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Research Article

Resilient Finite-Time Controller Design of a Class of Stochastic Nonlinear Systems

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This paper deals with the problem of resilient finite-time control for a class of stochastic nonlinear systems. The notion of finite-time annular domain stability of stochastic nonlinear systems is first introduced. Then, some sufficient conditions are given for the existence of resilient state feedback finite-time annular domain stabilizing controller, which are expressed in terms of matrix inequalities. A double-parameter searching algorithm is proposed to solve these obtained matrix inequalities. Finally, an example is given to illustrate the effectiveness of the developed method.

1. Introduction

Finite-time stability is a concept that was first introduced in the 1950s, which plays an important role in the study of the transient behavior of systems. Roughly speaking, a system is said to be finite-time stable (FTS) if, given a bound on the initial condition, its state does not exceed a certain threshold during a specified time interval. Various developments and extensions in the field of FTS have been implemented, most of which have been applied to linear systems [1–4]and nonlinear systems [5–7]. Nevertheless, the FTS in [1–7] not only requires the state trajectory does not exceed a given upper bound during a prespecified time interval, but it has no requirement for the lower bound of state trajectory. Recently, [8] gave a new "finite-time stability" for linear Itô stochastic systems. In fact, this kind of stability is called "finite-time annular domain stability" (FTADS for short) more precisely. Roughly speaking, a system is FTAD-stable if its state trajectories do not exceed an upper bound c_2 and are not less than a lower bound c_1 ($c_1 < c_2$) during the specific time interval. The FTADS can be used to solve some problems not only from engineering practice, such as chemical reaction temperature controlled systems and electronic circuit systems [8], but also from medicine. For example, the body's normal systolic blood pressure is 90~ 130 mmHg. If the body's systolic blood pressure is less than 90 mmHg, then one suffers from low blood pressure disease [9].

On the other hand, stochastic nonlinear systems have attracted considerable attention and have become a popular research field of modern control theory [10–13]. Reference [10] investigates H_∞ control problem for a class of stochastic nonlinear systems with both state and disturbance-dependent noise. References [11, 12] studied the finite/infinite horizon mixed H_2/H_∞ control problem for the stochastic nonlinear systems with (x, u, v)-dependent noise, respectively. Reference [13] addressed stochastic passivity, feedback equivalence, and global stabilization for a class of stochastic nonlinear systems.

In the implement of state feedback control, there are often some perturbations appearing in controller gain, which may result in either the actuator degradations or the requirements for readjustment of controller gains during the controller implementation stage. Therefore, it is necessary and reasonable that any controller should be able to tolerate some levels of its gain variations, which motivates us to study the resilient (nonfragile) state feedback controller problems. Although there have been some study on designing the resilient (nonfragile) controller [14, 15], up to date, to the author's knowledge, the issue of resilient finite-time control for stochastic nonlinear systems has not been investigated.

In this paper, we consider the problem of resilient FTAD-stabilization for a class of stochastic nonlinear systems with norm-bounded and time-varying uncertainties. By stochastic analysis technology, Gronwall inequality, and neural network method, some sufficient conditions are obtained for the existence of resilient state feedback finite-time stabilizing controller. The contributions of this paper lie in the following two aspects. (1) The concept of FTADS is extended to a class of stochastic nonlinear systems with norm-bounded and time-varying uncertainties. More precisely, a system is said to be FTAD-stable if, given a bound on the initial state of the system, the state trajectories of the system do not exceed an upper bound c_2 and are not less than a lower bound c_1 ($c_1 < c_2$) in the mean square sense during a prespecified time interval *for all admissible uncertainties*. (2) The problem of resilient FTAD-stabilization is investigated and a resilient state feedback controller is designed such that the resulting closed-loop system is FTAD-stable for all admissible uncertainties.

The paper is organized as follows. In Section 2, system description along with necessary assumption is given. Section 3 provides main results. An example is analyzed to illustrate the results of the paper in Section 4. Section 5 gives the conclusion.

Notation. A^T is transpose of a matrix or vector A. $A > 0$ ($A \geq 0$) is positive definite (positive semidefinite) symmetric matrix. $L^2_{\mathcal{F}}([0,T],\mathbf{R}^l)$ is space of nonanticipative stochastic process $y(t) \in \mathbf{R}^l$ with respect to an increasing *σ-algebra* $\mathcal{F}_t$ $(t \geq 0)$ satisfying $\mathbb{E}\int_0^T \|y(t)\|^2 dt < \infty$. $I_{n\times n}$ is $n \times n$ identity matrix. $\operatorname{tr}(A)$ is trace of a matrix A. $\lambda_{\max}(A)(\lambda_{\min}(A))$ is the maximum (minimum) eigenvalue of a real matrix A. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. The asterisk "$*$" in a matrix is used to represent the term which is induced by symmetry.

2. Preliminaries and Problem Statement

Consider the following stochastic nonlinear system:

$$\begin{aligned} dx(t) = {} & \left[\left(A_1 + \Delta A_1(t)\right)x(t) + B_1 u(t) + f(x(t))\right]dt \\ & + \left[\left(A_2 + \Delta A_2(t)\right)x(t) + B_2 u(t)\right]dw(t), \\ & f(0) = 0, \qquad x(0) = x_0 \in \mathbf{R}^n, \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in L^2_{\mathcal{F}}(\mathbf{R}_+,\mathbf{R}^m)$ are called the system state, control input, respectively. x_0 is the initial state. Without loss of generality, throughout this paper, we assume $w(t)$ to be one-dimensional standard Wiener process defined on the probability space $(\Omega,\mathcal{F},\mathcal{F}_t,P)$ with $\mathcal{F}_t = \sigma\{w(s) : 0 \leq s \leq t\}$. $f(x(t))$ is assumed to be Borel measurable functions of suitable dimensions such that (1) has a unique strong solution on any finite interval $[0,T]$; see [16]. $A_1, A_2 \in \mathbf{R}^{n\times n}$, B_1, $B_2 \in \mathbf{R}^{n\times m}$ are constant matrices. $\Delta A_1(t)$, $\Delta A_2(t)$ are unknown matrices with time-varying uncertainties and satisfy the following conditions:

$$\left[\Delta A_1 \;\; \Delta A_2\right] = MF(t)\left[N_1 \;\; N_2\right], \tag{2}$$

where M, N_1, and N_2 are known matrices with appropriate dimensions; $F(t) : \mathbf{R} \rightarrow \mathbf{R}^{k\times l}$ is an unknown time-varying matrix function, which satisfies

$$F^T(t)F(t) \leq I, \quad \forall t > 0. \tag{3}$$

The parameter uncertainties are said to be admissible if (2) and (3) hold.

Remark 1. This kind of model (1) contains a large class of practical systems and has been widely investigated in control, filtering, and stability analysis [17–20].

Next, using LDI technique mentioned, nonlinear function $f(x(t))$ is to be parameterized by multilayer neural networks (MNNs). Here, we use the method in [21–23]. For the readers' convenience, the concrete process is as follows. Let the single hidden layer perceptron $\mathcal{N}(x(t),\mathcal{W}_1,\mathcal{W}_2)$ be suitably trained to approximate the nonlinear term $f(x(t))$, which is described in matrix-vector notation as

$$\mathcal{N}\left(x(t),\mathcal{W}_1(x(t)),\mathcal{W}_2(x(t))\right) = \phi_2\left[\mathcal{W}_2\phi_1\left[\mathcal{W}_1 x(t)\right]\right], \tag{4}$$

where $\mathcal{W}_i \in R^{n_h\times n}$, $i = 1,2$, denote the connecting weight matrices of neurons, and $\phi_i(\cdot)$ denotes the activation function vector of the NNs, which is defined as

$$\phi_i[\gamma] = \left[\psi_1(\mu_1),\psi_2(\mu_2),\ldots,\psi_n(\mu_n)\right]^T \tag{5}$$

in which we let

$$\psi_j(\mu_j) = \chi_j\left(\frac{1-e^{-\mu/q_j}}{1+e^{-\mu/q_j}}\right), \quad q_j,\chi_j > 0, \; j = 1,2,\ldots,n. \tag{6}$$

The maximum and minimum derivatives of activation function ψ_j are defined as follows:

$$h_j(k,\psi_j) = \begin{cases} \min\limits_{\gamma_j}\dfrac{\partial\psi_j(\gamma_j)}{\partial\gamma_j}, & k = 0, \\ \max\limits_{\gamma_j}\dfrac{\partial\psi_j(\gamma_j)}{\partial\gamma_j}, & k = 1. \end{cases} \tag{7}$$

The activation function ψ_j can be rewritten in the following min-max form:

$$\psi_j = \eta_j(0)h_j(0,\psi_j) + \eta_j(1)h_j(1,\psi_j), \tag{8}$$

where $\eta_j(k)$, $k = 0,1$, is a set of positive real numbers associated with ψ_j satisfying $\eta_j(k) > 0$ and $\eta_j(0)+\eta_j(1) = 1$.

According to the approximation theorem, for a given accuracy $\rho > 0$, there exist constant weight matrices $\mathcal{W}_i^*$ defined as

$$\begin{aligned} &(\mathcal{W}_1^*,\mathcal{W}_2^*) \\ &\quad = \arg\min_{(\mathcal{W}_1^*,\mathcal{W}_2^*)}\left\{\max_{x(t)\in D}\left\|f(x(t)) - \mathcal{N}(x(t),\mathcal{W}_1,\mathcal{W}_2)\right\|\right\}, \end{aligned} \tag{9}$$

where $D \in \mathbf{R}^m$ is a compact set, such that

$$\max_{x(t)\in D} \left\| f(x(t)) - \mathcal{N}(x(t), \mathscr{W}_1^*, \mathscr{W}_2^*) \right\| \le \rho \|x(t)\|. \quad (10)$$

Denote a set of n_i-dimensional index vectors of the ith layer ($i = 1, 2$) as

$$\kappa_{n_i} = \kappa_{n_i}(\sigma) = \left\{ \sigma \in \mathbf{R}^{n_i} \mid \sigma_j \in \{0, 1\}, j = 1, \ldots, n_i \right\}, \quad (11)$$

where σ is used as binary indicator. The ith layer with n_i neutrons has 2^{n_i} combinations of binary indicator with $k = 0, 1$, and the elements of index vectors for two-layer NNs have $2^{n_2} \times 2^{n_1}$ combinations in the $\Theta = \kappa_{n_2} \oplus \kappa_{n_1}$.

By using (7) and adopting the compact representation [21], the NNs (4) can be expressed as follows:

$$\begin{aligned} &\mathcal{N}(x(t), \mathscr{W}_1^*, \mathscr{W}_2^*) \\ &= \phi_2 \left[\mathscr{W}_2^* \left[\begin{array}{c} \sum_{k=0}^{1} \eta_{1,1}(k) h_{1,1}(k, \psi_{1,1} \times (\mathscr{W}_1^* x)_1) \\ \vdots \\ \sum_{k=0}^{1} \eta_{1,n_1}(k) h_{1,n_1}\left(k, \psi_{1,n_1} \times (\mathscr{W}_1^* x)_{n_1}\right) \end{array} \right] \right] \\ &= \sum_{\sigma\in\Theta} \nu_\sigma A_\sigma(\sigma, \phi, \mathscr{W}^*) x(t), \end{aligned} \quad (12)$$

where

$$A_\sigma = \operatorname{diag}\left[h_{2,j}\left(\sigma_{2,j}, \psi_{2,j}\right)\right] \mathscr{W}_2^* \operatorname{diag}\left[h_{1,j}\left(\sigma_{1,j}, \psi_{1,j}\right)\right] \mathscr{W}_1^* \quad (13)$$

$$\begin{aligned} &\sum_{\sigma\in\kappa_{n_2}\oplus\kappa_{n_1}} \nu_\sigma \\ &= \sum_{k_{2,n_2}=0,\ldots,k_{2,1}=0}^{1} \sum_{k_{2,n_1}=0,\ldots,k_{1,1}=0}^{1} \eta_{2,n_2}\left(k_{2,n_2}\right) \cdots \eta_{2,1}\left(k_{2,1}\right) \\ &\quad \times \eta_{1,n_1}\left(k_{1,n_1}\right) \cdots \eta_{1,1}\left(k_{1,1}\right) = 1 \\ &\qquad \eta_{i,j}\left(\sigma_{i,j}\right) \ge 0, \quad \sigma_{ij} = 0, 1, \\ &\quad \eta_{i,j}(0) + \eta_{i,j}(1) = 1, \quad i = 1, 2, \ j = n_1, n_2. \end{aligned} \quad (14)$$

Thus, by means of NNs, the resulting system (1) is transformed into a group of LDIs with error bound; that is,

$$\begin{aligned} dx(t) = &\left[\left(A_1 + \Delta A_1(t) + \sum_{\sigma\in\Theta} \nu_\sigma A_\sigma \right) x(t) \right. \\ &\left. + B_1 u(t) + \Delta f(x(t)) \right] dt \\ &+ \left[\left(A_2 + \Delta A_2(t) \right) x(t) + B_2 u(t) \right] dw(t), \\ &f(0) = 0, \qquad x(0) = x_0 \in \mathbf{R}^n, \end{aligned} \quad (15)$$

where

$$\begin{aligned} \Delta f(x(t)) &= \max_{x(t)\in D} \left\| f(x(t)) - \mathcal{N}(x(t), \mathscr{W}_1^*, \mathscr{W}_2^*) \right\| \\ &\le \rho \|x(t)\| \end{aligned} \quad (16)$$

denotes the approximation errors of the NNs.

Remark 2. Such parameterization makes sense because any continuous nonlinear function can be approximated arbitrarily well on a compact interval by NNs.

In the following, we will extend FTADS in [8] to stochastic nonlinear systems. It is formalized through the following definition.

Definition 3. Given positive real scalars c_1, c_2, c_3, c_4, and T, with $0 < c_1 < c_3 < c_4 < c_2$, and a positive definite matrix R. Stochastic nonlinear system (1) with $u(t) = 0$ is said to be FTAD-stable with respect to $(c_1, c_2, c_3, c_4, T, R)$ for all admissible uncertainties, if

$$\begin{aligned} c_3 \le \mathbb{E}\left[x_0^T R x_0\right] \le c_4 \Longrightarrow c_1 < \mathbb{E}\left[x^T(t) R x(t)\right] < c_2, \\ \forall t \in [0, T]. \end{aligned} \quad (17)$$

Remark 4. The FTADS requires the state trajectory not only not to exceed a given upper bound, but also not to be less than a given lower bound, which is different from FTS in [1–7]. The FTS only requires the state trajectory not to exceed a given upper bound. It is noted that a system which is FTS may not be FTADS. This point can be verified as follows. Although a system is FTS, its state trajectory may cross the region $\{x(t) \mid \mathbb{E}[x^T(t) R x(t)] < c_1\}$.

Next, we construct the following resilient state feedback controller for system (1):

$$u(t) = K(t) x(t), \quad (18)$$

where $K(t) = K + \Delta K(t)$ and K is a constant and $\Delta K(t)$ is a perturbed matrix which is assumed to be

$$\Delta K(t) = D_3 F(t) N_3, \quad (19)$$

where D_3 and N_3 are known real constant matrices with appropriate dimensions and the time-varying uncertain matrix $F(t)$ satisfies (3).

Remark 5. The uncertainty part of the resilient controller (18) is supposed to be 2-norm-bounded which is fit for general parameter perturbation case.

The aim of this paper is to design resilient controller (18) such that the following closed-loop system,

$$\begin{aligned} dx(t) = \left[\overline{\overline{A_1}} x(t) + \Delta f(x(t)) \right] dt + \overline{A_2} x(t) dw(t), \\ f(0) = 0, \qquad x(0) = x_0 \in \mathbf{R}^n, \end{aligned} \quad (20)$$

is FTAD-stable with respect to $(c_1, c_2, c_3, c_4, T, R)$, where $\overline{\overline{A_1}} = \overline{A_1} + \Delta\overline{A_1}$, $\overline{A_1} = \sum_{\sigma\in\Theta} \nu_\sigma A_\sigma + A_1 + B_1 K$, $\Delta\overline{A_1} = \Delta A_1 + B_1 \Delta K$,

$\overline{A_2} = A_2 + B_2K + \Delta A_2 + B_2\Delta K$, and $\overline{C}_1 = C_1 + D_1K + D_1\Delta K$ and

$$\begin{aligned}\Delta f(x(t)) &= \max_{x(t)\in D} \left\| f(x(t)) - \mathcal{N}(x(t), \mathcal{W}_1^*, \mathcal{W}_2^*) \right\| \\ &\leq \rho \|x(t)\|\end{aligned} \tag{21}$$

denotes the approximation errors of the NNs.

In the following, we give some lemmas which will be used in the next sections.

Lemma 6 (Itô-type formula). *For a given $V(x) \in C^2(R^n)$, associated with the following stochastic system:*

$$dx(t) = f(x)\,dt + g(x)\,dw(t), \tag{22}$$

the infinitesimal generator operator is defined by

$$\mathcal{L}V(x) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2}\operatorname{Tr}\left[g^T(x)\frac{\partial^2 V}{\partial x^2} g(x)\right]. \tag{23}$$

Lemma 7 (Gronwall inequality). *Let $\theta(t)$ be a nonnegative function such that*

$$\theta(t) \leq a + b\int_0^t \theta(s)\,ds, \quad 0 \leq t \leq T \tag{24}$$

for some constants $a, b \geq 0$, and then one has

$$\theta(t) \leq a\exp(bt), \quad 0 \leq t \leq T. \tag{25}$$

Lemma 8 (see [8]). *Let $\theta(t)$ be a nonnegative function such that*

$$\theta(t) \geq a + b\int_0^t \theta(s)\,ds, \quad 0 \leq t \leq T \tag{26}$$

for some constants $a, b \geq 0$, and then one has

$$\theta(t) \geq a\exp(bt), \quad 0 \leq t \leq T. \tag{27}$$

Lemma 9. *Let L, M, F, and N be real matrices of appropriate dimension with $F^T(t)F(t) \leq I$. Then, for a positive scalar $\epsilon > 0$, one has*

$$\begin{aligned}&L + MF(t)N + N^TF^T(t)M^T \\ &\quad\leq L + \epsilon MM^T + \epsilon^{-1}N^TN.\end{aligned} \tag{28}$$

3. Resilient Finite-Time Controller Design

In this section, we consider resilient FTAD-stabilization for system (1). First, an important lemma is given.

Lemma 10. *If there exist $\alpha \geq 0$, $\beta \geq 0$, a symmetric positive definite Q, and a matrix K such that*

$$\begin{bmatrix} \coprod & \widetilde{Q}\overline{A_2^T} \\ * & -\widetilde{Q}\end{bmatrix} < 0, \tag{29}$$

$$\begin{bmatrix} \sqcup & \widetilde{Q}\overline{A_2^T} \\ * & -\widetilde{Q}\end{bmatrix} < 0, \tag{30}$$

$$\frac{c_1}{\lambda_{\min}(Q)} < \frac{c_2}{\lambda_{\max}(Q)} e^{-\alpha T}, \tag{31}$$

$$c_1\lambda_{\max}(Q) - c_3\lambda_{\min}(Q) < 0, \tag{32}$$

then system (20) is FTAD-stable with respect to $(c_1, c_2, c_3, c_4, T, R)$, where $\widetilde{Q} = R^{-1/2}QR^{-1/2}$, $\coprod = \widetilde{Q}\overline{\overline{A_1^*}}^T + \overline{\overline{A_1^*}}\widetilde{Q} + 2\rho\widetilde{Q} - \alpha\widetilde{Q}$, $\sqcup = \beta\widetilde{Q} - \widetilde{Q}\overline{\overline{A_1^*}}^T - \overline{\overline{A_1^*}}\widetilde{Q}$, and $\overline{\overline{A_1^*}} = A_\sigma + A_1 + B_1K + \Delta A_1 + B_1\Delta K$.

Proof. Step 1. $\mathbb{E}[x^T(0)Rx(0)] < c_4 \Rightarrow \mathbb{E}[x^T(t)Rx(t)] < c_2$.

Take a quadratic function $V(x(t)) = x^T(t)\widetilde{Q}^{-1}x(t)$, where $\widetilde{Q} = R^{-1/2}QR^{-1/2}$ with $Q > 0$ being a solution to (29)–(32). Applying Itô formula for $V(x(t))$ along the trajectory of the system (20) and considering $\Delta f(x(t)) \leq \rho\|x(t)\|$, $\sum_{\sigma\in\Theta}\nu_\sigma = 1$, we obtain

$$\begin{aligned}\mathcal{L}V(x(t)) &= \left[\overline{\overline{A_1}}x(t) + \Delta f(x(t))\right]^T \widetilde{Q}^{-1}x(t) \\ &\quad + x^T(t)\widetilde{Q}^{-1}\left[\overline{\overline{A_1}}x(t) + \Delta f(x(t))\right] \\ &\quad + x^T(t)\overline{A_2^T}\widetilde{Q}^{-1}\overline{A_2}x(t) \\ &\leq x^T(t)\left[\overline{\overline{A_1^T}}\widetilde{Q}^{-1} + \widetilde{Q}^{-1}\overline{\overline{A_1}} + 2\rho\widetilde{Q}^{-1}\right. \\ &\quad \left. + \overline{A_2^T}\widetilde{Q}^{-1}\overline{A_2}\right]x(t) \\ &= x^T(t)\sum_{\sigma\in\Theta}\nu_\sigma\left[\overline{\overline{A_1^*}}^T\widetilde{Q}^{-1} + \widetilde{Q}^{-1}\overline{\overline{A_1^*}}\right. \\ &\quad \left. + 2\rho\widetilde{Q}^{-1} + \overline{A_2^T}\widetilde{Q}^{-1}\overline{A_2}\right]x(t),\end{aligned} \tag{33}$$

where $\overline{\overline{A_1^*}} = A_\sigma + A_1 + B_1K + \Delta A_1 + B_1\Delta K$.

Before and after multiplying (29) by

$$\begin{bmatrix}\widetilde{Q}^{-1} & 0 \\ * & \widetilde{Q}^{-1}\end{bmatrix}, \tag{34}$$

(29) becomes

$$\begin{bmatrix}\overline{\overline{A_1^*}}^T\widetilde{Q}^{-1} + \widetilde{Q}^{-1}\overline{\overline{A_1^*}} + 2\rho\widetilde{Q}^{-1} - \alpha\widetilde{Q}^{-1} & \overline{A_2^T}\widetilde{Q}^{-1} \\ * & -\widetilde{Q}^{-1}\end{bmatrix} < 0. \tag{35}$$

According to Schur complement, (35) is equivalent to the following inequality:

$$\begin{aligned}&\overline{\overline{A_1^*}}^T\widetilde{Q}^{-1} + \widetilde{Q}^{-1}\overline{\overline{A_1^*}} + 2\rho\widetilde{Q}^{-1} \\ &\quad + \overline{A_2^T}\widetilde{Q}^{-1}\overline{A_2} - \alpha\widetilde{Q}^{-1} < 0.\end{aligned} \tag{36}$$

From (33) and (36), it is easy to obtain that

$$\mathcal{L}V(x(t)) < \alpha V(x(t)). \tag{37}$$

Integrating both sides of (37) from 0 to t with $t \in [0, T]$ and then taking the expectation, it yields

$$\mathbb{E}V(x(t)) < \mathbb{E}V(x(0)) + \alpha\int_0^t \mathbb{E}V(x(s))\,ds. \tag{38}$$

By Lemma 7, we obtain

$$\mathbb{E}V(x(t)) < \mathbb{E}V(x(0))e^{\alpha t}. \tag{39}$$

According to given conditions, it follows that

$$\begin{aligned}\mathbb{E}V(x(t)) &= \mathbb{E}\left[x^T(t)R^{1/2}Q^{-1}R^{1/2}x(t)\right]\\ &\geq \lambda_{\min}\left(Q^{-1}\right)\mathbb{E}\left[x^T(t)Rx(t)\right],\\ V(x(0))e^{\alpha t} &= \mathbb{E}\left[x^T(0)R^{1/2}Q^{-1}R^{1/2}x(0)\right]e^{\alpha t}\\ &\leq \lambda_{\max}\left(Q^{-1}\right)\mathbb{E}\left[x^T(0)Rx(0)\right]e^{\alpha t}\\ &\leq \lambda_{\max}\left(Q^{-1}\right)c_1e^{\alpha T}.\end{aligned} \tag{40}$$

From (40), we easily obtain

$$\mathbb{E}\left[x^T(t)Rx(t)\right] \leq \lambda_{\max}(Q)e^{\alpha T}\frac{c_1}{\lambda_{\min}(Q)}. \tag{41}$$

By the condition (31), it is obvious that $\mathbb{E}[x^T(t)Rx(t)] < c_2$.

Step 2. $c_3 < \mathbb{E}[x^T(0)Rx(0)] \Rightarrow c_1 < \mathbb{E}[x^T(t)Rx(t)]$.

By Schur complement, (30) is equivalent to

$$\beta\widetilde{Q} - \widetilde{Q}\overline{\overline{A_1^*}}^T - \overline{\overline{A_1^*}}\widetilde{Q} + \widetilde{Q}\overline{\overline{A_2^*}}^T\widetilde{Q}^{-1}\overline{\overline{A_2^*}}^T\widetilde{Q} < 0. \tag{42}$$

Before and after multiplying (42) by $\widetilde{Q}^{-1}$, we obtain

$$\beta\widetilde{Q}^{-1} - \overline{\overline{A_1^*}}^T\widetilde{Q}^{-1} - \widetilde{Q}^{-1}\overline{\overline{A_1^*}} + \overline{\overline{A_2^*}}^T\widetilde{Q}^{-1}\overline{\overline{A_2^*}} < 0. \tag{43}$$

Consider (33), and (43) implies

$$\mathscr{L}V(x(t)) > \beta V(x(t)). \tag{44}$$

Integrating both sides of (44) from 0 to t with $t \in [0, T]$ and then taking the expectation, it yields

$$\mathbb{E}V(x(t)) > \mathbb{E}V(x(0)) + \beta\int_0^t \mathbb{E}V(x(s))\,ds. \tag{45}$$

By Lemma 8, we conclude that

$$\mathbb{E}V(x(t)) > \mathbb{E}V(x(0))e^{\beta t}. \tag{46}$$

According to the given conditions, it follows that

$$\begin{aligned}&\mathbb{E}\left[x^T(0)R^{1/2}Q^{-1}R^{1/2}x(0)\right]e^{\beta t}\\ &\quad< \mathbb{E}\left[x^T(t)R^{1/2}Q^{-1}R^{1/2}x(t)\right]\\ &\quad< \lambda_{\max}\left(Q^{-1}\right)\mathbb{E}\left[x^T(t)Rx(t)\right],\\ c_3\lambda_{\min}\left(Q^{-1}\right) &< \lambda_{\min}\left(Q^{-1}\right)\mathbb{E}\left[x^T(0)Rx(0)\right]e^{\beta t}\\ &\quad< \mathbb{E}\left[x^T(0)R^{1/2}Q^{-1}R^{1/2}x(0)\right]e^{\beta t}.\end{aligned} \tag{47}$$

Because of condition (32), we obtain

$$c_1 < \mathbb{E}\left[x^T(t)Rx(t)\right]. \tag{48}$$

From (48), it readily follows that $c_3 < \mathbb{E}[x^T(0)Rx(0)]$ implies that $c_1 < \mathbb{E}[x^T(t)Rx(t)]$. □

The following theorem gives a sufficient condition for resilient FTAD-stabilization of system (1).

Theorem 11. *If there exist scalars $\alpha \geq 0$, $\beta \geq 0$, and positive scalars ϵ_i $(i = 1, \ldots, 4)$, λ_1, λ_2, a symmetric positive definite Q, and a matrix L such that*

$$\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & 0\\ * & \Lambda_{22} & 0\\ * & * & \Lambda_{33}\end{bmatrix} < 0, \tag{49}$$

$$\begin{bmatrix}\Phi_{11} & \Lambda_{12} & 0\\ * & \Lambda_{22} & 0\\ * & * & \Lambda_{33}\end{bmatrix} < 0, \tag{50}$$

$$\lambda_1 I < Q < \lambda_2 I, \tag{51}$$

$$c_4\lambda_2e^{\alpha T} - c_2\lambda_1 < 0, \tag{52}$$

$$c_1\lambda_2 - c_3\lambda_1 < 0, \tag{53}$$

then system (20) is FTAD-stable with respect to $(c_1, c_2, c_3, c_4, T, R)$, where $\Lambda_{11} = \Upsilon_{11} + 2\rho\widetilde{Q} - \alpha\widetilde{Q}$, $\Phi_{11} = \beta\widetilde{Q} - \Upsilon_{11}$, $\Lambda_{12} = \left[N_1\widetilde{Q}\ \ \widetilde{Q}N_3^T\ \ \widetilde{Q}N_2^T\ \ \ \Theta_{15}\right]$, $\Lambda_{22} = \operatorname{diag}\{-\epsilon_1 I,\ -\epsilon_2 I,\ \epsilon_3 I\}$, $\Lambda_{33} = \begin{bmatrix}\Theta_{55} & \widetilde{Q}N_3^T\\ N_3\widetilde{Q} & -\epsilon_3 I\end{bmatrix}$, $\Theta_{15} = \widetilde{Q}A_2^T + L^TB_2^T$, $\Theta_{55} = -\widetilde{Q} + \epsilon_3B_2D_3D_3^TB_2^T + \epsilon_4MM^T$, $\Upsilon_{11} = \overline{\overline{A_{11}^*}}\widetilde{Q} + \widetilde{Q}\overline{\overline{A_{11}^*}}^T + B_1L + L^TB_1^T + \epsilon_1MM^T + \epsilon_2B_1D_3D_3^TB_1^T$, *and* $\overline{\overline{A_{11}^*}} = A_\sigma + A_1$. *In this case, a desired controller gain is given by $K = L\widetilde{Q}^{-1}$.*

Proof. Substitute $\overline{\overline{A_{11}^*}} = \overline{A_1^*} + \Delta\overline{A_1}$, $\overline{A_1^*} = A_\sigma + A_1 + B_1K$, $\Delta\overline{A_1} = \Delta A_1 + B_1\Delta K$, and $\overline{A_2} = A_2 + B_2K + \Delta A_2 + B_2\Delta K$ into (29) and (30), and let $\overline{\overline{A_{11}^*}} = A_\sigma + A_1$, (29), and (30), respectively, become

$$\begin{aligned}Z &= \begin{bmatrix}\Xi_{11} + \Delta\Xi_{11} & \widetilde{Q}A_2^T + \widetilde{Q}K^TB_2^T + \Delta\Xi_{12}\\ * & -\widetilde{Q}\end{bmatrix} < 0,\\ \overline{Z} &= \begin{bmatrix}\beta\widetilde{Q} - \Pi + \Delta\Xi_{11} & \widetilde{Q}A_2^T + \widetilde{Q}K^TB_2^T + \Delta\Xi_{12}\\ * & -\widetilde{Q}\end{bmatrix} < 0,\end{aligned} \tag{54}$$

where $\Pi = \overline{\overline{A_{11}^*}}\widetilde{Q} + \widetilde{Q}\overline{\overline{A_{11}^*}}^T + B_1K\widetilde{Q} + \widetilde{Q}K^TB_1^T$, $\Xi_{11} = \Pi + 2\rho\widetilde{Q} - \alpha\widetilde{Q}$, $\Delta\Xi_{11} = MF(t)N_1\widetilde{Q} + \widetilde{Q}N_1^TF^T(t)M^T + B_1D_3F(t)N_3\widetilde{Q} + \widetilde{Q}N_3^TF^T(t)D_3^TB_1^T$, and $\Delta\Xi_{12} = \widetilde{Q}N_2^TF^T(t)M^T + \widetilde{Q}N_3^TF^T(t)D_3^TB_2^T$.

In order to deal with the uncertainties described as the form in (2), we use the following approach:

$$Z = \Xi + \Delta\Xi, \tag{55}$$

where

$$\begin{aligned}\Xi &= \begin{bmatrix}\Xi_{11} & \widetilde{Q}A_2^T + \widetilde{Q}K^TB_2^T\\ * & -\widetilde{Q}\end{bmatrix},\\ \Delta\Xi &= \begin{bmatrix}\Delta\Xi_{11} & \Delta\Xi_{12}\\ * & 0\end{bmatrix}.\end{aligned} \tag{56}$$

According to Lemma 9, we obtain the following:

$$\begin{aligned}\Delta\Xi &= \mathscr{F}_1 + \mathscr{F}_1^T + \mathscr{F}_2 + \mathscr{F}_2^T + \mathscr{F}_3 + \mathscr{F}_3^T + \mathscr{F}_4 + \mathscr{F}_4^T \\ &\le \gimel_1 + \gimel_2 + \gimel_3 + \gimel_4 + \gimel_5 + \gimel_6 + \gimel_7 + \gimel_8 \\ &= \begin{bmatrix} \Gamma_{11} & 0 \\ * & \Gamma_{22} \end{bmatrix},\end{aligned} \tag{57}$$

where

$$\begin{aligned}\mathscr{F}_1 &= \begin{bmatrix} M^T & 0 \end{bmatrix}^T F(t) \begin{bmatrix} N_1\widetilde{Q} & 0 \end{bmatrix}, \\ \mathscr{F}_2 &= \begin{bmatrix} D_3^T B_1^T & 0 \end{bmatrix}^T F(t) \begin{bmatrix} N_3\widetilde{Q} & 0 \end{bmatrix}, \\ \mathscr{F}_3 &= \begin{bmatrix} 0 & D_3^T B_2^T \end{bmatrix}^T F(t) \begin{bmatrix} N_3\widetilde{Q} & 0 \end{bmatrix}, \\ \mathscr{F}_4 &= \begin{bmatrix} 0 & M^T \end{bmatrix}^T F(t) \begin{bmatrix} N_2\widetilde{Q} & 0 \end{bmatrix}, \\ \gimel_1 &= \epsilon_1 \begin{bmatrix} M^T & 0 \end{bmatrix}^T \begin{bmatrix} M^T & 0 \end{bmatrix}, \\ \gimel_2 &= \epsilon_1^{-1} \begin{bmatrix} \widetilde{Q}N_1^T & 0 \end{bmatrix}^T \begin{bmatrix} \widetilde{Q}N_1^T & 0 \end{bmatrix}, \\ \gimel_3 &= \epsilon_2 \begin{bmatrix} D_3^T B_1^T & 0 \end{bmatrix}^T \begin{bmatrix} D_3^T B_1^T & 0 \end{bmatrix}, \\ \gimel_4 &= \epsilon_2^{-1} \begin{bmatrix} N_3\widetilde{Q} & 0 \end{bmatrix}^T \begin{bmatrix} N_3\widetilde{Q} & 0 \end{bmatrix}, \\ \gimel_5 &= \epsilon_3 \begin{bmatrix} 0 & D_3^T B_2^T \end{bmatrix}^T \begin{bmatrix} 0 & D_3^T B_2^T \end{bmatrix}, \\ \gimel_6 &= \epsilon_3^{-1} \begin{bmatrix} 0 & N_3\widetilde{Q} \end{bmatrix}^T \begin{bmatrix} 0 & N_3\widetilde{Q} \end{bmatrix}, \\ \gimel_7 &= \epsilon_4 \begin{bmatrix} 0 & M^T \end{bmatrix}^T \begin{bmatrix} 0 & M^T \end{bmatrix}, \\ \gimel_8 &= \epsilon_4^{-1} \begin{bmatrix} N_2\widetilde{Q} & 0 \end{bmatrix}^T \begin{bmatrix} N_2\widetilde{Q} & 0 \end{bmatrix}, \\ \Gamma_{11} &= \epsilon_1 MM^T + \epsilon_1^{-1} N_1\widetilde{Q}\widetilde{Q}N_1^T + \epsilon_2 B_1 D_3 D_3^T B_1^T \\ &\quad + \epsilon_2^{-1}\widetilde{Q}N_3^T N_3\widetilde{Q} + \epsilon_4^{-1}\widetilde{Q}N_2^T N_2\widetilde{Q}, \\ \Gamma_{22} &= \epsilon_3 B_2 D_3 D_3^T B_2^T + \epsilon_3^{-1}\widetilde{Q}N_3^T N_3\widetilde{Q} + \epsilon_4 MM^T.\end{aligned} \tag{58}$$

From the above procedure and by Schur complement,

$$Z \le \begin{bmatrix} \Lambda_{11}^* & \Lambda_{12} & 0 \\ * & \Lambda_{22} & 0 \\ * & * & \Lambda_{33} \end{bmatrix}, \tag{59}$$

where $\Theta_{11} = \overline{\overline{A_{11}^*}}\widetilde{Q} + \widetilde{Q}\overline{\overline{A_{11}^*}}^T + B_1K\widetilde{Q} + \widetilde{Q}K^TB_1^T + 2\rho\widetilde{Q} - \alpha\widetilde{Q} + \epsilon_1 MM^T + \epsilon_2 B_1D_3D_3^TB_1^T$, $\Theta_{15} = \widetilde{Q}A_2^T + \widetilde{Q}K^TB_2^T$, $\Theta_{55} = -\widetilde{Q} + \epsilon_3 B_2D_3D_3^TB_2^T + \epsilon_4 MM^T$, $\Lambda_{11}^* = \Theta_{11} + 2\rho\widetilde{Q} - \alpha\widetilde{Q}$, $\Lambda_{12} = \begin{bmatrix} N_1\widetilde{Q} & \widetilde{Q}N_3^T & \widetilde{Q}N_2^T & \Theta_{15} \end{bmatrix}$, $\Lambda_{22} = \mathrm{diag}\{-\epsilon_1 I, -\epsilon_2 I, -\epsilon_3 I\}$, and $\Lambda_{33} = \begin{bmatrix} \Theta_{55} & \widetilde{Q}N_3^T \\ N_3\widetilde{Q} & -\epsilon_3 I \end{bmatrix}$. Let $K\widetilde{Q} = L$, and the right side of (59) becomes (49), which guarantees $Z < 0$. Using the same procedure, (50) guarantees $\overline{Z} < 0$.

On the other hand, it is easy to check that (51) and (52) can guarantee (31), and (53) can guarantee (32).

So, according to Lemma 10, system (20) is FTAD-stable with respect to $(c_1, c_2, c_3, c_4, T, R)$. □

In the special case, when $\Delta K = 0$, Theorem 11 reduces the following corollary.

Corollary 12. *If there exist scalars $\alpha \ge 0$, $\beta \ge 0$, and positive scalars ϵ_1, ϵ_2, λ_1, λ_2, a symmetric positive definite Q, and a matrix L such that* (51)–(53) *and*

$$\begin{gathered}\begin{bmatrix} \Delta_{11} & \Delta_{12} & 0 \\ * & -\epsilon_2 I & 0 \\ * & * & \Theta_{55}^* \end{bmatrix} < 0, \\ \begin{bmatrix} \Phi_{11}^* & \Delta_{12} & 0 \\ * & -\epsilon_2 I & 0 \\ * & * & \Theta_{55}^* \end{bmatrix} < 0,\end{gathered} \tag{60}$$

then system (20) *is FTAD-stable with respect to* $(c_1, c_2, c_3, c_4, T, R)$, *where* $\Delta_{11} = \Upsilon_{11} + 2\rho\widetilde{Q} - \alpha\widetilde{Q}$, $\Phi_{11}^* = \beta\widetilde{Q} - \Upsilon_{11}$, $\Delta_{12} = \begin{bmatrix} N_1\widetilde{Q} & \widetilde{Q}N_2^T & \Theta_{15}^* \end{bmatrix}$, $\Theta_{15}^* = \widetilde{Q}A_2^T + L^TB_2^T$, $\Theta_{55}^* = -\widetilde{Q} + \epsilon_2 MM^T$, $\Upsilon_{11} = \overline{\overline{A_{11}^*}}\widetilde{Q} + \widetilde{Q}\overline{\overline{A_{11}^*}}^T + B_1L + L^TB_1^T + \epsilon_1 MM^T$, *and* $\overline{\overline{A_{11}^*}} = A_\sigma + A_1$. *In this case, a desired controller gain is given by* $K = L\widetilde{Q}^{-1}$.

Remark 13. It is easy to see that the values of α and β determine the feasibility of Theorem 11 and Corollary 12. The procedure how to choose α and β is given in the next subsection.

Next, a double-parameter searching algorithm is given to solve the matrix inequalities in Theorem 11. Similar algorithm can be applied to Corollary 12.

Algorithm 14. Step 1. Give c_1, c_2, c_3, c_4, T, and R.
Step 2. Take a series of α_i $(i = 1, \ldots, n)$ by a step size d_1 and a series of β_j $(j = 1, \ldots, m)$ by a step size d_2.
Step 3. Set $i = 1$, and take a α_i.
Step 4. Set $j = 1$, and take a β_j.
Step 5. If (α_i, β_j) makes (49)–(53) have feasible solutions, then store (α_i, β_j) into $(X(i), Y(j))$ and $\beta_j = \beta_{j+1}$ and go to Step 5; otherwise, go to Step 6.
Step 6. If $i + 1 < n$, then $\alpha_i = \alpha_{i+1}$ and take β_1 and go to Step 5. Otherwise, go to Step 7.
Step 7. Stop. If $(X, Y) = (0, 0)$, then we cannot find (α, β) which makes (49)–(53) have feasible solution; otherwise, there exists (α, β) which makes (49)–(53) have feasible solution.

Remark 15. By Algorithm 14, we can obtain a region surrounded by α and β, if it exists, which is used to select α and β for appropriate conditions.

4. Numerical Example

In this section, we provide an illustrative example to demonstrate the effectiveness and advantages of the proposed method.

Example 1. Consider stochastic nonlinear system (1) with

$$A_1 = \begin{bmatrix} -11 & 2 \\ 1 & -15 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.3 \\ 0.4 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.4 & 0.5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.5 \\ -0.4 \end{bmatrix},$$
$$M = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad N_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}, \tag{61}$$
$$N_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.15 \end{bmatrix}, \qquad f(x(t)) = \begin{bmatrix} e^{x_1(t)} \sin x_1(t) \\ 0 \end{bmatrix}.$$

The initial value is taken as $x(0) = [1.5 - 1.2]^T$ and the parameters $c_1 = 1$, $c_2 = 25$, $c_3 = 3$, $c_4 = 5$, $T = 0.25$, and $R = I$ are given. Now, we consider a single hidden layer neural network with three hidden neurons to approximate the nonlinear function $e^{x(t)} \sin x(t)$. All parameters of activation functions (5) associated with the hidden layer are chosen to be $q_j = 0.5$, $\chi_j = 1$. For these activation functions, we have $h_j(0, \psi_j) = 0$, $h_j(1, \psi_j) = 1$. The connection weights are trained offline by using the back propagation algorithm. The initial weights and state vector are placed by uniformly distributed random numbers in $[-1 \;\; 1]$. After 1000 training steps, the optimal approximation weights are as follows:

$$W_1^* = [3.3892 \;\; -4.4106 \;\; -4.4786]^T,$$
$$W_2^* = [3.9513 \;\; -1.4257 \;\; -0.5506]. \tag{62}$$

The upper bound of approximation error is estimated as $\rho = 2 \times 10^{-5}$. Obviously, in this case, we have $\Theta = 2^3 \times 2^1$. According to (13), A_σ can be obtained as follows:

$$A_1 = A_2 = A_3 = A_4 = A_5$$
$$= A_6 = A_7 = A_8 = A_9$$
$$= A_{1\oplus[0,0,0]^T} = A_{0\oplus[i,j,k]^T} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$(i, j, k \in \{0, 1\})$$
$$A_{10} = A_{0\oplus[1,0,0]^T} = \begin{bmatrix} 13.3917 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{11} = A_{0\oplus[0,1,0]^T} = \begin{bmatrix} 6.2882 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{12} = A_{0\oplus[0,0,1]^T} = \begin{bmatrix} 2.4659 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{13} = A_{0\oplus[1,1,0]^T} = \begin{bmatrix} 19.68 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{14} = A_{0\oplus[1,0,1]^T} = \begin{bmatrix} 15.8576 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{15} = A_{0\oplus[0,1,1]^T} = \begin{bmatrix} 8.7541 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{16} = A_{0\oplus[1,1,1]^T} = \begin{bmatrix} 22.1548 & 0 \\ 0 & 0 \end{bmatrix}. \tag{63}$$

Here, we design the following resilient state feedback controller:

$$u(t) = [k_1 + \Delta k_1(t) \;\; k_2 + \Delta k_2(t)] \, x(t), \tag{64}$$

where k_1 and k_2 will be determined and $-0.5 \le \Delta k_1(t) \le 0.5$ and $-0.5 \le \Delta k_2(t) \le 0.5$ represent some variations in the gains of the controller. Then, we have

$$D_3 = [1 \;\; 1],$$
$$F(t) = \begin{bmatrix} 2\Delta k_1(t) & 0 \\ 0 & 2\Delta k_2(t) \end{bmatrix}, \tag{65}$$
$$N_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

Applying Algorithm 14 to Theorem 11, a region surrounded by α and β is obtained, which is illustrated by Figure 1. Selecting $\alpha = 4$, $\beta = 0$ and solving (49)–(53), we get

$$Q = \begin{bmatrix} 0.0769 & 0.0116 \\ 0.0116 & 0.1178 \end{bmatrix}, \qquad \epsilon_1 = 0.3291, \qquad \epsilon_2 = 0.3679,$$
$$\epsilon_3 = 0.5085, \qquad \epsilon_4 = 0.2028, \qquad \lambda_1 = 0.0699,$$
$$\lambda_2 = 0.1247, \qquad L = [0.2373 \;\; 0.3035],$$
$$K = [2.7388 \;\; 2.3069]. \tag{66}$$

Therefore, the following resilient state feedback controller,

$$u(t) = [2.7388 + \Delta k_1(t) \;\; 2.3069 + \Delta k_2(t)] \, x(t) \tag{67}$$

is obtained.

When $\Delta K = 0$, a nonresilient controller will be obtained. Applying Algorithm 14 to Corollary 12, a region surrounded by α and β is obtained, which is illustrated by Figure 2. Selecting $\alpha = 4$, $\beta = 0$ and solving (51)–(53) and (60), we get

$$Q = \begin{bmatrix} 2.9461 & 0.4550 \\ 0.4550 & 4.5492 \end{bmatrix}, \qquad \epsilon_1 = 11.9272,$$
$$\epsilon_2 = 13.2296,$$
$$\lambda_1 = 2.7195, \qquad \lambda_2 = 4.8390, \tag{68}$$
$$L = [8.5249 \;\; 12.2147],$$
$$K = [2.5178 \;\; 2.4332].$$

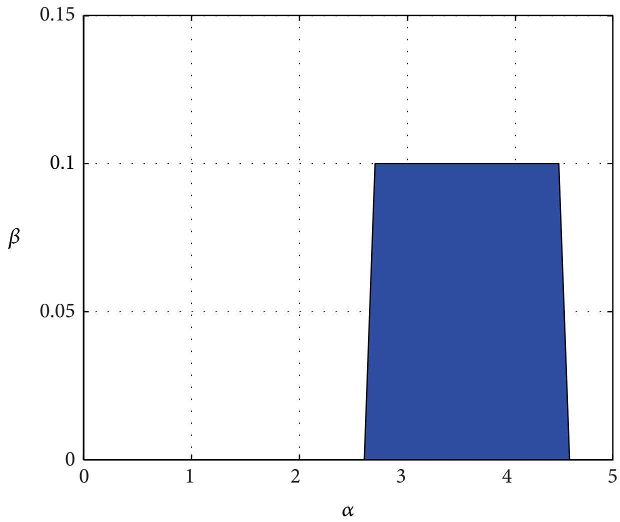

Figure 1: A region by α and β.

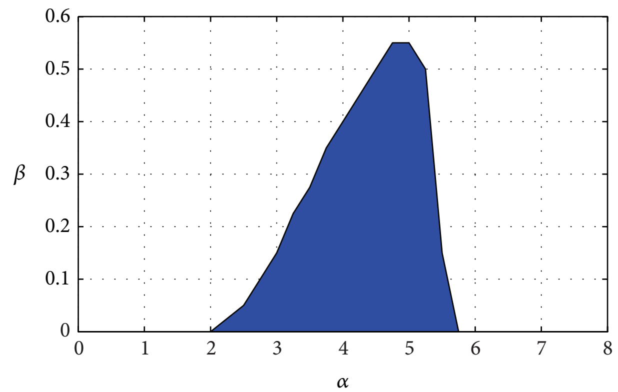

Figure 2: A region by α and β.

So, the following nonresilient state feedback controller,

$$u(t) = [2.5178 \quad 2.4332]\, x(t) \tag{69}$$

is obtained.

Next, a concrete response of closed-loop system of (1) is presented in the resilient control design case. When $F(t) = I$, the parameter perturbations are specific for $\Delta A_1 = MN_1$, $\Delta A_2 = MN_2$, and $\Delta K = D_3N_3$. We, respectively, apply resilient controller (67) and nonresilient controller (69) to system (1). The evolutions of $\mathbb{E}[x^T(t)Rx(t)]$ of closed-loop system (20) are obtained, which show that the closed-loop system of (1) is FTAD-stable with respect to $(1, 3, 5, 25, 0.25, I)$. The evolution of $\mathbb{E}[x^T(t)Rx(t)]$ using resilient controller is lower than that using nonresilient controller in Figure 3, which shows that resilient controller is superior to nonresilient controller.

5. Conclusion

In this study, we have studied the problem of resilient controller design for a class of stochastic nonlinear systems.

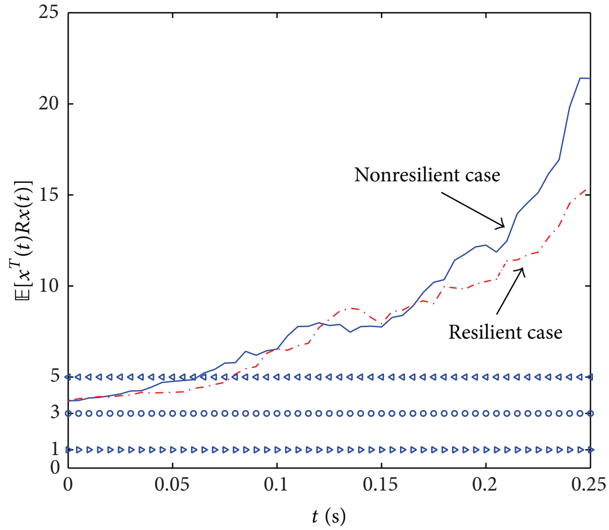

Figure 3: The response of system (1) for $\mathbb{E}[x^T(t)Rx(t)]$.

Some sufficient conditions for the existence of resilient state feedback finite-time stabilizing controller have been obtained, which are expressed in terms of matrix inequalities. A double-parameter searching algorithm is proposed to solve these obtained matrix inequalities. One example is presented to illustrate the effectiveness of the proposed results. In addition, we can also refer to [24–27] and extend the results of this paper to networked systems, Markovian jumping systems, sampled nonlinear systems, and so on.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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