Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2014, Article ID 853415, 9 pages
http://dx.doi.org/10.1155/2014/853415

Research Article

Singular LQ Problem for Irregular Singular Systems

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Received 26 April 2014; Accepted 25 June 2014; Published 8 July 2014

Academic Editor: Jong Hae Kim

This paper is concerned with the singular LQ problem for irregular singular systems with persistent disturbances. The full information feedback control method is employed to achieve the optimal control. By restricted system equivalence transformation, the system state is decomposed into free state and restricted state and the input is decomposed into free input and forced input. Some sufficient conditions for the unique existence of optimal control-state pair are derived and these conditions are all described unitedly with matrix rank equalities. The optimal control-state pair can be explicitly formulated via solving an algebraic Riccati equation and a Sylvester equation. Moreover, under the optimal control-state pair, the resulting system has no free state.

1. Introduction

In practical control systems there are unavoidably various external disturbances affecting the performances of systems. Many control problems involve designing a controller capable of stabilizing a given system while minimizing the worst-case response to some additive disturbances. The application areas of interest include, for example, the flight control through wind shear where disturbances arise from a model for wind shear based on harmonic oscillations [1], the active control for offshore structures where external disturbances are mainly from the wind or ocean wave forces [2], the noise reduction in vehicles and transformers, and the control of the linear course of ships [3, 4], and so forth. Therefore, the optimal control for systems with persistent disturbances is one of the very important control problems. Up to now, there has been much research work about the optimal control of linear standard state space systems with persistent disturbances (see, e.g., [5–7]).

Since, among all output feedback plants, it requires the weakest assumption, the full information feedback (FI, also called feedforward and feedback [8]) plant has received a great attention in the literature and has been extensively applied to control systems with disturbances. For example, Rusli et al. [9] design a low-order robust nonlinear FI controller for a multiscale system that dynamically couples Kinetic Monte Carlo and finite difference codes. Tang [5] proposes a FI control for linear systems affected by sinusoidal disturbances with known frequency and unknown magnitude and phase. Later, in his joint papers [6, 10–12] he brings forward a united description of disturbances including periodic disturbances as the special case and gives the FI controller designs for the continuous, discrete, time-delay, and continuous time-varying linear systems, respectively. Chiu [13] proposes a mixed FI based adaptive fuzzy control design for a class of MIMO uncertain nonlinear systems. Yu et al. [14] investigate the complete parametric approach for output regulation problems of matrix 2-order systems via FI.

Singular systems have comprehensive practical background [15–17]. Great progress has been made in the theory and its applications since 1970s [18–22]. However, there are few investigations on optimal control problems for irregular singular systems with disturbances, besides Chen [23, 24] where the singular LQ suboptimal and optimal control problems for irregular singular systems with disturbances are considered.

In this paper, the singular LQ problem for irregular singular systems with persistent disturbances is discussed. It is shown that the singular LQ problem for irregular singular systems with persistent disturbances can be transformed to the optimal problem for standard state space systems by restricted system equivalence transformation. The system state is decomposed into free state and restricted state and the input is decomposed into free input and forced input. Then, under some general conditions, FI optimal control-state pair and the optimal performance index are derived via solving an algebraic Riccati equation and a Sylvester equation.

The remainder of the paper is organized as follows. In Section 2, the singular LQ problem for irregular singular systems with persistent disturbances is transformed to the optimal problem for standard state space systems by restricted system equivalence. In Section 3, we deal with the FI optimal control problem for irregular singular systems and obtain the sufficient conditions for the unique existence of optimal control-state pair with regard to the cases when the disturbance is damped and not damped. A simulation example is exploited to demonstrate the effectiveness of the proposed results in Section 4. In Section 5, we give the brief conclusion of this paper.

Notation 1. Throughout the paper, the superscript "T" stands for matrix transposition; R^n denotes the n-dimensional Euclidean space; $R^{n\times m}$ is the set of $n\times m$ real matrices; I_n is the $n\times n$ identity matrix; $\Re\lambda$ stands for the real part of λ; for real symmetric matrix A, $A>0$ means that A is a definite-positive matrix and $A\geq 0$ means that A is a semidefinite-positive matrix. All of the matrices in the context, if not explicitly stated, are assumed to have compatible dimensions.

2. Statement and Transformation of LQ Problem For Singular Systems

Consider the singular system with disturbances:

$$E\dot{x} = Ax + B_1 w + B_2 u, \qquad Ex(0) = x_0, \tag{1}$$

where $E, A \in R^{m\times n}$, $B_1 \in R^{m\times l}$, $B_2 \in R^{m\times r}$, x, w, and u are state, disturbance, and input, respectively; $\operatorname{rank} E = p < n$. The disturbance w is governed by the exosystem [12]:

$$\dot{w} = Gw, \qquad w(0) = w_0, \tag{2}$$

where $G \in R^{l\times l}$ is stable.

System (1) is said to be regular if $m = n$ and $\det(sE - A) \not\equiv 0$; otherwise, it is irregular. For the regular singular system, Ishihara et al. [8] considered the FI and state feedback (SF) H_2 control problems. For the irregular singular system without disturbances, Zhu et al. [25] discussed the SF LQ control problem for system (1) with $B_1 = 0$.

In this paper, the performance index is selected as follows.

In the case when G is asymptotically stable, the quadratic performance index is

$$J(u,x) = \frac{1}{2}\int_0^\infty \left(x^T Q x + u^T R u\right) dt, \tag{3}$$

and the corresponding admissible control-state pair set is

$$\Omega = \{(u,x) \mid (u,x) \text{ is piecewise continuous, satisfies (1) and } J(u,x) < \infty\}, \tag{4}$$

where $Q \geq 0$ and $R \geq 0$. In the case when G is stable but not asymptotically stable, the disturbance w will have oscillation behaviour, the state x and the control u may not tend to zero at the same time, which may cause the quadratic performance index (3) tending to be infinite. So, in this case we adopt the quadratic average performance index

$$J'(u,x) = \lim_{t_f\to\infty}\frac{1}{2t_f}\int_0^{t_f}\left(x^T Q x + u^T R u\right) dt. \tag{5}$$

The corresponding admissible control-state pair set is

$$\Omega' = \{(u,x) \mid (u,x) \text{ is piecewise continuous, satisfies (1) and } J'(u,x) < \infty\}. \tag{6}$$

The control objective of this paper is stated as follows: when G is asymptotically stable, the control objective is to find an optimal control-state pair $(u^*, x^*) \in \Omega$ such that

$$J(u^*, x^*) = \min_{(u,x)\in\Omega} J(u,x). \tag{$\mathbb{P}$}$$

When G is stable but not asymptotically stable, the control objective is to find an optimal control-state pair $(u^*, x^*) \in \Omega'$ such that

$$J'(u^*, x^*) = \min_{(u,x)\in\Omega'} J'(u,x). \tag{$\mathbb{P}'$}$$

First of all, it is necessary to discuss some properties of irregular singular systems.

Definition 1. The irregular singular system with disturbances

$$\widetilde{E}\dot{\widetilde{x}} = \widetilde{A}\widetilde{x} + \widetilde{B}_1 w + \widetilde{B}_2 u, \qquad \widetilde{E}\widetilde{x}(0) = \widetilde{x}_0, \tag{7}$$

is said to be restricted system equivalent (r.s.e.) to the system (1) if there exist two nonsingular matrices $M \in R^{m\times m}$ and $N \in R^{n\times n}$ such that $x = N\widetilde{x}$, $MEN = \widetilde{E}$, $MAN = \widetilde{A}$, $MB_1 = \widetilde{B}_1$, $MB_2 = \widetilde{B}_2$, and $Mx_0 = \widetilde{x}_0$.

Obviously, restricted system equivalence is an equivalent relationship and it is consistent with the definition in [25] for the systems without disturbances.

Denote $Q = C^T C$, $R = D^T D$ and $y = [(Cx)^T, (Du)^T]^T$, where $C \in R^{q_1\times n}$ and $D \in R^{q_2\times r}$, and then $x^T Q x + u^T R u = y^T y$.

Since $\operatorname{rank} E = p < n$, there exist nonsingular matrices $M_1 \in R^{m\times m}$ and $N_1 \in R^{n\times n}$ such that $M_1 E N_1 = \operatorname{diag}([I_p, 0])$. Accordingly, let

$$M_1 A N_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad M_1 B_1 = \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix}, \qquad M_1 B_2 = \begin{bmatrix} B_{21} \\ B_{22} \end{bmatrix}, \qquad CN_1 = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \tag{8}$$

where $A_{11} \in R^{p\times p}$, $B_{11} \in R^{p\times l}$, $B_{21} \in R^{p\times r}$, $C_1 \in R^{q_1\times p}$.

Definition 2. System (1) is impulse controllable if, for every initial condition and disturbance governed by the system (2), there exists a smooth (impulse-free) control-state pair of system (1).

Definition 2 is consistent with the definition in [26] for the systems without disturbances.

Obviously, it is necessary for the solvability of the problem $(\mathbb{P})$ and $(\mathbb{P}')$ that system (1) is impulse controllable. The following lemma establishes two necessary and sufficient conditions for the impulse controllability of system (1).

Lemma 3. *System* (1) *is impulse controllable if and only if*

$$\operatorname{rank}\begin{bmatrix} A_{21} & A_{22} & B_{12} & B_{22} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} A_{22} & B_{22} \end{bmatrix}, \quad (9)$$

or

$$\operatorname{rank}\begin{bmatrix} 0 & E & 0 \\ E & A & B_2 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} E & A & B_1 & B_2 \end{bmatrix} + \operatorname{rank} E. \quad (10)$$

The proof is similar to that of Theorem 13 in [26], and it is presented in the Appendix.

In the following discussions, we always assume that (10) holds.

Denote $\operatorname{rank} A_{22} = d$ and $\operatorname{rank}[A_{22}, B_{22}] = s$; then there exist matrices $M_2 \in R^{p\times d}$, $M_3 \in R^{p\times(s-d)}$, $M_4 \in R^{d\times(s-d)}$, $M_5 \in R^{q_1\times d}$, $M_6 \in R^{q_1\times(s-d)}$, and $M_7 \in R^{q_2\times(s-d)}$ and nonsingular matrices $M_8 \in R^{s\times s}$, $M_9 \in R^{(m-p)\times(m-p)}$, $N_2 \in R^{(n-p)\times(n-p)}$, and $N_3 \in R^{r\times r}$ such that

$$M_{10}\begin{bmatrix} A_{11} & A_{12} & B_{11} & B_{21} \\ A_{21} & A_{22} & B_{12} & B_{22} \\ C_1 & C_2 & 0 & 0 \\ 0 & 0 & 0 & D \end{bmatrix} \times N_4 = \begin{bmatrix} \overline{A}_{11} & 0 & \overline{A}_{13} & \overline{B}_{11} & 0 & \overline{B}_{13} \\ \overline{A}_{21} & I_d & 0 & \overline{B}_{21} & 0 & \overline{B}_{23} \\ \overline{A}_{31} & 0 & 0 & \overline{B}_{31} & I_{s-d} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_{11} & 0 & C_{13} & D_{11} & 0 & D_{13} \\ C_{21} & 0 & 0 & D_{21} & 0 & D_{23} \end{bmatrix}, \quad (11)$$

where $\overline{A}_{11} \in R^{p\times p}$, $\overline{A}_{13} \in R^{p\times(n-p-d)}$, $\overline{B}_{11} \in R^{p\times l}$, $\overline{B}_{13} \in R^{p\times(r-s+d)}$, $C_{11} \in R^{q_1\times p}$, $C_{21} \in R^{q_2\times p}$, $N_4 = \operatorname{diag}([I_p, N_2, I_l, N_3])$, and

$$M_{10} = \begin{bmatrix} I_p & M_2 & M_3 & 0 & 0 & 0 \\ 0 & I_d & M_4 & 0 & 0 & 0 \\ 0 & 0 & I_{s-d} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{m-p-s} & 0 & 0 \\ 0 & M_5 & M_6 & 0 & I_{q_1} & 0 \\ 0 & 0 & M_7 & 0 & 0 & I_{q_2} \end{bmatrix} \cdot \operatorname{diag}\left(\left[I_p, M_8, I_{m-p-s+q_1+q_2}\right]\right)\operatorname{diag}\left(\left[I_p, M_9, I_{q_1+q_2}\right]\right). \quad (12)$$

Let

$$\left[x_1^T, x_2^T, x_3^T, u_1^T, u_2^T\right]^T = N^{-1}\left[x^T, u^T\right]^T, \quad (13)$$

where $x_1 \in R^p$, $x_2 \in R^d$, $x_3 \in R^{n-p-d}$, $u_1 \in R^{s-d}$, $u_2 \in R^{r-s+d}$, and

$$N = \operatorname{diag}\left(\left[N_1, I_r\right]\right)\operatorname{diag}\left(\left[I_p, N_2, N_3\right]\right), \quad (14)$$

and then the system (1) is r.s.e to the following system

$$\begin{aligned} \dot{x}_1 &= \overline{A}_{11}x_1 + \overline{A}_{13}x_3 + \overline{B}_{11}w + \overline{B}_{13}u_2, \quad x_1(0) = M_{11}x_0, \\ 0 &= \overline{A}_{21}x_1 + x_2 + \overline{B}_{21}w + \overline{B}_{23}u_2, \\ 0 &= \overline{A}_{31}x_1 + \overline{B}_{31}w + u_1, \end{aligned} \quad (15)$$

$$y = \begin{bmatrix} C_{11} \\ C_{21} \end{bmatrix} x_1 + \begin{bmatrix} C_{13} \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix} w + \begin{bmatrix} D_{13} \\ D_{23} \end{bmatrix} u_2, \quad (16)$$

where $M_{11} = [I_p, 0]M_1$.

The second and third equations in system (15) can be written as

$$\begin{aligned} x_2 &= -\overline{A}_{21}x_1 - \overline{B}_{21}w - \overline{B}_{23}u_2, \\ u_1 &= -\overline{A}_{31}x_1 - \overline{B}_{31}w. \end{aligned} \quad (17)$$

Therefore, in the system (15), the state variables x_1, x_2 and input variable u_1 are determined uniquely by x_3, u_2, and w. Thus the state variable x_3 is free and the input variable u_1 is not free. We call x_3 the free state and u_1 the forced input. Accordingly, x_1 and x_2 are called restricted state and u_2 is called free input.

Remark 4. System (1) has no free state if and only if

$$\operatorname{rank}\begin{bmatrix} 0 & E \\ E & A \end{bmatrix} = n + \operatorname{rank} E. \quad (18)$$

Remark 5. System (1) has no forced input if and only if

$$\operatorname{rank}\begin{bmatrix} 0 & E \\ E & A \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 0 & E & 0 \\ E & A & B_2 \end{bmatrix}. \quad (19)$$

Let $z = [x_3^T, u_2^T]^T$ and $\overline{B}_{12} = [\overline{A}_{13}, \overline{B}_{13}]$; then the dynamic equation of x_1, z, and w is

$$\begin{aligned} \dot{x}_1 &= \overline{A}_{11}x_1 + \overline{B}_{11}w + \overline{B}_{12}z, \qquad x_1(0) = M_{11}x_0, \\ \dot{w} &= Gw, \qquad w(0) = w_0. \end{aligned} \quad (20)$$

From (16),

$$y^T y = \left[x_1^T, w^T, z^T\right]\overline{Q}\left[x_1^T, w^T, z^T\right]^T, \quad (21)$$

where $\overline{Q} = (\overline{Q}_{ij})_{3\times 3}$ is a symmetric matrix, and

$$\begin{aligned} \overline{Q}_{11} &= \left[C_{11}^T, C_{21}^T\right]\left[C_{11}^T, C_{21}^T\right]^T, \\ \overline{Q}_{12} &= \left[C_{11}^T, C_{21}^T\right]\left[D_{11}^T, D_{21}^T\right]^T, \\ \overline{Q}_{13} &= \left[C_{11}^T, C_{21}^T\right]\begin{bmatrix} C_{13} & D_{13} \\ 0 & D_{23} \end{bmatrix}, \\ \overline{Q}_{22} &= \left[D_{11}^T, D_{21}^T\right]\left[D_{11}^T, D_{21}^T\right]^T, \\ \overline{Q}_{23} &= \left[D_{11}^T, D_{21}^T\right]\begin{bmatrix} C_{13} & D_{13} \\ 0 & D_{23} \end{bmatrix}, \\ \overline{Q}_{33} &= \begin{bmatrix} C_{13} & D_{13} \\ 0 & D_{23} \end{bmatrix}^T\begin{bmatrix} C_{13} & D_{13} \\ 0 & D_{23} \end{bmatrix}. \end{aligned} \quad (22)$$

Let

$$J_1(z,x_1)=\frac{1}{2}\int_0^{\infty}\left[x_1^T,w^T,z^T\right]\overline{Q}\left[x_1^T,w^T,z^T\right]^T dt,$$
$$J_1'(z,x_1)=\lim_{t_f\to\infty}\frac{1}{2t_f}\int_0^{t_f}\left[x_1^T,w^T,z^T\right]\overline{Q}\left[x_1^T,w^T,z^T\right]^T dt, \tag{23}$$

$$\begin{aligned}\Omega_1=\{(z,x_1)\mid (z,x_1)&\text{ is piecewise continuous,}\\ &\text{satisfies (20) and } J_1(z,x_1)<\infty\},\\ \Omega_1'=\{(z,x_1)\mid (z,x_1)&\text{ is piecewise continuous,}\\ &\text{satisfies (20) and } J_1'(z,x_1)<\infty\},\end{aligned} \tag{24}$$

and then the LQ problem ($\mathbb{P}$) is substantially transformed to the optimal problem, denoted by $\mathbb{P}_1$, of finding an optimal solution $(z^*,x_1^*)\in\Omega_1$ at which the performance index $J_1(z,x_1)$ achieves the minimum, and the LQ problem $(\mathbb{P}')$ is substantially transformed to the optimal problem, denoted by $\mathbb{P}_1'$, of finding an optimal solution $(z^*,x_1^*)\in\Omega_1'$ at which the performance index $J_1'(z,x_1)$ achieves the minimum.

Obviously, $\mathbb{P}_1$ and $\mathbb{P}_1'$ are optimal problems of the standard state space system, and the singularity of $\mathbb{P}_1$ and $\mathbb{P}_1'$ is determined by the property of $\overline{Q}_{33}$. Now, we give a necessary and sufficient condition for $\overline{Q}_{33}>0$.

Theorem 6. $\overline{Q}_{33}>0$ *if and only if*

$$\operatorname{rank}\begin{bmatrix}0&E&0\\E&A&B_2\\0&Q&0\\0&0&R\end{bmatrix}=n+r+\operatorname{rank}E. \tag{25}$$

Proof. $\overline{Q}_{33}>0$ is equivalent to $\operatorname{rank}\left[\begin{smallmatrix}C_{13}&D_{13}\\0&D_{23}\end{smallmatrix}\right]=n-p+r-s$, which is $\operatorname{rank}\Gamma=n+r+\operatorname{rank}E$, where

$$\Gamma=\begin{bmatrix}0&0&I_p&0&0&0&0\\0&0&0&0&0&0&0\\I_p&0&\overline{A}_{11}&0&\overline{A}_{13}&0&\overline{B}_{13}\\0&0&\overline{A}_{21}&I_d&0&0&\overline{B}_{23}\\0&0&\overline{A}_{31}&0&0&I_{s-d}&0\\0&0&0&0&0&0&0\\0&0&C_{11}&0&C_{13}&0&D_{13}\\0&0&C_{21}&0&0&0&D_{23}\end{bmatrix}. \tag{26}$$

From (8) and (11),

$$\Gamma=\operatorname{diag}\left([I_n,M_{10}]\right)\operatorname{diag}\left([M_1,M_1,I_q]\right)\begin{bmatrix}0&E&0\\E&A&B_2\\0&C&0\\0&0&D\end{bmatrix}\cdot\operatorname{diag}\left([N_1,N_1,I_r]\right)\operatorname{diag}\left([I_n,I_p,N_2,N_3]\right), \tag{27}$$

which finishes the proof. □

Remark 7. It is a routine matter to show that if (E,A) is regular and $R>0$, then (25) is equivalent to that (E,A,Q) is impulse observable.

3. Design of the FI Controller

In this section, we solve the problem ($\mathbb{P}$) and $(\mathbb{P}')$ via solving $\mathbb{P}_1$ and $\mathbb{P}_1'$, respectively.

3.1. Solution of the Problem ($\mathbb{P}$). Denoting $v=[x_1^T,w^T]^T$, (20) can be written as

$$\dot v=\overline{A}v+\overline{B}z,\qquad v(0)=\left[x_1^T(0),w^T(0)\right]^T, \tag{28}$$

where $\overline{A}=\left[\begin{smallmatrix}\overline{A}_{11}&\overline{B}_{11}\\0&G\end{smallmatrix}\right]$, $\overline{B}=\left[\begin{smallmatrix}\overline{B}_{12}\\0\end{smallmatrix}\right]$, and

$$y^Ty=\left[v^T,z^T\right]\widetilde{Q}\left[v^T,z^T\right]^T, \tag{29}$$

where $\widetilde{Q}=(\widetilde{Q}_{ij})_{2\times2}$ is a symmetric matrix, and

$$\widetilde{Q}_{11}=\begin{bmatrix}\overline{Q}_{11}&\overline{Q}_{12}\\\overline{Q}_{12}^T&\overline{Q}_{22}\end{bmatrix},\qquad \widetilde{Q}_{12}=\begin{bmatrix}\overline{Q}_{13}\\\overline{Q}_{23}\end{bmatrix},\qquad \widetilde{Q}_{22}=\overline{Q}_{33}. \tag{30}$$

Let

$$J_2(z,v)=\frac{1}{2}\int_0^{\infty}\left[v^T,z^T\right]\widetilde{Q}\left[v^T,z^T\right]^T dt, \tag{31}$$

$$\begin{aligned}\Omega_2=\{(z,v)\mid (z,v)&\text{ is piecewise continuous,}\\ &\text{satisfies (28) and } J_2(z,v)<\infty\},\end{aligned} \tag{32}$$

and then the optimal problem $\mathbb{P}_1$ is transformed to the optimal problem, denoted by $\mathbb{P}_2$, of finding an optimal solution $(z^*,v^*)\in\Omega_2$ at which the performance index $J_2(z,v)$ achieves the minimum.

Denote $\widetilde{A}=\overline{A}-\overline{B}\widetilde{Q}_{22}^{-1}\widetilde{Q}_{12}^T$. According to the Maximum Principle, if $(\widetilde{A},\overline{B})$ is stabilizable and $(\widetilde{A},\widetilde{Q}_{11}-\widetilde{Q}_{12}\widetilde{Q}_{22}^{-1}\widetilde{Q}_{12}^T)$ is detectable, then $\mathbb{P}_2$ has a unique solution (z^*,v^*), and the optimal value is $J_2^*=v^T(0)Pv(0)/2$, where

$$z^*=-\widetilde{Q}_{22}^{-1}\left(\widetilde{Q}_{12}^T+\overline{B}^TP\right)v^*, \tag{33}$$

v^* satisfies the equation

$$\dot v=\left(\widetilde{A}-\overline{B}\widetilde{Q}_{22}^{-1}\overline{B}^TP\right)v, \tag{34}$$

P is the unique positive semidefinite solution of the Riccati equation

$$P\widetilde{A}+\widetilde{A}^TP-P\overline{B}\widetilde{Q}_{22}^{-1}\overline{B}^TP+\widetilde{Q}_{11}-\widetilde{Q}_{12}\widetilde{Q}_{22}^{-1}\widetilde{Q}_{12}^T=0, \tag{35}$$

and the matrix $\widetilde{A}-\overline{B}\widetilde{Q}_{22}^{-1}\overline{B}^TP$ is asymptotically stable.

Before further discussion, we first give two lemmas.

Lemma 8. *When G is asymptotically stable, $(\widetilde{A},\overline{B})$ is stabilizable if and only if*

$$\operatorname{rank}\left[\lambda E-A\quad B_2\right]=\operatorname{rank}\left[E\quad A\quad B_2\right],\quad \forall\lambda,\ \Re\lambda\ge0. \tag{36}$$

Proof. From (8) and (11),

$$\operatorname{rank}\left[\lambda E - A \quad B_2\right] = \operatorname{rank}\left[\lambda I_p - \overline{A}_{11} \quad \overline{B}_{12}\right] + s. \tag{37}$$

When G is asymptotically stable, $\forall\lambda, \Re\lambda \geq 0$, the matrix $\lambda I_l - G$ is nonsingular, so

$$\operatorname{rank}\left[\lambda I_{p+l} - \widetilde{A} \quad \overline{B}\right] = \operatorname{rank}\left[\lambda I_p - \overline{A}_{11} \quad \overline{B}_{12}\right] + l. \tag{38}$$

Since $\operatorname{rank}\left[E \quad A \quad B_2\right] = p + s$, (36) holds if and only if

$$\operatorname{rank}\left[\lambda I_{p+l} - \widetilde{A} \quad \overline{B}\right] = p + l, \quad \forall\lambda, \Re\lambda \geq 0. \tag{39}$$

□

Lemma 9. *When G is asymptotically stable, $(\widetilde{A}, \widetilde{Q}_{11} - \widetilde{Q}_{12}\widetilde{Q}_{22}^{-1}\widetilde{Q}_{12}^T)$ is detectable if and only if*

$$\operatorname{rank}\begin{bmatrix} \lambda E - A & B_2 \\ Q & 0 \\ 0 & R \end{bmatrix} = n + r, \quad \forall\lambda, \Re\lambda \geq 0. \tag{40}$$

Proof. From (8) and (11),

$$\operatorname{rank}\begin{bmatrix} \lambda E - A & -B_2 \\ C & 0 \\ 0 & D \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \lambda I_p - \overline{A}_{11} & -\overline{A}_{13} & -\overline{B}_{13} \\ C_{11} & C_{13} & D_{13} \\ C_{21} & 0 & D_{23} \end{bmatrix} + s. \tag{41}$$

When G is asymptotically stable, $\forall\lambda, \Re\lambda \geq 0$, we have

$$\operatorname{rank}\begin{bmatrix} \lambda E - A & -B_2 \\ C & 0 \\ 0 & D \end{bmatrix} = \operatorname{rank}\begin{bmatrix} \lambda I_{p+l} - \overline{A} & -\overline{B} \\ \widetilde{Q}_{11} & \widetilde{Q}_{12} \\ \widetilde{Q}_{12}^T & \widetilde{Q}_{22} \end{bmatrix} + s - l. \tag{42}$$

Since $\widetilde{Q}_{22}$ is nonsingular, an easy computation shows that (40) is equivalent to

$$\operatorname{rank}\begin{bmatrix} \lambda I_{p+l} - \widetilde{A} \\ \widetilde{Q}_{11} - \widetilde{Q}_{12}\widetilde{Q}_{22}^{-1}\widetilde{Q}_{12}^T \end{bmatrix} = p + l, \quad \forall\lambda, \Re\lambda \geq 0. \tag{43}$$

□

Remark 10. In the case when (E, A) is regular and $R > 0$, (40) equivalently implies that (E, A, Q) is R-detectable.

Definition 11 (see [19]). The finite s's satisfying $\det(sE - A) = 0$ are called finite poles for the singular system $E\dot{x} = Ax$.

In the following, we give the conclusion concerning the problem $(\mathbb{P})$.

Theorem 12. *Assume that G is asymptotically stable and the rank equalities* (10), (25), (36), *and* (40) *hold, then there exists a unique FI optimal control-state pair for* $(\mathbb{P})$. *Under the optimal control-state pair, the finite poles of resulting system all are located on the left-half complex plane and the optimal value is*

$$J^* = \frac{1}{2}\begin{bmatrix} x_0 \\ w_0 \end{bmatrix}^T \begin{bmatrix} M_{11}^T P_{11} M_{11} & M_{11}^T P_{12} \\ P_{12}^T M_{11} & P_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ w_0 \end{bmatrix}, \tag{44}$$

where $P = (P_{ij})_{2\times 2}$ is the unique positive semidefinite solution of the Riccati equation (35) *and $P_{11} \in R^{p\times p}$.*

Proof. From Lemmas 8 and 9, the problem $\mathbb{P}_2$ has a unique solution given by (33) and (34) and every eigenvalue of $\widetilde{A} - \overline{B}\widetilde{Q}_{22}^{-1}\widetilde{Q}_{12}^T - \overline{B}\widetilde{Q}_{22}^{-1}\overline{B}^T P$ has negative real part.

In accord with (13), (14), (17), (33), and (34), it follows that the unique optimal control-state pair (u^*, x^*) of $(\mathbb{P})$ is

$$\begin{aligned} x^* &= N_1 \begin{bmatrix} I_p & 0 \\ 0 & N_2 \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} I_p & 0 \\ \overline{B}_{23}\left[0 \quad I_{r-s+d}\right] Q_1 - \overline{A}_{21} & \overline{B}_{23}\left[0 \quad I_{r-s+d}\right] Q_2 - \overline{B}_{21} \\ -\left[I_{n-p-d} \quad 0\right] Q_1 & -\left[I_{n-p-d} \quad 0\right] Q_2 \end{bmatrix} \\ &\quad \times \begin{bmatrix} x_1^* \\ w \end{bmatrix}, \\ u^* &= N_3 \begin{bmatrix} -\overline{A}_{31} & -\overline{B}_{31} \\ -\left[0 \quad I_{r-s+d}\right] Q_1 & -\left[0 \quad I_{r-s+d}\right] Q_2 \end{bmatrix} \begin{bmatrix} x_1^* \\ w \end{bmatrix}, \end{aligned} \tag{45}$$

and x_1^* satisfies the equation

$$\dot{x}_1 = \left(\overline{A}_{11} - \overline{B}_{12} Q_1\right) x_1 + \left(\overline{B}_{11} - \overline{B}_{12} Q_2\right) w, \tag{46}$$

where $Q_1 = \overline{Q}_{33}^{-1}(\overline{Q}_{13}^T + \overline{B}_{12}^T P_{11})$, $Q_2 = \overline{Q}_{33}^{-1}(\overline{Q}_{23}^T + \overline{B}_{12}^T P_{12})$.

Since the matrix $\widetilde{A} - \overline{B}\widetilde{Q}_{22}^{-1}\overline{B}^T P$ is asymptotically stable, the matrix $\overline{A}_{11} - \overline{B}_{12} Q_1$ is asymptotically stable, too. Substituting (45) into (1) leads to (46), so, under the control-state pair (u^*, x^*), the finite poles of resulting system all are the eigenvalues of the matrix $\overline{A}_{11} - \overline{B}_{12} Q_1$. Hence, the finite poles of resulting system all are located on the left-half complex plane.

It is obvious that (44) holds from $J_2^* = v^T(0) P v(0)/2$. □

3.2. Solution of the Problem $(\mathbb{P}')$. When G has an eigenvalue locating on imaginary axis, the Riccati equation (35) has no unique positive semidefinite solution, thereby solving the problem $(\mathbb{P}')$ cannot use the same method as the problem $(\mathbb{P})$.

Theorem 13. *Assume that G is stable but not asymptotically stable and the rank equalities* (10), (25), (36), *and* (40) *hold; then there exists a unique FI optimal control-state pair for* $(\mathbb{P}')$. *Under the optimal control-state pair, the finite poles of resulting system all are located on the left-half complex plane and the optimal value is*

$$J'^* = \lim_{t_f \to \infty} \frac{1}{2t_f} \int_0^{t_f} w^T Q_3 w \, dt, \tag{47}$$

where $Q_3 = P_2^T(\overline{B}_{11} - \overline{B}_{12}\overline{Q}_{33}^{-1}\overline{Q}_{23}^T) + (\overline{B}_{11} - \overline{B}_{12}\overline{Q}_{33}^{-1}\overline{Q}_{23}^T)^T P_2 - P_2^T\overline{B}_{12}\overline{Q}_{33}^{-1}\overline{B}_{12}^T P_2 + \overline{Q}_{22} - \overline{Q}_{23}\overline{Q}_{33}^{-1}\overline{Q}_{23}^T$, P_2 *is the unique solution of the Sylvester equation*

$$P_2 G + \left(\overline{A}_{11} - \overline{B}_{12}Q_4\right)^T P_2 + P_1\left(\overline{B}_{11} - \overline{B}_{12}\overline{Q}_{33}^{-1}\overline{Q}_{23}^T\right) + Q_5 = 0, \tag{48}$$

and P_1 *is the unique positive semidefinite solution of the Riccati equation*

$$P_1\widetilde{A}_{11} + \widetilde{A}_{11}^T P_1 - P_1\overline{B}_{12}\overline{Q}_{33}^{-1}\overline{B}_{12}^T P_1 + Q_6 = 0, \tag{49}$$

with $\widetilde{A}_{11} = \overline{A}_{11} - \overline{B}_{12}\overline{Q}_{33}^{-1}\overline{Q}_{13}^T$, $Q_4 = \overline{Q}_{33}^{-1}(\overline{Q}_{13}^T + \overline{B}_{12}^T P_1)$, $Q_5 = \overline{Q}_{12} - \overline{Q}_{13}\overline{Q}_{33}^{-1}\overline{Q}_{23}^T$, *and* $Q_6 = \overline{Q}_{11} - \overline{Q}_{13}\overline{Q}_{33}^{-1}\overline{Q}_{13}^T$.

Proof. Consider the problem $\mathbb{P}_1'$. It follows that $w(t) = e^{Gt}w(0)$ from $\dot{w} = Gw$.

Construct the Hamilton function

$$\begin{aligned} H(x_1, z, \xi) = {} & \frac{\left[x_1^T, \left(e^{Gt}w(0)\right)^T, z^T\right]\overline{Q}\left[x_1^T, \left(e^{Gt}w(0)\right)^T, z^T\right]^T}{2} \\ & + \xi^T\left(\overline{A}_{11}x_1 + \overline{B}_{11}e^{Gt}w(0) + \overline{B}_{12}z\right), \end{aligned} \tag{50}$$

and then from $\partial H/\partial z = 0$,

$$z^* = -\overline{Q}_{33}^{-1}\left(\overline{Q}_{13}^T x_1 + \overline{Q}_{23}^T w + \overline{B}_{12}^T\xi\right). \tag{51}$$

Since

$$\frac{\partial^2 H}{\partial z^2} = \overline{Q}_{33} > 0, \tag{52}$$

the Hamilton function $H(x_1, z, \xi)$ achieves minimum at z^*.

By (20) and (51), the two-point boundary value problem is

$$\begin{aligned} \dot{x}_1 &= \widetilde{A}_{11}x_1 + \left(\overline{B}_{11} - \overline{B}_{12}\overline{Q}_{33}^{-1}\overline{Q}_{23}^T\right)w - \overline{B}_{12}\overline{Q}_{33}^{-1}\overline{B}_{12}^T\xi, \\ \dot{\xi} &= -\frac{\partial H}{\partial x_1} = -Q_6x_1 - Q_5w - \widetilde{A}_{11}^T\xi, \\ x_1(0) &= M_{11}x_0, \qquad \xi(\infty) = 0. \end{aligned} \tag{53}$$

Let

$$\xi = P_1x_1 + P_2w. \tag{54}$$

Then

$$\begin{aligned} \dot{\xi} &= P_1\dot{x}_1 + P_2\dot{w} \\ &= P_1\left(\overline{A}_{11} - \overline{B}_{12}Q_4\right)x_1 + \left(P_1\left(\overline{B}_{11} - \overline{B}_{12}Q_7\right) + P_2G\right)w, \end{aligned} \tag{55}$$

where $Q_7 = \overline{Q}_{33}^{-1}(\overline{Q}_{23}^T + \overline{B}_{12}^T P_2)$.

On the other hand, by the second equation of (53),

$$\dot{\xi} = -\left(Q_6 + \widetilde{A}_{11}^T P_1\right)x_1 - \left(Q_5 + \widetilde{A}_{11}^T P_2\right)w. \tag{56}$$

Combine (55), (56), and the randomicity of x_1 and w; it follows that P_1 and P_2 satisfy (49) and (48), respectively.

It is obvious that $(\widetilde{A}_{11}, \overline{B}_{12})$ is stabilizable if and only if (36) holds, and $(\widetilde{A}_{11}, Q_6)$ is detectable if and only if (40) holds. So when (36) and (40) hold, the Riccati equation (49) has a unique positive semidefinite solution P_1 such that $\overline{A}_{11} - \overline{B}_{12}Q_4$ is asymptotically stable. Since G is stable,

$$\begin{gathered} \lambda_i\left(\overline{A}_{11} - \overline{B}_{12}Q_4\right) + \lambda_j(G) \neq 0, \\ i = 1, 2, \ldots, p, \quad j = 1, 2, \ldots, l, \end{gathered} \tag{57}$$

where $\lambda_i(\overline{A}_{11} - \overline{B}_{12}Q_4)$ and $\lambda_j(G)$ are eigenvalues of $\overline{A}_{11} - \overline{B}_{12}Q_4$ and G, respectively. Therefore, the Sylvester equation (48) has a unique solution P_2 ([27]).

In accord with (13), (14), (17), (51), (53), and (54), it follows that the unique optimal control-state pair (u^*, x^*) of $\left(\mathbb{P}'\right)$ is

$$\begin{aligned} x^* &= N_1\begin{bmatrix} I_p & 0 \\ 0 & N_2 \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} I_p & 0 \\ \overline{B}_{23}\left[0 \;\; I_{r-s+d}\right]Q_4 - \overline{A}_{21} - \left[I_{n-p-d} \;\; 0\right]Q_4 & \overline{B}_{23}\left[0 \;\; I_{r-s+d}\right]Q_7 - \overline{B}_{21} - \left[I_{n-p-d} \;\; 0\right]Q_7 \end{bmatrix} \\ &\quad \times \begin{bmatrix} x_1^* \\ w \end{bmatrix}, \\ u^* &= N_3\begin{bmatrix} -\overline{A}_{31} & -\overline{B}_{31} \\ -\left[0 \;\; I_{r-s+d}\right]Q_4 & -\left[0 \;\; I_{r-s+d}\right]Q_7 \end{bmatrix}\begin{bmatrix} x_1^* \\ w \end{bmatrix}, \end{aligned} \tag{58}$$

and x_1^* satisfies the equation

$$\dot{x}_1 = \left(\overline{A}_{11} - \overline{B}_{12}Q_4\right)x_1 + \left(\overline{B}_{11} - \overline{B}_{12}Q_7\right)w. \tag{59}$$

Obviously, under the control-state pair (u^*, x^*), the finite poles of resulting system all are the eigenvalues of the matrix $\overline{A}_{11} - \overline{B}_{12}Q_4$. Hence, the finite poles of resulting system all are located on the left-half complex plane.

By (48), (49), (51), (54), and (59), one can obtain that

$$\begin{aligned} & \left[x_1^{*T}, w^T, z^{*T}\right]\overline{Q}\left[x_1^{*T}, w^T, z^{*T}\right]^T \\ & \quad = -\frac{d}{dt}\left(x_1^{*T}P_1x_1^* + 2x_1^{*T}P_2w\right) + w^TQ_3w, \end{aligned} \tag{60}$$

and then

$$\begin{aligned} J'^* = J_1'^* = {} & \lim_{t_f \to \infty}\frac{1}{2t_f}\left(x_1^{*T}P_1x_1^* + 2x_1^{*T}P_2w\right)\Big|_{t_f}^{0} \\ & + \lim_{t_f \to \infty}\frac{1}{2t_f}\int_0^{t_f} w^TQ_3w\,dt, \end{aligned} \tag{61}$$

which indicates that (47) holds. □

4. A Simulation Example

In this section, we give a simple example to illuminate the design method of FI control law and demonstrate its feasibility.

Consider the irregular singular system (1) and exosystem (2), where

$$E=\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix},\qquad A=\begin{bmatrix}1&2&1\\2&1&0\\0&0&0\end{bmatrix},\qquad B_1=\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix},$$
$$B_2=\begin{bmatrix}10&5\\2&9\\6&8\end{bmatrix},\qquad x_0=[0,0,0]^T,\qquad w_0=[1,0]^T. \tag{62}$$

In the performance indices (3) and (5), we choose

$$Q=\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix},\qquad R=\begin{bmatrix}0&0\\0&1\end{bmatrix}. \tag{63}$$

Obviously, $d=1$, $s=2$, and we can choose $q_1=2$ and $q_2=1$. Let

$$\begin{gathered}M_1=I_3,\qquad M_2=-2,\qquad M_3=-1,\\ M_4=-\frac{1}{3},\qquad M_5=[0,-1]^T,\qquad M_6=\left[0,\frac{1}{3}\right]^T,\\ M_7=0,\qquad M_8=I_2,\qquad M_9=I_2,\\ N_1=I_3,\qquad N_2=I_2,\qquad N_3=\begin{bmatrix}\frac{1}{6}&-\frac{4}{3}\\0&1\end{bmatrix},\end{gathered} \tag{64}$$

and then the system (1) is r.s.e to the following system:

$$\begin{gathered}\dot{x}_1=-3x_1+x_3+\begin{bmatrix}1&-2\end{bmatrix}w-21u_2,\ x_1(0)=0,\\ 0=2x_1+x_2+\begin{bmatrix}0&1\end{bmatrix}w+\frac{19}{3}u_2,\\ 0=u_1,\end{gathered} \tag{65}$$

$$y=\begin{bmatrix}1\\-2\\0\end{bmatrix}x_1+\begin{bmatrix}1\\0\\0\end{bmatrix}x_3+\begin{bmatrix}0&0\\0&-1\\0&0\end{bmatrix}w+\begin{bmatrix}0\\-\frac{19}{3}\\1\end{bmatrix}u_2,$$

under the transformation (13).

(i) Let $G=\begin{bmatrix}-0.1&1\\-1&-0.2\end{bmatrix}$; then $w(t)$ is damped. We consider the performance index (3). The unique positive semidefinite solution of the Riccati equation (35) is

$$P=\begin{bmatrix}0.4401&0.0750&0.2308\\0.0750&0.0175&0.0403\\0.2308&0.0403&0.1259\end{bmatrix}. \tag{66}$$

The simulation results are displayed in Figure 1 and the optimal performance index is $J=0.008752$.

(ii) Let $G=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$; then $w(t)$ is sinusoidal perturbation. We consider the performance index (5). The unique positive semidefinite solution of the Riccati equation (49) is $P_1=0.4401$, and the unique solution of the Sylvester equation (48) is $P_2=[0.0719,0.2468]$. The simulation results are displayed in Figure 2 and the optimal performance index is $J=0.0007377$.

5. Conclusion

In this paper the singular LQ problem for irregular singular systems with persistent disturbances has been investigated. By restricted system equivalence transformation, we transformed the singular LQ problem for irregular singular systems with persistent disturbances to the optimal problem for standard state space systems. Consequently, based on optimization theory for standard state space systems, we have derived FI optimal control-state pair under some matrix rank equality conditions by solving an algebraic Riccati equation and a Sylvester equation. Under the optimal control-state pair, the finite poles of resulting system are all located on the left-half complex plane.

The significance of the paper, we think, can be summarized as follows: (1) to our knowledge, it seems that the present paper is the first to apply restricted system equivalence transformation to decompose system state into free state and restricted state and decompose input into free input and forced input; (2) it seems that the present paper is the first to apply the full information feedback (FI) method to the singular LQ problem for irregular singular systems with persistent disturbances; (3) all the conditions adopted in this paper are unitedly described by matrix rank equalities; (4) the optimal performance indices are formulated explicitly.

However, there are many problems unsolved about the singular LQ problem of the irregular singular systems. For example, in this paper only the case where the performance index is nonnegative was treated, and the indefinite LQ problem for irregular singular systems with persistent disturbances has not been involved. More importantly, in the controller design, we need to solve an algebraic Riccati equation, which is still a challenge. Therefore, we think that the significance of the paper exists in theory more than in practice.

Appendix

The Proof of Lemma 3

First of all, we prove that system (1) is impulse controllable if and only if (9) holds.

Necessity. By the transformation $x=N_1[\overline{x}_1^T,\overline{x}_2^T]^T$, the system (1) is r.s.e to the following system:

$$\begin{gathered}\dot{\overline{x}}_1=A_{11}\overline{x}_1+A_{12}\overline{x}_2+B_{11}w+B_{21}u,\quad \overline{x}_1(0)=M_{11}x_0,\\ 0=A_{21}\overline{x}_1+A_{22}\overline{x}_2+B_{12}w+B_{22}u,\end{gathered} \tag{A.1}$$

where $M_{11}=[I_p,0]M_1$.

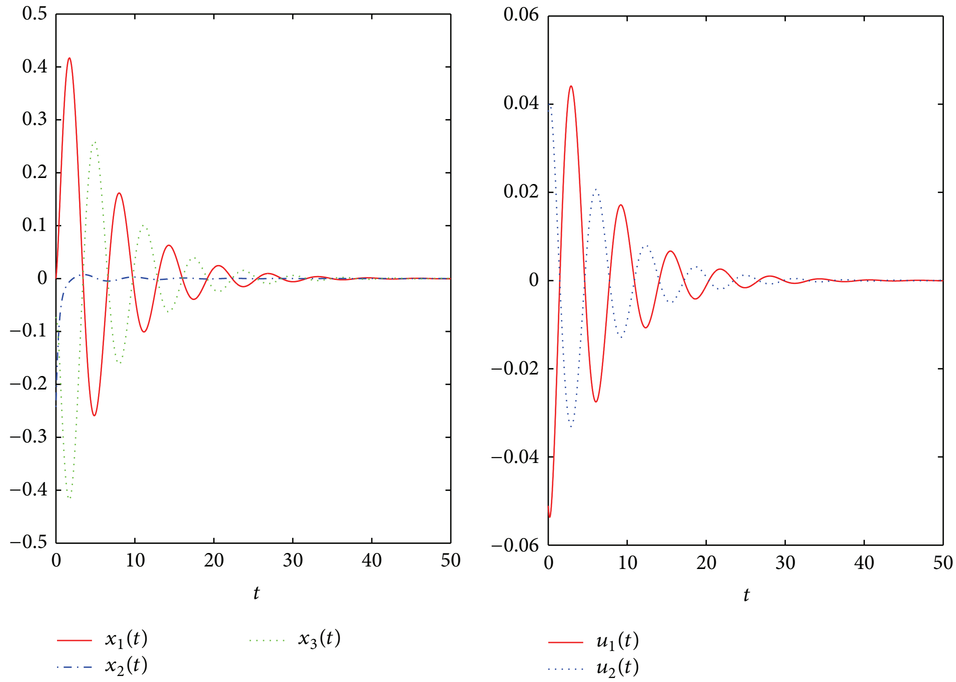

Figure 1: Curves of the state and control variables when exosystem is asymptotically stable.

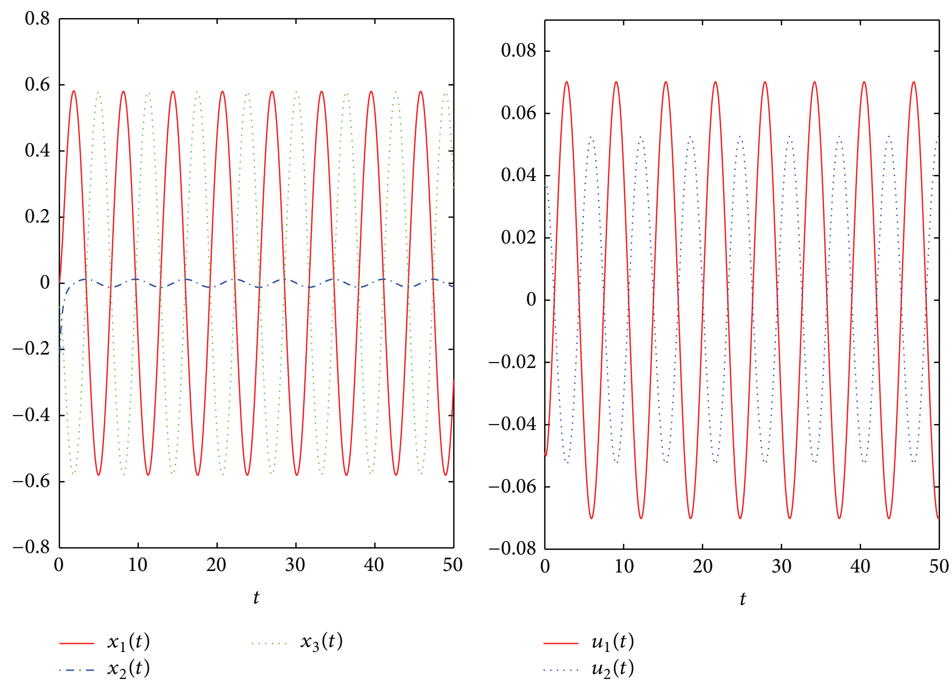

Figure 2: Curves of the state and control variables when exosystem is not asymptotically stable.

Premultiplying the second equation of (A.1) by v^T, where $v \in \operatorname{Ker}[A_{22}, B_{22}]^T$, we have that

$$v^T [A_{21}, B_{12}] [\overline{x}_1^T, w^T]^T = 0. \tag{A.2}$$

From the randomicity of $\overline{x}_1$ and w, it follows that $v^T[A_{21}, B_{12}] = 0$, which implies that $\operatorname{Ker}[A_{22}, B_{22}]^T \subset \operatorname{Ker}[A_{21}, B_{12}]^T$, so (9) holds.

Sufficiency. When (9) holds, there exists a matrix $\Phi \in R^{(n-p+r)\times(p+l)}$ such that $[A_{21}, B_{12}] = [A_{22}, B_{22}]\Phi$. Denote $\Phi =$

$(\Phi_{ij})_{2\times 2}$, $\overline{x}_2 = -\Phi_{11}\overline{x}_1 - \Phi_{12}w$ and $u = -\Phi_{21}\overline{x}_1 - \Phi_{22}w$; then first equation of (A.1) is changed to

$$\dot{\overline{x}}_1 = \left(A_{11} - A_{12}\Phi_{11} - B_{21}\Phi_{21}\right)\overline{x}_1 + \left(B_{11} - A_{12}\Phi_{12} - B_{21}\Phi_{22}\right)w, \qquad \overline{x}_1(0) = M_{11}x_0. \tag{A.3}$$

Obviously, for every initial condition and disturbance governed by the system (2) there exists a smooth solution $\overline{x}_1$ of (A.3). Then $(u = -\Phi_{21}\overline{x}_1 - \Phi_{22}w, \overline{x}_1, \overline{x}_2 = -\Phi_{11}\overline{x}_1 - \Phi_{12}w)$ is the smooth control-state pair of system (A.1), which finishes the proof of sufficiency.

From (8), we can prove that (9) is equivalent to (10) by easy computation.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work was supported by Natural Science Foundation of Zhejiang Province under Grant Y1110036 and Research Project of Zhejiang Province Education Department under Grant Y201018827.

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