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Research Article

Discussion for H-Matrices and It's Application

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Nonsingular H-matrices and positive stable matrices play an important role in the stability of neural network system. In this paper, some criteria for nonsingular H-matrices are obtained by the theory of diagonally dominant matrices and the obtained result is introduced into identifying the stability of neural networks. So the criteria for nonsingular H-matrices are expanded and their application on neural network system is given. Finally, the effectiveness of the results is illustrated by numerical examples.

1. Introduction

The research on data mining based on neural networks has a great significance. Recently, as one kind of artificial neural networks, Hopfield neural network is used for association rules mining and remarkable results are obtained. Nonsingular H-matrices and positive stable matrices play an important role in the stability of neural network system. However, it is rather difficult in practice to determine whether a matrix is a nonsingular H-matrix or not. Therefore, it is of a great theoretical and practical value to study the numerical methods for judging the nonsingular H-matrices, to provide the concise and practical criteria. Up to now, within the scope of the field, many researchers have done a lot of in-depth studies and acquired some very valuable results in many respects, such as nonsingular H-matrix properties and criteria (see [1–9]). In this paper, some criteria for nonsingular H-matrices are obtained by the theory of diagonally dominant matrices and the obtained result is introduced into identifying the stability of neural networks. So the criteria for nonsingular H-matrices are expanded and their application on neural network system is given. Effectiveness of the results is illustrated by numerical examples. For convenience, we are dealing with nonsingular H-matrices, calling them shortly H-matrices.

Next, we will introduce some notations.

Let $N = \{1, 2, \ldots, n\}$, and let $M = \{(i, j) \mid i \neq j;\ i, j \in N\}$. $C^{n,n}$ denotes the set of all n by n complex matrices: $R_i(A) = \sum_{j\neq i} |a_{ij}|$ and $C_i(A) = \sum_{j\neq i} |a_{ji}|$ (for all $i \in N$).

If $|a_{ii}| \geq (>) R_i(A)$ (for all $i \in N$), then A is said to be a (strictly) diagonally dominant matrix and is denoted by $A \in D_0$ $(A \in D)$; if $|a_{ii}a_{jj}| \geq (>) R_i(A)R_j(A)$ (for all $(i, j) \in M$), then A is said to be a (strictly) double diagonally dominant matrix and is denoted by $A \in DD_0$ $(A \in DD)$. It is well known that an equivalent definition of H-matrices is given by demanding that there exist positive numbers $x_1, x_2, \ldots, x_n$ such that $x_i|a_{ii}| > \sum_{j\neq i} x_j|a_{ij}|$ (for all $i \in N$); that is, there exists a positive diagonal matrix $X = \operatorname{diag}(x_1, \ldots, x_n)$ such that $AX \in D$ (see [1]). So, we always assume that $|a_{ii}| \neq 0$ (for all $i \in N$).

2. Definitions and Lemmas

It is learned that the class of α-double diagonally dominant matrices play a central role in identifying H-matrices. So, we will start with its definition and some background results.

Definition 1 (see [2]). Let $A = (a_{ij}) \in C^{n,n}$; if there exists some $\alpha \in [0, 1]$, satisfying

$$\left|a_{ii}a_{jj}\right| \geq (>) \left[R_i(A) R_j(A)\right]^{\alpha} \left[C_i(A) C_j(A)\right]^{1-\alpha} \quad (\forall (i, j) \in M), \tag{1}$$

then A is called a (strictly) α-double diagonally dominant matrix and is denoted by $A \in DD(\alpha_0)$ $(A \in DD(\alpha))$.

Lemma 2 (see [2]). *Let $A = (a_{ij}) \in C^{n,n}$; if $A \in DD(\alpha)$, then A is an H-matrix.*

Lemma 3 (see [3]). *Let $A = (a_{ij}) \in C^{n,n}$, if there exists some $\alpha \in [0,1]$, satisfying*

$$\left|a_{ii}a_{jj}\right| \geq \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha} \quad (\forall (i,j) \in M), \tag{2}$$

and, for every $(i,j) \in M$ with $|a_{ii}a_{jj}| = [R_i(A)R_j(A)]^{\alpha} \times [C_i(A)C_j(A)]^{1-\alpha}$, there exists a nonzero elements chain $a_{i_0i_1}, a_{i_1i_2}, \ldots, a_{i_rj_0}$ or $a_{j_0j_1}, a_{j_1j_2}, \ldots, a_{j_ti_0}$ such that $i_0 = i$ or $i_0 = j$, $j_0 \in J(A)$, where

$$J(A) = \left\{i \,\middle|\, \left|a_{ii}a_{jj}\right| > \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}, (i,j) \in M\right\} \neq \emptyset, \tag{3}$$

then A is an H-matrix.

Let $S(A)$ denote the set of all circuits of length $p \geq 2$ in $\Gamma(A)$ (directed graph of the matrix A). Recall that a circuit in $\Gamma(A)$ is an ordered sequence γ of vertices $i_1, i_2, \ldots, i_p, i_{p+1} = i_1$ $(p \geq 1)$, where $i_1, i_2, \ldots, i_p$ are all distinct and $e_{i_ji_{j+1}}$ $(j = 1, 2, \ldots, p)$ are arcs of $\Gamma(A)$. Let $E(A)$ denote the set of all arcs.

Lemma 4 (see [4]). *Let A be an irreducible complex matrix. Suppose there exists some $\alpha \in [0,1]$, satisfying*

$$\left|a_{ii}a_{jj}\right| \geq \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha} \quad (\forall (i,j) \in M). \tag{4}$$

If there exists some arc $e_{i_*j_*} \in E(A)$ and $(i_*, j_*) \in M$ such that

$$\left|a_{i_*i_*}a_{j_*j_*}\right| > \left[R_{i_*}(A)R_{j_*}(A)\right]^{\alpha}\left[C_{i_*}(A)C_{j_*}(A)\right]^{1-\alpha}, \tag{5}$$

then A is an H-matrix.

3. Criteria for H-Matrices

In the rest of the paper, we will use the notations:

$$\begin{gathered}
M_1 = \left\{(i,j) \mid R_i(A)R_j(A) < \left|a_{ii}a_{jj}\right| < C_i(A)C_j(A)\right\}; \\
M_2 = \left\{(i,j) \mid C_i(A)C_j(A) < \left|a_{ii}a_{jj}\right| < R_i(A)R_j(A)\right\}; \\
M_3 = \left\{(i,j) \mid \left|a_{ii}a_{jj}\right| \geq C_i(A)C_j(A) > R_i(A)R_j(A)\right\}; \\
M_4 = \left\{(i,j) \mid \left|a_{ii}a_{jj}\right| \geq R_i(A)R_j(A) > C_i(A)C_j(A)\right\}; \\
M_5 = \left\{(i,j) \mid \left|a_{ii}a_{jj}\right| > R_i(A)R_j(A) = C_i(A)C_j(A)\right\}; \\
M_0 = \left\{(i,j) \mid \left|a_{ii}a_{jj}\right| \leq R_i(A)R_j(A), \left|a_{ii}a_{jj}\right| \leq C_i(A)C_j(A)\right\}.
\end{gathered} \tag{6}$$

It is obvious to deduce that $M = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 \cup M_0$.

Let

$$\begin{gathered}
\alpha_{st} = \frac{\left|a_{ss}a_{tt}\right|}{R_s(A)R_t(A)}, \qquad \beta_{st} = \frac{C_s(A)C_t(A)}{\left|a_{ss}a_{tt}\right|}, \\
\gamma_{st} = \frac{C_s(A)C_t(A)}{R_s(A)R_t(A)}, \qquad \gamma_{st} = \alpha_{st}\beta_{st}, \\
\forall (s,t) \in M_1; \\
x_{ij} = \frac{\left|a_{ii}a_{jj}\right|}{C_i(A)C_j(A)}, \qquad y_{ij} = \frac{R_i(A)R_j(A)}{\left|a_{ii}a_{jj}\right|}, \\
z_{ij} = \frac{R_i(A)R_j(A)}{C_i(A)C_j(A)}, \qquad z_{ij} = x_{ij}y_{ij}, \\
\forall (i,j) \in M_2.
\end{gathered} \tag{7}$$

It is obvious to observe

$$\begin{gathered}
\gamma_{st} > \alpha_{st} > 1, \qquad \gamma_{st} > \beta_{st} > 1; \\
z_{ij} > x_{ij} > 1, \qquad z_{ij} > y_{ij} > 1.
\end{gathered} \tag{8}$$

The following are our main results. First, we give an equivalent representation for strictly α-double diagonally dominant matrices.

Lemma 5. *Let $A = (a_{ij}) \in C^{n,n}$; then $A \in DD(\alpha)$ if and only if $M_0 = \emptyset$ and for any $(s,t) \in M_1$, $(i,j) \in M_2$, satisfying*

$$\log_{\gamma_{st}}\beta_{st} + \log_{z_{ij}}y_{ij} < 1. \tag{9}$$

Proof. Sufficiency. From inequality (9), for any $(s,t) \in M_1$, $(i,j) \in M_2$, it follows that

$$\log_{z_{ij}}y_{ij} < 1 - \log_{\gamma_{st}}\beta_{st}. \tag{10}$$

Recalling that $\gamma_{st} > \beta_{st} > 1$, for any $(s,t) \in M_1$, we have $0 < \log_{\gamma_{st}}\beta_{st} < 1$. So there exists some positive number ε such that

$$0 < \log_{\gamma_{st}}\beta_{st} + \varepsilon < 1, \tag{11}$$

$$\log_{z_{ij}}y_{ij} < 1 - \left(\log_{\gamma_{st}}\beta_{st} + \varepsilon\right). \tag{12}$$

Let $\alpha = \log_{\gamma_{st}}\beta_{st} + \varepsilon$; it is easy to see $0 < \alpha < 1$ and $\log_{\gamma_{st}}\beta_{st} < \alpha$; that is,

$$\beta_{st} < \left(\alpha_{st}\beta_{st}\right)^{\alpha}. \tag{13}$$

By both ends of inequality (13) multiplied by $\beta_{st}^{-\alpha}$, we have $\alpha_{st}^{\alpha} > \beta_{st}^{1-\alpha}$; that is,

$$\left[\frac{\left|a_{ss}a_{tt}\right|}{R_s(A)R_t(A)}\right]^{\alpha} > \left[\frac{C_s(A)C_t(A)}{\left|a_{ss}a_{tt}\right|}\right]^{1-\alpha}. \tag{14}$$

The inequality above implies that

$$\left|a_{ss}a_{tt}\right| > \left[R_s(A)R_t(A)\right]^{\alpha}\left[C_s(A)C_t(A)\right]^{1-\alpha}. \tag{15}$$

By inequality (12) again, for any $(i, j) \in M_2$, it is obvious that $\log_{z_{ij}} y_{ij} < 1 - \alpha$; that is,

$$y_{ij} < \left(x_{ij} y_{ij}\right)^{1-\alpha}. \quad (16)$$

By both ends of inequality (16) multiplied by $y_{ij}^{\alpha-1}$, we have $x_{ij}^{1-\alpha} > y_{ij}^{\alpha}$; that is,

$$\left[\frac{|a_{ii}a_{jj}|}{C_i(A)C_j(A)}\right]^{1-\alpha} > \left[\frac{R_i(A)R_j(A)}{|a_{ii}a_{jj}|}\right]^{\alpha}. \quad (17)$$

The inequality above implies that

$$|a_{ii}a_{jj}| > \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}. \quad (18)$$

Moreover, for any $(l, m) \in M_3 \cup M_4 \cup M_5$, and any $\alpha \in (0, 1)$, it is obvious that

$$|a_{ll}a_{mm}| > \left[R_l(A)R_m(A)\right]^{\alpha}\left[C_l(A)C_m(A)\right]^{1-\alpha}. \quad (19)$$

Recalling that $M_0 = \emptyset$, for any $(i, j) \in M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 = M$, there exists some $\alpha \in [0, 1]$ such that

$$|a_{ii}a_{jj}| > \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}. \quad (20)$$

Therefore, we have $A \in DD(\alpha)$ by Definition 1.

Necessity. Suppose $A \in DD(\alpha)$; then $M_0 = \emptyset$, and, for any $(s, t) \in M_1$, there exists some $\alpha \in [0, 1]$ such that

$$|a_{ss}a_{tt}| > \left[R_s(A)R_t(A)\right]^{\alpha}\left[C_s(A)C_t(A)\right]^{1-\alpha}; \quad (21)$$

that is,

$$\left[\frac{|a_{ss}a_{tt}|}{R_s(A)R_t(A)}\right]^{\alpha} > \left[\frac{C_s(A)C_t(A)}{|a_{ss}a_{tt}|}\right]^{1-\alpha}. \quad (22)$$

Then by the notations of α_{st} and β_{st}, we have $\beta_{st}^{1-\alpha} < \alpha_{st}^{\alpha}$. Furthermore, by both ends of the inequality multiplied by β_{st}^{α}, we get $\beta_{st} < (\alpha_{st}\beta_{st})^{\alpha} = \gamma_{st}^{\alpha}$. Therefore, it can be seen that

$$\log_{\gamma_{st}} \beta_{st} < \log_{\gamma_{st}} \gamma_{st}^{\alpha} = \alpha. \quad (23)$$

Following a similar argument for any $(i, j) \in M_2$, we have

$$\log_{z_{ij}} y_{ij} < \log_{z_{ij}} z_{ij}^{1-\alpha} = 1 - \alpha. \quad (24)$$

Combining inequalities (23) and (24), we obtain inequality (9). The proof is completed. □

As its application, some new practical criteria for H-matrices are obtained.

Theorem 6. *Let $A = (a_{ij}) \in C^{n,n}$, $M_0 = \emptyset$, and, for any $(s, t) \in M_1$, $(i, j) \in M_2$, satisfying*

$$\log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} < 1; \quad (25)$$

then A is an H-matrix.

Proof. By Lemma 5, we obtain $A \in DD(\alpha)$, and further using Lemma 2, we conclude that A is an H-matrix. □

Theorem 7. *$A = (a_{ij}) \in C^{n,n}$ is an H-matrix if A satisfies either of the conditions:*

(1) *$M_0 \cup M_1 = \emptyset$;*

(2) *$M_0 \cup M_2 = \emptyset$.*

Proof. (1) Suppose $M_0 \cup M_1 = \emptyset$; then, for any $(i, j) \in M_2$, by $0 < \log_{z_{ij}} y_{ij} < 1$, there exists some positive number ε, such that

$$0 < \log_{z_{ij}} y_{ij} + \varepsilon < 1. \quad (26)$$

Let $\alpha = 1 - (\log_{z_{ij}} y_{ij} + \varepsilon) \subset (0, 1)$; then we have $\log_{z_{ij}} y_{ij} < 1 - \alpha$, which implies that

$$|a_{ii}a_{jj}| > \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}. \quad (27)$$

For any $(l, m) \in M_3 \cup M_4 \cup M_5$, and any $\alpha \in (0, 1)$, it is obvious that

$$|a_{ll}a_{mm}| > \left[R_l(A)R_m(A)\right]^{\alpha}\left[C_l(A)C_m(A)\right]^{1-\alpha}. \quad (28)$$

Next, similarly as in the proof of Sufficiency of Lemma 5, we conclude that A is an H-matrix.

(2) Suppose $M_0 \cup M_2 = \emptyset$; then for any $(s, t) \in M_1$, by $0 < \log_{\gamma_{st}} \beta_{st} < 1$, there exists some positive number ε such that

$$0 < \log_{\gamma_{st}} \beta_{st} + \varepsilon < 1. \quad (29)$$

Let $\alpha = \log_{\gamma_{st}} \beta_{st} + \varepsilon \subset (0, 1)$; then we have $\log_{\gamma_{st}} \beta_{st} < \alpha$, which implies that

$$|a_{ss}a_{tt}| \geq \left[R_s(A)R_t(A)\right]^{\alpha}\left[C_s(A)C_t(A)\right]^{1-\alpha}. \quad (30)$$

Similarly, we conclude that A is an H-matrix. □

Theorem 8. *Let $A = (a_{ij}) \in C^{n,n}$, $M_0 = \emptyset$, and, for any $(s, t) \in M_1$, $(i, j) \in M_2$, satisfying*

$$\log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} \leq 1. \quad (31)$$

If, for every pair of indices $(s, t) \in M_1$, $(i, j) \in M_2$ with

$$\log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} = 1, \quad (32)$$

there exists two nonzero elements chains $a_{s_0s_1}, a_{s_1s_2}, \ldots, a_{s_ht_0}$ or $a_{t_0t_1}, a_{t_1t_2}, \ldots, a_{t_ks_0}$ and $a_{i_0i_1}, a_{i_1i_2}, \ldots, a_{i_pj_0}$ or $a_{j_0j_1}, a_{j_1j_2}, \ldots, a_{j_qi_0}$ with $s_0 = s$ or $s_0 = t$, $t_0 \in G(A)$ and $i_0 = i$ or $i_0 = j$, $j_0 \in G(A)$, where

$$G(A) = \left\{i \mid \log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} < 1, \ (i, j) \in M_2\right\} \neq \emptyset, \quad (33)$$

then A is an H-matrix.

Proof. Similarly as in the proof of Sufficiency of Lemma 5 combined with inequality (31), we can prove that for any $(s,t) \in M_1$, and $(i,j) \in M_2$, there exists some $\alpha \in [0,1]$ such that

$$\begin{aligned} \left|a_{ss}a_{tt}\right| &\geq \left[R_s(A)R_t(A)\right]^{\alpha}\left[C_s(A)C_t(A)\right]^{1-\alpha}; \\ \left|a_{ii}a_{jj}\right| &\geq \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}. \end{aligned} \tag{34}$$

Moreover, for any $(l,m) \in M_3 \cup M_4 \cup M_5$, and any $\alpha \in (0,1)$, it is obvious that

$$\left|a_{ll}a_{mm}\right| > \left[R_l(A)R_m(A)\right]^{\alpha}\left[C_l(A)C_m(A)\right]^{1-\alpha}. \tag{35}$$

Recalling that $G(A) \neq \emptyset$, we conclude that there exists some $(s,t) \in M_1$, $(i,j) \in M_2$ such that

$$\begin{aligned} \left|a_{ss}a_{tt}\right| &> \left[R_s(A)R_t(A)\right]^{\alpha}\left[C_s(A)C_t(A)\right]^{1-\alpha}; \\ \left|a_{ii}a_{jj}\right| &> \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}. \end{aligned} \tag{36}$$

By equality (32), we know that, for every pair of indices $(s,t) \in M_1$, $(i,j) \in M_2$ with

$$\begin{aligned} \left|a_{ss}a_{tt}\right| &= \left[R_s(A)R_t(A)\right]^{\alpha}\left[C_s(A)C_t(A)\right]^{1-\alpha}; \\ \left|a_{ii}a_{jj}\right| &= \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}, \end{aligned} \tag{37}$$

there exists two nonzero elements chains $a_{s_0s_1}, a_{s_1s_2}, \ldots, a_{s_ht_0}$ or $a_{t_0t_1}, a_{t_1t_2}, \ldots, a_{t_ks_0}$ and $a_{i_0i_1}, a_{i_1i_2}, \ldots, a_{i_pj_0}$ or $a_{j_0j_1}, a_{j_1j_2}, \ldots, a_{j_qi_0}$ with $s_0 = s$ or $s_0 = t$, $t_0 \in J'(A)$ and $i_0 = i$ or $i_0 = j$, $j_0 \in J'(A)$, where

$$\begin{aligned} J'(A) = \Big\{ i \;\Big|\; &\left|a_{ii}a_{jj}\right| > \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}, \\ &(i,j) \in M_2 \Big\} \neq \emptyset. \end{aligned} \tag{38}$$

By Lemma 3, it follows that A is an H-matrix. □

Similarly as in the proof of Theorem 8, we can obtain the following result.

Theorem 9. *Let $A = (a_{ij}) \in C^{n,n}$, $M_0 = \emptyset$, and, for any $(s,t) \in M_1$, $(i,j) \in M_2$, satisfying*

$$\log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} \leq 1. \tag{39}$$

If, for every pair of indices $s \in L_1$, $i \in L_2$, there exists two nonzero elements chains $a_{s_0s_1}, a_{s_1s_2}, \ldots, a_{s_ht_0}$ or $a_{t_0t_1}, a_{t_1t_2}, \ldots, a_{t_ks_0}$ and $a_{i_0i_1}, a_{i_1i_2}, \ldots, a_{i_pj_0}$ or $a_{j_0j_1}, a_{j_1j_2}, \ldots, a_{j_qi_0}$ with $s_0 = s$ or $s_0 = t$ and $i_0 = i$ or $i_0 = j$ such that $t_0, j_0 \in N \setminus (L_1 \cup L_2) \neq \emptyset$, where

$$\begin{aligned} L_1 &= \left\{ s \mid \log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} = 1,\ (s,t) \in M_1 \right\}; \\ L_2 &= \left\{ i \mid \log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} = 1,\ (i,j) \in M_2 \right\}, \end{aligned} \tag{40}$$

then A is an H-matrix.

Theorem 10. *Let A be an irreducible complex matrix, $M_0 = \emptyset$, and, for any $(s,t) \in M_1$, $(i,j) \in M_2$, satisfying*

$$\log_{\gamma_{st}} \beta_{st} + \log_{z_{ij}} y_{ij} \leq 1. \tag{41}$$

If there exists some arc $e_{i_*j_*} \in E(A)$ and $(i_*, j_*) \in M_2$ such that

$$\log_{\gamma_{st}} \beta_{st} + \log_{z_{i_*j_*}} y_{i_*j_*} < 1, \tag{42}$$

then A is an H-matrix.

Proof. With the same argument as in the proof of Theorem 8, we can obtain that, for any $(i,j) \in M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 = M$, there exists some $\alpha \in [0,1]$ such that

$$\left|a_{ii}a_{jj}\right| \geq \left[R_i(A)R_j(A)\right]^{\alpha}\left[C_i(A)C_j(A)\right]^{1-\alpha}. \tag{43}$$

By inequality (42), we know that there exists some arc $e_{i_*j_*} \in E(A)$ and $(i_*, j_*) \in M_2$ such that

$$\left|a_{i_*i_*}a_{j_*j_*}\right| > \left[R_{i_*}(A)R_{j_*}(A)\right]^{\alpha}\left[C_{i_*}(A)C_{j_*}(A)\right]^{1-\alpha}. \tag{44}$$

Recalling that A is irreducible, it follows that A is an H-matrix by Lemma 4. □

4. Algorithm and Program

Algorithm for Theorem 6.

(1) Input matrix A;

(2) calculate $R_i(A)$ and $C_i(A)$ (for all $i \in N$) (denoted in the Introduction of the paper);

(3) define indices M_1, M_2, and M_0;

(4) if $M_0 \neq \emptyset$, then the criterion is invalid;

(5) if $M_0 = \emptyset$, then calculate $\alpha_{ij}, \beta_{ij}, \gamma_{ij}$ (for all $(i,j) \in M_1$) and x_{ij}, y_{ij}, z_{ij} (for all $(i,j) \in M_2$);

(6) calculate and verify the condition of Theorem 6. If the condition is satisfied, then output "A is an H-matrix."

We write the related program by the above algorithm using MATLAB Software. All the results are calculated by MATLAB 7.0. The procedures are shown in Procedure 1.

5. Numerical Examples

Example 1. Let

$$A = \begin{bmatrix} 3.3 & -0.5 & -0.5 & -0.4 & -0.1 \\ -0.5 & 2.5 & -1 & -1 & -0.5 \\ -2.2 & -0.5 & 3 & -0.5 & 0 \\ -1 & -0.3 & -0.5 & 10 & -1 \\ -0.5 & -1.2 & 0 & -1 & 10 \end{bmatrix}. \tag{45}$$

```
A=input("please input a matrix")
M1=[];M2=[];M6=[];F=[];B=[];
n=size(A,1);RA=zeros(n,1);CA=zeros(n,1);
for k=1:n
    A=abs(A);
    RA(k)=[sum(A(k,:))-A(k,k)];
    CA(k)=[sum(A(:,k))-A(k,k)];
end
for i=1:n-1
    for j=i+1:n
    RR=RA(i)*RA(j);
    aa=abs(A(i,i)*A(j,j));
    CC=CA(i)*CA(j);
    if RR<aa&aa<CC
       M1=[M1;i,j];
       alpha=aa/RR;beta=CC/aa;gamma=alpha*beta;
       F=[F,alpha,beta,gamma];
    elseif CC<aa&aa<RR
       M2=[M2;i,j];
       x=aa/CC;y=RR/aa;z=x*y;
       B=[B,x,y,z];
    elseif RR>=aa&CC>=aa
       M6=1;break;
       Show="the criterion is invalid";
       end
    end
end
if M6==1
    "the criterion is invalid";
    elseif size(M1,1)==0|size(M2,1)==0
       "A is an H-matrix"
       else
          k1=size(F,1);k2=size(B,1);
          for i=1:k1
          F2(i)=log(F(i,2))/log(F(i,3));
          end
          for i=1:k2
          B2(i)=log(B(i,2))/log(B(i,3));
          end
          if max(B2)+max(F2)<1
           show="A is an H-matrix"
       end
end
```

Procedure 1

Then we have

$$\begin{gathered} R_1(A)=1.5, \qquad R_2(A)=3, \qquad R_3(A)=3.2, \\ R_4(A)=2.8, \qquad R_5(A)=2.7; \\ C_1(A)=4.2, \qquad C_2(A)=2.5, \qquad C_3(A)=2, \\ C_4(A)=2.9, \qquad C_5(A)=1.6; \\ |a_{11}|=3.3, \qquad |a_{22}|=2.5, \qquad |a_{33}|=3, \\ |a_{44}|=10, \qquad |a_{55}|=10. \end{gathered} \tag{46}$$

But, we notice $|a_{22}| = 2.5 = C_2(A) < R_2(A) = 3$. The condition does not satisfy either Theorem 2 or Theorem 3 in [5], so we cannot obtain that A is an H-matrix.

According to the notations of this paper, we have

$$M_1 = \{(1,2)\}, \qquad M_2 = \{(2,3)\}, \qquad M_0 = \emptyset. \tag{47}$$

By calculating, we obtain

$$\log_{\gamma_{12}}\beta_{12} = 0.2846; \qquad \log_{z_{23}} y_{23} = 0.3784, \tag{48}$$

and then

$$\log_{\gamma_{12}}\beta_{12} + \log_{z_{23}} y_{23} = 0.2846 + 0.3784 = 0.6630 < 1. \tag{49}$$

It satisfies the condition of Theorem 6, and then A is an H-matrix.

We consider the following Hopfield type continuous neural networks:

$$C_i \frac{du_i}{dt} = -\frac{u_i}{R_i} + \sum_{j=1}^{5} T_{ij} g_j \left(u_j (t-\tau)\right) + I_i \quad (i = 1,2,3,4,5), \tag{50}$$

where,

$$\begin{gathered} g_i (u_i) > 0, \quad u_i \neq 0, \ 0 < g_i \leq 1, \\ g_i (\pm\infty) = \pm 1, \quad C_i = 1 \quad (i = 1,2,3,4,5); \\ R_1 = \frac{1}{4.3}, \qquad R_2 = \frac{1}{3.5}, \qquad R_3 = \frac{1}{4}, \qquad R_4 = R_5 = \frac{1}{11}; \\ \left(T_{ij}\right)_{5\times 5} = \begin{bmatrix} -1 & 0.5 & 0.5 & -0.4 & 0.1 \\ 0.5 & 1 & 1 & -1 & 0.5 \\ 2.2 & -0.5 & 1 & 0.5 & 0 \\ -1 & 0.3 & -0.5 & 1 & -1 \\ 0.5 & -1.2 & 0 & 1 & -1 \end{bmatrix}. \end{gathered} \tag{51}$$

Notice that $\operatorname{diag}(1/R_1, 1/R_2, 1/R_3, 1/R_4, 1/R_5) - (|T_{ij}|)_{5\times 5} = A$ is an H-matrix, and then A is a nonsingular M-matrix, which ensures existence, uniqueness, and global exponential stability of the equilibrium point of the above neural networks by [10].

Example 2. Let

$$A = \begin{bmatrix} 4 & 1 & 0.5 \\ 2 & 2 & 1 \\ 0.5 & 2 & 3 \end{bmatrix}. \tag{52}$$

By calculating, we have

$$M_2 = \{(2,3)\}, \qquad M_1 = M_0 = \emptyset. \tag{53}$$

It satisfies the condition (1) of Theorem 7, and then A is an H-matrix.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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