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Research Article

Robust Control for Uncertain Linear System Subject to Input Saturation

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A robust control scheme using composite nonlinear feedback (CNF) technology is proposed to improve tracking control performance for the uncertain linear system with input saturation and unknown external disturbances. A disturbance observer is presented to estimate the unknown disturbance generated by a linear exogenous system. The designed gain matrix of the disturbance observer is determined by solving linear matrix inequalities (LMIs). Based on the output of the designed disturbance observer, a robust CNF controller including a linear feedback control item and a nonlinear item is developed to follow the desired tracking signals. The linear feedback controller is designed using LMIs and the stability of the closed-loop system is proved via rigorous Lyapunov analysis. Finally, the extensive simulation results are presented to illustrate the effectiveness of the proposed control scheme.

1. Introduction

As is well known, almost all practical control systems have limitations on the amplitudes or rates of the control input [1]. Therefore, the input saturation which can cause the nonlinearity usually appears in most of the physical systems in our real life, such as aircraft, robot [2, 3], and industry control systems [4–6]. The input saturation problem is of great importance because it may lead to performance degradation and even destroy the stability of the control systems if they are ignored in the process of controller design [7, 8]. In general, it is hard to overcome the effect of input saturation through traditional linear control technologies because of the nonlinear characteristic of input saturation. Meanwhile, the linear system usually possesses unmodelled dynamics, modeling error, system parameter perturbations, and other uncertainties [9]. Generally speaking, the control performance of linear systems is severely affected by uncertainties. Thus, the task that designs high performance feedback control schemes for systems with input saturation and parametric uncertainties is theoretically challenging and critical for practical applications [10, 11].

Over the past years, several research methods on the input saturation problem have been reported in the literature, for example, antiwindup schemes in [12, 13], predictive control in [14, 15], positively invariant sets method in [16, 17], low gain technology in [18, 19], variable structure control in [20, 21], and adaptive fuzzy control in [22–24]. Among these, a composite nonlinear feedback (CNF) control scheme, as an effective method to solve this problem, has been exclusively studied. The CNF method was proposed for a class of second-order linear systems in [25]. Then, a CNF control technique was developed for general single-input single-output (SISO) systems with measurement feedback and successfully applied to a hard disk drive (HDD) servo system in [26]. In [27], the design and implementation of a dual-stage actuated HDD servo system were studied via composite nonlinear control approach. Inspired by these works, the CNF control technique was extended to a general multi-input multioutput (MIMO) system under state feedback in [28] and a class of cascade nonlinear systems with input saturation in [29]. However, in the process of control design, the external disturbance has not been explicitly considered in above-mentioned literature. Considering the transient tracking performance and external disturbances, disturbance estimator was introduced into CNF control framework to propose a control strategy for servo system subject to actuator saturation and disturbances which was assumed to be an unknown constant and applied to discrete-time systems in [30] and continuous systems in [31], respectively. In [32], the CNF control technique was extended to design a robust flight control system for an unmanned helicopter system with a wind gust disturbance. From above analysis, most of disturbances were assumed to be constant in these research works and parametric uncertainties are not explicitly considered in the control design. However, disturbances including external disturbances and parameter perturbations widely exist in practical systems, such as aircrafts, missiles, satellites, and many other systems [33, 34]. Thus, to solve this problem, disturbance observer-based control (DOBC) as a promising approach to handle system disturbances and to improve robustness can be employed.

The disturbance observer as an effective method which is extensively used to approximate unknown external disturbance has been attracting increasing attention [35, 36]. A two-stage design procedure was developed to improve disturbance attenuation ability of current linear/nonlinear controllers, where the disturbance observer design is separated from the controller design in [37]. In [38], a new DOBC was presented for a class of MIMO nonlinear systems to attenuate and reject the disturbances. A novel fuzzy-observer-design approach was presented for Takagi-Sugeno fuzzy models with unknown output disturbances in [39], where the disturbance was supposed to be an auxiliary state vector by an augmented fuzzy descriptor model and can be in any form. A disturbance observer-based multivariable control (DOMC) scheme was developed to control a two-input-two-output ball mill grinding circuit in [40]. Various control schemes based on the output of the disturbance observer can also be exclusively studied. In [41], a novel type of control scheme combining the DOBC with H_∞ control was proposed for a class of complex continuous models with disturbances. In [42], a sliding mode control (SMC) scheme was developed for a class of nonlinear systems based on disturbance observers. In [43], a nonlinear output disturbance observer based on the model of ocean wave was proposed to eliminate the disturbance of depth signal. A new DOBC technique for mismatched disturbances/uncertainties was presented in [44]. However, these research results did not consider the system subject to input saturation.

In [45], the system with input saturation and external disturbance has been studied, but the parametric uncertainties problem has not been considered, and the tracking signals are assumed to be constant. Thus, this paper is motivated by the robust control for the uncertain system with parametric uncertainties, input saturation, and external disturbance. A robust control design scheme based on disturbance observer will be proposed for the uncertain system subject to input saturation and unknown external disturbance. A disturbance observer is developed to estimate disturbances generated by an exogenous system via solving linear matrix inequality (LMI). Then, based on the output of the disturbance observer, a robust control scheme is proposed and the stability of the closed-loop system is proved by the Lyapunov function method. The outline of this paper is as follows. Section 2 gives the description and formulation of the problem. In Section 3, the design of disturbances observer is presented. In Section 4, the robust CNF control method will be described and the developed control method is applied to design a tracking controller for a control system. Finally, simulation results will be given in Section 5 to illustrate the effectiveness of the proposed control scheme, followed by drawing some concluding remarks in Section 6.

2. Problem Formulation

Considering a class of linear systems with parametric uncertainties, unknown disturbances and input saturation are described as

$$\begin{aligned} \dot{x} &= (A + \Delta A)\, x + (B_1 + \Delta B_1)\, \mathrm{sat}(u) + B_2 d, \\ y &= C_1 x, \\ z &= C_2 x + D_2 d, \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$, $z \in R^m$, and $d \in R^l$ are the state, control input, measurement output, controlled output, and external disturbance of system, respectively. d is norm bounded by a nonnegative scalar η, that is, $\|d\| \le \eta$, and $A, B_1, B_2, C_1, C_2, D_2$ are appropriate dimensional constant matrices. The function $\mathrm{sat}(\cdot)$ represents the input saturation of system defined as

$$\mathrm{sat}(u_i) = \mathrm{sign}(u_i) \min\{u_{\max i}, |u_i|\}, \quad i = 1, 2, \ldots, m, \tag{2}$$

where $u_{\max i}$ represents the saturation level of the ith input and is known. ΔA and ΔB_1 representing the parametric uncertainties of system (1) are assumed to be in the following form:

$$\begin{bmatrix} \Delta A & \Delta B_1 \end{bmatrix} = DF(t) \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \tag{3}$$

where D, E_1, and E_2 are appropriate dimensional constant matrices. $F(t)$ is an unknown, real, and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$F^T(t) F(t) \le I, \quad \forall t. \tag{4}$$

To continue the composite nonlinear feedback control design, the following assumptions and the lemma for the given system (1) are required [31].

Assumption 1. (A, B_1) is stabilizable.

Assumption 2. (A, C_1) is detectable.

Assumption 3. (A, B_1, C_2) is invertible and has no invariant zero at $s = 0$.

Assumption 4. Control gain matrix B_1 is row full rank.

Lemma 5. *Assume that U and V are vectors or matrices with appropriate dimension; then, for any positive constant α, the following inequality holds:*

$$U^T V + V^T U \le \alpha U^T U + \alpha^{-1} V^T V. \tag{5}$$

Remark 1 (see [31]). Note that Assumption 1 means that there exists state feedback matrix K which satisfies that $A + B_1K$ is an asymptotically stable matrix. Assumption 2 denotes that the states variables can be detected by the output y of system (1). Assumption 3 implies that the matrix $C_2(A + B_1K)^{-1}B_1$ is invertible and will be used in the control design. For the convenience of the robust controller design, Assumption 4 is adopted to avoid the control singularity. Thus, all these assumptions are fairly standard for the tracking control.

In this paper, the control objective is that the robust CNF controller based on disturbance observer will be designed for uncertain systems (1) with input saturation and disturbances such that the closed-loop system is asymptotically stable and the controlled output z can well track the reference signal r.

3. Design of Disturbance Observer

In this section, a disturbance observer is developed to estimate the unknown disturbance of the system (1). To design the robust controller, suppose that the disturbance d is generated by a linear exogenous system [46]:

$$\begin{aligned} \dot{w} &= W_1 w, \\ d &= V_1 w, \end{aligned} \tag{6}$$

where $w \in R^p$, $d \in R^l$, W_1, and V_1 are matrices with corresponding dimensions.

The disturbance observer is formulated as

$$\begin{aligned} \dot{v} &= (W_1 + LB_2V_1)(v - Lx) + L(Ax + B_1 \operatorname{sat}(u)), \\ \widehat{w} &= v - Lx, \\ \widehat{d} &= V_1 \widehat{w}, \end{aligned} \tag{7}$$

where $\widehat{d}$ is the estimate of d and v is the auxiliary design vector of the disturbance observer. $L \in R^{p\times n}$ is a designed gain matrix and will be given by solving linear matrix inequalities (LMIs). The estimation error is defined as $\widetilde{w} = w - \widehat{w}$; based on (6) and (7), it is shown that the error dynamic satisfies

$$\begin{aligned} \dot{\widetilde{w}} &= W_1 w - (W_1 + LB_2V_1)(v - Lx) \\ &\quad - L(Ax + B_1 \operatorname{sat}(u)) + L\dot{x} \\ &= (W_1 + LB_2V_1)\widetilde{w} + L\Delta B_1 \operatorname{sat}(u) + L\Delta Ax. \end{aligned} \tag{8}$$

In this case, it is obvious that the designed observer gain matrix L not only needs to satisfy the desired stability of the disturbance observer (7), that is, $W_1 + LB_2V_1 < 0$, but also achieves robustness performance under the uncertainties $L\Delta B_1 \operatorname{sat}(u)$ and $L\Delta Ax$.

Remark 2 (see [42]). As it is known, a wide class of real engineering disturbances can be presented by this disturbance model such as unknown constant load disturbances and harmonic disturbances. For example, unknown constant disturbance can be presented with $W_1 = 0$ and $V_1 = 1$, and a harmonic disturbance with known frequency ω but unknown phase and magnitude can be represented with

$$W_1 = \begin{bmatrix} 0 & \omega \\ \omega & 0 \end{bmatrix}, \qquad V_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{9}$$

4. Robust CNF Control Design Using the Disturbance Observer

In this section, we will design robust CNF control law using the disturbance observer. The sate feedback robust CNF control law can be designed by the following step-by-step procedure.

Step 1. A linear state feedback control law with a disturbance compensation term is designed as

$$u_L = Kx + K_d w + \Lambda r + \Lambda_r \dot{r}, \tag{10}$$

where K is the designed state feedback matrix, and satisfies that $A + B_1K$ is an asymptotically stable matrix. $K_d w$ is the disturbance compensation term and r is the tracking reference signal differing from previous CNF method in which the tracking reference signal must be constant. Next, Λ is chosen as

$$\Lambda = -\left[C_2(A + B_1K)^{-1}B_1\right]^{-1}. \tag{11}$$

It is apparent that (11) is well defined when Assumption 3 is given. Considering Assumption 4, the matrix K_d is given by

$$\begin{aligned} K_d &= B_1^T\left(B_1B_1^T\right)^{-1}\Lambda_d V_1 W_1, \\ \Lambda_d &= -(A + B_1K)^{-1}B_2. \end{aligned} \tag{12}$$

At the same time, Λ_r is chosen as

$$\Lambda_r = B_1^T\left(B_1B_1^T\right)^{-1}\Lambda_e r, \tag{13}$$

where

$$\Lambda_e = -(A + B_1K)^{-1}B_1\Lambda. \tag{14}$$

Step 2. The nonlinear feedback law u_N is constructed as

$$u_N = \begin{cases} -\dfrac{B_1^T\left(B_1B_1^T\right)^{-1}P_2^{-1}(x - x_e)K_a}{\|x - x_e\|^2}, & \|x - x_e\| \geq \varepsilon \\ 0, & \|x - x_e\| < \varepsilon, \end{cases} \tag{15}$$

where $P_2 > 0$ is a positive-definite matrix, K_a is a designed matrix, ε is a minimal positive design constant, and x_e is defined as

$$x_e = \Lambda_e r + \Lambda_d d. \tag{16}$$

In (15), K_a is designed as

$$K_a = \frac{1}{2q}\left(\left(\alpha_5^{-1} + \alpha_6^{-1}\right)u_{\max}^2 E_2^T E_2 + \left(\alpha_4^{-1} + \alpha_4^{-1}\right)x_e^T E_1^T E_1 x_e\right), \tag{17}$$

where α_4, α_5, and α_6 will be defined in Theorem 6.

Step 3. The linear and nonlinear feedback laws designed in above steps are now combined as a robust CNF controller:

$$u = u_L + u_N = Kx + K_d w + \Lambda r + \Lambda_r \dot{r} + u_N. \tag{18}$$

If the disturbance is replaced by the estimated one, the CNF controller is given by

$$u = Kx + K_d \widehat{w} + \Lambda r + \Lambda_r \dot{r} + \widehat{u}_N, \tag{19}$$

$$\widehat{u}_N = \begin{cases} -\dfrac{B_1^T \left(B_1 B_1^T\right)^{-1} P_2^{-1} \left(x - \widehat{x}_e\right) K_a}{\left\|x - \widehat{x}_e\right\|^2}, & \left\|x - \widehat{x}_e\right\| \geq \epsilon \\ 0, & \left\|x - \widehat{x}_e\right\| < \epsilon, \end{cases} \tag{20}$$

where $\widehat{x}_e = \Lambda_e r + \Lambda_d \widehat{d}$ is the estimation of x_e, and K_a is rewritten as

$$K_a = \frac{1}{2q}\left(\left(\alpha_5^{-1} + \alpha_6^{-1}\right) u_{\max}^2 E_2^T E_2 + \left(\alpha_2^{-1} + \alpha_4^{-1}\right) \widehat{x}_e^T E_1^T E_1 \widehat{x}_e\right). \tag{21}$$

This completes the robust CNF controller design procedure.

The main objective of the designed robust CNF controller is to ensure not only the asymptotical stability of the closed-loop system and the disturbance observer estimate error, but also that the controlled output z can track the reference signal r as smooth as possible. Thus, the stability and tracking condition is given in the following theorem.

Theorem 6. *Considering the given uncertain system* (1) *with external disturbance and input saturation and provided that the following conditions are satisfied:*

(1) *for any $\tau \in (0, 1)$, let $\rho_\tau > 0$ be the largest positive scalar such that for all $x \in X_\tau$, the following property holds:*

$$\left\|\left[K, K_1\right]\begin{pmatrix} x \\ w \end{pmatrix}\right\| \leq (1 - \tau)\, u_{\max}, \tag{22}$$

where

$$X_\tau = \left\{\begin{pmatrix} x \\ w \end{pmatrix} \mid \begin{pmatrix} x \\ w \end{pmatrix}^T \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}\begin{pmatrix} x \\ w \end{pmatrix} \leq \rho_\tau\right\} \tag{23}$$

and $K_1 = -K\Lambda_d V_1$, $P_1, P_2 > 0$;

(2) *the initial conditions x_0 satisfy*

$$\begin{pmatrix} x_0 - \widehat{x}_{e0} \\ w_0 - \widehat{w}_0 \end{pmatrix} \in X_\tau; \tag{24}$$

(3) *the reference signal r satisfies*

$$\|Mr\| + \left\|\Lambda_r \dot{r}\right\| \leq \tau u_{\max} - \eta \left\|K_2\right\| - \left\|K_d \widehat{w}\right\|, \tag{25}$$

where

$$\begin{aligned} M &= K\Lambda_e + \Lambda, \\ K_2 &= K\Lambda_d; \end{aligned} \tag{26}$$

(4) *for given positive constants α_2, α_4, and α_6, there exist constants $\alpha_1 > 0$, $\alpha_3 > 0$, and $\alpha_5 > 0$ and some matrices $X \in R^{n\times n} > 0$, $Y \in R^{m\times n}$, $P_1 \in R^{m\times m} > 0$, and $T \in R^{m\times n}$ such that the following LMI holds:*

$$\begin{pmatrix} \Gamma_{11} & \Gamma_{12} & XE_1^T & XE_1^T & 0 \\ \Gamma_{12}^T & \Gamma_{22} & 0 & 0 & TD \\ E_1 X & 0 & -\alpha_1 I & 0 & 0 \\ E_1 X & 0 & 0 & -\alpha_3 I & 0 \\ 0 & D^T T^T & 0 & 0 & -\left(\alpha_1 + \alpha_3 + \alpha_4\right)^{-1} I \end{pmatrix} < 0, \tag{27}$$

where

$$\begin{aligned} \Gamma_{11} &= AX + XA^T + B_1 Y + YB_1^T + \left(\alpha_1 + \alpha_2 + \alpha_5\right) DD^T, \\ \Gamma_{12} &= B_2 V_1, \\ \Gamma_{22} &= P_1 W_1 + W_1^T P_1 + TB_2 V_1 + V_1^T B_2^T T^T, \end{aligned} \tag{28}$$

then the disturbance observer approximation error is asymptotically stable and the controlled output z can track the reference r asymptotically under the developed CNF control law (19)*, where $K = YX^{-1}$, $L = P_1^{-1} T$.*

Proof. Let us define a new state variable $\overline{x} = x - \widehat{x}_e$, and $\widetilde{d} = d - \widehat{d}$. Invoking the definition of $\widetilde{w}$, the CNF control law of (19) can be rewritten as

$$\begin{aligned} u = {} & \left[K, -K\Lambda_d V_1\right]\begin{pmatrix} \overline{x} \\ \widetilde{w} \end{pmatrix} \\ & + \left[K\Lambda_e + \Lambda, K\Lambda_d\right]\begin{pmatrix} r \\ d \end{pmatrix} + K_d \widehat{w} + \Lambda_r \dot{r} + \widehat{u}_N. \end{aligned} \tag{29}$$

Considering (22) and (26), we obtain

$$u = \left[K, K_1\right]\begin{pmatrix} \overline{x} \\ \widetilde{w} \end{pmatrix} + \left[M, K_2\right]\begin{pmatrix} r \\ d \end{pmatrix} + K_d \widehat{w} + \Lambda_r \dot{r} + \widehat{u}_N. \tag{30}$$

Invoking (1) and the the definition of variables $\overline{x}$, the time derivative of $\overline{x}$ can be written as

$$\dot{\overline{x}} = \dot{x} - \dot{\widehat{x}}_e = (A + \Delta A)\, x + \left(B_1 + \Delta B_1\right) \operatorname{sat}(u) + B_2 d - \dot{\widehat{x}}_e. \tag{31}$$

Next, we note that

$$\left(A + B_1 K\right) x_e = \left(A + B_1 K\right)\left(\Lambda_e r + \Lambda_d d\right) = -B_1 \Lambda r - B_2 d. \tag{32}$$

According to (32), we have

$$\left(A + B_1 K\right) x_e + B_1 \Lambda r + B_2 d = 0. \tag{33}$$

Based on the the definition of variables $\widehat{x}_e$ and substituting (33) into (31), we obtain

$$\begin{aligned} \dot{\overline{x}} = {} & Ax + B_1 \operatorname{sat}(u) - \left(A + B_1 K\right) x_e - B_1 \Lambda r \\ & - B_1 K_d \widehat{w} - \Lambda_r \dot{r} + \Delta A x + \Delta B_1 \operatorname{sat}(u). \end{aligned} \tag{34}$$

Then, according to $\bar{x} = x - \hat{x}_e$, we have

$$\begin{aligned}\dot{\bar{x}} =& \left(A + B_1K\right)\bar{x} + B_1 \operatorname{sat}(u) + A\hat{x}_e - B_1K\bar{x} - \left(A + B_1K\right)x_e \\ &- B_1\Lambda r - B_1K_d\widetilde{w} - B_1\Lambda_r\dot{r} + \Delta Ax + \Delta B_1 \operatorname{sat}(u) \\ =& \left(A + B_1K\right)\bar{x} + B_1 \operatorname{sat}(u) + \left(A + B_1K\right)\hat{x}_e \\ &- \left(A + B_1K\right)x_e - B_1Kx \\ &- B_1\Lambda r - B_1K_d\widetilde{w} - B_1\Lambda_r\dot{r} + \Delta Ax + \Delta B_1 \operatorname{sat}(u).\end{aligned} \tag{35}$$

Considering the definition of variables $\widetilde{d}$ and $\bar{x}$ and substituting $\hat{x}_e = \Lambda_e r + \Lambda_d\hat{d}$ into (35) yield

$$\begin{aligned}\dot{\bar{x}} =& \left(A + B_1K\right)\bar{x} + B_1 \operatorname{sat}(u) - \left(A + B_1K\right)\Lambda_d\widetilde{d} \\ &+ B_1K\Lambda_d\widetilde{d} - B_1K\bar{x} \\ &- \left(B_1K\Lambda_e r + B_1\Lambda\right)r - B_1K\Lambda_d d - B_1K_d\widetilde{w} - B_1\Lambda_r\dot{r} \\ &+ \Delta Ax + \Delta B_1 \operatorname{sat}(u).\end{aligned} \tag{36}$$

Considering the definition of variables $\bar{x}$, we have

$$\begin{aligned}\dot{\bar{x}} =& \left(A + B_1K + \Delta A\right)\bar{x} + B_1 \operatorname{sat}(u) \\ &- \left(A + B_1K\right)\Lambda_d\widetilde{d} + B_1K\Lambda_d\widetilde{d} \\ &- B_1K\bar{x} - \left(B_1K\Lambda_e r + B_1\Lambda\right)r - B_1K\Lambda_d d \\ &- B_1K_d\widetilde{w} - B_1\Lambda_r\dot{r} - \Lambda_r\dot{r} + \Delta A\hat{x}_e + \Delta B_1 \operatorname{sat}(u) \\ =& \left(A + B_1K + \Delta A\right)\bar{x} - \left(A + B_1K\right)\Lambda_d\widetilde{d} \\ &+ B_1\left(\operatorname{sat}(u) - \left[K, -K\Lambda_d V_1\right]\begin{pmatrix}\bar{x}\\ \widetilde{w}\end{pmatrix}\right. \\ &\left.\qquad - \left[K\Lambda_e + \Lambda, K\Lambda_d\right]\begin{pmatrix}r\\ d\end{pmatrix} - K_d\widetilde{w} - \Lambda_r\dot{r}\right) \\ &+ \Delta A\hat{x}_e + \Delta B_1 \operatorname{sat}(u).\end{aligned} \tag{37}$$

Substituting (11), (22), and (26) into (37) gives

$$\begin{aligned}\dot{\bar{x}} =& \left(A + B_1K + \Delta A\right)\bar{x} + B_2V_1\widetilde{w} \\ &+ B_1\nu + \Delta A\hat{x}_e + \Delta B_1 \operatorname{sat}(u),\end{aligned} \tag{38}$$

where

$$\sigma = \operatorname{sat}(u) - \left[K, K_1\right]\begin{pmatrix}\bar{x}\\ \widetilde{w}\end{pmatrix} - \left[H, K_2\right]\begin{pmatrix}r\\ d\end{pmatrix} - K_d\widetilde{w} - \Lambda_r\dot{r}. \tag{39}$$

Note that for all $\left(\begin{smallmatrix}\bar{x}\\ \widetilde{w}\end{smallmatrix}\right) \in X_\tau$ and $\|Mr\| + \|\Lambda_r\dot{r}\| \le \tau u_{\max} - \eta\|K_2\| - \|K_d\widetilde{w}\|$, we have

$$\begin{aligned}&\left\|\left[K, K_1\right]\begin{pmatrix}\bar{x}\\ \widetilde{w}\end{pmatrix} + \left[M, K_2\right]\begin{pmatrix}r\\ d\end{pmatrix} + K_d\widetilde{w} + \Lambda_r\dot{r}\right\|, \\ &\quad\le \left\|\left[K, K_1\right]\begin{pmatrix}\bar{x}\\ \widetilde{w}\end{pmatrix}\right\| + \|Mr\| + \left\|\Lambda_r\dot{r}\right\| \\ &\qquad + \eta\left\|K_2\right\| + \left\|K_d\widetilde{w}\right\| \le u_{\max}.\end{aligned} \tag{40}$$

Thus, the value of σ can be determined via (30) and (39) for three different values of saturation function:

$$\begin{aligned}&\hat{u}_N < \sigma < 0, \qquad u < -u_{\max}, \\ &\sigma = \hat{u}_N, \qquad |u| \le u_{\max}, \\ &0 < \sigma < \hat{u}_N, \qquad u > u_{\max}.\end{aligned} \tag{41}$$

From above analysis, we can obtain that

$$\sigma = q\hat{u}_N, \tag{42}$$

where $q \in [0, 1]$.

Substituting (42) into system (38), we have

$$\begin{aligned}\dot{\bar{x}} =& \left(A + B_1K + \Delta A\right)\bar{x} + B_2V_1\widetilde{w} \\ &+ qB_1\hat{u}_N + \Delta A\hat{x}_e + \Delta B_1 \operatorname{sat}(u).\end{aligned} \tag{43}$$

Then, the error dynamic equation (8) can be rewritten as

$$\dot{\widetilde{w}} = \left(W_1 + LB_2V_1\right)\widetilde{w} + L\Delta B_1 \operatorname{sat}(u) + L\Delta A\bar{x} + L\Delta A\hat{x}_e. \tag{44}$$

Choose the Lyapunov function as

$$V = \bar{x}^T P_2\bar{x} + \widetilde{w}^T P_1\widetilde{w}. \tag{45}$$

Invoking (43) and (44), the time derivative of V along the trajectory of the system (45) is

$$\begin{aligned}\dot{V} =& \bar{x}^T\left(\left(A + B_1K + \Delta A\right)^T P_2 + P_2\left(A + B_1K + \Delta A\right)\right)\bar{x} \\ &+ 2q\bar{x}^T P_2B_1\hat{u}_N + \bar{x}^T P_2B_2V_1\widetilde{w} + \widetilde{w}^T V_1^T B_2^T P_2\bar{x} \\ &+ \bar{x}^T P_2\Delta A\hat{x}_e + \hat{x}_e^T\Delta A^T P_2\bar{x} \\ &+ \widetilde{w}^T\left(\left(W_1 + LB_2V_1\right)^T P_1 + P_1\left(W_1 + LB_2V_1\right)\right)\widetilde{w} \\ &+ \widetilde{w}^T P_1L\Delta A\bar{x} + \bar{x}^T\Delta A^T L^T P_1\widetilde{w} \\ &+ \widetilde{w}^T P_1L\Delta A\hat{x}_e + \hat{x}_e^T\Delta A^T L^T P_1\widetilde{w} \\ &+ 2\bar{x}^T P_2\Delta B_1 \operatorname{sat}(u) + 2\widetilde{w}^T P_1L\Delta B_1 \operatorname{sat}(u).\end{aligned} \tag{46}$$

Recalling (3), it obtains that

$$\begin{aligned}\dot{V} =& \bar{x}^T\left(\left(A + B_1K\right)^T P_2 + P_2\left(A + B_1K\right)\right)\bar{x} \\ &+ \bar{x}^T\left(E_1^T F^T D^T P_2 + P_2DFE_1\right)\bar{x} \\ &+ 2q\bar{x}^T P_2B_1\hat{u}_N + \bar{x}^T P_2B_2V_1\widetilde{w} + \widetilde{w}^T V_1^T B_2^T P_2\bar{x} \\ &+ \bar{x}^T P_2DFE_1\hat{x}_e + \hat{x}_e^T E_1^T F^T D^T P_2\bar{x} \\ &+ \widetilde{w}^T\left(\left(W_1 + LB_2V_1\right)^T P_1 + P_1\left(W_1 + LB_2V_1\right)\right)\widetilde{w} \\ &+ \widetilde{w}^T P_1LDFE_1\bar{x} + \bar{x}^T E_1^T F^T D^T L^T P_1\widetilde{w} \\ &+ \widetilde{w}^T P_1LDFE_1\hat{x}_e + \hat{x}_e^T E_1^T F^T D^T L^T P_1\widetilde{w} \\ &+ 2\bar{x}^T P_2DFE_2 \operatorname{sat}(u) + 2\widetilde{w}^T P_1LDFE_2 \operatorname{sat}(u).\end{aligned} \tag{47}$$

Using Lemma 5, we have

$$\begin{aligned}
&\overline{x}^T\left(E_1^TF^TD^TP_2+P_2DFE_1\right)\overline{x} \\
&\quad\le \alpha_1\overline{x}^TP_2DD^TP_2\overline{x}+\alpha_1^{-1}\overline{x}^TE_1^TE_1\overline{x}, \\
&\overline{x}^TP_2DFE_1\widehat{x}_e+\widehat{x}_e^TE_1^TF^TD^TP_2 \\
&\quad\le \alpha_2\overline{x}^TP_2DD^TP_2\overline{x}+\alpha_2^{-1}\widehat{x}_e^TE_1^TE_1\widehat{x}_e, \\
&\widetilde{w}^TP_1LDFE_1\overline{x}+\overline{x}^TE_1^TF^TD^TL^TP_1\widetilde{w} \\
&\quad\le \alpha_3\widetilde{w}^TP_1LDD^TL^TP_1\widetilde{w}+\alpha_3^{-1}\overline{x}^TE_1^TE_1\overline{x}, \\
&\widetilde{w}^TP_1LDFE_1\widehat{x}_e+\widehat{x}_e^TE_1^TF^TD^TL^TP_1\widetilde{w} \\
&\quad\le \alpha_4\widetilde{w}^TP_1LDD^TL^TP_1\widetilde{w}+\alpha_4^{-1}\widehat{x}_e^TE_1^TE_1\widehat{x}_e, \\
&2\overline{x}^TP_2DFE_2\,\mathrm{sat}(u) \\
&\quad\le \alpha_5\overline{x}^TP_2DD^TP_2\overline{x}+\alpha_5^{-1}\,\mathrm{sat}(u)^TE_2^TE_2\,\mathrm{sat}(u), \\
&2\widetilde{w}^TP_1LDFE_2\,\mathrm{sat}(u) \\
&\quad\le \alpha_6\widetilde{w}^TP_1LDD^TL^TP_1\widetilde{w}+\alpha_6^{-1}\,\mathrm{sat}(u)^TE_2^TE_2\,\mathrm{sat}(u).
\end{aligned} \tag{48}$$

Substituting (48) into (47) and considering (20) yield

$$\begin{aligned}
\dot{V}\le{}& \overline{x}^T\left((A+B_1K)^TP_2+P_2(A+B_1K)\right)\overline{x} \\
&+\alpha_1\overline{x}^TP_2DD^TP_2\overline{x} \\
&+\alpha_1^{-1}\overline{x}^TE_1^TE_1\overline{x}-2qK_a+\overline{x}^TP_2B_2V_1\widetilde{w}+\widetilde{w}^TV_1^TB_2^TP_2\overline{x} \\
&+\alpha_2\overline{x}^TP_2DD^TP_2\overline{x}+\alpha_2^{-1}\widehat{x}_e^TE_1^TE_1\widehat{x}_e \\
&+\widetilde{w}^T\left((W_1+LB_2V_1)^TP_1+P_1(W_1+LB_2V_1)\right)\widetilde{w} \\
&+\alpha_3\widetilde{w}^TP_1LDD^TL^TP_1\widetilde{w}+\alpha_3^{-1}\overline{x}^TE_1^TE_1\overline{x} \\
&+\alpha_4\widetilde{w}^TP_1LDD^TL^TP_1\widetilde{w}+\alpha_4^{-1}\widehat{x}_e^TE_1^TE_1\widehat{x}_e \\
&+\alpha_5\overline{x}^TP_2DD^TP_2\overline{x}+\alpha_6\widetilde{w}^TP_1LDD^TL^TP_1\widetilde{w} \\
&+\left(\alpha_5^{-1}+\alpha_6^{-1}\right)\mathrm{sat}(u)^TE_2^TE_2\,\mathrm{sat}(u).
\end{aligned} \tag{49}$$

Considering the definition of $\mathrm{sat}(u)$, we have

$$\left\|\mathrm{sat}(u)^TE_2^TE_2\,\mathrm{sat}(u)\right\|\le u_{\max}^2E_2^TE_2. \tag{50}$$

Substituting (21) and (50) into (49) yields

$$\begin{aligned}
\dot{V}\le{}& \overline{x}^T\left((A+B_1K)^TP_2+P_2(A+B_1K)\right)\overline{x} \\
&+(\alpha_1+\alpha_2+\alpha_5)\overline{x}^TP_2DD^TP_2\overline{x}+\left(\alpha_1^{-1}+\alpha_3^{-1}\right)\overline{x}^TE_1^TE_1\overline{x} \\
&+\overline{x}^TP_2B_2V_1\widetilde{w}+\widetilde{w}^TV_1^TB_2^TP_2\overline{x} \\
&+\widetilde{w}^T\left((W_1+LB_2V_1)^TP_1+P_1(W_1+LB_2V_1)\right)\widetilde{w} \\
&+(\alpha_3+\alpha_4+\alpha_6)\widetilde{w}^TP_1LDD^TL^TP_1\widetilde{w}.
\end{aligned} \tag{51}$$

Equation (51) can be rewritten as

$$\dot{V}\le\begin{pmatrix}\overline{x}\\ \widetilde{w}\end{pmatrix}^T\overline{\Gamma}\begin{pmatrix}\overline{x}\\ \widetilde{w}\end{pmatrix}, \tag{52}$$

where

$$\begin{aligned}
\overline{\Gamma}&=\begin{bmatrix}\overline{\Gamma}_{11}&\overline{\Gamma}_{12}\\ \overline{\Gamma}_{12}^T&\overline{\Gamma}_{22}\end{bmatrix}, \\
\overline{\Gamma}_{11}&=(A+B_1K)^TP_2+P_2(A+B_1K) \\
&\quad+(\alpha_1+\alpha_2+\alpha_5)P_2DD^TP_2+\left(\alpha_1^{-1}+\alpha_3^{-1}\right)E_1^TE_1 \\
\overline{\Gamma}_{12}&=P_2B_2V_1, \\
\overline{\Gamma}_{22}&=(W_1+LB_2V_1)^TP_1+P_1(W_1+LB_2V_1) \\
&\quad+(\alpha_3+\alpha_4+\alpha_6)P_1LDD^TL^TP_1.
\end{aligned} \tag{53}$$

Let $P_2^{-1}=X$, $K=YX^{-1}$, and $L=P_1^{-1}T$. Both sides of (53), multiplying by $\mathrm{diag}(P_2^{-1},I)$, yield

$$\widetilde{\Gamma}=\begin{bmatrix}\widetilde{\Gamma}_{11}&\widetilde{\Gamma}_{12}\\ \widetilde{\Gamma}_{12}^T&\widetilde{\Gamma}_{22}\end{bmatrix}, \tag{54}$$

where

$$\begin{aligned}
\widetilde{\Gamma}_{11}&=AX+XA^T+B_1Y+YB_1^T \\
&\quad+(\alpha_1+\alpha_2+\alpha_5)DD^T+\left(\alpha_1^{-1}+\alpha_3^{-1}\right)XE_1^TE_1X, \\
\widetilde{\Gamma}_{12}&=B_2V_1, \\
\widetilde{\Gamma}_{22}&=P_1W_1+W_1^TP_1+TB_2V_1+V_1^TB_2^TT^T \\
&\quad+(\alpha_3+\alpha_4+\alpha_6)TDD^TT^T.
\end{aligned} \tag{55}$$

Equation (51) can be rewritten as

$$\widetilde{\Gamma}=\begin{bmatrix}\Gamma_{11}&\Gamma_{12}\\ \Gamma_{12}^T&\Gamma_{22}\end{bmatrix}+\overline{E}\Delta\overline{F}, \tag{56}$$

where

$$\begin{aligned}
\overline{E}&=\begin{bmatrix}XE_1^T&XE_1^T&0\\ 0&0&TD\end{bmatrix}, \\
\Delta&=\begin{bmatrix}\alpha_1^{-1}I&0&0\\ 0&\alpha_3^{-1}I&0\\ 0&0&(\alpha_3+\alpha_4+\alpha_6)I\end{bmatrix}, \\
\overline{F}&=\begin{bmatrix}E_1X&0\\ E_1X&0\\ 0&D^TT^T\end{bmatrix}.
\end{aligned} \tag{57}$$

Considering (27) and using the Schur complement theorem, we have

$$\widetilde{\Gamma} \leq 0. \tag{58}$$

Thus, combining (52)–(56) with (57), we obtain

$$\dot{V} \leq 0, \quad \forall \begin{pmatrix} \overline{x} \\ \widetilde{w} \end{pmatrix} \in X_\tau. \tag{59}$$

From (59), we can know that X_τ is an invariant set of the closed-loop system (43) and (44) and all the trajectories of the closed-loop system starting from inside X_τ will converge to the origin; meanwhile, the disturbance observer estimate error is asymptotically stable. Thus, we have

$$\lim_{t\to\infty} \begin{pmatrix} \overline{x} \\ \widetilde{w} \end{pmatrix} = 0 \Longrightarrow \lim_{t\to\infty} \widetilde{w} = \lim_{t\to\infty} \overline{x} = 0 \Longrightarrow \lim_{t\to\infty} x = x_e. \tag{60}$$

Furthermore, if $D_2 = C_2(A + B_1K)^{-1}B_2$, we obtain

$$\lim_{t\to\infty} z = \lim_{t\to\infty} C_2 x_e = r. \tag{61}$$

This completes the proof of Theorem 6. □

Remark 3. It can be seen from Theorem 6 that the closed-loop system and the disturbance observer estimate error for the studied plant in (1) under the developed robust CNF controller of (20) and disturbance observer (7) are asymptotically stable.

Remark 4. To handle the nonlinear terms $\mathrm{sat}(u)^T E_2^T E_2 \,\mathrm{sat}(u)$ and $\widehat{x}_e^T E_1^T E_1 \widehat{x}_e$ in (49), the nonlinear feedback law $\widehat{u}_N$ is designed as the form of (20). It can been seen from (60) and (61) that $x = x_e$ when $t \to \infty$. From above analysis, we can obtain that $z = r$. Thus, $x - \widehat{x}_e = 0$ means that the controlled output z can track the reference r asymptotically under the control of the CNF control law of (20). Therefore, $\widehat{u}_N = 0$ is reasonable when the difference between x and $\widehat{x}_e$ is small enough; that is, $x - \widehat{x}_e < \epsilon$.

Remark 5. It can been seen from [26] that the CNF control can actually improve the transient performance of output response of the closed-loop system by introducing nonlinear feedback portion in which the desired trajectory is normally assumed to be constant. However, as the desired tracking trajectory used in this paper is time-varying, which differs from [26], our control objective is to ensure that output of the closed-loop system can track the time-variant trajectory in the presence of input saturation, external disturbance, and uncertainties and the tracking errors are asymptomatically stable under the control of the proposed robust CNF controller. Thus, the improvement of the transient performance is not investigated in the paper, and this study is our future research work.

5. Simulation Results

In this section, the extensive simulation results are given to demonstrate the effectiveness of the proposed robust CNF control techniques by using two simulation examples.

5.1. Numerical Example. Consider an uncertain system [47] characterized by (1) with

$$\begin{gathered} A = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -3 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}, \\ B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad C_1 = C_2 = D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ E_1 = \mathrm{diag}\,(0.2, 0.2), \qquad E_2 = \mathrm{diag}\,(0.3, 0.3), \\ F = \mathrm{diag}\,(0.5\sin(t), 0.5\cos(t)). \end{gathered} \tag{62}$$

The references signals are $r = (3, 0.12)^T$. The system disturbance d is generated by a linear exogenous system described by (3) with

$$W_1 = \begin{pmatrix} 0 & 1.5 \\ -1.5 & 0 \end{pmatrix}, \qquad V_1 = \begin{pmatrix} 1 & 0 \end{pmatrix}. \tag{63}$$

Here, the given disturbance represents an external harmonic disturbance with known frequency but without any information of its magnitude and phase. Choosing $\alpha_2 = \alpha_4 = \alpha_6 = 1$ and solving LMI (27) give

$$\begin{gathered} K = \begin{bmatrix} -1.0561 & -0.0355 \\ -0.0108 & -0.1691 \end{bmatrix}, \qquad L = \begin{bmatrix} -0.1297 & -0.0613 \\ -0.0652 & -0.0079 \end{bmatrix}, \\ P_2 = \begin{bmatrix} 2.9955 & 0.0353 \\ 0.0353 & 4.3282 \end{bmatrix}, \qquad \alpha_1 = 3.0227, \\ \alpha_3 = 3.0227, \qquad \alpha_5 = 3.0227. \end{gathered} \tag{64}$$

The initial state values are $x_0 = [0, 0]^T$ and $u_{\max} = [5, 2]^T$, the initial generated disturbance value is $d_0 = 0.12$, and the disturbance observer initial value is $\widehat{d}_0 = 0.2$. The CNF controller is designed according to (19).

The simulation results for the system using the developed CNF controller are presented in Figures 1, 2, 3, 4, and 5. Figure 1 indicates that the output of disturbance can effectively approximate the unknown external harmonic disturbance. It is shown in Figures 2 and 3 that the control output z can track the references r asymptotically under the control of (19). Figures 4 and 5 show that the control input does not exceed the limitation of input. Thus, the developed composite nonlinear feedback control (CNF) scheme is valid for the uncertain linear system with input saturation and unknown external disturbances.

5.2. Chaotic System. A chaotic system with disturbance is described as follows:

$$\dot{x} = (A + \Delta A)\, x + B_2 d, \tag{65}$$

where $x = [x_1, x_2, x_3, x_4]^T$ is the system state, $d = \sin(10t)$ is the external disturbance, $\Delta A = DF(t)E_1$ represents the system uncertainties, and $D = I_4$, $E_1 = 0.2I_4$, and

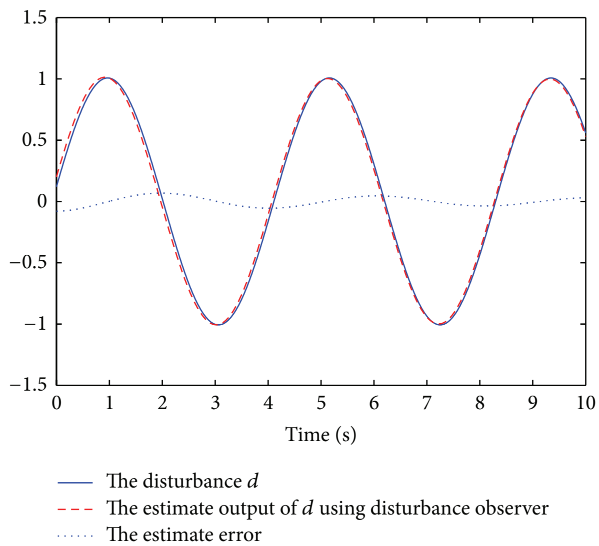

FIGURE 1: The disturbance d and the approximation output of $\hat{d}$.

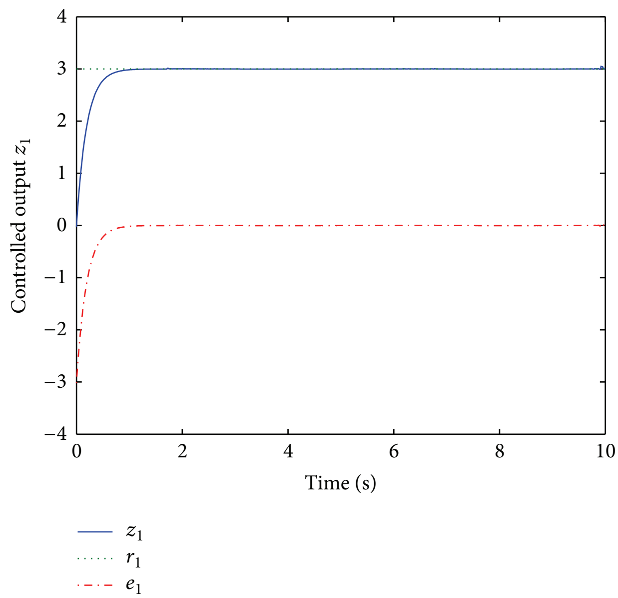

FIGURE 2: Controlled output z_1 and tracking error e_1.

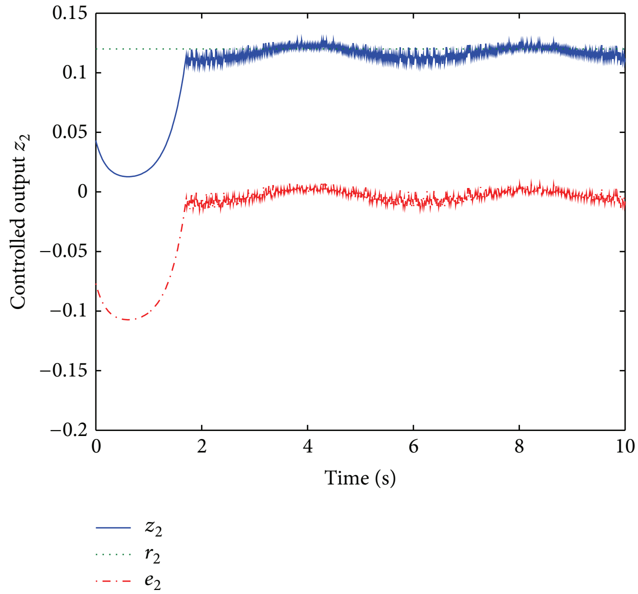

FIGURE 3: Controlled output z_2 and tracking error e_2.

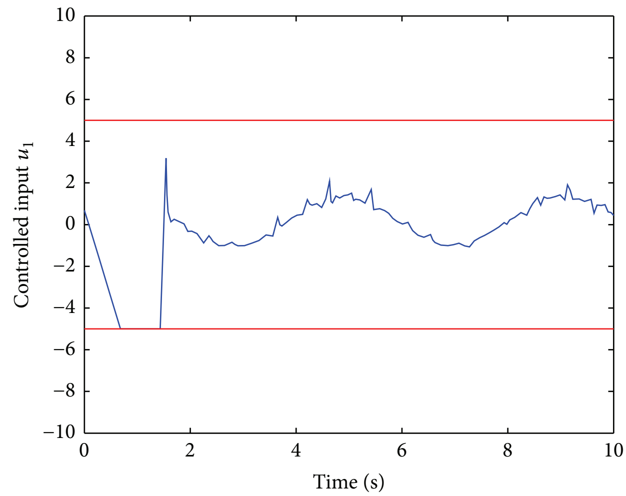

FIGURE 4: Control input u_1.

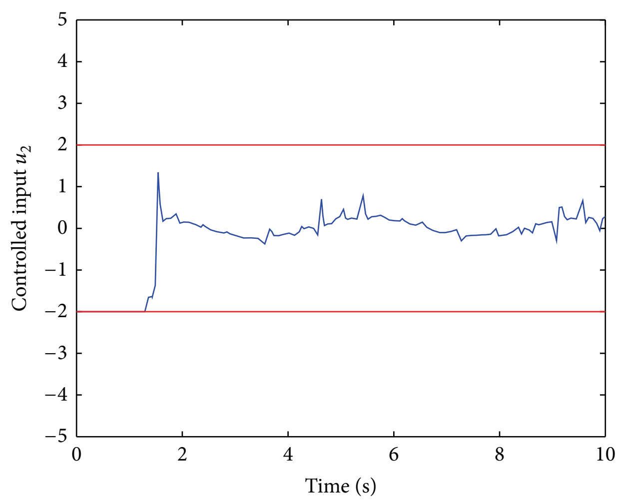

FIGURE 5: Control input u_2.

$F = \mathrm{diag}(0, 10\cos(t), 100\sin(t), 0)$; the system matrices are given by

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -20 & 0 & 0 & -20 \\ -2 & 6 & -66 & 0 \\ 0 & 1.5 & -1 & 50 \end{bmatrix}, \qquad B_2 = [1, 1, 1, 1]^T. \quad (66)$$

The system state responses without control are shown in Figure 6. It can be seen that it is a typical chaotic system.

To control the chaotic system (65), a controller u is introduced. Thus, the system (65) can be transformed into another system as the form of plant (1) used to synchronize with the chaotic system, and the references signals r of this system are obtained from system (65), where the system matrixes A, B_2

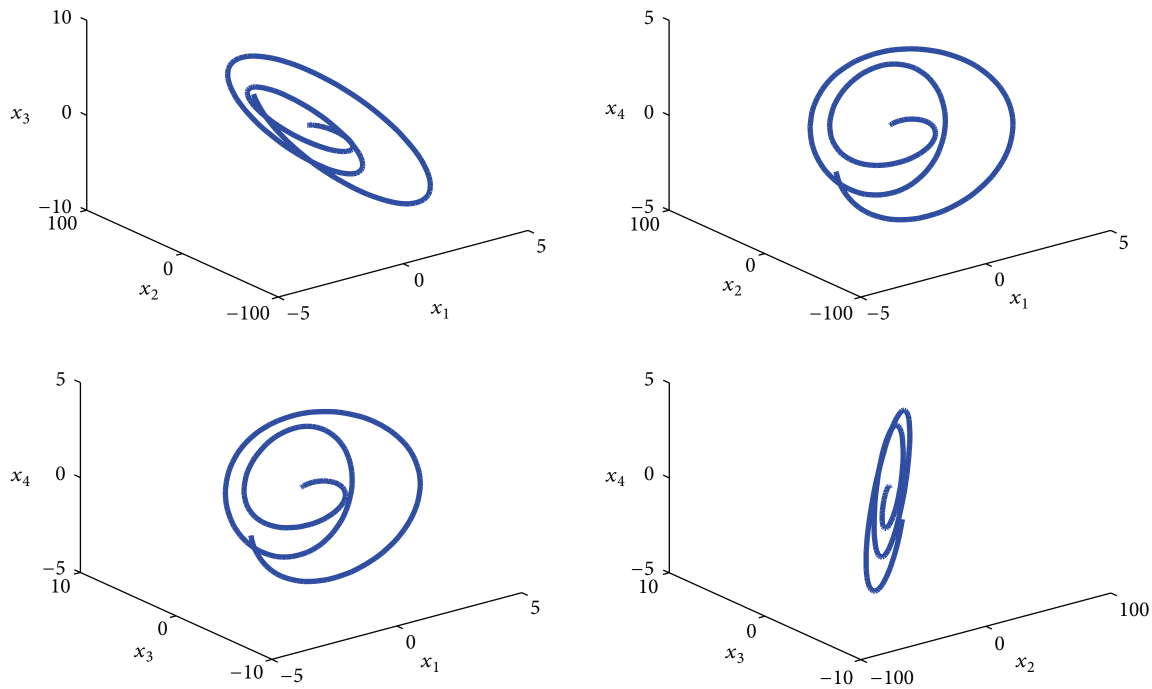

Figure 6: The states response of chaotic system.

are the same as those of the chaotic system (65), $B_1 = \mathrm{diag}(10, 10, 10, 10)$ and $C_1 = C_2 = \mathrm{diag}(1, 1, 1, 1)$. It is obvious that (A, B_1) is stabilizable. The system disturbance d is equal to that of last section and uncertainties ΔB_1 are given by $E_2 = 0.3I_4$.

Choosing $\alpha_2 = \alpha_4 = \alpha_6 = 1$ and solving LMI (27) give

$$K = \begin{bmatrix} -1.6871 & 0.8152 & 0.0741 & -0.6320 \\ 0.7644 & -0.9456 & -0.1257 & 0.7273 \\ 0.0847 & -0.1188 & -0.1133 & 0.2485 \\ -0.6227 & 0.7553 & 0.3379 & -1.5494 \end{bmatrix},$$

$$L = \begin{bmatrix} -0.0704 & -0.0704 & -0.0704 & -0.0704 \\ -0.0029 & -0.0029 & -0.0029 & -0.0029 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.5327 & -0.1736 & -0.0169 & 0.1476 \\ -0.1736 & 0.4143 & 0.0183 & -0.1619 \\ -0.0169 & 0.0183 & 0.3090 & -0.0714 \\ 0.1476 & -0.1619 & -0.0714 & 0.5091 \end{bmatrix}, \tag{67}$$

$$\alpha_1 = 2.6417, \qquad \alpha_3 = 2.6417, \qquad \alpha_5 = 2.6417.$$

The initial state values of the synchronize system are $x_0 = [1, 1, 1, 1]^T$ and $u_{\max} = 1$, the initial generated disturbance value is $d_0 = 1$, and the disturbance observer initial value is $\hat{d}_0 = 1.16$. The CNF controller is designed according to (19).

The simulation results for the the synchronization of chaotic circuit system and designed system using the developed CNF controller are presented in Figures 7, 8, and 9. Figure 7 indicates that the output of disturbance can effectively approximate the unknown external harmonic disturbance. It is shown in Figure 8 that the control output z can track the references r asymptotically under the control of (19) and the tracking errors are asymptotically stable. Therefore, the outputs of the chaotic system and designed system are asymptotically synchronized. Figure 9 shows that the control input does not exceed the limitation of input.

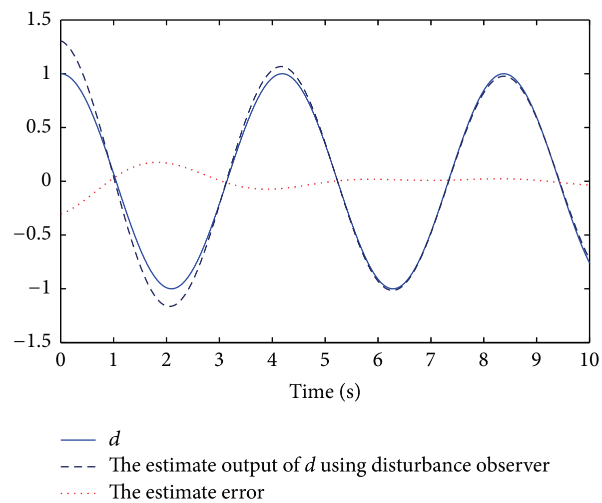

Figure 7: The disturbance d and the approximation output of $\hat{d}$.

It can be shown from these simulation results of the numerical example and uncertain system that the disturbance observer can well estimate the system disturbance, and the closed-loop system for the linear system with input saturation and parametric uncertainties under the the designed robust control scheme using the disturbance observer is asymptotically stable. Thus, the proposed robust control method is valid.

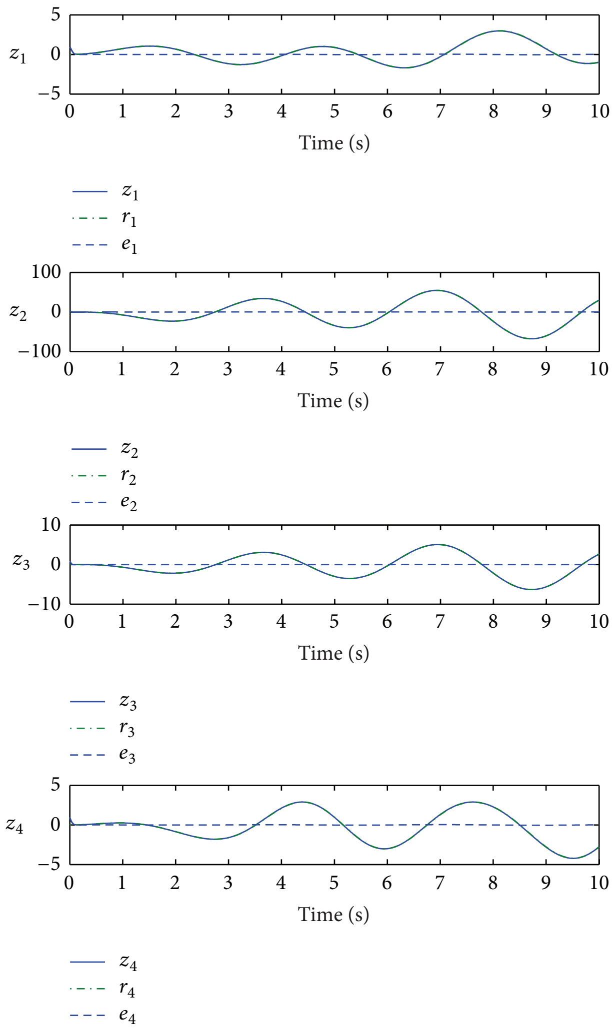

Figure 8: Controlled output z, reference r, and tracking error e.

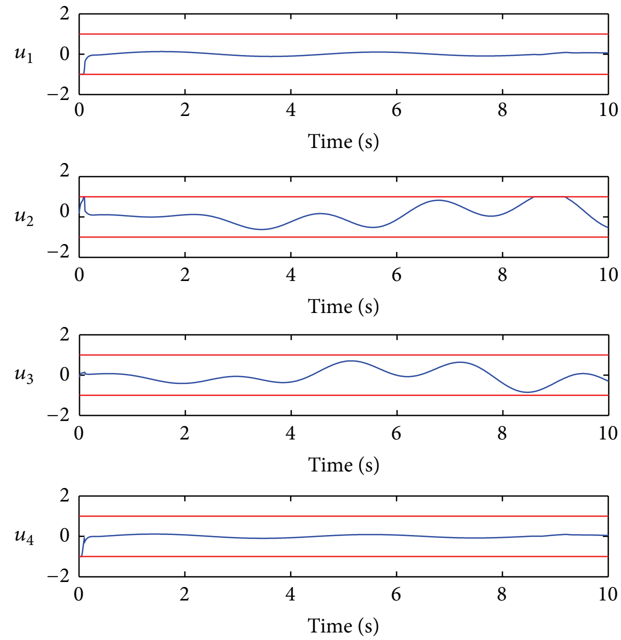

Figure 9: Control input u.

6. Conclusion

In this paper, a CNF control scheme based on the disturbance observer has been proposed to achieve satisfactory tracking control performance for the linear system subject to input saturation, parametric uncertainties, and unknown external disturbance. The disturbance observer has been designed to approximate the system disturbance generated by a linear exogenous system. Based on the output of the disturbance observer, a CNF controller has been developed for the uncertain system subject to input saturation; then, the stability of the closed-loop system under the designed controller has been rigorously proved. Finally, the control method has been applied to the uncertain linear system to illustrate the effectiveness of the proposed control scheme. The simulation results have suggested that the designed CNF control scheme is valid. The direction of future research is to make further improvement of transient tracking performance and extend our results to other MIMO systems, such as near space vehicles (NSV), helicopters, and aircrafts.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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