right] - \left[\frac{dz_2(t)}{z_2(t)} - \frac{(dz_2(t))^2}{2z_2^2(t)}\right]\right\} \\
&= \operatorname{sgn}(x_1(t) - x_2(t)) \times \left[-b_1(x_1(t) - x_2(t)) - \left(\frac{c_2 y_1(t)}{1 + \alpha_2 x_1(t) + \beta_2 y_1(t)} - \frac{c_2 y_2(t)}{1 + \alpha_2 x_2(t) + \beta_2 y_2(t)}\right) - \left(\frac{c_3 z_1(t)}{1 + \alpha_3 x_1(t) + \beta_3 z_1}(t) - \frac{c_3 z_2(t)}{1 + \alpha_3 x_2(t) + \beta_3 z_2(t)}\right)\right] dt \\
&+ \operatorname{sgn}(y_1(t) - y_2(t)) \times \left[-b_2(y_1(t) - y_2(t)) - \left(\frac{h_2 y_1(t)}{f_2 + g_2 z_1(t)} - \frac{h_2 y_2(t)}{f_2 + g_2 z_2(t)}\right) + \left(\frac{d_2 x_1(t)}{1 + \alpha_2 x_1(t) + \beta_2 y_1}(t) - \frac{d_2 x_2(t)}{1 + \alpha_2 x_2(t) + \beta_2 y_2(t)}\right)\right] dt \\
&+ \operatorname{sgn}(z_1(t) - z_2(t)) \times \left[-b_3(z_1(t) - z_2(t)) - \left(\frac{h_3 z_1(t)}{f_3 + g_3 y_1(t)} - \frac{h_3 z_2(t)}{f_3 + g_3 y_2(t)}\right) + \left(\frac{d_3 x_1(t)}{1 + \alpha_3 x_1(t) + \beta_3 z_1}(t) - \frac{d_3 x_2(t)}{1 + \alpha_2 x_2(t) + \beta_2 z_2(t)}\right)\right] dt.
\end{aligned} \quad (63)$$

Integrating from 0 to t and taking expectations yields

$$\begin{aligned}
E[V(t)] - E[V(0)] &= E\left[\int_0^t \left[\operatorname{sgn}(x_1(s) - x_2(s)) \times \left(-b_1(x_1(s) - x_2(s)) - \left(\frac{c_2 y_1(s)}{1 + \alpha_2 x_1(s) + \beta_2 y_1(s)} - \frac{c_2 y_2(s)}{1 + \alpha_2 x_2(s) + \beta_2 y_2(s)}\right) - \left(\frac{c_3 z_1(s)}{1 + \alpha_3 x_1(s) + \beta_3 z_1}(s) - \frac{c_3 z_2(s)}{1 + \alpha_3 x_2(s) + \beta_3 z_2(s)}\right)\right)\right.\right. \\
&\quad + \operatorname{sgn}(y_1(s) - y_2(s))
\end{aligned}$$

$$\times\Bigg(-b_2\left(y_1(s)-y_2(s)\right)$$
$$-\left(\frac{h_2y_1(s)}{f_2+g_2z_1(s)}-\frac{h_2y_2(s)}{f_2+g_2z_2(s)}\right)$$
$$+\left(\frac{d_2x_1(s)}{1+\alpha_2x_1(s)+\beta_2y_1}(s)\right.$$
$$\left.-\frac{d_2x_2(s)}{1+\alpha_2x_2(s)+\beta_2y_2(s)}\right)\Bigg)$$
$$+\operatorname{sgn}\left(z_1(s)-z_2(s)\right)$$
$$\times\Bigg(-b_3\left(z_1(s)-z_2(s)\right)$$
$$-\left(\frac{h_3z_1(s)}{f_3+g_3y_1(s)}-\frac{h_3z_2(s)}{f_3+g_3y_2(s)}\right)$$
$$+\left(\frac{d_3x_1(s)}{1+\alpha_3x_1(s)+\beta_3z_1}(s)\right.$$
$$\left.\left.-\frac{d_3x_2(s)}{1+\alpha_2x_2(s)+\beta_2z_2(s)}\right)\Bigg)\right]ds\Bigg]. \tag{64}$$

So

$$\frac{dE[V(t)]}{dt}$$
$$=E\Bigg[\operatorname{sgn}\left(x_1(t)-x_2(t)\right)$$
$$\times\Bigg(-b_1\left(x_1(t)-x_2(t)\right)$$
$$-\left(\frac{c_2y_1(t)}{1+\alpha_2x_1(t)+\beta_2y_1(t)}\right.$$
$$\left.-\frac{c_2y_2(t)}{1+\alpha_2x_2(t)+\beta_2y_2(t)}\right)$$
$$-\left(\frac{c_3z_1(t)}{1+\alpha_3x_1(t)+\beta_3z_1}(t)\right.$$
$$\left.-\frac{c_3z_2(t)}{1+\alpha_3x_2(t)+\beta_3z_2(t)}\right)\Bigg)$$
$$+\operatorname{sgn}\left(y_1(t)-y_2(t)\right)$$
$$\times\Bigg(-b_2\left(y_1(t)-y_2(t)\right)$$
$$-\left(\frac{h_2y_1(t)}{f_2+g_2z_1(t)}-\frac{h_2y_2(t)}{f_2+g_2z_2(t)}\right)$$
$$+\left(\frac{d_2x_1(t)}{1+\alpha_2x_1(t)+\beta_2y_1}(t)\right.$$
$$\left.-\frac{d_2x_2(t)}{1+\alpha_2x_2(t)+\beta_2y_2(t)}\right)\Bigg)$$
$$+\operatorname{sgn}\left(z_1(t)-z_2(t)\right)$$
$$\times\Bigg(-b_3\left(z_1(t)-z_2(t)\right)$$
$$-\left(\frac{h_3z_1(t)}{f_3+g_3y_1(t)}-\frac{h_3z_2(t)}{f_3+g_3y_2(t)}\right)$$
$$+\left(\frac{d_3x_1(t)}{1+\alpha_3x_1(t)+\beta_3z_1}(t)\right.$$
$$\left.-\frac{d_3x_2(t)}{1+\alpha_2x_2(t)+\beta_2z_2(t)}\right)\Bigg)\Bigg]$$
$$\leqslant -b_1E\left[\left|x_1(t)-x_2(t)\right|\right]$$
$$-c_2E\Bigg[\operatorname{sgn}\left(x_1(t)-x_2(t)\right)$$
$$\times\left(\frac{y_1(t)}{1+\alpha_2x_1(t)+\beta_2y_1(t)}\right.$$
$$\left.-\frac{y_2(t)}{1+\alpha_2x_2(t)+\beta_2y_2(t)}\right)\Bigg]$$
$$-c_3E\Bigg[\operatorname{sgn}\left(x_1(t)-x_2(t)\right)$$
$$\times\left(\frac{z_1(t)}{1+\alpha_3x_1(t)+\beta_3z_1}(t)\right.$$
$$\left.-\frac{z_2(t)}{1+\alpha_3x_2(t)+\beta_3z_2(t)}\right)\Bigg]$$
$$-b_2E\left[\left|y_1(t)-y_2(t)\right|\right]$$
$$-h_2E\Bigg[\operatorname{sgn}\left(y_1(t)-y_2(t)\right)$$
$$\times\left(\frac{y_1(t)}{f_2+g_2z_1(t)}-\frac{y_2(t)}{f_2+g_2z_2(t)}\right)\Bigg]$$
$$+d_2E\Bigg[\operatorname{sgn}\left(y_1(t)-y_2(t)\right)$$
$$\times\left(\frac{x_1(t)}{1+\alpha_2x_1(t)+\beta_2y_1}(t)\right.$$
$$\left.-\frac{x_2(t)}{1+\alpha_2x_2(t)+\beta_2y_2(t)}\right)\Bigg]$$
$$-b_3E\left[\left|z_1(t)-z_2(t)\right|\right]$$
$$-h_3E\Bigg[\operatorname{sgn}\left(z_1(t)-z_2(t)\right)$$
$$\times\left(\frac{z_1(t)}{f_3+g_3y_1(t)}-\frac{z_2(t)}{f_3+g_3y_2(t)}\right)\Bigg]$$
$$+d_3E\Bigg[\operatorname{sgn}\left(z_1(t)-z_2(t)\right)$$

$$\times\left(\frac{x_1(t)}{1+\alpha_3 x_1(t)+\beta_3 z_1}(t) - \frac{x_2(t)}{1+\alpha_2 x_2(t)+\beta_2 z_2(t)}\right)\Bigg]$$

$$\begin{aligned}
&\leqslant -b_1 E\left[\left|x_1(t)-x_2(t)\right|\right] \\
&\quad + c_2 E\left[\left|\frac{y_1(t)}{1+\alpha_2 x_1(t)+\beta_2 y_1(t)} - \frac{y_2(t)}{1+\alpha_2 x_2(t)+\beta_2 y_2(t)}\right|\right] \\
&\quad + c_3 E\left[\left|\frac{z_1(t)}{1+\alpha_3 x_1(t)+\beta_3 z_1}(t) - \frac{z_2(t)}{1+\alpha_3 x_2(t)+\beta_3 z_2(t)}\right|\right] \\
&\quad - b_2 E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad + h_2 E\left[\left|\frac{y_1(t)}{f_2+g_2 z_1(t)} - \frac{y_2(t)}{f_2+g_2 z_2(t)}\right|\right] \\
&\quad + d_2 E\left[\left|\frac{x_1(t)}{1+\alpha_2 x_1(t)+\beta_2 y_1}(t) - \frac{x_2(t)}{1+\alpha_2 x_2(t)+\beta_2 y_2(t)}\right|\right] \\
&\quad - b_3 E\left[\left|z_1(t)-z_2(t)\right|\right] \\
&\quad + h_3 E\left[\left|\frac{z_1(t)}{f_3+g_3 y_1(t)} - \frac{z_2(t)}{f_3+g_3 y_2(t)}\right|\right] \\
&\quad + d_3 E\left[\left|\frac{x_1(t)}{1+\alpha_3 x_1(t)+\beta_3 z_1}(t) - \frac{x_2(t)}{1+\alpha_2 x_2(t)+\beta_2 z_2(t)}\right|\right] \\
&\leqslant -b_1 E\left[\left|x_1(t)-x_2(t)\right|\right] + c_2 E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad + \frac{c_2\alpha_2}{\beta_2} E\left[\left|x_1(t)-x_2(t)\right|\right] + c_3 E\left[\left|z_1(t)-z_2(t)\right|\right] \\
&\quad + \frac{c_3\alpha_3}{\beta_3} E\left[\left|x_1(t)-x_2(t)\right|\right] - b_2 E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad + \frac{h_2}{f_2} E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad + \frac{h_2 g_2}{f_2^2} E\left[y_1(t)\right] E\left[\left|z_1(t)-z_2(t)\right|\right] \\
&\quad + \frac{h_2}{f_2} E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad + d_2 E\left[\left|x_1(t)-x_2(t)\right|\right] + \frac{d_2\beta_2}{\alpha_2} E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad - b_3 E\left[\left|z_1(t)-z_2(t)\right|\right] + \frac{h_3}{f_3} E\left[\left|z_1(t)-z_2(t)\right|\right] \\
&\quad + \frac{h_3 g_3}{f_3^2} E\left[z_1(t)\right] E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad + \frac{h_3}{f_3} E\left[\left|z_1(t)-z_2(t)\right|\right] + d_3 E\left[\left|x_1(t)-x_2(t)\right|\right] \\
&\quad + \frac{d_3\beta_3}{\alpha_3} E\left[\left|z_1(t)-z_2(t)\right|\right] \\
&\leqslant \left(-b_1 + \frac{c_2\alpha_2}{\beta_2} + \frac{c_3\alpha_3}{\beta_3} + d_2 + d_3\right) \\
&\quad \times E\left[\left|x_1(t)-x_2(t)\right|\right] \\
&\quad + \left(-b_2 + c_2 + \frac{2h_2}{f_2} + \frac{d_2\beta_2}{\alpha_2} + \frac{h_3 g_3}{f_3^2}\left(E\left[z_1^3(t)\right]\right)^{1/3}\right) \\
&\quad \times E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\quad + \left(-b_3 + c_3 + \frac{2h_3}{f_3} + \frac{d_3\beta_3}{\alpha_3} + \frac{h_2 g_2}{f_2^2}\left(E\left[y_1^3(t)\right]\right)^{1/3}\right) \\
&\quad \times E\left[\left|z_1(t)-z_2(t)\right|\right].
\end{aligned} \tag{65}$$

By Lemma 4

$$\begin{aligned}
\left(E\left[x_1^3(t)\right]\right)^{1/3} &\leqslant \frac{a_1+\sigma_1^2}{b_1}, \\
\left(E\left[y_1^3(t)\right]\right)^{1/3} &\leqslant \frac{a_2+\left(d_2/\alpha_2\right)+\sigma_2^2}{b_2}, \\
\left(E\left[z_1^3(t)\right]\right)^{1/3} &\leqslant \frac{a_3+\left(d_3/\alpha_3\right)+\sigma_3^2}{b_3}.
\end{aligned} \tag{66}$$

Thus

$$\begin{aligned}
&\frac{dE\left[V(t)\right]}{dt} \\
&\quad \leqslant \left(-b_1 + \frac{c_2\alpha_2}{\beta_2} + \frac{c_3\alpha_3}{\beta_3} + d_2 + d_3\right) E\left[\left|x_1(t)-x_2(t)\right|\right] \\
&\qquad + \left(-b_2 + c_2 + \frac{2h_2}{f_2} + \frac{d_2\beta_2}{\alpha_2} + \frac{h_3 g_3\left(a_2 + d_2/\alpha_2 + \sigma_2^2\right)}{b_2 f_3^2}\right) \\
&\qquad \times E\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\qquad + \left(-b_3 + c_3 + \frac{2h_3}{f_3} + \frac{d_3\beta_3}{\alpha_3} + \frac{h_2 g_2\left(a_3 + d_3/\alpha_3 + \sigma_3^2\right)}{b_3 f_2^2}\right) \\
&\qquad \times E\left[\left|z_1(t)-z_2(t)\right|\right] \\
&\quad = AE\left[\left|x_1(t)-x_2(t)\right|\right] + BE\left[\left|y_1(t)-y_2(t)\right|\right] \\
&\qquad + CE\left[\left|z_1(t)-z_2(t)\right|\right].
\end{aligned} \tag{67}$$

Integrating both sides leads to

$$E[V(t)] \leqslant E[V(0)] + \int_0^t AE[|x_1(s) - x_2(s)|] + BE[|y_1(s) - y_2(s)|] + CE[|z_1(s) - z_2(s)|]\,ds. \quad (68)$$

Therefore

$$\begin{aligned} 0 &\leqslant E[V(t)] + \int_0^t AE[|x_1(s) - x_2(s)|] + BE[|y_1(s) - y_2(s)|] + CE[|z_1(s) - z_2(s)|]\,ds \\ &\leqslant E[V(0)] < \infty. \end{aligned} \quad (69)$$

Then we get

$$\begin{aligned} E&[|(x_1(t), y_1(t), z_1(t)) - (x_2(t), y_2(t), z_2(t))|] \\ &= E\left[\left(|x_1(t) - x_2(t)|^2 + |y_1(t) - y_2(t)|^2 + |z_1(t) - z_2(t)|^2\right)^{1/2}\right] \\ &\leqslant E[|x_1(t) - x_2(t)|] + E[|y_1(t) - y_2(t)|] + E[|z_1(t) - z_2(t)|] \in L^1[0, \infty). \end{aligned} \quad (70)$$

Therefore from Lemmas 11 and 12, we have

$$\lim_{t \to +\infty} E[|(x_1(t), y_1(t), z_1(t)) - (x_2(t), y_2(t), z_2(t))|] = 0. \quad (71)$$

This completes the proof of Theorem 13. □

6. Extinction

Theorem 14. *For any initial value* $(x_0, y_0, z_0) \in R_+^3$, *the solution* $(x(t), y(t), z(t))$ *of system* (4) *obeys*

$$\begin{aligned} \limsup_{t\to\infty} \frac{\ln x(t)}{t} &\leqslant a_1 - \frac{\sigma_1^2}{2}, \\ \limsup_{t\to\infty} \frac{\ln y(t)}{t} &\leqslant a_2 + \frac{d_2}{\alpha_2} - \frac{\sigma_2^2}{2}, \\ \limsup_{t\to\infty} \frac{\ln z(t)}{t} &\leqslant a_3 + \frac{d_3}{\alpha_3} - \frac{\sigma_3^2}{2}. \end{aligned} \quad (72)$$

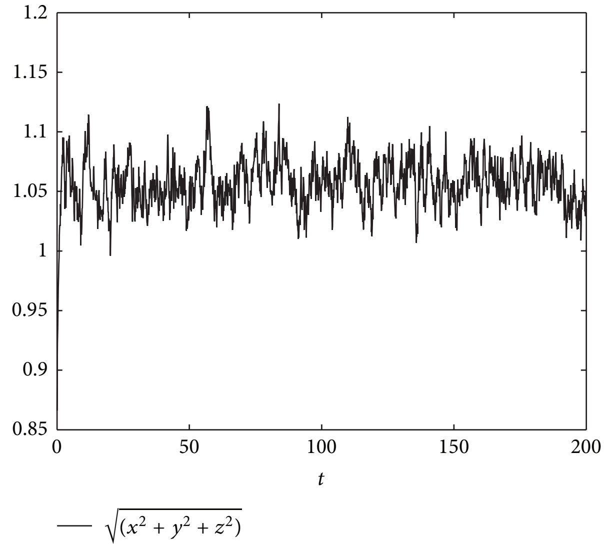
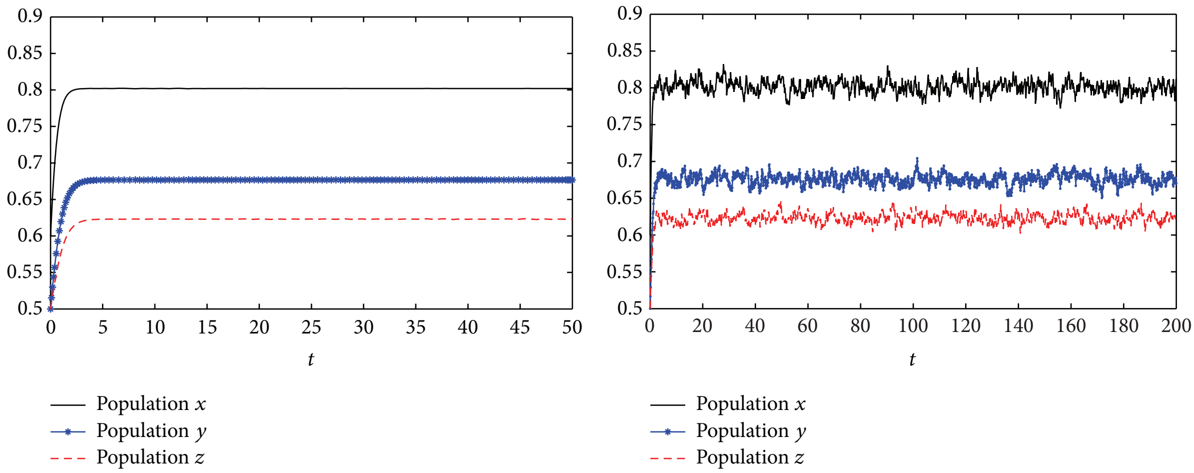


Figure 1: Solutions of system (4) for $(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$, $a_1 = 2$; $a_2 = 1$; $a_3 = 1.5$; $b_1 = 3$; $b_2 = 2$; $b_3 = 2$; $c_2 = 0.2$; $c_3 = 0.1$; $d_2 = 1$; $d_3 = 1.5$; $\alpha_2 = 1$; $\beta_2 = 0.5$; $\alpha_3 = 0.6$; $\beta_3 = 0.8$; $h_2 = 0.5$; $h_3 = 1$; $f_2 = 1$; $f_3 = 0.3$; $g_2 = 0.5$; $g_3 = 1$; $\sigma_1 = 0.05$; $\sigma_2 = 0.05$; $\sigma_3 = 0.05$.

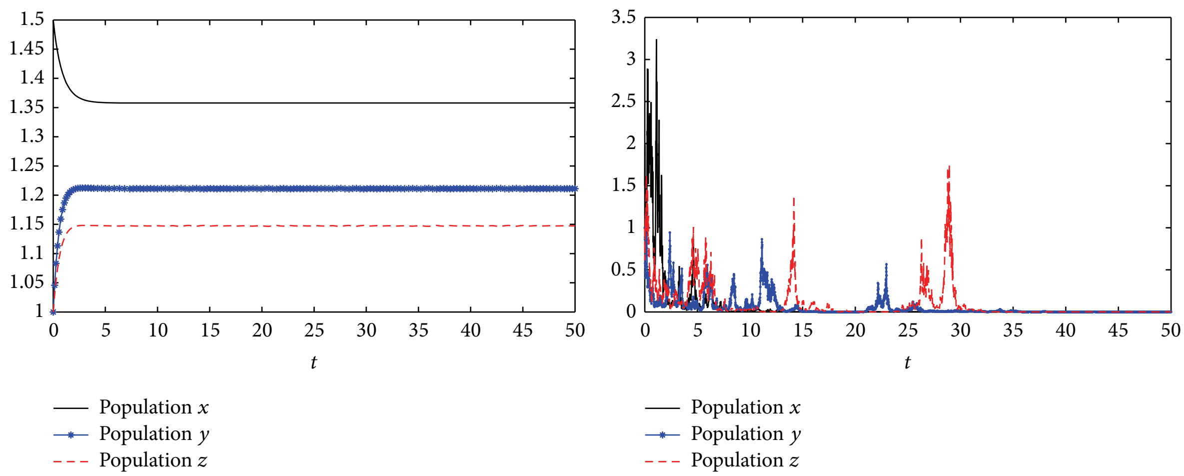
Proof. Define Lyapunov functions $\ln x$, $\ln y$, and $\ln z$, respectively, hence by Itô's formula, we get

$$\begin{aligned} \ln x(t) &= \ln x_0 + \left(a_1 - \frac{\sigma_1^2}{2}\right)t + \int_0^t -b_1 x(s) - \frac{c_2 y(s)}{1 + \alpha_2 x(s) + \beta_2 y(s)} - \frac{c_3 z(s)}{1 + \alpha_3 x(s) + \beta_3 z(s)}\,ds, \\ \ln y(t) &= \ln y_0 + \left(a_2 - \frac{\sigma_2^2}{2}\right)t + \int_0^t -b_2 y(s) - \frac{h_2 y(s)}{f_2 + g_2 z(s)}\,ds + \int_0^t \frac{d_2 x(s)}{1 + \alpha_2 x(s) + \beta_2 y(s)}\,ds, \\ \ln z(t) &= \ln z_0 + \left(a_3 - \frac{\sigma_3^2}{2}\right)t + \int_0^t -b_3 z(s) - \frac{h_3 z(s)}{f_3 + g_3 y(s)}\,ds + \int_0^t \frac{d_3 x(s)}{1 + \alpha_3 x(s) + \beta_3 z(s)}\,ds. \end{aligned} \quad (73)$$

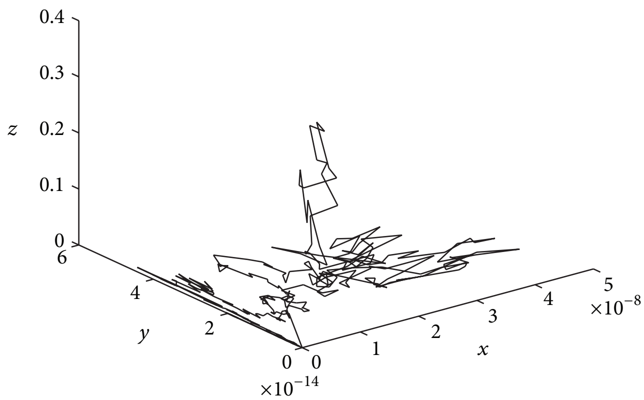


(a) (b)

Figure 2: Solutions of system (4) for $(x_0, y_0, z_0) = (0.6, 0.5, 0.5)$, $a_1 = 2$; $a_2 = 1$; $a_3 = 1.5$; $b_1 = 2.5$; $b_2 = 2$; $b_3 = 1.5$; $c_2 = 0.2$; $c_3 = 0.1$; $d_2 = 1.5$; $d_3 = 1$; $\alpha_2 = 1$; $\beta_2 = 0.5$; $\alpha_3 = 1$; $\beta_3 = 0.8$; $h_2 = 0.5$; $h_3 = 1$; $f_2 = 1$; $f_3 = 0.3$; $g_2 = 1$; $g_3 = 0.5$; $\sigma_1 = 0.05$; $\sigma_2 = 0.05$; $\sigma_3 = 0.05$.



(a) (b)



(c)

Figure 3: Solutions of system (4) for $(x_0, y_0, z_0) = (1.5, 1, 1)$, $a_1 = 1.1$; $a_2 = 1.2$; $a_3 = 1.5$; $b_1 = 0.8$; $b_2 = 1.1$; $b_3 = 1.2$; $c_2 = 0.02$; $c_3 = 0.01$; $d_2 = 1.2$; $d_3 = 1$; $\alpha_2 = 0.7$; $\beta_2 = 0.5$; $\alpha_3 = 0.8$; $\beta_3 = 0.5$; $h_2 = 0.8$; $h_3 = 0.5$; $f_2 = 1$; $f_3 = 0.3$; $g_2 = 0.8$; $g_3 = 0.5$; $\sigma_1 = 1.8$; $\sigma_2 = 1.8$; $\sigma_3 = 1.8$.

Then we have

$$\ln x(t) \leqslant \ln x_0 + \left(a_1 - \frac{\sigma_1^2}{2}\right)t + \sigma_1 B_1(t),$$
$$\ln y(t) \leqslant \ln y_0 + \left(a_2 + \frac{d_2}{\alpha_2} - \frac{\sigma_2^2}{2}\right)t + \sigma_2 B_2(t), \quad (74)$$
$$\ln z(t) \leqslant \ln z_0 + \left(a_3 + \frac{d_3}{\alpha_3} - \frac{\sigma_3^2}{2}\right)t + \sigma_3 B_3(t).$$

Dividing t on the both sides and letting $t \to \infty$, we can derive

$$\limsup_{t\to\infty} \frac{\ln x(t)}{t} \leqslant a_1 - \frac{\sigma_1^2}{2}, \quad (75)$$

$$\limsup_{t\to\infty} \frac{\ln y(t)}{t} \leqslant a_2 + \frac{d_2}{\alpha_2} - \frac{\sigma_2^2}{2}, \quad (76)$$

$$\limsup_{t\to\infty} \frac{\ln z(t)}{t} \leqslant a_3 + \frac{d_3}{\alpha_3} - \frac{\sigma_3^2}{2}, \quad (77)$$

as required. □

So we can obtain that if $a_1 - (\sigma_1^2/2) < 0$, $a_2 + (d_2/\alpha_2) - (\sigma_2^2/2) < 0$, and $a_3 + (d_3/\alpha_3) - (\sigma_3^2/2) < 0$ hold, then for any initial value $(x_0, y_0, z_0) \in R_+^3$, the solution $(x(t), y(t), z(t))$ of system (4) will be extinct exponentially with probability one.

7. Numerical Simulations

At last, we numerically simulate the solution $(x(t), y(t), z(t))$ of system (4) to substantiate the analytical findings. By the method mentioned in [14], we consider the discretization equation:

$$\begin{aligned} x_{k+1} &= x_k + x_k \times \left(a_1 - b_1 x_k - \frac{c_2 y_k}{1 + \alpha_2 x_k + \beta_2 y_k} - \frac{c_3 z_k}{1 + \alpha_3 x_k + \beta_3 z_k}\right)\Delta t \\ &\quad + \sigma_1 x_k \xi_k \sqrt{\Delta t} + \frac{1}{2}\sigma_1^2 x_k \left(\xi_k^2 - 1\right)\Delta t, \\ y_{k+1} &= y_k + y_k \times \left(a_2 - b_2 y_k - \frac{h_2 y_k}{f_2 + g_2 z_k} + \frac{d_2 x_k}{1 + \alpha_2 x_k + \beta_2 y_k}\right)\Delta t \\ &\quad + \sigma_2 y_k \eta_k \sqrt{\Delta t} + \frac{1}{2}\sigma_2^2 y_k \left(\eta_k^2 - 1\right)\Delta t, \\ z_{k+1} &= z_k + z_k \times \left(a_3 - b_3 z_k - \frac{h_3 z_k}{f_3 + g_3 y_k} + \frac{d_3 x_k}{1 + \alpha_3 x_k + \beta_3 z_k}\right)\Delta t \\ &\quad + \sigma_3 z_k \zeta_k \sqrt{\Delta t} + \frac{1}{2}\sigma_3^2 z_k \left(\zeta_k^2 - 1\right)\Delta t, \end{aligned} \quad (78)$$

where ξ_k, η_k, and ζ_k are Gaussian random variables that follow $N(0, 1)$.

For example, in Figure 1, we choose initial value $(x_0, y_0, z_0) = (0.5, 0.5, 0.5)$ and parameters satisfying conditions of Theorem 7; the system is stochastically permanent. In Figure 2, (a) is the solution to the deterministic system (4), (b) is the solution to the stochastic system (4), and (c) is the phase diagram of the stochastic system (4). Comparing (a) with (b), we find that with small environmental noise, the stochastic system is getting more similar to the deterministic. In Figure 3, (a) is the solution to the deterministic system (4) and (b) is the solution to the stochastic system (4). Comparing (a) with (b), we find that the sufficiently large environmental noise make the stochastic system extinct. The phase diagram of (x, y, z) is displayed in (c).

8. Discussion

This paper has been devoted to dynamics of a stochastic cooperative predator-prey system with Beddington-DeAngelis functional. Firstly we show that, although the coefficients in the model neither satisfy the linear growth condition nor local Lipschitz continuity, the stochastic model has a globally positive solution. Then we know that the positive solution is stochastically bounded. Moreover, under some conditions, we analyze global asymptotic stability of the positive solutions. We can find that the stochastic model will preserve this nice property provided that the noise is sufficiently small. Some meaningful questions deserve further investigation. One way we can consider colored noise in the models owing to sudden environmental changes caused by seasons or other reasons. Moreover, it is worth to rebuild our model with some parameters, not the ones studied above, which are also subject to stochastic excitation. The ecosystem we considered has some limitations, for example, both predators (y and z) with the common prey (x) and we only established a model with a mutual cooperation between both predators. It is worth to discuss mechanics of the population contained predators with a mutual competition in other ecosystems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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