

Research Article

Nontrivial Solutions of a Class of Kirchhoff Type Problems via Local Linking Method

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The existence of nontrivial solutions of Kirchhoff type problems is proved by using the local linking method, and the new results do not require classical compactness conditions.

1. Introduction

This paper is concerned with the existence of nontrivial solutions of the following nonlocal Kirchhoff type problem:

$$\begin{aligned} -\left(a + b \int_{\Omega} |\nabla u|^2\right) \Delta u &= f(x, u) \quad \text{in } \Omega; \\ u &= 0 \quad \text{on } \partial\Omega, \end{aligned} \quad (1)$$

where Ω is a smooth bounded domain in R^n ($n = 1, 2$), $a, b > 0$, and $f(x, u): \bar{\Omega} \times R$ is a continuous real function. Similar nonlocal problems model several biological systems where u describes a process which depends on the average of itself, for example, that of the population density; see [1].

Problem (1) is related to the stationary analogue of the equation

$$u_{tt} - \left(a + b \int_{\Omega} |\nabla u|^2\right) \Delta u = g(x, u) \quad (2)$$

proposed by Kirchhoff [2] as an extension of the classical D'Alembert wave equation for free vibrations of elastic strings. Kirchhoff's model takes into account the changes in length of the string produced by transverse vibrations. Some early classical studies of Kirchhoff equations were those of Bernstein [3] and Pohožaev [4]. However, (2) received great attention only after Lions [5] proposed an abstract framework for the problem. Some interesting results can be found in [6] and the references theorem. In recent years, Alves et al. [7]

and Ma and Rivera [8] have obtained positive solutions of such problems by variational methods.

Later Perera and Zhang [9, 10] obtained nontrivial solutions of Kirchhoff type problems with asymptotically 4-linear term via Yang index. In [9] using the invariant sets of descent flow, Perera and Zhang got a positive, a negative, and a sign-changing solution under the 4-sublinear case, asymptotically 4-linear case, and 4-superlinear case. In [10], the authors considered the 4-superlinear case:

$$(AR) : \exists \nu > 4 : \nu F(x, t) \leq t f(x, t), \quad |t| \text{ large}, \quad (3)$$

where $F(x, t) = \int_0^t f(x, s) ds$, which implies

$$F(x, t) \geq C(|t|^\nu - 1). \quad (4)$$

In [11], Mao and Zhang used minimax methods and invariant sets of decent flow to prove 4-superlinear Kirchhoff type problems without the PS condition and got multiple solutions. In [12], the paper studies the problem (1) by means of the Morse theory and local linking.

Motivated by [12–15] and by the new information concerning local linking, we study Kirchhoff type problems 4-superlinear growth as $|u| \rightarrow \infty$. We make the following assumptions.

(S₁) There exist $\delta > 0$, $k \geq 1$, $c > 0$, and $2 < \alpha < 4$ such that

$$cu^\alpha + \frac{\lambda_k}{2} u^2 < F(x, u) < \frac{\lambda_{k+1}}{2} u^2 \quad \text{for } |u| < \delta. \quad (5)$$

(S₂) $(F(x, u)/u^4) \rightarrow +\infty$ as $|u| \rightarrow +\infty$ uniformly in $x \in \Omega$.

(S₃) There exist $r > 4$ and $\mu > r - 4$ such that

$$\limsup_{|u| \rightarrow \infty} \frac{F(x, u)}{|u|^r} < \infty \quad \text{uniformly for a.e. } x \in \Omega, \quad (6)$$

$$\liminf_{|u| \rightarrow \infty} \frac{(f(x, u)u - 4F(x, u))}{|u|^\mu} > 0 \quad (7)$$

uniformly for a.e. $x \in \Omega$.

We are ready to state our main results.

Theorem 1. *Assume that (S₁)–(S₃) hold. Then problem (1) has at least one nontrivial solution.*

Remark 2. Most of the results on the existence and multiplicity of solutions of (1) were obtained under the above superlinear condition (AR) with or without the evenness assumption. Roughly speaking the role of (AR) is to ensure the boundedness of all (PS) (or (PS*)) sequences for the corresponding functional. However, there are many functions which are superlinear but it is not necessary to satisfy (AR) even if $1 < \theta \leq 2$. For example,

$$f(x, u) = \mu |u|^{\mu-2} u + (\mu - 1) |u|^{\mu-3} u \sin^2 u + |u|^{\mu-1} \sin 2u, \quad u \in R \setminus \{0\}, \quad (8)$$

where $\mu > 2$. Then it is easy to check that (AR) does not hold even for any $\theta > \mu - 1 > 1$. On the other hand, in order to verify (AR), it is usually an annoying task to compute the primitive function of f and sometimes it is almost impossible. For example,

$$f(x, u) = |u| u \left(1 + e^{(1+|\sin u|)^\alpha} + |\cos u|^\alpha \right), \quad u \in R, \quad (9)$$

where $\alpha > 0$.

The aim of this paper is to deal with superlinear problems which do not satisfy (AR). (S₂) is weaker than (AR) used in [10]. Without (AR), it becomes more complicated. We do not know in our situations whether the Palais-Smale sequence is bounded. We replace (PS) (or (PS*)) sequences with weaker Cerami sequence or (C)*-sequence (the definition given in the next section).

Remark 3. Compared with the method of invariant sets of descent flow and Morse theory, Yang index, our method is more simple and direct.

2. Preliminaries

Let X be a real Banach space with a direct sum decomposition:

$$X = X^1 \oplus X^2. \quad (10)$$

Consider two sequences of subspace:

$$X_0^1 \subset X_1^1 \subset \dots \subset X^1, \quad X_0^2 \subset X_1^2 \subset \dots \subset X^2 \quad (11)$$

such that $X^j = \bigcup_{n \in N} X_n^j$, $j = 1, 2$, where $\dim X_n^j < \infty$, $j = 1, 2, n \in N$.

For every multi-index $\alpha = (\alpha_1, \alpha_2) \in N \times N$, we denote by X_α the space

$$X_{\alpha_1}^1 \oplus X_{\alpha_2}^2. \quad (12)$$

We can know

$$\alpha \leq \beta \iff \alpha_1 \leq \beta_1, \quad \alpha_2 \leq \beta_2. \quad (13)$$

A sequence $\{\alpha_n\} \subset N^2$ is admissible if, for every $\alpha \in N^2$, there is $m \in N$ such that

$$n \geq m \implies \alpha_n \geq \alpha. \quad (14)$$

For every $\varphi : X \rightarrow R$ we denote by φ_α the functional φ restricted to X_α .

Definition 4 (see [14]). Let $\varphi \in C^1(X, R)$. The function φ satisfies (C)* condition if every sequence $\{u_{\alpha_n}\}$ such that α_n is admissible and

$$u_{\alpha_n} \in X_{\alpha_n}, \quad \sup_n \varphi(u_{\alpha_n}) < \infty, \quad \varphi'_{\alpha_n}(u_{\alpha_n}) \rightarrow 0 \quad (15)$$

contains a subsequence which converges to a critical point of φ .

Definition 5 (see [15]). Let X be Banach space with a direct sum decomposition $X = X^1 \oplus X^2$. The functional $\varphi \in C^1(X, R)$ has a local linking at 0 if, for some $r > 0$,

$$\begin{aligned} \varphi(u) &\geq 0 & u \in X^1, \quad \|u\| \leq r, \\ \varphi(u) &\leq 0 & u \in X^2, \quad \|u\| \leq r. \end{aligned} \quad (16)$$

Lemma 6 (see [14]). *Let X be Banach space with a direct sum decomposition $X = X^1 \oplus X^2$. Suppose that $\varphi \in C^1(X, R)$ satisfies the following assumptions.*

- (I₁) φ has a local linking at 0 with respect to (X^1, X^2) .
- (I₂) φ satisfies (C)* condition.
- (I₃) φ maps bounded sets into bounded sets.
- (I₄) For every $m \in N$, $\varphi(u) \rightarrow -\infty$ as $u \in X_m^1 \oplus X^2$ and $\|u\| \rightarrow \infty$.

Then φ has at least one nontrivial critical point.

Next we let $H : H_0^1(\Omega)$ be the Sobolev space equipped with the inner product and the norm

$$\langle u, v \rangle = \int_{\Omega} \nabla u \nabla v, \quad \|u\| = (u, u)^{1/2}. \quad (17)$$

Denote by $0 < \lambda_1 < \lambda_2 < \dots$ the distinct Dirichlet eigenvalues of $-\Delta$ on Ω , and denote by $\varphi_1, \varphi_2, \varphi_3, \dots$ the eigenfunction corresponding to the eigenvalues; then

$$\lambda_1(\Omega) := \inf_{u \in H, \|u\|_2 = 1} |\nabla u|_2^2 \quad (18)$$

is achieved by $\varphi_1 > 0$. Let

$$E_j := \bigoplus_{i \leq j} \ker(-\Delta - \lambda_i), \quad j \in \mathbb{N}. \quad (19)$$

Recall that a function $u \in H$ is called a weak solution of (1) if

$$(a + b\|u\|^2) \int_{\Omega} \nabla u \cdot \nabla v = \int_{\Omega} f(x, u) v, \quad \forall v \in H. \quad (20)$$

Weak solutions are the critical points of C^1 functional:

$$\varphi(u) := \frac{a}{2} \|u\|^2 + \frac{b}{4} \|u\|^4 - \int_{\Omega} F(x, u), \quad \forall u \in H. \quad (21)$$

Then

$$\begin{aligned} \varphi'(u)v &= (a + b\|u\|^2) \int_{\Omega} \nabla u \cdot \nabla v - \int_{\Omega} f(x, u) v, \\ &\quad \forall u, v \in H. \end{aligned} \quad (22)$$

3. The Existence of the Solutions

Lemma 7. *Let F satisfies (S_1) ; the function φ has a local linking with respect to $H = X^1 \oplus X^2$, where $X^1 = E_k, X^2 = (X^1)^\perp$.*

Proof. Let $X^1 = E_k, X^2 = (X^1)^\perp$. If $u \in X^1$, then $\|u\|^2 \leq \lambda_k |u|_2^2$; and if $u \in X^2$, then $\|u\|^2 \geq \lambda_{k+1} |u|_2^2$. If $u \in X^1$, from (S_1) , for the given δ , there exists a $\rho_1 = \delta/c_1 > 0$ such that $\|u\| \leq \rho_1 \Rightarrow |u| \leq c_1 \|u\| \leq \delta$ and since $\dim X^1$ is finite $c|u|_\alpha^\alpha \geq c_2 \|u\|^\alpha$; by (S_1) , we have

$$\begin{aligned} \varphi(u) &= \frac{a}{2} \|u\|^2 + \frac{b}{4} \|u\|^4 - \int_{\Omega} F(x, u) dx \\ &\leq \frac{a}{2} \|u\|^2 + \frac{b}{4} \|u\|^4 - \frac{a\lambda_k}{2} |u|_2^2 - c|u|_\alpha^\alpha \\ &\leq \frac{b}{4} \|u\|^4 - c_2 \|u\|^\alpha. \end{aligned} \quad (23)$$

Noting that $2 < \alpha < 4$, we can choose $\rho_2 \leq \rho_1$ small enough such that $\varphi(u) < 0$ for $\|u\| \leq \rho_2$.

For $u \in X^2$, from $(S_1), (S_3)$, we have

$$F(x, u) \leq \frac{a\lambda_{k+1}}{2} u^2 + c_3 |u|^r; \quad (24)$$

then

$$\begin{aligned} \varphi(u) &= \frac{a}{2} \|u\|^2 + \frac{b}{4} \|u\|^4 - \int_{\Omega} F(x, u) dx \\ &> \frac{a}{2} \|u\|^2 + \frac{b}{4} \|u\|^4 - \frac{a\lambda_{k+1}}{2} |u|_2^2 - c_3 |u|_r^r \\ &\geq \frac{b}{4} \|u\|^4 - c_4 \|u\|^r. \end{aligned} \quad (25)$$

For $r > 4$, choosing ρ_3 small enough such that $(b/4)\|u\|^4 - c_4\|u\|^r > 0$ for $\|u\| \leq \rho_3$, we have $\varphi(u) > 0$ for $u \in X^2, \|u\| \leq \rho_3$.

Choose $\rho = \min\{\rho_2, \rho_3\}$; it is very clear that φ has a local linking at 0 with respect to $H = X^1 \oplus X_2$. \square

Lemma 8. *Suppose (S_3) holds. Then function φ satisfies condition $(C)^*$; that is, for every sequence $\{u_{\alpha_n}\} \subset X_{\alpha_n}, \{u_{\alpha_n}\}$ has a convergent subsequence if $\{\varphi(u_{\alpha_n})\}$ is bounded and $(1 + \|u_n\|)\|\varphi'(u_{\alpha_n})\| \rightarrow 0$ as $n \rightarrow \infty$, where $X_{\alpha_n} = X_{\alpha_n}^1 \oplus X^2, X_{\alpha_n}^1 = \overline{\text{span}\{e_0, e_1, \dots, e_n\}}$, and $(e_i)_{i \geq 0}$ is a Hilbert basis for X^1 .*

Proof. Suppose that $\{u_{\alpha_n}\} \subset X_{\alpha_n}, \varphi(u_{\alpha_n})$ is bounded and $(1 + \|u_{\alpha_n}\|)\|\varphi'(u_{\alpha_n})\| \rightarrow 0$ as $n \rightarrow \infty$; then there exists a constant $M > 0$ such that

$$|\varphi(u_{\alpha_n})| \leq M, \quad (1 + \|u_{\alpha_n}\|)\|\varphi'(u_{\alpha_n})\| \leq M \quad (26)$$

for every $n \in \mathbb{N}$. On the one hand, by (6), there is constant $c_5 > 0$ such that

$$\lim_{|u| \rightarrow \infty} \sup_{|u| > u_0} \frac{F(x, u)}{|u|^r} < c_5; \quad (27)$$

then there exists $\delta_1 > 0, u_0 > \delta_1$, such that

$$\sup_{|u| > u_0} \frac{F(x, u)}{|u|^r} < c_5, \quad \frac{F(x, u)}{|u|^r} < c_5 \quad \text{when } |u| \geq \delta_1. \quad (28)$$

Hence

$$F(x, u) \leq c_5 |u|^r \quad \forall |u| \geq \delta_1, \text{ a.e. } x \in \Omega. \quad (29)$$

We know

$$|F(x, u)| \leq M_1 \quad \forall |u| \leq \delta_1, \text{ a.e. } x \in \Omega. \quad (30)$$

So for all $u \in R^n$ ($n = 1, 2$) and a.e. $x \in \Omega$, we obtain

$$F(x, u) \leq c_5 |u|^r + M_1. \quad (31)$$

It follows from (26) and (31)

$$\begin{aligned} \frac{b}{4} \|u_{\alpha_n}\|^4 &= \varphi(u_{\alpha_n}) + \int_{\Omega} F(x, u_{\alpha_n}) dx - \frac{a}{2} \|u_{\alpha_n}\|^2 \\ &\leq \varphi(u_{\alpha_n}) + \int_{\Omega} F(x, u_{\alpha_n}) dx \\ &< M + M_1 |\Omega| + c_5 \int_{\Omega} |u_{\alpha_n}|^r dx. \end{aligned} \quad (32)$$

On the other hand, by (7),

$$\lim_{|u| \rightarrow \infty} \inf_{|u| > u_0} \frac{f(x, u)u - 4F(x, u)}{|u|^\mu} > 0; \quad (33)$$

then there are constants $c_6 > 0$ and $\delta_2 > 0, u_0 > \delta_2$, such that

$$\begin{aligned} \inf_{|u| > u_0} \frac{f(x, u)u - 4F(x, u)}{|u|^\mu} &> c_6, \\ \frac{f(x, u)u - 4F(x, u)}{|u|^\mu} &> c_6 \end{aligned} \quad (34)$$

when $|u| \geq \delta_2$.

Hence

$$f(x, u)u - 4F(x, u) \geq c_6 |u|^\mu > 0 \quad \forall |u| \geq \delta_2, \text{ a.e. } x \in \Omega. \tag{35}$$

By $F \in C^1$, one has

$$\begin{aligned} |f(x, u)u - 4F(x, u)| &\leq M_2 \\ \text{for every } |u| &\leq \delta_2, \text{ a.e. } x \in \Omega. \end{aligned} \tag{36}$$

Hence we obtain that

$$f(x, u)u - 4F(x, u) \geq c_6 |u|^\mu - M_2 - c_6 \delta_2^\mu \tag{37}$$

for all $u \in R^n$ and a.e. $x \in \Omega$. Then we have

$$\begin{aligned} 5M &\geq 4\varphi(u_{\alpha_n}) - (\varphi'(u_{\alpha_n}), u_{\alpha_n}) \\ &= 2a \|u_{\alpha_n}\|^2 + b \|u_{\alpha_n}\|^4 \\ &\quad - 4 \int_{\Omega} F(x, u_{\alpha_n}) - a \|u_{\alpha_n}\|^2 - b \|u_{\alpha_n}\|^4 \\ &\quad + \int_{\Omega} f(x, u_{\alpha_n})u_{\alpha_n} \\ &= \int_{\Omega} f(x, u_{\alpha_n})u_{\alpha_n} - 4F(x, u_{\alpha_n})dx + a \|u_{\alpha_n}\|^2 \\ &\geq c_6 \int_{\Omega} |u_{\alpha_n}|^\mu dx - M_2 |\Omega| + a \|u_{\alpha_n}\|^2 - c_6 \delta_2^\mu. \end{aligned} \tag{38}$$

So $\int_{\Omega} |u_{\alpha_n}|^\mu dx$ is bounded. If $\mu \geq r$, by (32) and Hölder inequality,

$$\int_{\Omega} |u_{\alpha_n}|^r dx \leq |\Omega|^{(\mu-r)/\mu} \left(\int_{\Omega} |u_{\alpha_n}|^\mu dx \right)^{r/\mu}. \tag{39}$$

Thus we get that $\{\|u_{\alpha_n}\|\}$ is bounded. If $\mu \leq r$, we obtain

$$\begin{aligned} \int_{\Omega} |u_{\alpha_n}|^r dx &= \int_{\Omega} |u_{\alpha_n}|^{r-\mu} \cdot |u_{\alpha_n}|^\mu \\ &\leq \|u_{\alpha_n}\|_\infty^{r-\mu} \int_{\Omega} |u_{\alpha_n}|^\mu dx \\ &\leq c_7 \|u_{\alpha_n}\|^{r-\mu} \int_{\Omega} |u_{\alpha_n}|^\mu dx. \end{aligned} \tag{40}$$

Then by (32) and $r - \mu < 4$, we know that $\{\|u_{\alpha_n}\|\}$ is bounded too. Hence $\{\|u_{\alpha_n}\|\}$ is bounded. Since $\{\|u_{\alpha_n}\|\}$ is bounded, for a subsequence, $\{u_{\alpha_n}\}$ converges to some u_α weakly in H and strongly in $L^s(\Omega)$, $1 \leq s < +\infty$, and a.e. in Ω :

$$\begin{aligned} &\varphi'(u_{\alpha_n})(u_\alpha - u_{\alpha_n}) \\ &= (a + b \|u_{\alpha_n}\|^2) \int_{\Omega} \nabla u_{\alpha_n} \cdot \nabla (u_\alpha - u_{\alpha_n}) \\ &\quad - \int_{\Omega} f(x, u_{\alpha_n})(u_\alpha - u_{\alpha_n}) \\ &\longrightarrow 0. \end{aligned} \tag{41}$$

Noting that $p \in (1, +\infty)$, $q = p/(p - 1)$,

$$\begin{aligned} \int_{\Omega} |f(x, u_{\alpha_n})(u_\alpha - u_{\alpha_n})| &\leq \left(\int_{\Omega} |f(x, u_{\alpha_n})|^q \right)^{1/q} \\ &\quad \times \left(\int_{\Omega} |u_\alpha - u_{\alpha_n}|^p \right)^{1/p} \\ &\leq c_7 \cdot \|u_\alpha - u_{\alpha_n}\|_p \longrightarrow 0 \end{aligned} \tag{42}$$

as $n \rightarrow +\infty$. In addition,

$$\begin{aligned} &(a + b \|u_{\alpha_n}\|^2) \int_{\Omega} \nabla u_{\alpha_n} \cdot \nabla (u_\alpha - u_{\alpha_n}) \\ &= (a + b \|u_{\alpha_n}\|^2) [(u_{\alpha_n}, u_\alpha) - (u_{\alpha_n}, u_{\alpha_n})] \\ &= (a + b \|u_{\alpha_n}\|^2) \\ &\quad \times [(u_{\alpha_n}, u_\alpha) - (u_\alpha, u_\alpha) + (u_\alpha, u_\alpha) - (u_{\alpha_n}, u_{\alpha_n})]. \end{aligned} \tag{43}$$

Since $\{u_{\alpha_n}\} \rightarrow u_\alpha$, then $(u_{\alpha_n}, u_\alpha) - (u_\alpha, u_\alpha) = (u_{\alpha_n} - u_\alpha, u_\alpha) \rightarrow 0$. The combination of (32)-(44) implies that

$$(u_{\alpha_n}, u_{\alpha_n}) \longrightarrow (u_\alpha, u_\alpha), \quad j \longrightarrow +\infty. \tag{44}$$

Thus, $\{u_{\alpha_n}\} \rightarrow u_\alpha$ in H . □

Lemma 9. φ maps bounded sets into bounded sets.

Proof. From (S_3) , we know

$$|F(x, u)| \leq c_5 |u|^r + M_1, \tag{45}$$

so

$$\begin{aligned} \varphi(u) &= \frac{a}{2} \|u\|^2 + \frac{b}{4} \|u\|^4 - \int_{\Omega} F(x, u) dx \\ &\leq \frac{a}{2} \|u\|^2 + \frac{b}{4} \|u\|^4 + \int_{\Omega} (M_1 + c_5 |u|^r) \\ &\leq \frac{a}{2} \|u\|^2 + \frac{b}{2} \|u\|^4 + M_1 |\Omega| + c_8 \|u\|^r. \end{aligned} \tag{46}$$

Hence φ maps bounded sets into bounded sets. □

Lemma 10. For every $m \in N$, $\varphi(u) \rightarrow -\infty$, $\|u\| \rightarrow \infty$, $u \in X_m^1 \oplus X^2$.

Proof. By condition (S_2) and (2), there exist enough constants $M_3 > 0$, $M_4 > 0$, for all $(x, u) \in \Omega \times R^n$; we have

$$F(x, u) \geq M_3 |u|^4 - M_4. \tag{47}$$

So, for all $u_0 \in X_m^1 \oplus X^2$, $\|u_0\| = 1$, one has

$$\begin{aligned} \varphi(su_0) &= \frac{a}{2} \|su_0\|^2 + \frac{b}{4} \|su_0\|^4 - \int_{\Omega} F(x, su_0) dx \\ &\leq \frac{a}{2} s^2 + \frac{b}{4} s^4 - M_3 s^4 \int_{\Omega} |u_0|^4 + M_4 |\Omega| \\ &\leq \frac{a}{2} s^2 + \frac{b}{4} s^4 - M_3 c_9 s^4 + M_4 |\Omega| \\ &\leq \frac{a}{2} s^2 + s^4 \left(\frac{b}{4} - M_3 c_9 \right) + M_4 |\Omega| \\ &\longrightarrow -\infty. \end{aligned} \quad (48)$$

Hence, for every $m \in N$, $\forall u \in X_m^1 \oplus X^2$, we have

$$\varphi(u) \longrightarrow -\infty \quad \text{as } \|u\| \longrightarrow \infty. \quad (49)$$

Then the conclusion is obtained. \square

Proof of Theorem 1. By Lemmas 7–10 and Lemma 6, we complete the proof. \square

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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