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Research Article

Exponential Synchronization of Two Nonlinearly Coupled Complex Networks with Time-Varying Delayed Dynamical Nodes

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This paper investigates the exponential synchronization between two nonlinearly coupled complex networks with time-varying delay dynamical nodes. Based on the Lyapunov stability theory, some criteria for the exponential synchronization are derived with adaptive control method. Moreover, the presented results here can also be applied to complex dynamical networks with single time delay case. Finally, numerical analysis and simulations for two nonlinearly coupled networks which are composed of the time-delayed Lorenz chaotic systems are given to demonstrate the effectiveness and feasibility of the proposed complex network synchronization scheme.

1. Introduction

Generally speaking, a complex network consists of a large number of interconnected nodes by edges, where a node is a fundamental unit having specific contents and exhibiting dynamical behavior, typically. As we all have known, the complex network models widely exist in real world, such as Internet, World Wide Web, biological neural networks, social connection networks, global economic markets, and ecosystems. Since the discovery of some typical complex networks such as the random networks, small-world networks and scale-free networks, many scientists and engineers from various fields, for instance, mathematics, physics, engineering, and biology, have paid increasing attention to the studies of complex networks.

In past few decades, the control and synchronization problem of networks coupled with complex dynamical systems, especially chaotic systems, has been extensively investigated in various fields due to its many potential applications [1–10]. Many kinds of synchronization have been proposed, such as complete synchronization, lag synchronization, cluster synchronization, projective synchronization, and generalized synchronization. Since chaotic systems defy synchronization, how to design effective controllers for synchronizing coupled chaotic systems becomes an important and challenging problem. Many effective methods including pinning control [11–14], adaptive control [15–20], impulsive control [21–26], and intermittent control [27–29] have been adopted to design proper controllers. Inner synchronization, that is, the synchronization of all the nodes within a network, has been investigated recently. As a matter of fact, there exist other kinds of network synchronization in real world, for example, the synchronization phenomenon between two or more complex networks, which was called outer synchronization [30]. Therefore, how to synchronize between different networks is an interesting and challenging work. Li et al. [30] pioneered in studying the outer synchronization between two unidirectionally coupled complex networks and derived analytically a criterion for them having the identical topological structures. Tang et al. [31] discussed the synchronization between two complex dynamical networks with nonidentical topological structures via using adaptive control method. Li et al. [32] studied the synchronization between two networks with different topology structures and different dynamical behaviors with open-plus-closed-loop method. Wu et al. [33] studied the problem of generalized outer synchronization between two complex dynamical networks with different topologies and diverse node dynamics. Li et al. [34] discussed the outer synchronization of coupled discrete-time network. The adaptive-impulsive synchronization between two complex networks with nondelayed and delayed coupling was discussed in [35], but the delay is constant; moreover, the inner connecting function is linear and the delay in the dynamical nodes is ignored. In the real world, time delays are ubiquitous in natural and artificial systems. In much of the literature, time delays in the couplings are considered; however, the time delays in the dynamical nodes, which are more complex, are still relatively unexplored. To the best of our knowledge, the problem of synchronization between two nonlinearly coupled complex networks with time-delayed dynamical nodes is seldom discussed.

Motivated by the above discussions, the aim of this paper is to discuss the problem of exponential synchronization between two nonlinearly coupled dynamical networks with identical time-delayed dynamical nodes via adaptive control. Particularly, the coupling matrices are not assumed to be symmetric or irreducible. Based on the Lyapunov function method and mathematical analysis, synchronization criteria are derived analytically. Numerical examples are used finally to illustrate the usefulness of synchronization conditions. The above proposed scheme herein will be very useful for practical engineering applications.

The rest of this paper is organized as follows. In Section 2, model description and preliminaries are given. In Section 3, some sufficient conditions for the exponential synchronization are derived with the adaptive method. In Section 4, one illustrative example is given for supporting the theoretical results. Finally, conclusions are given in Section 5.

2. Model Description and Preliminaries

In this paper, consider two complex dynamical networks each consisting of N nonlinearly and diffusively coupled identical nodes, with each node being an n-dimensional dynamical system, respectively. One of the networks is characterized by

$$\begin{aligned}\dot{x}_i(t) = f\left(t, x_i(t), x_i(t-\tau(t))\right) + \sum_{j=1}^{N} b_{ij} h\left(x_j(t)\right) \\ + \sum_{j=1}^{N} c_{ij} g\left(x_j(t-\tau(t))\right),\end{aligned} \tag{1}$$

where, $i = 1, 2, \ldots, N$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state vector of the ith node, $f : R \times R^n \times R^n \to R^n$ is a smooth nonlinear function, $\tau(t)$ is the time-varying delay. $h(\cdot) \in R^n$ and $g(\cdot) \in R^n$ are the inner connecting functions in each node. While $B = (b_{ij})_{N\times N} \in R^{N\times N}$, $C = (c_{ij})_{N\times N} \in R^{N\times N}$ are the weight configuration matrices. If there is a connection from node i to node j $(j \neq i)$, then the coupling $b_{ij} \neq 0$, $c_{ij} \neq 0$; otherwise, $b_{ij} = c_{ij} = 0$ $(j \neq i)$, and the diagonal elements of matrix B, C are defined as $b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}$, $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}$. Here, the configuration matrices are not assumed to be symmetric or irreducible.

We refer to model (1) as the drive complex dynamical network, and the response complex network with control can be rewritten in the following form:

$$\begin{aligned}\dot{y}_i(t) = f\left(t, y_i(t), y_i(t-\tau(t))\right) + \sum_{j=1}^{N} b_{ij} h\left(y_j(t)\right) \\ + \sum_{j=1}^{N} c_{ij} g\left(y_j(t-\tau(t))\right) + u_i,\end{aligned} \tag{2}$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t))^T \in R^n$ is the response state vector of the ith node and u_i $(i = 1, 2, \ldots, N)$ are the controllers to be designed later.

Before presenting the derivation of the main results, the definition, the assumptions, and lemmas are introduced as follows.

Definition 1 (see [36]). The networks (1) and (2) are said to be globally exponentially asymptotically synchronous if there exist constants $M > 0$ and $\rho > 0$, such that for any initial condition

$$\left\| y_i(t) - x_i(t) \right\| = \left\| e_i(t) \right\| \leq M e^{-\rho t}. \tag{3}$$

Assumption 2. Time delay $\tau(t)$ is a differential function with $0 \leq \tau(t) \leq \tau_M$ and $0 \leq \dot{\tau}(t) \leq \varepsilon < 1$. Clearly, this assumption includes constant delay as a special case.

Assumption 3 (see [37]). For the vector valued function $f(t, x_i(t), x_i(t-\tau(t)))$, assume that there exist positive constants $\gamma_1 > 0$, $\gamma_2 > 0$ such that f satisfies the semi-Lipschitz condition

$$\begin{aligned}&(x(t) - y(t))^T \left(f\left(t, x(t), x(t-\tau(t))\right) \right. \\ &\qquad \left. - f\left(t, y(t), y(t-\tau(t))\right) \right) \\ &\leq \gamma_1 (x(t) - y(t))^T \left(x(t) - y(t)\right) \\ &\quad + \gamma_2 (x(t-\tau(t)) - y(t-\tau(t)))^T \left(x(t-\tau(t)) - y(t-\tau(t))\right),\end{aligned} \tag{4}$$

for all $x, y \in R^n$ and $t \geq 0$.

Remark 4. Assumption 3 gives some requirements for the dynamics of isolated node in network. It is easy to verify that many chaotic systems with delays or without delays satisfy Assumption 3, for example, Chua's oscillator, Rössler's system, the Lorenz system, Chen's system, and Lü's system as well as the delayed Lorenz system, delayed Hopfield neural networks, and delayed cellular neural networks, and so on.

Lemma 5 (see [38]). *For any vectors $x, y \in R^n$ and positive definite matrix $Q \in R^{n\times n}$, the following matrix inequality holds: $x^T y \leq (1/2)(x^T Q x + y^T Q^{-1} y)$.*

3. Exponential Synchronization Criteria

In this section, we will make drive-response complex dynamical networks with nondelayed and delayed coupling achieving global exponential synchronization by using adaptive controlling method.

Define the synchronization errors $e_i(t) = y_i(t) - x_i(t)$ $(i = 1, 2, \ldots, N)$; the following errors dynamics system is obtained:

$$\dot{e}_i = f(t, y_i(t), y_i(t-\tau(t))) - f(t, x_i(t), x_i(t-\tau(t))) + \sum_{j=1}^{N} b_{ij}\overline{h}(e_j(t)) + \sum_{j=1}^{N} c_{ij}\overline{g}(e_j(t-\tau(t))) + u_i, \tag{5}$$

where $\overline{h}(e_j(t)) = h(y_j(t)) - h(x_j(t))$, $\overline{g}(e_j(t-\tau(t))) = g(y_j(t-\tau(t))) - g(x_j(t-\tau(t)))$.

Theorem 6. *Suppose Assumptions 2 and 3 hold. The response network* (2) *can globally exponentially asymptotically synchronize with the driven network* (1) *if the controllers are designed as follows:*

$$u_i = \begin{cases} -d_i e_i(t) - \dfrac{N\left\|\overline{h}(e_i(t))\right\|^2}{2\left\|e_i(t)\right\|^2} e_i(t) \\ \quad -\dfrac{N e^{\rho\tau_M}\left\|\overline{g}(e_i(t))\right\|^2}{2(1-\varepsilon)\left\|e_i(t)\right\|^2} e_i(t), & \left\|e_i(t)\right\| \neq 0, \\ 0, & \left\|e_i(t)\right\| = 0, \end{cases} \tag{6}$$

where

$$\dot{d}_i = k_i e^{\rho(t-\tau_M)}\left\|e_i(t)\right\|^2 \tag{7}$$

with k_i being the known adjustable positive constants.

Proof. We construct the Lyapunov function as follows:

$$\begin{aligned} V(t) &= \frac{1}{2} e^{\rho(t-\tau_M)} \sum_{i=1}^{N} e_i^T(t) e_i(t) \\ &+ \frac{1}{2}\sum_{i=1}^{N}\frac{1}{k_i}(d_i - d_i^*)^2 \\ &+ \frac{\gamma_2}{1-\varepsilon}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e^{\rho\theta} e_i^T(\theta) e_i(\theta)\, d\theta \\ &+ \frac{N}{2(1-\varepsilon)}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e^{\rho\theta}\overline{g}^T(e_i(\theta))\,\overline{g}(e_i(\theta))\, d\theta, \end{aligned} \tag{8}$$

where d_i^* is sufficiently large positive constants to be determined.

Calculate the derivative of (8) along the trajectories of (5), and with adaptive controllers (6). Thus, we obtain

$$\begin{aligned} \dot{V} &= \frac{1}{2}\rho e^{\rho(t-\tau_M)}\sum_{i=1}^{N} e_i^T(t) e_i(t) \\ &+ e^{\rho(t-\tau_M)}\sum_{i=1}^{N} e_i^T(t)\Bigg[f(t, y_i(t), y_i(t-\tau(t))) \\ &\qquad - f(t, x_i(t), x_i(t-\tau(t))) \\ &\qquad + \sum_{j=1}^{N} b_{ij}\overline{h}(e_j(t)) \\ &\qquad + \sum_{j=1}^{N} c_{ij}\overline{g}(e_j(t-\tau(t))) + u_i \Bigg] \\ &+ \sum_{i=1}^{N}\frac{1}{k_i}(d_i - d_i^*)\dot{d}_i + \frac{\gamma_2}{1-\varepsilon}\sum_{i=1}^{N} e^{\rho t} e_i^T(t) e_i(t) \\ &- \frac{\gamma_2(1-\dot{\tau}(t))}{1-\varepsilon}\sum_{i=1}^{N} e^{\rho(t-\tau(t))} e_i^T(t-\tau(t)) e_i(t-\tau(t)) \\ &+ \frac{N}{2(1-\varepsilon)}\sum_{i=1}^{N} e^{\rho t}\overline{g}^T(e_i(t))\,\overline{g}(e_i(t)) \\ &- \frac{N(1-\dot{\tau}(t))}{2(1-\varepsilon)}\sum_{i=1}^{N} e^{\rho(t-\tau(t))}\overline{g}^T(e_i(t-\tau(t)))\,\overline{g}(e_i(t-\tau(t))). \end{aligned} \tag{9}$$

From Assumption 2, we get

$$\frac{1}{2} \leq \frac{1-\dot{\tau}(t)}{2(1-\varepsilon)}, \qquad e^{\rho(\tau_M - \tau(t))} \geq 1. \tag{10}$$

We have

$$\begin{aligned} \dot{V} \leq e^{\rho(t-\tau_M)} &\Bigg[\frac{1}{2}\rho\sum_{i=1}^{N} e_i^T(t) e_i(t) + \gamma_1\sum_{i=1}^{N} e_i^T(t) e_i(t) \\ &+ \frac{\gamma_2}{1-\varepsilon}\sum_{i=1}^{N} e^{\rho\tau_M} e_i^T(t) e_i(t) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij} e_i^T(t)\overline{h}(e_j(t)) \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij} e_i^T(t)\overline{g}(e_j(t-\tau(t))) \\ &- \sum_{i=1}^{N} d_i^*\left\|e_i(t)\right\|^2 - \frac{N}{2}\sum_{i=1}^{N}\overline{h}^T(e_i(t))\,\overline{h}(e_i(t)) \\ &- \frac{N}{2}\sum_{i=1}^{N}\overline{g}^T(e_i(t-\tau(t)))\,\overline{g}(e_i(t-\tau(t))) \Bigg]. \end{aligned} \tag{11}$$

From Lemma 5, we have

$$\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij} e_i^T(t)\,\overline{h}\left(e_j(t)\right) \le \frac{N^2}{2}\max_{1\le i\le N}\left(b_{ii}^2\right)\sum_{i=1}^{N} e_i^T(t)\,e_i(t) + \frac{N}{2}\sum_{i=1}^{N}\left\|\overline{h}\left(e_i(t)\right)\right\|^2,$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij} e_i^T(t)\,\overline{g}\left(e_j(t-\tau(t))\right) \le \frac{N^2}{2}\max_{1\le i\le N}\left(c_{ii}^2\right)\sum_{i=1}^{N} e_i^T(t)\,e_i(t) + \frac{N}{2}\sum_{i=1}^{N}\left\|\overline{g}\left(e_i(t-\tau(t))\right)\right\|^2. \tag{12}$$

Thus, we obtain

$$\dot{V} \le e^{\rho(t-\tau_M)}\sum_{i=1}^{N}\left[\frac{1}{2}\rho + \gamma_1 + \frac{\gamma_2 e^{\rho\tau_M}}{1-\varepsilon} + \frac{N^2}{2}\max_{1\le i\le N}\left(a_{ii}^2\right) + \frac{N^2}{2}\max_{1\le i\le N}\left(c_{ii}^2\right) - d_i^*\right] e_i^T(t)\,e_i(t). \tag{13}$$

It is obvious that the constants d_i^* $(i = 1, 2, \ldots, N)$ can be properly chosen to make $(1/2)\rho + \gamma_1 + \gamma_2 e^{\rho\tau_M}/(1-\varepsilon) + (N^2/2)\max_{1\le i\le N}(a_{ii}^2) + (N^2/2)\max_{1\le i\le N}(c_{ii}^2) - d_i^* < 0$, namely, $\dot{V} \le 0$, and then $V(t) \le V(0)$, for any $t \ge 0$.

On the other hand, we have

$$e^{\rho(t-\tau_M)}\frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\,e_i(t) \le V(t) \le V(0). \tag{14}$$

Therefore, one has

$$\left\|e_i(t)\right\| \le \sqrt{2V(0)}\,e^{-(\rho/2)(t-\tau_M)}. \tag{15}$$

Thus, the errors vector $e(t) \to 0$, that is, the network (1) and network (2), are globally exponentially asymptotically synchronous.

In a special case, when the networks are all linearly coupled, that is, $h(x_i(t)) = \Gamma_1 x_i(t)$, $g(x_i(t-\tau(t))) = \Gamma_2 x_j(t-\tau(t))$, the network (1) degenerates into the following:

$$\dot{x}_i(t) = f\left(t, x_i(t), x_i(t-\tau(t))\right) + \sum_{j=1}^{N} b_{ij}\Gamma_1 x_j(t) + \sum_{j=1}^{N} c_{ij}\Gamma_2 x_j(t-\tau(t)) \tag{16}$$

and the response complex network with control is given by

$$\dot{y}_i(t) = f\left(t, y_i(t), y_i(t-\tau(t))\right) + \sum_{j=1}^{N} b_{ij}\Gamma_1 y_j(t) + \sum_{j=1}^{N} c_{ij}\Gamma_2 y_j(t-\tau(t)) + u_i. \tag{17}$$

We have Corollary 7 for the networks (16) and (17), due to the inequality

$$\begin{aligned}\left\|h\left(x_i(t)\right)\right\| &\le \left\|\Gamma_1\right\|\left\|x_i(t)\right\|,\\ \left\|g\left(x_i(t-\tau(t))\right)\right\| &\le \left\|\Gamma_2\right\|\left\|x_i(t-\tau(t))\right\|.\end{aligned} \tag{18}$$

□

Corollary 7. *Consider the complex networks* (16) *and* (17), *if Assumptions 2 and 3 hold. Use the following adaptive controllers and updated laws:*

$$u_i = \begin{cases} -d_i e_i(t) - \dfrac{N}{2}\left\|\Gamma_1\right\|^2 e_i(t) - \dfrac{N e^{\rho\tau_M}}{2(1-\varepsilon)}\left\|\Gamma_2\right\|^2 e_i(t), & \left\|e_i(t)\right\| \ne 0,\\ 0, & \left\|e_i(t)\right\| = 0,\end{cases}$$
$$\dot{d}_i = k_i e^{\rho(t-\tau_M)}\left\|e_i(t)\right\|^2, \tag{19}$$

where d_i are the feedback strength and $k_i > 0$ are arbitrary constants. Then, the drive-response networks can globally exponentially asymptotically synchronize.

If the time-varying delay in the network (1) is the constant delay, we can obtain the following results.

Theorem 8. *Suppose Assumptions 2 and 3 hold. Use the following adaptive controllers and updated laws:*

$$u_i = \begin{cases} -d_i e_i(t) - \dfrac{N\left\|\overline{h}\left(e_i(t)\right)\right\|^2}{2\left\|e_i(t)\right\|^2} e_i(t) - \dfrac{N e^{\rho\tau}\left\|\overline{g}\left(e_i(t)\right)\right\|^2}{2\left\|e_i(t)\right\|^2} e_i(t), & \left\|e_i(t)\right\| \ne 0,\\ 0, & \left\|e_i(t)\right\| = 0,\end{cases}$$
$$\dot{d}_i = k_i e^{\rho(t-\tau)}\left\|e_i(t)\right\|^2, \tag{20}$$

where d_i are the feedback strength and $k_i > 0$ are arbitrary constants. Then, the drive-response networks can globally exponentially asymptotically synchronize.

Proof. We construct the Lyapunov function as follows:

$$\begin{aligned} V(t) = {} & \frac{1}{2}e^{\rho(t-\tau)}\sum_{i=1}^{N} e_i^T(t)\,e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{k_i}\left(d_i - d_i^*\right)^2 \\ & + \frac{\gamma_2}{1-\varepsilon}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e^{\rho\theta} e_i^T(\theta)\,e_i(\theta)\,d\theta \\ & + \frac{N}{2(1-\varepsilon)}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e^{\rho\theta}\,\overline{g}^T\left(e_i(\theta)\right)\overline{g}\left(e_i(\theta)\right)d\theta, \end{aligned} \tag{21}$$

where d_i^* is a positive constants to be determined. The rest of the proof is similar to that of Theorem 6 and is omitted here. □

Similar to Corollary 7, we have Corollary 9.

Corollary 9. *Suppose Assumptions 2 and 3 hold. Use the following adaptive controllers and updated laws:*

$$u_i = \begin{cases} -d_i e_i(t) - \dfrac{N}{2}\|\Gamma_1\|^2 e_i(t) \\ \quad -\dfrac{Ne^{\rho\tau}}{2}\|\Gamma_2\|^2 e_i(t), & \|e_i(t)\| \neq 0, \\ 0, & \|e_i(t)\| = 0, \end{cases} \tag{22}$$
$$\dot{d}_i = k_i e^{\rho(t-\tau)} \|e_i(t)\|^2,$$

where d_i are the feedback strength and $k_i > 0$ are arbitrary constants. Then, the drive-response network can globally exponentially asymptotically synchronize.

Remark 10. It is noted that the configuration matrices B and C do not need to be symmetric, diffusive, or irreducible. This means that the networks can be undirected or directed networks and may also contain isolated nodes or clusters. Therefore, the network structures here are very general and the results can be applied to a great many complex dynamical networks.

Remark 11. The feedback strengths d_i are automatically adapted to the suitable constant, which depends on the initial values. The constants k_i can be chosen properly to adjust the synchronization speed. The larger the constants k_i the faster the achievement of synchronization of the drive-response nonlinearly coupled networks with time-varying delayed dynamical nodes.

Remark 12. Some stability criteria for the exponential synchronization between drive and response nonlinearly coupled networks with time-varying delays are derived, which can also be applied to the complex network with single time delay. Thus, the results presented in this paper improve and generalize the corresponding results of recent works. Moreover, our designed synchronization controller is not only robust but also easy to implement.

4. Numerical Simulation

In this section, numerical simulation is given to verify and demonstrate the effectiveness of the proposed method for exponentially synchronizing two nonlinearly coupled complex networks with time-delayed dynamical nodes. Consider the time-delayed Lorenz chaotic system as node dynamics. It is described by

$$f(t, x_i(t), x_i(t-\tau(t))) = \begin{pmatrix} a(x_{i2} - x_{i1}) \\ rx_{i1} + (c-1)x_{i2} - x_{i1}x_{i3} + cx_{i2}(t-\tau(t)) \\ -bx_{i3} + x_{i1}x_{i2} \end{pmatrix}, \tag{23}$$

which has a chaotic attractor when $a = 10$, $b = 8/3$, $r = 28$, and $c = 5$. See Figure 1.

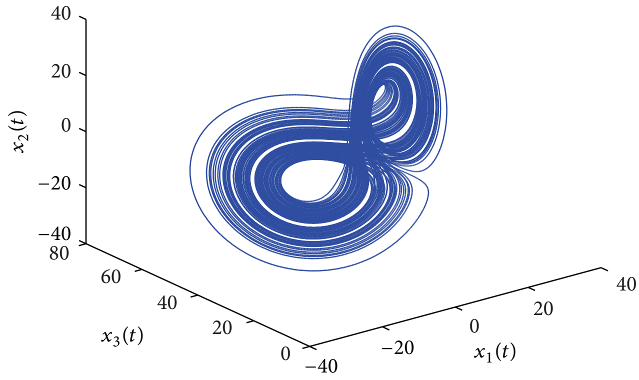

FIGURE 1: Chaos phase portrait of the time-delayed Lorenz system.

Rewrite (16) as

$$f(t, x_i(t), x_i(t-\tau(t))) = A_0 x_i(t) + \phi(t, x_i(t), x_i(t-\tau(t))), \tag{24}$$

where

$$A_0 = \begin{pmatrix} -a & a & 0 \\ r & c-1 & 0 \\ 0 & 0 & -b \end{pmatrix},$$
$$\phi(t, x_i(t), x_i(t-\tau(t))) = \begin{pmatrix} 0 \\ -x_{i1}x_{i3} + cx_{i2}(t-\tau(t)) \\ x_{i1}x_{i2} \end{pmatrix}. \tag{25}$$

For any state vectors x_i of the time-delayed Lorenz chaotic system, there exists a constant M satisfying $\|x_i\| \leq M$ since chaotic attractor is bounded.

To satisfy Assumption 3, consider that one can always find $\eta > 0$ such that $|xy| \leq \eta(x^2/2) + y^2/2\eta$, and then we have

$$\begin{aligned} & e_i^T(t)\left[f(t, y_i(t), y_i(t-\tau(t))) - f(t, x_i(t), x_i(t-\tau(t)))\right] \\ & = e_i^T A_0 e_i + e_{i1}e_{i3}x_{i2} - e_{i1}e_{i2}x_{i3} + 5e_{i2}e_{i2}(t-\tau(t)) \\ & \leq \left(-10 + \eta_1\frac{38+M}{2} + \frac{\eta_2 M}{2}\right)e_{i1}^2 \\ & \quad + \left(4 + \frac{38+M}{2\eta_1} + \frac{5\eta_3}{2}\right)e_{i2}^2 \\ & \quad + \left(\frac{M}{2\eta_2} - \frac{8}{3}\right)e_{i3}^2 + \frac{M}{2\eta_3 e_{i2}^2(t-\tau(t))} \\ & \leq \gamma_1 e_i^T e_i + \gamma_2 e_i^T(t-\tau(t)) e_i(t-\tau(t)), \end{aligned} \tag{26}$$

where γ_1, γ_2 can be determined by choosing appropriate parameters $\eta_i > 0$, $i = 1, 2, 3$.

Now, we consider two nonlinearly coupled complex dynamical networks (1) and (2) with coupling delay

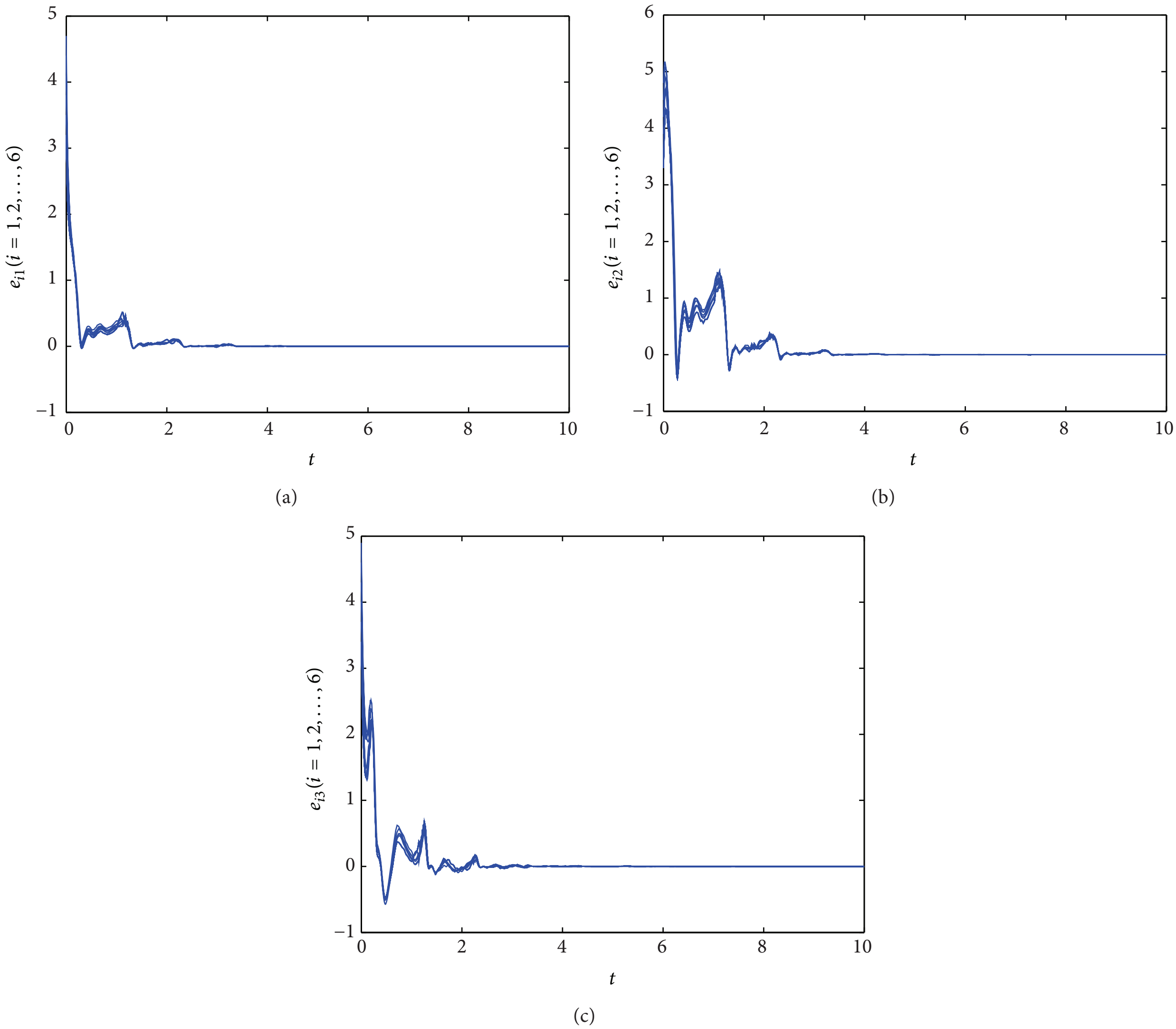

FIGURE 2: Errors of the drive-response coupled networks: (a) e_{i1}; (b) e_{i2}; (c) e_{i3}.

consisting of 6 identical time delayed chaotic systems. Take the weight configuration coupling matrices

$$B = \begin{pmatrix} -4 & 3 & 0 & 0 & 1 & 0 \\ 1 & -6 & 2 & 0 & 0 & 3 \\ 2 & 1 & -3 & 0 & 0 & 0 \\ 0 & 3 & 0 & -7 & 4 & 0 \\ 0 & 0 & 0 & 4 & -4 & 0 \\ 1 & 0 & 1 & 0 & 0 & -2 \end{pmatrix},$$
$$C = \begin{pmatrix} -6 & 3 & 2 & 0 & 1 & 0 \\ 3 & -4 & 1 & 0 & 0 & 0 \\ 2 & 1 & -3 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & -5 & 5 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix}. \quad (27)$$

For simplicity, in the numerical simulations, we assume the time-varying delay $\tau(t) = e^t/(1 + e^t)$; then $\dot{\tau}(t) = e^t/(1 + e^t)^2 \in (0, 1/2]$ satisfies Assumption 2. The initial values of the drive systems and the response systems are chosen as $x_i(0) = (0.1 + 0.1i, 0.2 + 0.2i, 0.3 + 0.3i)^T$, $y_i(0) = (-0.2 + 0.1i, -0.3 + 0.2i, -0.4 + 0.3i)^T$, the positive constants $k_i = 1$; let $h(x) = \sin(x) + 3x$ and $g(x) = -\cos(x) + 3x$. Based on Theorem 6, the global exponential synchronization can be achieved. The synchronization errors are shown, respectively, in Figure 2. Figure 3 displays the state subvariables for node $i = 3$ of the drive network and response networks. Figure 4 plots the total synchronization errors $\|e(t)\| = \sqrt{\sum_{i=1}^{6}[e_{i1}^2(t) + e_{i2}^2(t) + e_{i3}^2(t)]}$ with different k_i $(i = 1, 2, \ldots, 6)$. As described in Figure 4, the larger the constants k_i the faster the convergence to synchronization. The numerical results show that adaptive scheme for the exponential synchronization of the drive-response nonlinearly coupled complex networks is effective in all the theorems and corollaries.

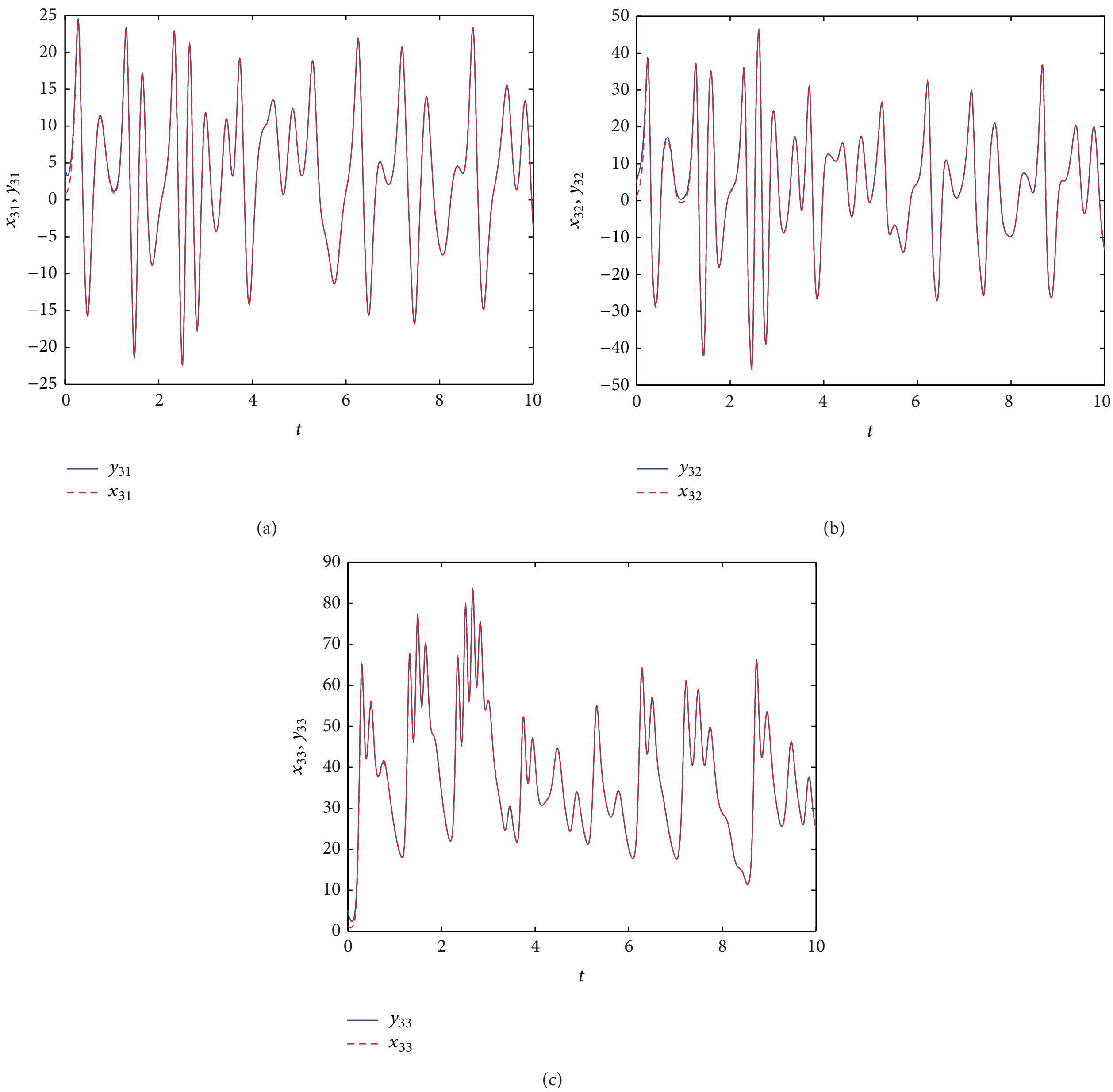

Figure 3: State subvariables for node $i = 3$ between the drive and response coupled networks.

5. Conclusion

In this paper, the adaptive controllers have been proposed to study the global exponential synchronization between two nonlinearly coupled complex networks with time-varying delay dynamical nodes. By constructing the appropriate Lyapunov functions, some criteria are derived. In particular, the coupling matrices are not symmetric and irreducible. Numerical results demonstrate that the proposed approach is effective and feasible. In the analysis and simulation study of this paper, we fully considered the impact of the time delay element to the exponential synchronization of the drive-response coupled networks. In order to obtain the synchronization criteria, we did not take into account the environment factors, for example, noise, on the networks, which often affects the synchronization process of the drive-response coupled dynamical networks. Therefore, in the near future work, we will further investigate the exponential synchronization problem of drive-response nonlinearly coupled dynamical network with noise.

Conflict of Interests

The author declares that he has no conflict of interests.

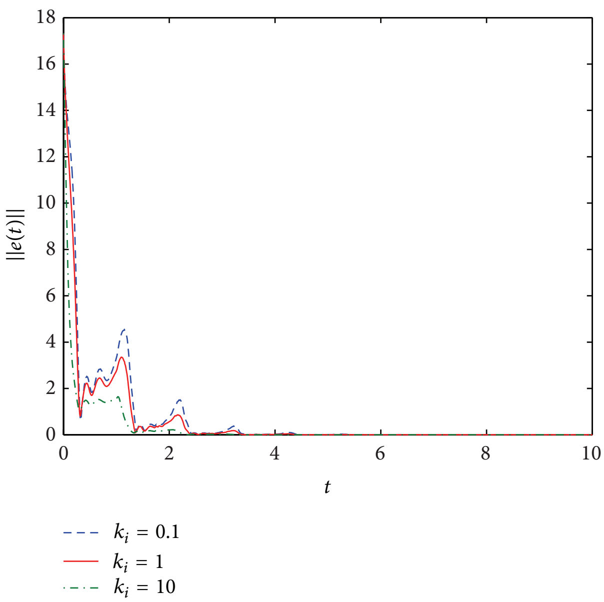

FIGURE 4: Total synchronization errors $\|e(t)\|$ with $k_i = 0.1, 1, 10$.

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