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Research Article

Iterative Algorithms for New General Systems of Set-Valued Variational Inclusions Involving (A, η)-Maximal Relaxed Monotone Operators

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We introduce and study a class of new general systems of set-valued variational inclusions involving (A, η)-maximal relaxed monotone operators in Hilbert spaces. By using the general resolvent operator technique associated with (A, η)-maximal relaxed monotone operators, we construct some new iterative algorithms for finding approximation solutions to the general system of set-valued variational inclusion problem and prove the convergence of this algorithm. Our results improve and extend some known results.

1. Introduction

It is well known that variational inequalities and variational inclusions, which have been extended and generalized in different directions by using novel and innovative techniques and ideas, provide mathematical models to some problems arising in economics, mechanics, engineering science, and other pure and applied sciences. Among these methods, the resolvent operator technique is very important. See, for example, [1–17] and the references therein.

Recently, Huang and Fang [18] introduced a system of order complementarity problems and established some existence results for the system using fixed-point theory. Verma [19] introduced and studied some systems of the system variational inequalities and developed some iterative algorithms for approximating the solutions of the systems of variational inequalities. Cho et al. [20] introduced and studied a new system of nonlinear variational inequalities in Hilbert spaces. The authors also proved some existence and uniqueness theorems of solutions for the system and also constructed an iterative algorithm for approximating the solution of the system of nonlinear variational inequalities. Further, Fang et al. [1], Yan et al. [2], Fang and Huang [3], and Cao [4] considered some new systems of variational inclusions involving H-monotone operators and (H, η)-monotone operators in Hilbert space, respectively. Using the corresponding resolvent operator associated with H-monotone operators and (H, η)-monotone operators, the authors proved the existence of solutions for these new systems of variational inclusions and constructed a new algorithm for approximating the solution of this system and discussed the convergence of the sequence of iterations generated by the algorithm.

Very recently, Lan et al. [5] and Peng and Zhao [7] introduced and studied a new system of nonlinear A-monotone multivalued variational inclusions in Hilbert spaces, respectively. By using the concept and properties of A-monotone operators and the resolvent operator technique associated with A-monotone operators due to Verma [8], the author constructed a new iterative algorithm for solving this system of nonlinear multivalued variational inclusions associated with A-monotone operators in Hilbert spaces and proved the existence of solutions for the nonlinear multivalued variational inclusion systems and the convergence of iterative sequences generated by the algorithm. For more details, see, for example, [1–5, 7, 8, 10–19, 21–25].

On the other hand, Lan [6] first introduced a new concept of (A,η)-monotone (the so-called (A,η)-maximal relaxed monotone [9]) operators, which generalizes the (H,η)-monotonicity, A-monotonicity, and other existing monotone operators as special cases, and studied some properties of (A,η)-monotone operators and defined resolvent operators associated with (A,η)-monotone operators.

Inspired and motivated by the above works, the purpose of this paper is to consider the following new general system of set-valued variational inclusions involving relative (A,η)-maximal monotone operators in Hilbert spaces: find $(x_1^*, x_2^*, \ldots, x_m^*) \in H_1 \times H_2 \times \cdots \times H_m$ and $u_{ij} \in U_{ij}(x_j^*)$ such that

$$0 \in F_i\left(u_{i1}, \ldots, u_{ii-1}, x_i^*, u_{ii+1}, \ldots, u_{im}\right) + M_i\left(x_i^*\right), \tag{1}$$

where m is a given positive integer, $F_i : H_1 \times H_2 \times \cdots \times H_m \to H_i$, $A_i : H_i \to H_i$, and $\eta_i : H_i \times H_i \to H_i$ are single-valued operators, $U_{ij} : H_j \to 2^{H_j}$ is a set-valued operator, $M_i : H_i \to 2^{H_i}$ is (A_i, η_i)-maximal relaxed monotone, and $i, j = 1, 2, \ldots, m$ and $i \neq j$.

Some special cases of the problem (1) had been studied by many authors. Here, we mention some of them as follows.

Case 1. If $m = 2$, then the problem (1) reduces to the problem of finding $(x_1^*, x_2^*) \in H_1 \times H_2$ and $u_1 \in U_1(x_1^*)$ and $u_2 \in U_2(x_2^*)$ such that

$$\begin{gathered} 0 \in F_1\left(x_1^*, u_2\right) + M_1\left(x_1^*\right), \\ 0 \in F_2\left(u_1, x_2^*\right) + M_2\left(x_2^*\right). \end{gathered} \tag{2}$$

The problem (2) is called a nonlinear set-valued variational inclusion system problem, which was considered and studied by Agarwal and Verma [9].

Case 2. When $m = 2$ and $M_i(x_i) = \partial\varphi_i(x_i)$, for all $x_i \in H_i$, $i = 1, 2$, where $\varphi_i : H_i \to R \cup \{+\infty\}$ is proper, convex, and lower semicontinuous functional and $\partial\varphi_i$ denotes the subdifferential operator of φ_i, then problem (1) becomes the following system of set-valued mixed variational inequalities: find $(x_1^*, x_2^*) \in H_1 \times H_2$ and $u_1 \in U_1(x_1^*)$ and $u_2 \in U_2(x_2^*)$ such that

$$\begin{gathered} \left\langle F_1\left(x_1^*, u_2\right), x - x_1^*\right\rangle + \varphi_1(x) - \varphi_1\left(x_1^*\right) \geq 0, \quad \forall x \in H_1, \\ \left\langle F_2\left(u_1, x_2^*\right), y - x_2^*\right\rangle + \varphi_2(y) - \varphi_2\left(x_2^*\right) \geq 0, \quad \forall y \in H_2. \end{gathered} \tag{3}$$

If $U_1 = U_2 \equiv I$, the identity operator, then the problem (3) reduces to the following problem of finding $(x_1^*, x_2^*) \in H_1 \times H_2$ such that

$$\begin{gathered} \left\langle F_1\left(x_1^*, x_2^*\right), x - x_1^*\right\rangle + \varphi_1(x) - \varphi_1\left(x_1^*\right) \geq 0, \quad \forall x \in H_1, \\ \left\langle F_2\left(x_1^*, x_2^*\right), y - x_2^*\right\rangle + \varphi_2(y) - \varphi_2\left(x_2^*\right) \geq 0, \quad \forall y \in H_2, \end{gathered} \tag{4}$$

which is called the system of nonlinear variational inequalities considered by Cho et al. [20]. Some specializations of problem (4) are dealt by Kim and Kim [21].

Case 3. If $m = 2$ and $U_1 = U_2 \equiv I$, then the problem (1) reduces to finding $(x_1^*, x_2^*) \in H_1 \times H_2$ such that

$$\begin{gathered} 0 \in F_1\left(x_1^*, x_2^*\right) + M_1\left(x_1^*\right), \\ 0 \in F_2\left(x_1^*, x_2^*\right) + M_2\left(x_2^*\right), \end{gathered} \tag{5}$$

which was considered by Fang et al. [1].

In brief, the problem (1) is the most general and unifying system form, so long as, for appropriate and suitable choices of positive integer m and operators F_i, A_i, η_i, M_i, and U_{ij} for $i, j = 1, 2, \ldots, m$, one can know that the problem (1) includes a number of known general problems of variational character, including variational inequality (system) problems and variational inclusion (system) problems as special cases. For more details, see [1–5, 7–25] and the reference therein.

Furthermore, in this paper, we will construct some new iterative algorithms to approximate the solution of the general system of set-valued variational inclusions and prove the convergence of the sequences generated by the algorithms in Hilbert spaces.

2. Preliminaries

Thereafter, let H, H_i $(i = 1, 2, \ldots, m)$ be real Hilbert spaces endowed with the norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$. Let 2^H and $C(H)$ denote the family of all the nonempty subsets of H and the family of all closed subsets of H, respectively.

In order to get the main results of the paper, we need the following concepts and lemmas.

Definition 1. Let $A : H \to H$ be a single-valued operator. Then the map A is said to be

(i) α-strongly monotone, if there exists a constant $\alpha > 0$ such that

$$\langle A(x) - A(y), x - y\rangle \geq \alpha\|x - y\|^2, \quad \forall x, y \in H; \tag{6}$$

(ii) β-Lipschitz continuous, if there exists a constant $\beta > 0$ such that

$$\|Ax - Ay\| \leq \beta\|x - y\|, \quad \forall x, y \in H. \tag{7}$$

Definition 2. Let $\eta : H \times H \to H$ and $A : H \to H$ be single-valued operators; let $M : H \to 2^H$ be set-valued operator. Then

(i) η is said to be τ-Lipschitz continuous, if there exists a constant $\tau > 0$ such that

$$\|\eta(x, y)\| \leq \tau\|x - y\|, \quad \forall x, y \in H; \tag{8}$$

(ii) A is said to be η-monotone, if

$$\langle A(x) - A(y), \eta(x, y)\rangle \geq 0, \quad \forall x, y \in H; \tag{9}$$

(iii) A is said to be strictly η-monotone, if A is η-monotone and

$$\langle A(x) - A(y), \eta(x, y)\rangle = 0 \quad \text{iff } x = y; \tag{10}$$

(iv) A is said to be (r,η)-strongly monotone, if there exists a constant $r>0$ such that

$$\langle A(x)-A(y),\eta(x,y)\rangle \geq r\|x-y\|^2, \quad \forall x,y\in H; \tag{11}$$

(v) M is said to be (m,η)-relaxed monotone, if there exists a constant $m>0$ such that

$$\langle u-v,\eta(x,y)\rangle \geq (-m)\|x-y\|^2, \quad \forall x,y\in H, \quad u\in M(x), \quad v\in M(y); \tag{12}$$

(vi) M is said to be (A,η)-maximal relaxed monotone, if M is (m,η)-relaxed monotone and

$$(A+\lambda M)(H)=H, \quad \forall\lambda>0. \tag{13}$$

Definition 3. For $i=1,2,\ldots,m$, let H_i be a Hilbert space and let $A_i : H_i \to H_i$ be a single-valued operator; then nonlinear operator $F_i : H_1\times H_2\times\cdots\times H_m \to H_i$ is said to be

(i) μ_i-strongly monotone with respect to A_i in the ith argument, if there exist constants $\mu_i>0$ such that, for $x_i^1, x_i^2\in H_i$, $i=1,2,\ldots,m$,

$$\left\langle F_i\left(\ldots,x_i^1,\ldots\right)-F_i\left(\ldots,x_i^2,\ldots\right), A_i\left(x_i^1\right)-A_i\left(x_i^2\right)\right\rangle \geq \mu_i\left\|x_i^1-x_i^2\right\|^2; \tag{14}$$

(ii) (c_i,μ_i)-relaxed cocoercive with respect to A_i (or relative (c_i,μ_i)-relaxed cocoercive) in the ith argument, if there exist constants $c_i,\mu_i>0$ such that, for $x_i^1,x_i^2\in H_i$, $i=1,2,\ldots,m$,

$$\left\langle F_i\left(\ldots,x_i^1,\ldots\right)-F_i\left(\ldots,x_i^2,\ldots\right), A_i\left(x_i^1\right)-A_i\left(x_i^2\right)\right\rangle \geq (-c_i)\left\|F_i\left(\ldots,x_i^1,\ldots\right)-F_i\left(\ldots,x_i^2,\ldots\right)\right\|^2+\mu_i\left\|x_i^1-x_i^2\right\|^2; \tag{15}$$

(iii) ζ_{ij}-Lipschitz continuous in the jth argument, if there exists constant $\zeta_{ij}>0$ such that, for $x_j,y_j\in H_j$, $j=1,2,\ldots,m$,

$$\left\|F_i\left(x_1,\ldots,x_{j-1},x_j,x_{j+1},\ldots,x_m\right) - F_i\left(x_1,\ldots,x_{j-1},y_j,x_{j+1},\ldots,x_m\right)\right\| \leq \zeta_{ij}\left\|x_j-y_j\right\|. \tag{16}$$

Remark 4. When $m=1$, then Definition 3 reduces to the corresponding concept of the relative strong monotonicity, relative relaxed cocoercive, and Lipschitz continuity.

Definition 5. Let $\eta : H\times H \to H$ be a single-valued operator, let $A : H \to H$ be a strictly η-monotone operator, and let $M : H \to 2^H$ be an (A,η)-maximal relaxed monotone operator. Then general resolvent operator $R_{M,\lambda}^{A,\eta} : H \to H$ is defined by

$$R_{M,\lambda}^{A,\eta}(z)=(A+\lambda M)^{-1}(z), \quad \forall z\in H, \tag{17}$$

where $\lambda>0$ is a constant.

Lemma 6 (see [6]). *Let $\eta : H\times H \to H$ be a τ-Lipschitz continuous operator, let $A : H \to H$ be an (r,η)-strongly monotone operator, and let $M : H \to 2^H$ be an (A,η)-maximal relaxed monotone operator. Then general resolvent operator $R_{M,\lambda}^{A,\eta} : H \to H$ is $\tau/(r-\lambda m)$-Lipschitz continuous; that is,*

$$\left\|R_{M,\lambda}^{A,\eta}(x)-R_{M,\lambda}^{A,\eta}(y)\right\| \leq \frac{\tau}{r-\lambda m}\|x-y\|, \quad \forall x,y\in H, \tag{18}$$

where $r-\lambda m>0$.

Next, we define the Hausdorff pseudometric $D : C(H)\times C(H) \to \mathbb{R}\cup\{+\infty\}$ as follows:

$$D(U,V)=\max\left\{\sup_{x\in U}\inf_{y\in V}\|x-y\|, \sup_{y\in V}\inf_{x\in U}\|x-y\|\right\}, \quad \forall U,V\in C(H). \tag{19}$$

Note that if $C(H)$ is restricted to closed bounded subsets of the family $CB(H)$, then the Hausdorff pseudometric D reduces to Hausdorff metric $\widehat{H} : CB(H)\times CB(H) \to \mathbb{R}$ defined by

$$\widehat{H}(U,V)=\max\left\{\sup_{x\in U}\inf_{y\in V}\|x-y\|, \sup_{y\in V}\inf_{x\in U}\|x-y\|\right\}, \quad \forall U,V\in CB(H). \tag{20}$$

Definition 7. A set-valued operator $U : H \to 2^H$ is said to be D-γ-Lipschitz continuous if there exists a constant $\gamma>0$ such that

$$D(U(x),U(y)) \leq \gamma\|x-y\|, \quad \forall x,y\in H. \tag{21}$$

Lemma 8. *Let θ be a constant and $0<\omega<1$; then function $f(\omega)=1-\omega+\omega\theta$, for $\omega\in[0,1]$, is nonnegative and strictly decreases and $f(\omega)\in[0,1]$. Further, if $\omega\neq 0$, then $f(\omega)\in(0,1)$.*

Proof. Since $f(\omega)$ is linear function, the conclusions immediately hold. □

3. Iterative Algorithm and Convergence

In this section, we first prove the equivalence between the problem (1) and the problem of finding the fixed points of the general resolvent operator $R_{M,\lambda}^{A,\eta}$ associated with (A,η)-maximal relaxed monotone operators. This equivalence is quite general and very important from a numerical point of view. Then, by using the equivalence, some new iterative algorithms for finding the approximation solutions of the problem (1) are analyzed. Further, the convergence criteria for the algorithms are also discussed.

Lemma 9. *Let $(x_1^*,x_2^*,\ldots,x_m^*)\in H_1\times H_2\times\cdots\times H_m$ and $u_{ij}\in U_{ij}(x_j^*)$ $(i,j=1,2,\ldots,m,\ j\neq i)$; then*

$(x_1^*, x_2^*, \ldots, x_m^*, u_{12}, \ldots, u_{1m}, \ldots, u_{m1}, \ldots, u_{mm-1})$ *(denoted by* $(*)$*) is a solution of the problem* (1) *if and only if* $(*)$ *satisfy*

$$x_i^* = R_{M_i,\rho_i}^{A_i,\eta_i}\left[A_i\left(x_i^*\right) - \rho_i F_i\left(u_{i1}, \ldots, u_{ii-1}, x_i^*, u_{ii+1}, \ldots, u_{im}\right)\right], \tag{22}$$

where $R_{M_i,\rho_i}^{A_i,\eta_i} = (A_i + \rho_i M_i)^{-1}$ *and* $\rho_i > 0$ *is a constant, for* $i = 1, 2, \cdots, m$.

Proof. Let $(*)$ satisfy the relation (22). By Definition 5 of general resolvent operator, the equality (22) holds if and only if

$$\begin{aligned} A_i\left(x_i^*\right) - \rho_i F_i\left(u_{i1}, \ldots, u_{ii-1}, x_i^*, u_{ii+1}, \ldots, u_{im}\right) \\ \in \left(A_i + \rho_i M_i\right)\left(x_i^*\right); \end{aligned} \tag{23}$$

that is,

$$0 \in F_i\left(u_{i1}, \ldots, u_{ii-1}, x_i^*, u_{ii+1}, \ldots, u_{im}\right) + M_i\left(x_i^*\right), \tag{24}$$

where $i = 1, 2, \ldots, m$. Hence $(*)$ are the solution of the problem (1). This completes the proof. □

By using formula (22) and Nadler [26], we can develop the following new iterative algorithms.

Algorithm 10. Consider the following.

Step 1. Choose $(x_1^0, x_2^0, \ldots, x_m^0) \in H_1 \times H_2 \times \cdots \times H_m$ and $u_{ij}^0 \in U_{ij}(x_j^0)$ for $i, j = 1, 2, \ldots, m,\ j \neq i$.

Step 2. Let

$$\begin{aligned} x_i^{n+1} = {} & \left(1 - \lambda_n - \delta_n\right) x_i^n \\ & + \lambda_n R_{M_i,\rho_i}^{A_i,\eta_i}\left[A_i\left(x_i^n\right)\right. \\ & \left. -\rho_i F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right)\right], \end{aligned} \tag{25}$$

for all $i = 1, 2, \ldots, m$ and $n = 0, 1, 2, \ldots$, where λ_n and δ_n are nonnegative constants such that $0 < \lambda_n + \delta_n \leq 1$ and $\liminf_{n \geq 0} \lambda_n > 0$.

Step 3. Choose $u_{ij}^{n+1} \in U_{ij}(x_j^{n+1})$ $(i, j = 1, 2, \ldots, m,\ j \neq i)$ such that

$$\left\|u_{ij}^{n+1} - u_{ij}^n\right\| \leq \left(1 + \frac{1}{n+1}\right) D_j\left(U_{ij}\left(x_j^{n+1}\right), U_{ij}\left(x_j^n\right)\right), \tag{26}$$

where $D_j(\cdot, \cdot)$ is the Hausdorff pseudometric on $C(H_j)$.

Step 4. If x_i^{n+1} and u_{ij}^{n+1} $(i, j = 1, 2, \ldots, m)$ satisfy (25) to sufficient accuracy, stop. Otherwise, set $n := n + 1$ and return to Step 2.

Algorithm 11. Consider the following.

Step 1. Choose $(x_1^0, x_2^0, \ldots, x_m^0) \in H_1 \times H_2 \times \cdots \times H_m$ and $u_{ij}^0 \in U_{ij}(x_j^0)$, for $i, j = 1, 2, \ldots, m,\ j \neq i$.

Step 2. Let

$$\begin{aligned} x_i^{n+1} = {} & (1 - \lambda - \delta) x_i^n \\ & + \lambda R_{M_i,\rho_i}^{A_i,\eta_i}\left[A_i\left(x_i^n\right)\right. \\ & \left. -\rho_i F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right)\right], \end{aligned} \tag{27}$$

for all $i = 1, 2, \ldots, m$ and $n = 0, 1, 2, \ldots$, where λ and δ are nonnegative constants such that $0 < \lambda + \delta \leq 1$.

Step 3. Choose $u_{ij}^{n+1} \in U_{ij}(x_j^{n+1})$ $(i, j = 1, 2, \ldots, m,\ j \neq i)$ such that

$$\left\|u_{ij}^{n+1} - u_{ij}^n\right\| \leq \left(1 + \frac{1}{n+1}\right) D_j\left(U_{ij}\left(x_j^{n+1}\right), U_{ij}\left(x_j^n\right)\right), \tag{28}$$

where $D_j(\cdot, \cdot)$ is the Hausdorff pseudometric on $C(H_j)$.

Step 4. If x_i^{n+1} and u_{ij}^{n+1} $(i, j = 1, 2, \ldots, m)$ satisfy (27) to sufficient accuracy, stop. Otherwise, set $n := n + 1$ and return to Step 2.

Algorithm 12. Consider the following.

Step 1. Choose $(x_1^0, x_2^0, \ldots, x_m^0) \in H_1 \times H_2 \times \cdots \times H_m$ and $u_{ij}^0 \in U_{ij}(x_j^0)$, for $i, j = 1, 2, \ldots, m,\ j \neq i$.

Step 2. Let

$$\begin{aligned} x_i^{n+1} = {} & (1 - \lambda) x_i^n \\ & + \lambda R_{M_i,\rho_i}^{A_i,\eta_i}\left[A_i\left(x_i^n\right)\right. \\ & \left. - \rho_i F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right)\right], \end{aligned} \tag{29}$$

for all $i = 1, 2, \ldots, m$ and $n = 0, 1, 2, \ldots$, where λ is a nonnegative constant such that $0 < \lambda \leq 1$.

Step 3. Choose $u_{ij}^{n+1} \in U_{ij}(x_j^{n+1})$ $(i, j = 1, 2, \ldots, m,\ j \neq i)$ such that

$$\left\|u_{ij}^{n+1} - u_{ij}^n\right\| \leq \left(1 + \frac{1}{n+1}\right) D_j\left(U_{ij}\left(x_j^{n+1}\right), U_{ij}\left(x_j^n\right)\right), \tag{30}$$

where $D_j(\cdot, \cdot)$ is the Hausdorff pseudometric on $C(H_j)$.

Step 4. If x_i^{n+1} and u_{ij}^{n+1} $(i, j = 1, 2, \ldots, m)$ satisfy (29) to sufficient accuracy, stop. Otherwise, set $n := n + 1$ and return to Step 2.

Remark 13. Let $m = 2$; then Algorithms 10–12 reduce to Algorithms 4.1–4.3 of Agarwal and Verma [9], respectively.

Now, we provide the main results concerning problem (1) with respect to Algorithms 10–12.

Theorem 14. *For $i = 1, 2, \ldots, m$, let $\eta_i : H_i \times H_i \to H_i$ be τ_i-Lipschitz continuous operator, let $A_i : H_i \to H_i$ be β_i-Lipschitz continuous and (r_i, η_i)-strongly monotone operator, and let $M_i : H_i \to 2^{H_i}$ be (A_i, η_i)-maximal relaxed monotone operator. Suppose that $U_{ij} : H_j \to C(H_j)$ is D_j-γ_{ij}-Lipschitz continuous for $j = 1, 2, \ldots, m$ and $j \neq i$ and nonlinear operator $F_i : H_1 \times H_2 \times \cdots \times H_m \to H_i$ is (c_i, μ_i)-relaxed cocoercive with respect to A_i in the ith argument and ζ_{ij}-Lipschitz continuous in the jth argument for $j = 1, 2, \ldots, m$. If there exists constant $\rho_i > 0$, for $i = 1, 2, \ldots, m$, such that*

$$\frac{\tau_j}{r_j - \rho_j m_j} \cdot \sqrt{\beta_j^2 - 2\rho_j\mu_j + 2\rho_j c_j \zeta_{jj}^2 + \rho_j^2 \zeta_{jj}^2} + \sum_{i=1, i\neq j}^{m} \frac{\rho_i \tau_i \zeta_{ij} \gamma_{ij}}{r_i - \rho_i m_i} < 1, \tag{31}$$

where $r_j - \rho_j m_j > 0$ for $j = 1, 2, \ldots, m$, then the problem (1) admits a solution $(x_1^*, x_2^*, \ldots, x_m^*, u_{12}, \ldots, u_{1m}, \ldots, u_{m1}, \ldots, u_{mm-1})$ (in short, $(*)$), where $x_i^* \in H_i$ and $u_{ij} \in U_{ij}(x_j^*)$, for any $i, j = 1, 2, \ldots, m$, $j \neq i$, and sequences $\{x_j^n\}$ and $\{u_{ij}^n\}$ generated by Algorithm 10 strongly converge to x_j^* and u_{ij} $(i, j = 1, 2, \ldots, m,\ j \neq i)$, respectively.

Proof. For $i = 1, 2, \ldots, m$, applying Algorithm 10 and Lemma 6, we have

$$\begin{aligned}
&\left\|x_i^{n+1} - x_i^n\right\| \\
&\leq \left(1 - \lambda_n - \delta_n\right)\left\|x_i^n - x_i^{n-1}\right\| \\
&\quad + \lambda_n \Big\| R_{M_i,\rho_i}^{A_i,\eta_i}\left[A_i\left(x_i^n\right) - \rho_i F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right)\right] \\
&\qquad - R_{M_i,\rho_i}^{A_i,\eta_i}\left[A_i\left(x_i^{n-1}\right) - \rho_i F_i\left(u_{i1}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^{n-1}, \ldots, u_{im}^{n-1}\right)\right]\Big\| \\
&\leq \left(1 - \lambda_n\right)\left\|x_i^n - x_i^{n-1}\right\| \\
&\quad + \frac{\lambda_n \tau_i}{r_i - \rho_i m_i}\Big\|A_i\left(x_i^n\right) - A_i\left(x_i^{n-1}\right) \\
&\qquad - \rho_i\left[F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right)\right]\Big\| \\
&\quad + \frac{\lambda_n \tau_i \rho_i}{r_i - \rho_i m_i}\Big\|F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right) \\
&\qquad - F_i\left(u_{i1}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^{n-1}, \ldots, u_{im}^{n-1}\right)\Big\|.
\end{aligned} \tag{32}$$

Since A_i is β_i-Lipschitz continuous, F_i is (c_i, μ_i)-relaxed cocoercive with respect to A_i in the ith argument, and F_i is ζ_{ij}-Lipschitz continuous in the j-th argument for $j = 1, 2, \ldots, m$, then we have

$$\begin{aligned}
&\Big\|A_i\left(x_i^n\right) - A_i\left(x_i^{n-1}\right) \\
&\quad - \rho_i\left[F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right)\right]\Big\|^2 \\
&= \left\|A_i\left(x_i^n\right) - A_i\left(x_i^{n-1}\right)\right\|^2 \\
&\quad - 2\rho_i\Big\langle F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right), \\
&\qquad A_i\left(x_i^n\right) - A_i\left(x_i^{n-1}\right)\Big\rangle \\
&\quad + \rho_i^2\left\|F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right)\right\|^2 \\
&\leq \beta_i^2\left\|x_i^n - x_i^{n-1}\right\|^2 \\
&\quad - 2\rho_i\Big[\left(-c_i\right)\left\|F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^n, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right)\right\|^2 \\
&\qquad + \mu_i\left\|x_i^n - x_i^{n-1}\right\|^2\Big] + \rho_i^2\zeta_{ii}^2\left\|x_i^n - x_i^{n-1}\right\|^2 \\
&\leq \left(\beta_i^2 - 2\rho_i\mu_i + 2\rho_i c_i \zeta_{ii}^2 + \rho_i^2\zeta_{ii}^2\right)\left\|x_i^n - x_i^{n-1}\right\|^2.
\end{aligned} \tag{33}$$

By D_j-γ_{ij}-Lipschitz continuity of U_{ij} and (26), we get

$$\begin{aligned}
&\Big\|F_i\left(u_{i1}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^{n-1}, \ldots, u_{im}^{n-1}\right)\Big\| \\
&\leq \Big\|F_i\left(u_{i1}^n, u_{i2}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^{n-1}, u_{i2}^n, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right)\Big\| \\
&\quad + \cdots + \Big\|F_i\left(u_{i1}^{n-1}, u_{i2}^{n-1}, \ldots, u_{ii-1}^n, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^{n-1}, u_{i2}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right)\Big\| \\
&\quad + \Big\|F_i\left(u_{i1}^{n-1}, u_{i2}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^n, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^{n-1}, u_{i2}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^{n-1}, \ldots, u_{im}^n\right)\Big\| \\
&\quad + \cdots + \Big\|F_i\left(u_{i1}^{n-1}, u_{i2}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^{n-1}, \ldots, u_{im}^n\right) - F_i\left(u_{i1}^{n-1}, u_{i2}^{n-1}, \ldots, u_{ii-1}^{n-1}, x_i^{n-1}, u_{ii+1}^{n-1}, \ldots, u_{im}^{n-1}\right)\Big\|
\end{aligned}$$

$$\begin{aligned}
&\leq \zeta_{i1}\left\|u_{i1}^{n}-u_{i1}^{n-1}\right\|+\cdots+\zeta_{ii-1}\left\|u_{ii-1}^{n}-u_{ii-1}^{n-1}\right\| \\
&\quad+\zeta_{ii+1}\left\|u_{ii+1}^{n}-u_{ii+1}^{n-1}\right\|+\cdots+\zeta_{im}\left\|u_{im}^{n}-u_{im}^{n-1}\right\| \\
&=\sum_{j=1, j\neq i}^{m}\zeta_{ij}\left\|u_{ij}^{n}-u_{ij}^{n-1}\right\| \\
&\leq \sum_{j=1, j\neq i}^{m}\zeta_{ij}\left(1+\frac{1}{n}\right)D_{j}\left(U_{ij}\left(x_{j}^{n}\right),U_{ij}\left(x_{j}^{n-1}\right)\right) \\
&\leq\left(1+\frac{1}{n}\right)\sum_{j=1, j\neq i}^{m}\zeta_{ij}\gamma_{ij}\left\|x_{j}^{n}-x_{j}^{n-1}\right\|.
\end{aligned} \tag{34}$$

It follows from (32)–(34) that

$$\begin{aligned}
&\left\|x_{i}^{n+1}-x_{i}^{n}\right\| \\
&\quad\leq\left(1-\lambda_{n}\right)\left\|x_{i}^{n}-x_{i}^{n-1}\right\| \\
&\qquad+\frac{\lambda_{n}\tau_{i}}{r_{i}-\rho_{i}m_{i}}\left[\sqrt{\beta_{i}^{2}-2\rho_{i}\mu_{i}+2\rho_{i}c_{i}\zeta_{ii}^{2}+\rho_{i}^{2}\zeta_{ii}^{2}}\left\|x_{i}^{n}-x_{i}^{n-1}\right\|\right. \\
&\qquad\left.+\left(1+\frac{1}{n}\right)\rho_{i}\sum_{j=1, j\neq i}^{m}\zeta_{ij}\gamma_{ij}\left\|x_{j}^{n}-x_{j}^{n-1}\right\|\right],
\end{aligned} \tag{35}$$

which implies that

$$\begin{aligned}
\sum_{j=1}^{m}\left\|x_{j}^{n+1}-x_{j}^{n}\right\| &= \sum_{i=1}^{m}\left\|x_{i}^{n+1}-x_{i}^{n}\right\| \\
&\leq\sum_{i=1}^{m}\left[\left(1-\lambda_{n}\right)\left\|x_{i}^{n}-x_{i}^{n-1}\right\|\right. \\
&\qquad+\frac{\lambda_{n}\tau_{i}}{r_{i}-\rho_{i}m_{i}}\left(\sqrt{\beta_{i}^{2}-2\rho_{i}\mu_{i}+2\rho_{i}c_{i}\zeta_{ii}^{2}+\rho_{i}^{2}\zeta_{ii}^{2}}\right. \\
&\qquad\times\left\|x_{i}^{n}-x_{i}^{n-1}\right\|+\left(1+\frac{1}{n}\right)\rho_{i} \\
&\qquad\left.\left.\times\sum_{j=1, j\neq i}^{m}\zeta_{ij}\gamma_{ij}\left\|x_{j}^{n}-x_{j}^{n-1}\right\|\right)\right] \\
&=\sum_{i=1}^{m}\left[\left(1-\lambda_{n}\right)+\frac{\lambda_{n}\tau_{i}}{r_{i}-\rho_{i}m_{i}}\right. \\
&\qquad\left.\times\sqrt{\beta_{i}^{2}-2\rho_{i}\mu_{i}+2\rho_{i}c_{i}\zeta_{ii}^{2}+\rho_{i}^{2}\zeta_{ii}^{2}}\right]\left\|x_{i}^{n}-x_{i}^{n-1}\right\| \\
&\quad+\left(1+\frac{1}{n}\right)\lambda_{n}\sum_{i=1}^{m}\sum_{j=1, j\neq i}^{m}\frac{\rho_{i}\tau_{i}\zeta_{ij}\gamma_{ij}}{r_{i}-\rho_{i}m_{i}}\left\|x_{j}^{n}-x_{j}^{n-1}\right\| \\
&=\sum_{j=1}^{m}\left[\left(1-\lambda_{n}\right)+\frac{\lambda_{n}\tau_{j}}{r_{j}-\rho_{j}m_{j}}\right. \\
&\qquad\left.\times\sqrt{\beta_{j}^{2}-2\rho_{j}\mu_{j}+2\rho_{j}c_{j}\zeta_{jj}^{2}+\rho_{j}^{2}\zeta_{jj}^{2}}\right]\left\|x_{j}^{n}-x_{j}^{n-1}\right\| \\
&\quad+\left(1+\frac{1}{n}\right)\lambda_{n}\sum_{j=1}^{m}\sum_{i=1, i\neq j}^{m}\frac{\rho_{i}\tau_{i}\zeta_{ij}\gamma_{ij}}{r_{i}-\rho_{i}m_{i}}\left\|x_{j}^{n}-x_{j}^{n-1}\right\| \\
&=\sum_{j=1}^{m}\left[\left(1-\lambda_{n}\right)+\lambda_{n}\right. \\
&\qquad\times\left(\frac{\tau_{j}}{r_{j}-\rho_{j}m_{j}}\sqrt{\beta_{j}^{2}-2\rho_{j}\mu_{j}+2\rho_{j}c_{j}\zeta_{jj}^{2}+\rho_{j}^{2}\zeta_{jj}^{2}}\right. \\
&\qquad\left.\left.+\left(1+\frac{1}{n}\right)\sum_{i=1, i\neq j}^{m}\frac{\rho_{i}\tau_{i}\zeta_{ij}\gamma_{ij}}{r_{i}-\rho_{i}m_{i}}\right)\right]\left\|x_{j}^{n}-x_{j}^{n-1}\right\| \\
&=\sum_{j=1}^{m}\left[\left(1-\lambda_{n}\right)+\lambda_{n}\theta_{j}^{n}\right]\left\|x_{j}^{n}-x_{j}^{n-1}\right\| \\
&\leq\sum_{j=1}^{m}\left[\left(1-\omega\right)+\omega\theta_{j}^{n}\right]\left\|x_{j}^{n}-x_{j}^{n-1}\right\| \\
&\leq f_{n}\left(\omega\right)\sum_{j=1}^{m}\left\|x_{j}^{n}-x_{j}^{n-1}\right\|,
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
&\omega=\liminf_{n\geq 0}\lambda_{n}>0,\qquad f_{n}\left(\omega\right)=\max_{1\leq j\leq m}\left\{\left(1-\omega\right)+\omega\theta_{j}^{n}\right\}, \\
&\theta_{j}^{n}=\frac{\tau_{j}}{r_{j}-\rho_{j}m_{j}}\sqrt{\beta_{j}^{2}-2\rho_{j}\mu_{j}+2\rho_{j}c_{j}\zeta_{jj}^{2}+\rho_{j}^{2}\zeta_{jj}^{2}} \\
&\qquad+\left(1+\frac{1}{n}\right)\sum_{i=1, i\neq j}^{m}\frac{\rho_{i}\tau_{i}\zeta_{ij}\gamma_{ij}}{r_{i}-\rho_{i}m_{i}}.
\end{aligned} \tag{37}$$

By condition (31), we know sequence $\{\theta_{j}^{n}\}$ is monotonely decreasing and $\theta_{j}^{n}\rightarrow\theta_{j}$ as $n\rightarrow\infty$. Therefore,

$$f\left(\omega\right)=\lim_{n\rightarrow\infty}f_{n}\left(\omega\right)=\max_{1\leq j\leq m}\left\{\left(1-\omega\right)+\omega\theta_{j}\right\}. \tag{38}$$

Since $0<\theta_{j}<1$, for $j=1,2,\ldots,m$, we get $\theta=\max_{1\leq j\leq m}\{\theta_{j}\}\in(0,1)$. By Lemma 8, we have $f(\omega)=1-\omega+\omega\theta\in(0,1)$. From (36), it follows that $\{x_{j}^{n}\}$ are Cauchy sequences and there exists $x_{j}^{*}\in H_{j}$ such that $x_{j}^{n}\rightarrow x_{j}^{*}$ as $n\rightarrow\infty$, for $j=1,2,\ldots,m$.

Next, we show that $u_{ij}^{n}\rightarrow u_{ij}\in U_{ij}(x_{j}^{*})$ as $n\rightarrow\infty$ for $i,j=1,2,\ldots,m$ and $j\neq i$.

It follows from (34) that $\{u_{ij}^n\}$ are also Cauchy sequences. Hence, there exists $u_{ij} \in H_j$ such that $u_{ij}^n \to u_{ij}$ as $n \to \infty$, for $i, j = 1, 2, \ldots, m$, $j \neq i$. Furthermore,

$$\begin{aligned} d\left(u_{ij}, U_{ij}\left(x_j^*\right)\right) &= \inf\left\{\left\|u_{ij} - t\right\| : t \in U_{ij}\left(x_j^*\right)\right\} \\ &\leq \left\|u_{ij} - u_{ij}^n\right\| + d\left(u_{ij}^n, U_{ij}\left(x_j^*\right)\right) \\ &\leq \left\|u_{ij} - u_{ij}^n\right\| + D_j\left(U_{ij}\left(x_j^n\right), U_{ij}\left(x_j^*\right)\right) \\ &\leq \left\|u_{ij} - u_{ij}^n\right\| + \gamma_{ij}\left\|x_j^n - x_j^*\right\| \to 0 \quad (n \to \infty). \end{aligned} \tag{39}$$

Since $U_{ij}(x_j^*)$ is closed, we have $u_{ij} \in U_{ij}(x_j^*)$, for $j = 1, 2, \ldots, m$, $j \neq i$.

Using continuity, $(*)$ that is, $(x_1^*, x_2^*, \ldots, x_m^*) \in H_1 \times H_2 \times \cdots \times H_m$ and $u_{ij} \in U_{ij}(x_j^*)$ $(i, j = 1, 2, \ldots, m,\ j \neq i)$ satisfy (22) and so, in light of Lemma 9, $(*)$, is a solution to problem (1). This completes the proof. □

Remark 15. If $m = 2$, then Theorem 14 reduces to Theorem 4.2 in [9].

From Theorem 14, we have the following results.

Corollary 16. *For $i, j = 1, 2, \ldots, m$ and $j \neq i$, assume that η_i, A_i, M_i, U_{ij}, and H_i are the same as in Theorem 14. Let $F_i : H_1 \times H_2 \times \cdots \times H_m \to H_i$ be μ_i-strongly monotone with respect to A_i in the ith argument and ζ_{ij}-Lipschitz continuous in the jth argument for $i, j = 1, 2, \ldots, m$ and $j \neq i$. If there exist constants $\rho_i > 0$, for $i = 1, 2, \ldots, m$, such that*

$$\frac{\tau_j}{r_j - \rho_j m_j} \cdot \sqrt{\beta_j^2 - 2\rho_j \mu_j + \rho_j^2 \zeta_{jj}^2} + \sum_{i=1, i \neq j}^{m} \frac{\rho_i \tau_i \zeta_{ij} \gamma_{ij}}{r_i - \rho_i m_i} < 1, \tag{40}$$

where $r_j - \rho_j m_j > 0$, for $j = 1, 2, \ldots, m$, then problem (1) admits a solution $(x_1^*, x_2^*, \ldots, x_m^*, u_{12}, \ldots, u_{1m}, \ldots, u_{m1}, \ldots, u_{mm-1})$, where, for any $i, j = 1, 2, \ldots, m$, $j \neq i$, $x_i^* \in H_i$ and $u_{ij} \in U_{ij}(x_j^*)$ and sequences $\{x_j^n\}$, $\{u_{ij}^n\}$ generated by Algorithm 11 strongly converge to x_j^* and u_{ij}, respectively.

Corollary 17. *For $i, j = 1, 2, \ldots, m$ and $j \neq i$, let η_i, A_i, M_i, F_i, and H_i be the same as in Corollary 16 and let $U_{ij} : H_j \to CB(H_j)$ be $\widehat{H}$-γ_{ij}-Lipschitz continuous. If condition (40) in Corollary 16 holds, then problem (1) admits a solution $(x_1^*, x_2^*, \ldots, x_m^*, u_{12}, \ldots, u_{1m}, \ldots, u_{m1}, \ldots, u_{mm-1})$, where $x_i^* \in H_i$ and $u_{ij} \in U_{ij}(x_j^*)$, and sequences $\{x_j^n\}$, $\{u_{ij}^n\}$ $(i, j = 1, 2, \ldots, m,\ j \neq i)$ generated by Algorithm 12 strongly converge to x_j^* and u_{ij}, for any $i, j = 1, 2, \ldots, m$, $j \neq i$, respectively.*

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] Y.-P. Fang, N.-J. Huang, and H.-B. Thompson, "A new system of variational inclusions with (H, η)-monotone operators in Hilbert spaces," *Computers & Mathematics with Applications*, vol. 49, no. 2-3, pp. 365–374, 2005.

[2] W.-Y. Yan, Y.-P. Fang, and N.-J. Huang, "A new system of set-valued variational inclusions with H-monotone operators," *Mathematical Inequalities & Applications*, vol. 8, no. 3, pp. 537–546, 2005.

[3] Y.-P. Fang and N.-J. Huang, "H-monotone operators and system of variational inclusions," *Communications on Applied Nonlinear Analysis*, vol. 11, no. 1, pp. 93–101, 2004.

[4] H.-W. Cao, "Sensitivity analysis for a system of generalized nonlinear mixed quasi variational inclusions with H-monotone operators," *Journal of Applied Mathematics*, vol. 2011, Article ID 921835, 15 pages, 2011.

[5] H.-Y. Lan, J.-H. Kim, and Y.-J. Cho, "On a new system of nonlinear A-monotone multivalued variational inclusions," *Journal of Mathematical Analysis and Applications*, vol. 327, no. 1, pp. 481–493, 2007.

[6] H.-Y. Lan, "A class of nonlinear (A, η)-monotone operator inclusion problems with relaxed cocoercive mappings," *Advances in Nonlinear Variational Inequalities*, vol. 9, no. 2, pp. 1–11, 2006.

[7] J.-W. Peng and L.-J. Zhao, "General system of A-monotone nonlinear variational inclusions problems with applications," *Journal of Inequalities and Applications*, vol. 2009, Article ID 364615, 13 pages, 2009.

[8] R.-U. Verma, "A-monotonicity and applications to nonlinear variational inclusion problems," *Journal of Applied Mathematics and Stochastic Analysis*, vol. 17, no. 2, pp. 193–195, 2004.

[9] R.-P. Agarwal and R.-U. Verma, "General system of (A, η)-maximal relaxed monotone variational inclusion problems based on generalized hybrid algorithms," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 2, pp. 238–251, 2010.

[10] R.-U. Verma, "Sensitivity analysis for generalized strongly monotone variational inclusions based on the (A, η)-resolvent operator technique," *Applied Mathematics Letters*, vol. 19, no. 12, pp. 1409–1413, 2006.

[11] X.-P. Ding and C.-L. Luo, "Perturbed proximal point algorithms for general quasi-variational-like inclusions," *Journal of Computational and Applied Mathematics*, vol. 113, no. 1-2, pp. 153–165, 2000.

[12] Y.-P. Fang and N.-J. Huang, "H-monotone operator and resolvent operator technique for variational inclusions," *Applied Mathematics and Computation*, vol. 145, no. 2-3, pp. 795–803, 2003.

[13] N.-J. Huang and Y.-P. Fang, "A new class of general variational inclusions involving maximal η-monotone mappings," *Publicationes Mathematicae Debrecen*, vol. 62, no. 1-2, pp. 83–98, 2003.

[14] M.-M. Jin, "Generalized nonlinear implicit quasivariational inclusions with relaxed monotone mappings," *Advances in Nonlinear Variational Inequalities*, vol. 7, no. 2, pp. 173–181, 2004.

[15] M.-M. Jin, "Iterative algorithm for a new system of nonlinear set-valued variational inclusions involving (H, η)-monotone mappings," *Journal of Inequalities in Pure and Applied Mathematics*, vol. 7, no. 2, article 72, 10 pages, 2006.

[16] R.-P. Agarwal, N.-J. Huang, and Y.-J. Cho, "Generalized nonlinear mixed implicit quasi-variational inclusions with set-valued mappings," *Journal of Inequalities and Applications*, vol. 7, no. 6, pp. 807–828, 2002.

[17] R.-U. Verma, "Approximation solvability of a class of nonlinear set-valued variational inclusions involving (A, η)-monotone mappings," *Journal of Mathematical Analysis and Applications*, vol. 337, no. 2, pp. 969–975, 2008.

[18] N.-J. Huang and Y.-P. Fang, "Fixed point theorems and a new system of multivalued generalized order complementarity problems," *Positivity*, vol. 7, no. 3, pp. 257–265, 2003.

[19] R.-U. Verma, "Projection methods, algorithms, and a new system of nonlinear variational inequalities," *Computers & Mathematics with Applications*, vol. 41, no. 7-8, pp. 1025–1031, 2001.

[20] Y.-J. Cho, Y.-P. Fang, N.-J. Huang, and H.-J. Hwang, "Algorithms for systems of nonlinear variational inequalities," *Journal of the Korean Mathematical Society*, vol. 41, no. 3, pp. 489–499, 2004.

[21] J.-K. Kim and D.-S. Kim, "A new system of generalized nonlinear mixed variational inequalities in Hilbert spaces," *Journal of Convex Analysis*, vol. 11, no. 1, pp. 235–243, 2004.

[22] R.-U. Verma, "Generalized system for relaxed cocoercive variational inequalities and projection methods," *Journal of Optimization Theory and Applications*, vol. 121, no. 1, pp. 203–210, 2004.

[23] G. Kassay and J. Kolumbán, "System of multi-valued variational inequalities," *Publicationes Mathematicae Debrecen*, vol. 56, no. 1-2, pp. 185–195, 2000.

[24] Y. Yao, Y.-C. Liou, C.-L. Li, and H.-T. Lin, "Extended extragradient methods for generalized variational inequalities," *Journal of Applied Mathematics*, vol. 2012, Article ID 237083, 14 pages, 2012.

[25] M.-A. Noor, K.-I. Noor, Z. Huang, and E. Al-Said, "Implicit schemes for solving extended general nonconvex variational inequalities," *Journal of Applied Mathematics*, vol. 2012, Article ID 646259, 10 pages, 2012.

[26] S.-B. Nadler Jr., "Multi-valued contraction mappings," *Pacific Journal of Mathematics*, vol. 30, no. 2, pp. 475–488, 1969.