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Research Article

A Novel Scheme Adaptive Hybrid Dislocated Synchronization for Two Identical and Different Memristor Chaotic Oscillator Systems with Uncertain Parameters

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The drive system can synchronize with the response system by the scaling factor in the traditional projective synchronization. This paper proposes a novel adaptive hybrid dislocated synchronization with uncertain parameters scheme for chaos synchronization using the Lyapunov stability theory. The drive system is synchronized by the sum of hybrid dislocated state variables for the response system. By designing effective hybrid dislocated adaptive controller and hybrid dislocated adaptive law of the parameters estimation, we investigate the synchronization of two identical memristor chaotic oscillator systems and two different memristor chaotic oscillator systems with uncertain parameters. Finally, the numerical simulation examples are provided to show the effectiveness of our method.

1. Introduction

A chaotic system has complex dynamical behaviors that possess some special features such as being extremely sensitive to tiny variations of initial conditions and having bounded trajectories in the phase space with a positive leading Lyapunov exponent and so on. Pecora and Carroll [1] have realized chaos synchronization in 1990, many types of synchronization methods have been investigated in the past 10 years. Based on the Lyapunov stability theory, some kinds of synchronization have been intensively studied and a lot of theoretical results have been obtained, such as complete synchronization [1], partial synchronization [2], anti-synchronization [3], generalized synchronization [4–7], phase synchronization [8], anti-phase synchronization [9], lag synchronization [10], projective synchronization [11–14], time scale synchronization [15], combination synchronization [16–19], and compound synchronization [20]. In this period, several theoretical methods have been developed to realize chaos synchronization such as OGY method [21], feedback control method [22–24], active control method [25], backstepping method [26], adaptive control method [27], sliding mode control method [28], impulsive control method [29, 30], coupling control method [31], and observer control method [32], and so on.

In recent years, projective synchronization received many attractions, which characterizes that the state vectors of synchronized systems become proportional with a scaling factor. Mainieri and Rehacek have studied projective synchronization in coupled partially linear chaotic systems such as the Lorenz system [11]. Projective synchronization in general class of chaotic systems including nonpartially-linear chaotic system has been achieved with nonlinear observer control [33]. More recently, a new synchronization method referred to as "modified projective synchronization" has been proposed in [34, 35], where the chaotic systems can synchronize up to a constant scaling matrix. The above projective synchronization is confined to three-dimensional chaotic systems, the projective synchronization problem for a class of four-dimensional chaotic systems is concerned [36, 37].

The above methods realize the projective synchronization whose scaling factor is a constant or a function for the corresponding state variable. Xu et al. has realized the general hybrid projective dislocated synchronization between two chaotic nonlinear systems, which includes complete dislocated synchronization, dislocated antisynchronization, and projective dislocated synchronization as its special items [38]. The transmitted signals are such complex and unpredictable that they may have stronger antiattack ability and antitranslated capability than that transmitted by the usual transmission model. In our paper, the drive system is synchronized by the sum of hybrid dislocated state variables for the response system. What is more, the memristor chaotic oscillator system is a new four-dimensional (4D) autonomous chaotic system and can produce novel rich and complex dynamic activities, which is different from the traditional chaotic systems for the unique memory of the memristor initial state. Motivated by the existing works, we focus on not only the identification of parameters but also the novel adaptive hybrid dislocated control synchronization.

In this paper, the problem of chaos synchronization to memristor chaotic oscillator system with uncertain parameters is considered. At first, we give a general scheme description for synchronization with uncertain parameters between two identical and two different chaotic systems. Then the chaos synchronization of the systems is proved by the Lyapunov stability theory. Finally, the numerical simulation examples are given to show the effectiveness of our method.

2. Chaos Synchronization

The section will discuss hybrid dislocated adaptive method to achieve synchronization for memristor chaotic oscillator systems with uncertain parameters. Synchronization between two identical chaotic systems and two different chaotic systems are considered, respectively.

2.1. Chaos Synchronization between Two Identical Chaotic Systems. A drive system is given by

$$\dot{x} = f(x) + g(x)\alpha, \tag{1}$$

and the corresponding response system is written by

$$\dot{y} = f(y) + g(y)\tilde{\alpha} + u, \tag{2}$$

where $x = (x_1, x_2, \dots, x_n)^T \in R^n$ and $y = (y_1, y_2, \dots, y_n)^T \in R^n$ are state vectors, $f : R^n \to R^n$ and $g : R^n \to R^{n\times m}$ are two continuous functions, the estimated parameter vector is $\tilde{\alpha} = (\tilde{\alpha}_1, \dots, \tilde{\alpha}_m)^T \in R^m$ for the parameter vector $\alpha = (\alpha_1, \dots, \alpha_m)^T \in R^m$, and u is the control law to be constructed.

Definition 1. If there exist non-zero constants $d_{ij} \neq 0$ ($i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$), the following condition

$$\lim_{t\to\infty} \left\| \sum_{j=1, i\neq j}^{n} d_{ij} y_j + x_i \right\| = 0 \tag{3}$$

can hold true, such that the drive system (1) and the corresponding response system (2) have realized hybrid projective complete dislocated synchronization with uncertain parameters.

Specifically, the error dynamics system of general hybrid projective complete dislocated synchronization with uncertain parameters is given by

$$\dot{e} = \left(\sum_{i=2}^{n} d_{1i}\dot{y}_i + \dot{x}_1, \sum_{i=1, i\neq 2}^{n} d_{2i}\dot{y}_i + \dot{x}_2, \dots, \sum_{i=1, i\neq n}^{n} d_{ni}\dot{y}_i + \dot{x}_n \right)^T,$$
$$\dot{e}_\alpha = \dot{\tilde{\alpha}} - \dot{\alpha} = \left(\dot{\tilde{\alpha}}_1 - \dot{\alpha}_1, \dots, \dot{\tilde{\alpha}}_m - \dot{\alpha}_m \right)^T. \tag{4}$$

Lyapunov function is designed as follows:

$$V = \frac{1}{2} e^T P e + \frac{1}{2} e_\alpha^T Q e_\alpha, \tag{5}$$

where P and Q are the positive definite constant matrices. The time derivative of V along the trajectories of (5) is written by

$$\dot{V} = \frac{1}{2}\left(\dot{e}^T P e + e^T P \dot{e}\right) + \frac{1}{2}\left(\dot{e}_\alpha^T Q e_\alpha + e_\alpha^T Q \dot{e}_\alpha\right). \tag{6}$$

If there exists suitable feedback control law $u(x, y) \in R^m$ and constants d_{ij} to make $\dot{V} < 0$ hold true, based on the Lyapunov stability theorem, the drive system (1) and the response system (2) have completed general hybrid projective complete dislocated synchronization with uncertain parameters.

Remark 2. If $d_{ij} = 1$ ($i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, $i \neq j$), the rest $d_{ij} = 0$, then hybrid projective complete dislocated synchronization will become into complete dislocated synchronization. If $d_{ij} = -1$ ($i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, $i \neq j$), the rest $d_{ij} = 0$, then hybrid projective complete dislocated synchronization will become into dislocated anti-synchronization. If $d_{ij} = -\lambda_i$ ($i = 1, 2, \dots, n$, λ_i is constant, $\lambda_i \neq 0, 1$, $i \neq j$), the rest $d_{ij} = 0$, then hybrid projective complete dislocated synchronization will become into projective dislocated synchronization.

2.1.1. Main Results. Memristor chaotic oscillator system [39] is described by

$$\begin{aligned} \dot{x}_1 &= a_1\left(x_3 - \varphi(x_4)x_1\right), \\ \dot{x}_2 &= a_2 x_2 - a_3 x_3, \\ \dot{x}_3 &= x_2 - x_1 - a_4 x_3, \\ \dot{x}_4 &= x_1, \end{aligned} \tag{7}$$

where x_1, x_2, x_3, and x_4 are state variables of the drive system (7), and a_1, a_2, a_3, and a_4 are the real constants of the drive system (7). $q(w)$ is a piecewise linear function of the form:

$$q(w) = 18w - 1.75\left(|w + 1| - |w - 1|\right), \tag{8}$$

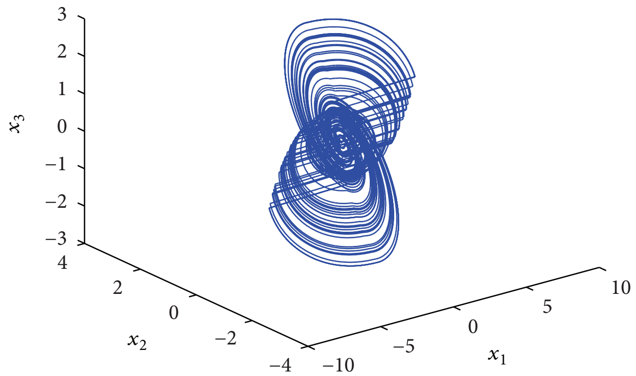

FIGURE 1: Chaotic attractors of memristor chaotic oscillator system described by (7).

where w is state variable. $\varphi(w)$ is given as the following expression:

$$\varphi(w) = \frac{dq(w)}{dw} = \begin{cases} 0.1, & |w| < 1, \\ 18, & |w| > 1. \end{cases} \quad (9)$$

Actually, system (7) shows chaotic when $a_1 = 0.31$, $a_2 = 0.35$, $a_3 = 0.29$, and $a_4 = 0.41$ as shown in Figure 1. Two identical memristor chaotic oscillator systems are given, where the drive system with the variable x drives the response system having identical equations denoted the variable y. The drive system is (7), the response system is described by

$$\begin{aligned} \dot{y}_1 &= b_1(y_3 - \varphi(y_4)y_1) + u_1, \\ \dot{y}_2 &= b_2 y_2 - b_3 y_3 + u_2, \\ \dot{y}_3 &= y_2 - y_1 - b_4 y_3 + u_3, \\ \dot{y}_4 &= y_1 + u_4, \end{aligned} \quad (10)$$

where b_1, b_2, b_3, and b_4 are parameters of the response system (10) which needs to be estimated u_1, u_2, u_3, and u_4 are the control laws to assure that two chaotic systems can be synchronized.

Let

$$\begin{aligned} e_1 &= x_1 + d_{12}y_2 + d_{13}y_3 + d_{14}y_4, \\ e_2 &= x_2 + d_{21}y_1 + d_{23}y_3 + d_{24}y_4, \\ e_3 &= x_3 + d_{31}y_1 + d_{32}y_2 + d_{34}y_4, \\ e_4 &= x_4 + d_{41}y_1 + d_{42}y_2 + d_{43}y_3, \end{aligned} \quad (11)$$

where d_{ij} $(i = 1,2,3,4,\ j = 1,2,3,4,\ i \neq j)$ are constants, and

$$\begin{aligned} E = {} & d_{13}d_{24}d_{31}d_{42} - d_{12}d_{24}d_{31}d_{43} - d_{14}d_{23}d_{31}d_{42} \\ & - d_{13}d_{21}d_{34}d_{42} - d_{14}d_{21}d_{32}d_{43} + d_{12}d_{21}d_{34}d_{43} \\ & - d_{13}d_{24}d_{32}d_{41} + d_{14}d_{23}d_{32}d_{41} - d_{12}d_{23}d_{34}d_{41}, \end{aligned} \quad (12)$$

$$\begin{aligned} A_1 = {} & A(d_{23}d_{34}d_{42} + d_{32}d_{24}d_{43}) \\ & + B(d_{12}d_{34}d_{43} - d_{14}d_{32}d_{43} - d_{13}d_{34}d_{42}) \\ & + C(d_{13}d_{24}d_{42} - d_{12}d_{24}d_{43} - d_{14}d_{23}d_{42}) \\ & + D(d_{14}d_{23}d_{32} - d_{12}d_{23}d_{34} - d_{13}d_{24}d_{32}), \\ A_2 = {} & A(d_{21}d_{34}d_{43} - d_{31}d_{24}d_{43} - d_{23}d_{34}d_{41}) \\ & + B(d_{13}d_{34}d_{41} + d_{14}d_{31}d_{43}) \\ & + C(d_{14}d_{41}d_{23} - d_{14}d_{21}d_{43} - d_{13}d_{24}d_{41}) \\ & + D(d_{13}d_{24}d_{31} - d_{13}d_{21}d_{34} - d_{14}d_{23}d_{31}), \\ A_3 = {} & -A(d_{24}d_{32}d_{41} + d_{21}d_{34}d_{42} - d_{24}d_{31}d_{42}) \\ & - B(d_{14}d_{31}d_{42} + d_{12}d_{34}d_{41} - d_{14}d_{32}d_{41}) \\ & + C(d_{12}d_{24}d_{41} + d_{14}d_{21}d_{42}) \\ & - D(d_{12}d_{24}d_{31} + d_{14}d_{21}d_{32} - d_{12}d_{21}d_{34}), \\ A_4 = {} & A(-d_{21}d_{32}d_{43} - d_{23}d_{31}d_{42} + d_{23}d_{32}d_{41}) \\ & + B(-d_{32}d_{13}d_{41} - d_{12}d_{31}d_{43} + d_{13}d_{31}d_{42}) \\ & + C(-d_{13}d_{21}d_{42} - d_{12}d_{23}d_{41} + d_{12}d_{21}d_{43}) \\ & + D(d_{13}d_{21}d_{32} + d_{12}d_{23}d_{31}), \end{aligned} \quad (13)$$

$$\begin{aligned} A = {} & (b_1 - a_1) - (x_1 + d_{12}y_2 + d_{13}y_3 + d_{14}y_4) \\ & - a_1[x_3 - \varphi(x_4)x_1] - d_{12}(b_2y_2 - b_3y_3) \\ & - d_{13}(y_2 - y_1 - b_4y_3) - d_{14}y_1, \\ B = {} & (b_2 - a_2) - (x_2 + d_{21}y_1 + d_{23}y_3 + d_{24}y_4) \\ & - (a_2x_2 - a_3x_3) - d_{21}b_1[y_3 - \varphi(y_4)y_1] \\ & - d_{23}(y_2 - y_1 - b_4y_3) - d_{24}y_1, \\ C = {} & (b_3 - a_3) - (x_3 + d_{31}y_1 + d_{32}y_2 + d_{34}y_4) \\ & - (x_2 - x_1 - a_4x_3) - d_{31}b_1[y_3 - \varphi(y_4)y_1] \\ & - d_{32}(b_2y_2 - b_3y_3) - d_{34}y_1, \\ D = {} & (b_4 - a_4) - (x_4 + d_{41}y_1 + d_{42}y_2 + d_{43}y_3) \\ & - x_1 - d_{41}b_1[y_3 - \varphi(y_4)y_1] \\ & - d_{42}(b_2y_2 - b_3y_3) - d_{43}(y_2 - y_1 - b_4y_3). \end{aligned} \quad (14)$$

The control law is given as follows:

$$\begin{aligned} u_1 &= \frac{A_1}{E}, \qquad u_2 = \frac{A_2}{E}, \\ u_3 &= \frac{A_3}{E}, \qquad u_4 = \frac{A_4}{E}. \end{aligned} \quad (15)$$

The update rules for unknown parameters b_1, b_2, b_3, and b_4 are given as follows:

$$\dot{b}_1 = -e_1, \qquad \dot{b}_2 = -e_2,$$
$$\dot{b}_3 = -e_3, \qquad \dot{b}_4 = -e_4. \tag{16}$$

Theorem 3. *The drive system* (7) *and response system* (10) *can complete the hybrid projective complete dislocated synchronization for any initial conditions* $(x_1(0), x_2(0), x_3(0), x_4(0))$ *and* $(y_1(0), y_2(0), y_3(0), y_4(0))$ *via the control law* (15) *and the update law* (16).

Proof. The error dynamics can be gained as follows:

$$\begin{aligned}
\dot{e}_1 &= \dot{x}_1 + d_{12}\left(b_2 y_2 - b_3 y_3 + u_2\right) \\
&\quad + d_{13}\left(y_2 - y_1 - b_4 y_3 + u_3\right) + d_{14}\left(y_1 + u_4\right), \\
\dot{e}_2 &= \dot{x}_2 + d_{21}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right] \\
&\quad + d_{23}\left(y_2 - y_1 - b_4 y_3 + u_3\right) + d_{24}\left(y_1 + u_4\right), \\
\dot{e}_3 &= \dot{x}_3 + d_{31}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right] \\
&\quad + d_{32}\left(b_2 y_2 - b_3 y_3 + u_2\right) + d_{34}\left(y_1 + u_4\right), \\
\dot{e}_4 &= \dot{x}_4 + d_{41}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right] \\
&\quad + d_{42}\left(b_2 y_2 - b_3 y_3 + u_2\right) + d_{43}\left(y_2 - y_1 - b_4 y_3 + u_3\right).
\end{aligned} \tag{17}$$

Substituting (12), (13), and (15) into (17), we get

$$\begin{aligned}
\dot{e}_1 &= \dot{x}_1 + d_{12}\left(b_2 y_2 - b_3 y_3\right) + d_{13}\left(y_2 - y_1 - b_4 y_3\right) \\
&\quad + d_{14} y_1 + A, \\
\dot{e}_2 &= \dot{x}_2 + d_{21} b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) \\
&\quad + d_{23}\left(y_2 - y_1 - b_4 y_3\right) + d_{24} y_1 + B, \\
\dot{e}_3 &= \dot{x}_3 + d_{31} b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) \\
&\quad + d_{32}\left(b_2 y_2 - b_3 y_3\right) + d_{34} y_1 + C, \\
\dot{e}_4 &= \dot{x}_4 + d_{41} b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) \\
&\quad + d_{42}\left(b_2 y_2 - b_3 y_3\right) + d_{43}\left(y_2 - y_1 - b_4 y_3\right) + D.
\end{aligned} \tag{18}$$

Substituting (7), (10), and (14) into (18), it is easy to gain the error dynamics as follows:

$$\begin{aligned}
\dot{e}_1 &= \left(b_1 - a_1\right) \\
&\quad - \left[x_1 + d_{12}\left(b_2 y_2 - b_3 y_3 + u_2\right)\right. \\
&\qquad \left. + d_{13}\left(y_2 - y_1 - b_4 y_3 + u_3\right) + d_{14}\left(y_1 + u_4\right)\right], \\
\dot{e}_2 &= \left(b_2 - a_2\right) \\
&\quad - \left\{x_2 + d_{21}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right]\right. \\
&\qquad \left. + d_{23}\left(y_2 - y_1 - b_4 y_3 + u_3\right) + d_{24}\left(y_1 + u_4\right)\right\}, \\
\dot{e}_3 &= \left(b_3 - a_3\right) \\
&\quad - \left\{x_3 + d_{31}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right]\right. \\
&\qquad \left. + d_{32}\left(b_2 y_2 - b_3 y_3 + u_2\right) + d_{34}\left(y_1 + u_4\right)\right\}, \\
\dot{e}_4 &= \left(b_4 - a_4\right) \\
&\quad - \left\{x_4 + d_{41}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right]\right. \\
&\qquad + d_{42}\left(b_2 y_2 - b_3 y_3 + u_2\right) \\
&\qquad \left. + d_{43}\left(y_2 - y_1 - b_4 y_3 + u_3\right)\right\}.
\end{aligned} \tag{19}$$

The following Lyapunov candidate is chosen by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_{a_1}^2 + e_{a_2}^2 + e_{a_3}^2 + e_{a_4}^2\right), \tag{20}$$

where

$$e_{a_1} = b_1 - a_1, \qquad e_{a_2} = b_2 - a_2,$$
$$e_{a_3} = b_3 - a_3, \qquad e_{a_4} = b_4 - a_4. \tag{21}$$

Then the differential of the Lyapunov function along the trajectory of error system (19) is

$$\begin{aligned}
&\dot{V}\left(e_1, e_2, e_3, e_4, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}\right) \\
&= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + \dot{e}_{a_1} e_{a_1} \\
&\quad + \dot{e}_{a_2} e_{a_2} + \dot{e}_{a_3} e_{a_3} + \dot{e}_{a_4} e_{a_4} \\
&= \left\{\left(b_1 - a_1\right)\right. \\
&\qquad - \left[x_1 + d_{12}\left(b_2 y_2 - b_3 y_3 + u_2\right)\right. \\
&\qquad\quad \left.\left. + d_{13}\left(y_2 - y_1 - b_4 y_3 + u_3\right) + d_{14}\left(y_1 + u_4\right)\right]\right\} e_1 \\
&\quad + \left\{\left(b_2 - a_2\right)\right. \\
&\qquad - \left\{x_2 + d_{21}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right]\right. \\
&\qquad\quad \left.\left. + d_{23}\left(y_2 - y_1 - b_4 y_3 + u_3\right) + d_{24}\left(y_1 + u_4\right)\right\}\right\} e_2 \\
&\quad + \left\{\left(b_3 - a_3\right)\right. \\
&\qquad - \left\{x_3 + d_{31}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right]\right. \\
&\qquad\quad \left.\left. + d_{32}\left(b_2 y_2 - b_3 y_3 + u_2\right) + d_{34}\left(y_1 + u_4\right)\right\}\right\} e_3 \\
&\quad + \left\{\left(b_4 - a_4\right)\right. \\
&\qquad - \left\{x_4 + d_{41}\left[b_1\left(y_3 - \varphi\left(y_4\right) y_1\right) + u_1\right]\right. \\
&\qquad\quad + d_{42}\left(b_2 y_2 - b_3 y_3 + u_2\right) \\
&\qquad\quad \left.\left. + d_{43}\left(y_2 - y_1 - b_4 y_3 + u_3\right)\right\}\right\} e_4 \\
&\quad - \left(b_1 - a_1\right) e_1 - \left(b_2 - a_2\right) e_2 \\
&\quad - \left(b_3 - a_3\right) e_3 - \left(b_4 - a_4\right) e_4
\end{aligned}$$

$$= (b_1 - a_1 - e_1) e_1 - (b_2 - a_2 - e_2) e_2 - (b_3 - a_3 - e_3) e_3 - (b_4 - a_4 - e_4) e_4 - (b_1 - a_1) e_1 - (b_2 - a_2) e_2 - (b_3 - a_3) e_3 - (b_4 - a_4) e_4 = -e_1^2 - e_2^2 - e_3^2 - e_4^2. \quad (22)$$

Since $\dot{V}$ is negative semidefinite, we cannot immediately obtain that the origin of error system (19) is asymptotically stable. In fact, as $\dot{V} \leq 0$, then $e_1, e_2, e_3, e_4 \in \ell_\infty$ and $e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4} \in \ell_\infty$. From the error system (10), we get $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \in \ell_\infty$, $\dot{e}_{a_1}, \dot{e}_{a_2}, \dot{e}_{a_3}, \dot{e}_{a_4} \in \ell_\infty$. So we have

$$\int_0^t \|e\|^2 dt \leq \int_0^t e^T e dt \leq \int_0^t -\dot{V} dt = V(0) - V(t) \leq V(0) < \infty, \quad (23)$$

where $e = [e_1, e_2, e_3, e_4, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}]^T$. Thus $e_1, e_2, e_3, e_4 \in \ell_2$ and $e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4} \in \ell_2$. According to the Barbalat's lemma, we have $e_1, e_2, e_3, e_4 \rightarrow 0$ and $e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4} \rightarrow 0$ $(t \rightarrow \infty)$. Therefore, the response system (10) synchronizes the drive system (7) by the controller (15). This completes the proof. □

Remark 4. A chaotic system with unknown parameters cannot be known in advance, but it has a given geometric topology structure. The control laws u_1, u_2, u_3, and u_4 are too complex to realize the synchronization control; the reasonable design of u_1, u_2, u_3, and u_4 are the key factors to a successful method.

2.1.2. Simulation and Results. In the numerical simulations, the fourth-order Runge-Kutta method is applied to solve the systems with time step size 0.001. It is assumed that the initial condition, $(x_1(0), x_2(0), x_3(0), x_4(0)) = (10^{-2}, 2 * 10^{-2}, 2 * 10^{-2}, 8 * 10^{-2})$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (10^{-2}, 2 * 10^{-2}, 2 * 10^{-2}, 8 * 10^{-2})$ are employed. Parameters are designed as $d_{11} = -0.002$, $d_{12} = 0.003$, $d_{13} = 0.001$, $d_{21} = -0.002$, $d_{22} = 0.003$, $d_{23} = 0.001$, $d_{31} = -0.002$, $d_{32} = 0.003$, $d_{33} = 0.001$, $d_{41} = -0.002$, $d_{42} = 0.003$, $d_{43} = 0.001$. Synchronization of the systems (7) and (10) by hybrid dislocated adaptive control law (15) and (16) with the initial estimated parameters $a_1 = 8$, $a_2 = 4$, $a_3 = 2$, and $a_4 = 1$ are displayed in Figures 2 and 3. Figure 2 displays synchronization errors of systems (7) and (10). Figure 3 shows that the estimated values of the unknown parameters a_1, a_2, a_3, and a_4 that can converge to $a_1 = 0.31$, $a_2 = 0.35$, $a_3 = 0.29$, and $a_4 = 0.41$.

Remark 5. In the simulation, parameters d_{ij} $(i = 1, 2, 3, 4,\ j = 1, 2, 3, 4,\ i \neq j)$ are chosen to make the following condition $d_{13}d_{24}d_{31}d_{42} - d_{12}d_{24}d_{31}d_{43} - d_{14}d_{23}d_{31}d_{42} - d_{13}d_{21}d_{34}d_{42} - d_{14}d_{21}d_{32}d_{43} + d_{12}d_{21}d_{34}d_{43} - d_{13}d_{24}d_{32}d_{41} + d_{14}d_{23}d_{32}d_{41} - d_{12}d_{23}d_{34}d_{41} \neq 0$ hold true.

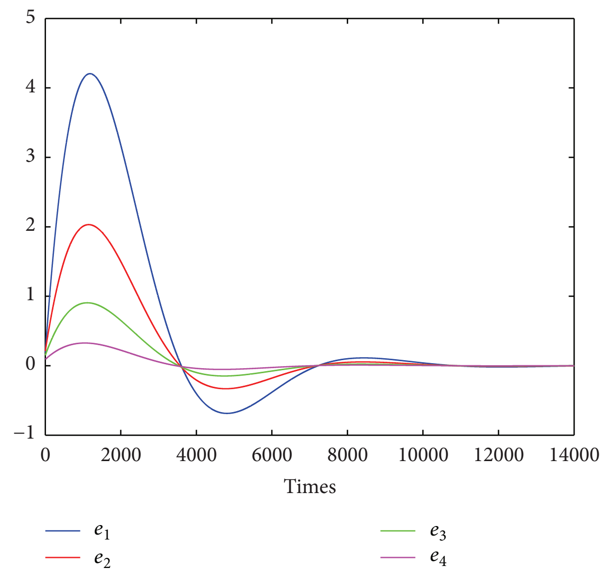

FIGURE 2: Synchronization errors e_1, e_2, e_3, and e_4 coupled memristor chaotic oscillator systems.

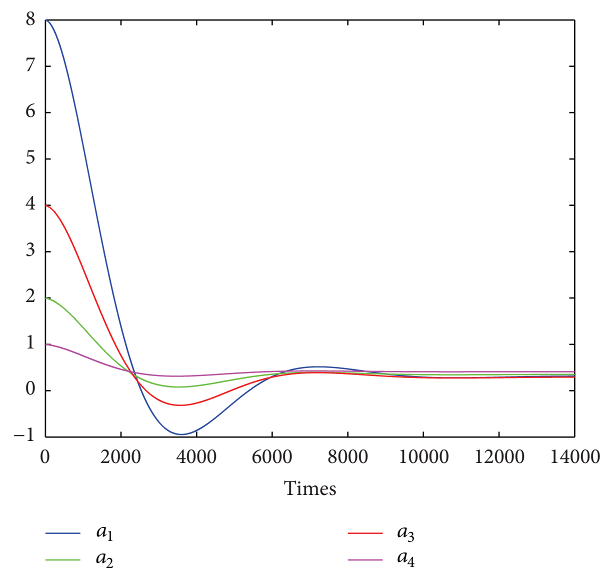

FIGURE 3: Estimated parameters of the controlled memristor chaotic oscillator systems.

2.2. Chaos Synchronization between Two Different Systems. A drive system is described by

$$\dot{x} = f_1(x) + g_1(x)\alpha, \quad (24)$$

and the corresponding response system is defined as follows:

$$\dot{y} = f_2(y) + g_2(y)\beta + u, \quad (25)$$

where $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ and $y = (y_1, y_2, \ldots, y_n)^T \in R^n$ are state vectors, $f : R^n \rightarrow R^n$ and $g : R^n \rightarrow R^{n\times m}$ are two continuous functions, the estimated parameter vectors of the vectors $\alpha = (\alpha_1, \ldots, \alpha_m)^T \in R^m$ and $\beta = (\beta_1, \ldots, \beta_m)^T \in R^m$

are $\tilde{\alpha} = (\tilde{\alpha}_1, \ldots, \tilde{\alpha}_m)^T \in R^m$ and $\tilde{\beta} = (\tilde{\beta}_1, \ldots, \tilde{\beta}_m)^T \in R^m$, and u is a control law to be designed.

Definition 6. For the drive system (24) and response system (25), let the vector error state be

$$e = AY + BX, \quad (26)$$

where $e = [e_1, e_2, \ldots, e_m]^T$, $Y = [y_1, y_2, \ldots, y_m]^T$, $X = [x_1, x_2, \ldots, x_n]^T$, $A = (d_{ij})_{m\times m}$, and $B = (d_{ij})_{m\times n}$.

Suppose

$$\begin{aligned} d_{is} &= 0, \qquad d_{it} \neq 0, \\ d_{is} &\neq 0, \qquad d_{it} = 0. \end{aligned} \quad (27)$$

Two kinds of cases (I) $n \geq m$ and (II) $n < m$ are discussed for the further research.

Case I ($n \geq m$). The order of the drive system is not lower than that of the response system. When $s \in S$, $S \subset M$, $M = \{1, 2, \ldots, m\}$, $t \in T$, $T \cap S = \phi$, and $T \cup S = M$, such that the system (24) and system (25) are general hybrid projective complete dislocated synchronization with uncertain parameters.

Case II ($n < m$). The order of the drive system is lower than that of the response system. When $s \in S$, $S \subset N$, $N = \{1, 2, \ldots, n\}$, $t \in T$, $T \cap S = \phi$, and $T \cup S = N$, such that the system (24) and system (25) are general hybrid projective complete dislocated synchronization with uncertain parameters.

Then there exists suitable feedback control law $u(x, y) \in R_m$ and $A = (d_{ij})_{m\times m}$, $B = (d_{ij})_{m\times n}$, so as to

$$\lim_{t\to\infty} \|e(t)\| = 0. \quad (28)$$

In the following, we will give a principle to find suitable feedback control law $u(x, y)$ such that the two chaotic systems are hybrid projective complete dislocated synchronization with uncertain parameters. Construct a dynamical Lyapunov function:

$$V = \frac{1}{2} e^T P e + \frac{1}{2} e_\alpha^T Q e_\alpha + \frac{1}{2} e_\beta^T R e_\beta, \quad (29)$$

where P, Q, and R are the positive definite constant matrices. The time derivative of V along the trajectories of (29) is

$$\begin{aligned} \dot{V} &= \frac{1}{2} \left(\dot{e}^T P e + e^T P \dot{e} \right) + \frac{1}{2} \left(\dot{e}_\alpha^T Q e_\alpha + e_\alpha^T Q \dot{e}_\alpha \right) \\ &\quad + \frac{1}{2} \left(\dot{e}_\beta^T R e_\beta + e_\beta^T R \dot{e}_\beta \right). \end{aligned} \quad (30)$$

A reasonable control law $u(x, y)$ is designed such that $\dot{V}$ is negative definite. Then based on the Lyapunov's function method, the general hybrid projective complete dislocated synchronization with uncertain parameters of chaotic systems (24) and (25) is realized by the given designed feedback control law $u(x, y)$.

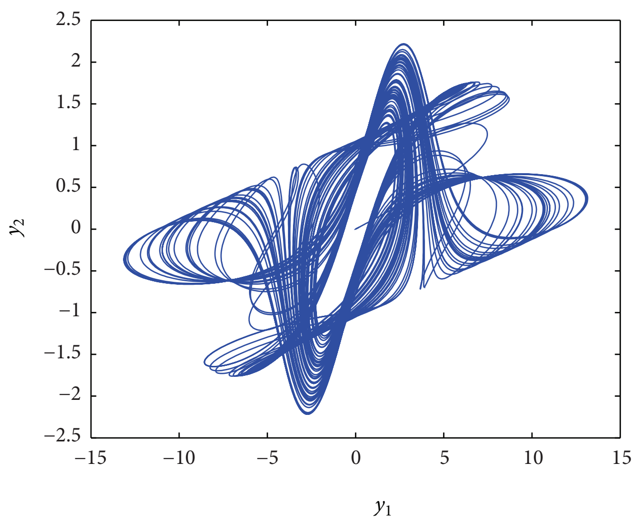

Figure 4: Chaotic attractors of memristor chaotic oscillator system described by (31).

2.2.1. Main Results. Here, an example is given to show the effectiveness of above method. The memristor chaotic oscillator system described by (7) drives the other memristor chaotic oscillator system [40]:

$$\begin{aligned} \dot{y}_1 &= b_1 y_2 + b_2 y_1 - y_1 y_4^2 + u_1, \\ \dot{y}_2 &= y_1 - y_2 + y_3 + u_2, \\ \dot{y}_3 &= -b_3 y_2 - b_4 y_3 + u_3, \\ \dot{y}_4 &= y_1 + u_4, \end{aligned} \quad (31)$$

where the memristor chaotic oscillator system exhibits a chaotic attractor at parameters $b_1 = 16.4$, $b_2 = 3.2$, $b_3 = 15$, and $b_4 = 0.5$ as Figure 4.

Let

$$\begin{aligned} e_1 &= d_{11} y_1 + d_{12} y_2 + d_{13} x_3 + d_{14} x_4, \\ e_2 &= d_{21} y_2 + d_{22} y_3 + d_{23} x_1 + d_{24} x_4, \\ e_3 &= d_{31} y_3 + d_{32} y_4 + d_{33} x_1 + d_{34} x_2, \\ e_4 &= d_{41} y_4 + d_{42} y_1 + d_{43} x_2 + d_{44} x_3, \end{aligned} \quad (32)$$

where d_{ij} ($i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$) are real constants.

Let the control law be as follows:

$$\begin{aligned} u_1 &= \frac{A_1}{E}, \qquad u_2 = \frac{A_2}{E}, \\ u_3 &= \frac{A_3}{E}, \qquad u_4 = \frac{A_4}{E}. \end{aligned} \quad (33)$$

Let

$$E = d_{11} d_{21} d_{31} d_{41} - d_{12} d_{22} d_{32} d_{42}, \quad (34)$$

$$\begin{aligned}A_1 &= d_{21}d_{31}d_{41}A - d_{12}d_{31}d_{41}B + d_{12}d_{22}d_{41}C\\&\quad - d_{12}d_{22}d_{32}D,\\A_2 &= -d_{22}d_{32}d_{42}A + d_{11}d_{31}d_{41}B - d_{11}d_{22}d_{41}C\\&\quad + d_{11}d_{22}d_{32}D,\\A_3 &= d_{21}d_{32}d_{21}A - d_{12}d_{32}d_{42}B + d_{11}d_{21}d_{41}C\\&\quad - d_{11}d_{21}d_{32}D,\\A_4 &= -d_{21}d_{31}d_{42}A + d_{12}d_{31}d_{41}B - d_{12}d_{22}d_{42}C\\&\quad + d_{11}d_{21}d_{31}D,\end{aligned} \tag{35}$$

$$\begin{aligned}A &= (a_1 - a_1^* + b_1 - b_1^*) - (d_{11}y_1 + d_{12}y_2 + d_{13}x_3 + d_{14}x_4)\\&\quad - d_{11}\left(b_1y_2 + b_2y_1 - y_1y_4^2\right) - d_{12}(y_1 - y_2 + y_3)\\&\quad - d_{13}(x_2 - x_1 - a_4x_3) - d_{14}x_1,\\B &= (a_2 - a_2^* + b_2 - b_2^*) - (d_{21}y_2 + d_{22}y_3 + d_{23}x_1 + d_{24}x_4)\\&\quad - d_{21}(y_1 - y_2 + y_3) - d_{22}(-b_3y_2 - b_4y_3)\\&\quad - d_{23}(a_1(x_3 - \varphi(w)x_1)) - d_{24}x_1,\\C &= (a_3 - a_3^* + b_3 - b_3^*) - (d_{31}y_3 + d_{32}y_4 + d_{33}x_1 + d_{34}x_2)\\&\quad - d_{31}(-b_3y_2 - b_4y_3) - d_{32}y_4 - d_{33}a_1(x_3 - \varphi(w)x_1)\\&\quad - d_{34}(a_2x_2 - a_3x_3),\\D &= (a_4 - a_4^* + b_4 - b_4^*) - (d_{41}y_4 + d_{42}y_1 + d_{43}x_1 + d_{44}x_3)\\&\quad - d_{41}y_4 - d_{42}\left(b_1y_2 + b_2y_1 - y_1y_4^2\right)\\&\quad - d_{43}(a_2x_2 - a_3x_3) - d_{44}(x_2 - x_1 - a_4x_3).\end{aligned} \tag{36}$$

The update laws for unknown parameters a_1^*, a_2^*, a_3^*, a_4^*, b_1^*, b_2^*, b_3^*, and b_4^* are given as follows:

$$\begin{aligned}\dot{a}_1^* &= -e_1, & \dot{a}_2^* &= -e_2,\\\dot{a}_3^* &= -e_3, & \dot{a}_4^* &= -e_4,\\\dot{b}_1^* &= -e_1, & \dot{b}_2^* &= -e_2,\\\dot{b}_3^* &= -e_3, & \dot{b}_4^* &= -e_4.\end{aligned} \tag{37}$$

Theorem 7. *The drive system* (7) *and the response system* (31) *can realize the hybrid projective complete dislocated synchronization for any initial conditions* $(x_1(0), x_2(0), x_3(0), x_4(0))$ *and* $(y_1(0), y_2(0), y_3(0), y_4(0))$ *by the control law* (33) *and the update laws* (37).

Proof. It is easy to see that the error dynamics can be obtained as follows:

$$\begin{aligned}\dot{e}_1 &= d_{11}\dot{y}_1 + d_{12}\dot{y}_2 + d_{13}\dot{x}_3 + d_{14}\dot{x}_4,\\\dot{e}_2 &= d_{21}\dot{y}_2 + d_{22}\dot{y}_3 + d_{23}\dot{x}_1 + d_{24}\dot{x}_4,\\\dot{e}_3 &= d_{31}\dot{y}_3 + d_{32}\dot{y}_4 + d_{33}\dot{x}_1 + d_{34}\dot{x}_2,\\\dot{e}_4 &= d_{41}\dot{y}_4 + d_{42}\dot{y}_1 + d_{43}\dot{x}_2 + d_{44}\dot{x}_3.\end{aligned} \tag{38}$$

Substituting (7) and (31) into (38), we have

$$\begin{aligned}\dot{e}_1 &= d_{11}\left(b_1y_2 + b_2y_1 - y_1y_4^2 + u_1\right) + d_{12}(y_1 - y_2 + y_3 + u_2)\\&\quad + d_{13}(x_2 - x_1 - a_4x_3) + d_{14}x_1,\\\dot{e}_2 &= d_{21}(y_1 - y_2 + y_3 + u_2) + d_{22}(-b_3y_2 - b_4y_3 + u_3)\\&\quad + d_{23}a_1(x_3 - \varphi(x_4)x_1) + d_{24}x_1,\\\dot{e}_3 &= d_{31}(-b_3y_2 - b_4y_3 + u_3) + d_{32}(y_1 + u_4)\\&\quad + d_{33}a_1(x_3 - \varphi(x_4)x_1) + d_{34}(a_2x_2 - a_3x_3),\\\dot{e}_4 &= d_{41}(y_1 + u_4) + d_{42}\left(b_1y_2 + b_2y_1 - y_1y_4^2 + u_1\right)\\&\quad + d_{43}(a_2x_2 - a_3x_3) + d_{44}(x_2 - x_1 - a_4x_3).\end{aligned} \tag{39}$$

Substituting (33), (34), and (35) into (39), we get

$$\begin{aligned}\dot{e}_1 &= d_{11}\left(b_1y_2 + b_2y_1 - y_1y_4^2\right) + d_{12}(y_1 - y_2 + y_3)\\&\quad + d_{13}(x_2 - x_1 - a_4x_3) + d_{14}x_1 + A,\\\dot{e}_2 &= d_{21}(y_1 - y_2 + y_3) + d_{22}(-b_3y_2 - b_4y_3)\\&\quad + d_{23}a_1(x_3 - \varphi(x_4)x_1) + d_{24}x_1 + B,\\\dot{e}_3 &= d_{31}(-b_3y_2 - b_4y_3) + d_{32}y_1\\&\quad + d_{33}a_1(x_3 - \varphi(x_4)x_1) + d_{34}(a_2x_2 - a_3x_3) + C,\\\dot{e}_4 &= d_{41}y_1 + d_{42}\left(b_1y_2 + b_2y_1 - y_1y_4^2\right)\\&\quad + d_{43}(a_2x_2 - a_3x_3) + d_{44}(x_2 - x_1 - a_4x_3) + D.\end{aligned} \tag{40}$$

Substituting (7), (31), and (36) into (40), it is easy to gain the error dynamics as follows:

$$\begin{aligned}\dot{e}_1 &= (a_1 - a_1^* + b_1 - b_1^*)\\&\quad - \left\{d_{11}\left(b_1y_2 + b_2y_1 - y_1y_4^2 + u_1\right)\right.\\&\quad\quad \left. + d_{12}(y_1 - y_2 + y_3 + u_2) + d_{13}x_3 + d_{14}x_4\right\},\\\dot{e}_2 &= (a_2 - a_2^* + b_2 - b_2^*)\\&\quad - \left[d_{21}(y_1 - y_2 + y_3 + u_2)\right.\\&\quad\quad \left. + d_{22}(y_1y_2 - b_3y_3 + u_3) + d_{23}x_1 + d_{24}x_4\right],\\\dot{e}_3 &= (a_3 - a_3^* + b_3 - b_3^*)\\&\quad - \left[d_{31}(y_1y_2 - b_3y_3 + u_3)\right.\\&\quad\quad \left. + d_{32}(y_1 + u_4) + d_{33}x_1 + d_{34}x_2\right],\\\dot{e}_4 &= (a_4 - a_4^* + b_4 - b_4^*)\\&\quad - \left\{d_{41}(y_1 + u_4) + d_{42}\left(b_1y_2 + b_2y_1 - y_1y_4^2 + u_1\right)\right.\\&\quad\quad \left. + d_{43}x_2 + d_{44}x_3\right\}.\end{aligned} \tag{41}$$

The following Lyapunov candidate is chosen:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_{a_1}^2 + e_{a_2}^2 + e_{a_3}^2 + e_{a_4}^2 + e_{b_1}^2 + e_{b_2}^2 + e_{b_3}^2 + e_{b_4}^2\right), \quad (42)$$

where

$$\begin{aligned} e_{a_1} &= a_1^* - a_1, & e_{a_2} &= a_2^* - a_2, \\ e_{a_3} &= a_3^* - a_3, & e_{a_4} &= a_4^* - a_4, \\ e_{b_1} &= b_1^* - b_1, & e_{b_2} &= b_2^* - b_2, \\ e_{b_3} &= b_3^* - b_3, & e_{b_4} &= b_4^* - b_4. \end{aligned} \quad (43)$$

Then, the differential of the Lyapunov function along the trajectory of error system (32) is gained by

$$\begin{aligned} &\dot{V}\left(e_1, e_2, e_3, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}, e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4}\right) \\ &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + \dot{e}_{a_1} e_{a_1} + \dot{e}_{a_2} e_{a_2} \\ &\quad + \dot{e}_{a_3} e_{a_3} + \dot{e}_{a_4} e_{a_4} + \dot{e}_{b_1} e_{b_1} + \dot{e}_{b_2} e_{b_2} + \dot{e}_{b_3} e_{b_3} + \dot{e}_{b_4} e_{b_4} \\ &= \{(a_1 - a_1^* + b_1 - b_1^*) \\ &\qquad - \{d_{11}\left(b_1 y_2 + b_2 y_1 - y_1 y_4^2 + u_1\right) \\ &\qquad\quad + d_{12}\left(y_1 - y_2 + y_3 + u_2\right) + d_{13} x_3 + d_{14} x_4\}\} e_1 \\ &\quad + \{(a_2 - a_2^* + b_2 - b_2^*) \\ &\qquad - [d_{21}\left(y_1 - y_2 + y_3 + u_2\right) + d_{22}\left(y_1 y_2 - b_3 y_3 + u_3\right) \\ &\qquad\quad + d_{23} x_1 + d_{24} x_4]\} e_2 \\ &\quad + \{(a_3 - a_3^* + b_3 - b_3^*) \\ &\qquad - [d_{31}\left(y_1 y_2 - b_3 y_3 + u_3\right) + d_{32}\left(y_1 + u_4\right) \\ &\qquad\quad + d_{33} x_1 + d_{34} x_2]\} e_3 \\ &\quad + \{(a_4 - a_4^* + b_4 - b_4^*) \\ &\qquad - \{d_{41}\left(y_1 + u_4\right) + d_{42}\left(b_1 y_2 + b_2 y_1 - y_1 y_4^2 + u_1\right) \\ &\qquad\quad + d_{43} x_2 + d_{44} x_3\}\} e_4 - \left(a_1^* - a_1\right) e_1 \\ &\quad - \left(a_2^* - a_2\right) e_2 - \left(a_3^* - a_3\right) e_3 - \left(a_4^* - a_4\right) e_4 - \left(b_1^* - b_1\right) e_1 \\ &\quad - \left(b_2^* - b_2\right) e_2 - \left(b_3^* - b_3\right) e_3 - \left(b_4^* - b_4\right) e_4 \\ &= \left(a_1^* - a_1 + b_1^* - b_1 - e_1\right) e_1 \\ &\quad + \left(a_2^* - a_2 + b_2^* - b_2 - e_2\right) e_2 \\ &\quad + \left(a_3^* - a_3 + b_3^* - b_3 - e_3\right) e_3 \\ &\quad + \left(a_4^* - a_4 + b_4^* - b_4 - e_4\right) e_4 \\ &\quad - \left(a_1^* - a_1\right) e_1 - \left(a_2^* - a_2\right) e_2 - \left(a_3^* - a_3\right) e_3 \\ &\quad - \left(a_4^* - a_4\right) e_4 - \left(b_1^* - b_1\right) e_1 - \left(b_2^* - b_2\right) e_2 \\ &\quad - \left(b_3^* - b_3\right) e_3 - \left(b_4^* - b_4\right) e_4 \\ &= -e_1^2 - e_2^2 - e_3^2 - e_4^2. \end{aligned} \quad (44)$$

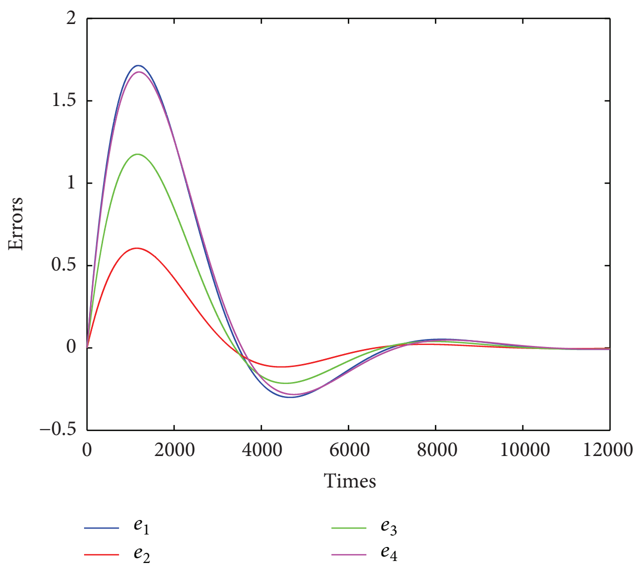

FIGURE 5: Synchronization errors e_1, e_2, e_3, and e_4 between memristor chaotic oscillator system and hyperchaotic Lü system.

Since $\dot{V}$ is negative semidefinite, we cannot immediately obtain that the origin of error system (32) is asymptotically stable. In fact, as $\dot{V} \leq 0$, then $e_1, e_2, e_3, e_4 \in \ell_\infty$, $e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4} \in \ell_\infty$, and $e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4} \in \ell_\infty$. From the error system (32), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \in \ell_\infty$, $\dot{e}_{a_1}, \dot{e}_{a_2}, \dot{e}_{a_3}, \dot{e}_{a_4} \in \ell_\infty$, and $\dot{e}_{b_1}, \dot{e}_{b_2}, \dot{e}_{b_3}, \dot{e}_{b_4} \in \ell_\infty$. Since $\dot{V} = -e^T e$, then we have

$$\begin{aligned} \int_0^t \|e\|^2 dt &\leq \int_0^t e^T e dt \\ &\leq \int_0^t -\dot{V} dt = V(0) - V(t) \leq V(0) < \infty, \end{aligned} \quad (45)$$

where $e = [e_1, e_2, e_3, e_4, e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4}, e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4}]^T$. Thus $e_1, e_2, e_3, e_4 \in \ell_2$, $e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4} \in \ell_2$, and $e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4} \in \ell_2$. According to the Barbalat's lemma, we have $e_1, e_2, e_3, e_4 \to 0$, $e_{a_1}, e_{a_2}, e_{a_3}, e_{a_4} \to 0$ and $e_{b_1}, e_{b_2}, e_{b_3}, e_{b_4} \to 0 (t \to \infty)$. Therefore, the response system (31) synchronizes the drive system (7) by the control law (33). This completes the proof. □

2.2.2. Simulation and Results. In the numerical simulations, the fourth-order Runge-Kutta method is also used to solve the systems with time step size 0.001. The initial condition, $(x_1(0), x_2(0), x_3(0), x_4(0)) = (10^{-2}, 2*10^{-2}, 2*10^{-2}, 8*10^{-2})$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (10^{-2}, 2*10^{-2}, 2*10^{-2})$, are employed. Parameters are chosen as $d_{11} = -0.002$, $d_{12} = 0.003$, $d_{13} = 0.002$, $d_{14} = 0.002$, $d_{21} = -0.002$, $d_{22} = 0.003$, $d_{23} = 0.002$, $d_{24} = 0.002$, $d_{31} = -0.002$, $d_{32} = 0.003$, $d_{33} = 0.002$, $d_{34} = 0.002$, $d_{41} = -0.002$, $d_{42} = 0.003$, $d_{43} = 0.002$, and $d_{44} = 0.002$. Synchronization of the systems (7) and (31) the control law (33) and the update laws (37) with the initial estimated parameters $a_1 = 0.5$, $a_2 = 0.5$, $a_3 = 0.5$, and $a_4 = 0.5$ and $b_1 = 17$, $b_2 = 9$, $b_3 = -1$, and $b_4 = -5.1$, are shown in Figures 5–7. Figure 5 displays synchronization

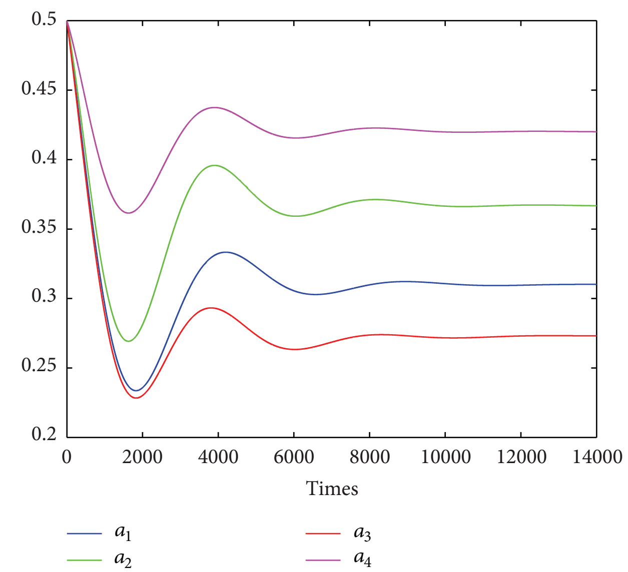

Figure 6: Estimated parameters of memristor chaotic oscillator system (7).

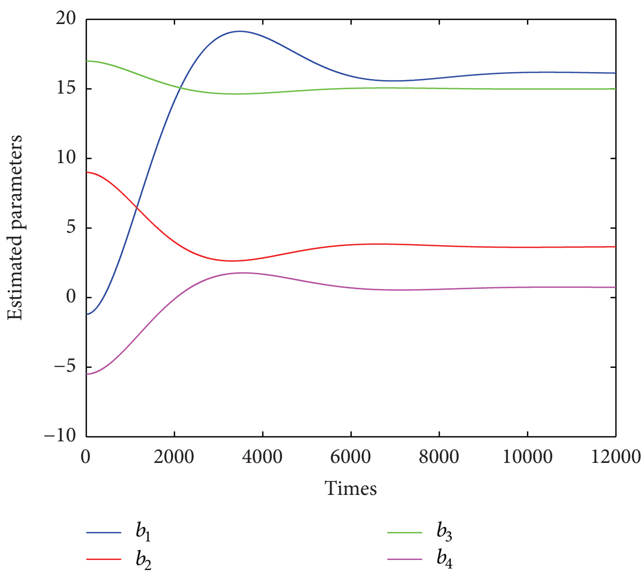

Figure 7: Estimated parameters of memristor chaotic oscillator system (31).

errors of systems (7) and (31). Figures 6 and 7 show that the estimated values of the unknown parameters a_1, a_2, a_3, and a_4 and b_1, b_2, b_3, and b_4 of the unknown parameters converge to $a_1 = 0.31$, $a_2 = 0.35$, $a_3 = 0.29$, and $a_4 = 0.41$ and $b_1 = 16.4$, $b_2 = 3.2$, $b_3 = 15$, and $b_4 = 0.5$, respectively.

Remark 8. In the simulations, d_{ij} $(i = 1, 2, 3, 4,\ \ j = 1, 2)$ are chosen to make $d_{11}d_{21}d_{31}d_{41} - d_{12}d_{22}d_{32}d_{42} \neq 0$ hold true.

3. Conclusion

In this paper, based on adaptive synchronization and general hybrid projective dislocated synchronization, we propose a novel hybrid dislocated adaptive synchronization scheme for asymptotic chaos synchronization using the Lyapunov stability theory. Complete dislocated synchronization, dislocated anti-synchronization, projective dislocated synchronization, and parameter identification are considered as its special items. In this way, we investigate the synchronization between two identical memristor chaotic oscillator systems and two different memristor chaotic oscillator systems with four uncertain parameters. Finally, two numerical simulation examples are provided to show the effectiveness and correctness of our method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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