

## Research Article

# The Nested Site Selection Model for Water Treatment Plants Based on the Optimization of Water Supply Radius

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This paper analyzes the site selection problems of water treatment plants by utilizing the set covering model. Fully consider the influence of the pipe network arrangement on the site selection when confirming the covering radius, analyze the best water supply radius of water treatment plants combining investment benefits and pipe network optimization theory, and take the best water supply radius as the covering radius. Make the pipe network optimization nest into the site selection problem meanwhile confirming the covering radius, which fully reflect the viewpoints of the integrated logistics arrangement system. Considering the multiple solutions of the set covering model, this paper introduces the model of which the cost's present value is minimum to make the quadratic optimization and get the best site selection results of water treatment plant. At last, this paper verifies the model by combining with the cases of water supply project construction of a county; the results prove that the model is feasible and effective. This paper expects to provide some reference for the planning design of regionally centralized water supply projects concerning villages and small towns and the site selection and construction of water treatment plant.

## 1. Introduction

Construction of the logistics system is of great significance to improve the level of economic development within the region and to meet the growing demand of people. Among them, the location of logistics distribution center has direct bearing on whether the logistics system is able to minimize transportation cost to the utmost, thereby reducing operating cost of the system. As a special logistics system, rural water supply project delivers special material which is water resource meeting certain health conditions for residents. This special "cargo" transportation requires special materials pipelines as transport carrier. The logistics system is composed of three subsystems, water intake engineering subsystem, the subsystem of pressuring and chlorine disinfection in water purification plant, and water distribution engineering subsystem. The section of water purification plant is equivalent to the distribution center or transfer station in general logistics system. As the location of the distribution center

in the logistics system is an important decision problem, in rural water supply system, water purification plant sitting is also related to that whether to the maximum extent it can meet water demand with minimal investment in pipeline construction in rural water supply system.

Common location problem is to determine the optimal number and best location of new facilities in the candidate set of points, and the goal is to minimize the construction investment and operating cost. Since a German Weber raised the issue in the 19th century, the study on site selection was in full swing. Scholars from various countries in this field have already achieved many important results. AIKENS.CH gave nine basic site location models in his research [1], including simple location model without capacity of present value, location model with limited capacity, location model without certain demand, dynamic location model, and random site selection model. Duan and Chen [2] consider competition issue in the site selection. Competition is not only from the existing logistics and distribution centers but

also from competitors that may arise in the future. Based on the “original distribution center—a new distribution center—join in the future distribution center,” double-layer programming model under this framework is established, providing decision-making basis for the new distribution center under competitive environment. Doerner and others [3] set facility construction cost, full coverage of schools in preset distance, the smallest distance between students and the nearest school, and tsunami risk as the goal and establish a multiobjective location selection model, using NSGA-II algorithm to solve and set location planning for schools in tsunami-prone coastal areas as case study in southern Sri Lanka tsunami-prone coastal area school sitting issues case study. Set covering pattern (SCP) is a classic location model in site selection problem. Because of its wide range in practice in the sitting and the result is able to meet the actual situation, it has been accepted by the researchers [4–8]. Hwang [9], in his study of the logistics supply chain system, firstly considers the use of set covering model to choose the minimum number of service points in a series of discrete facility sitting points. Liu et al. [10] established a multicriteria set covering model with certain kinds of decision criteria. Each criterion has attributes matrix and optimization goals. If a candidate point is selected, you will need to meet all the criteria. Eiselt and Marianov [11] will apply gradual covering to the set covering, taking into account the progressive coverage set covering sitting based on quality of service and giving the minimum acceptable level of service concepts and different coverage functions. In most studies on set covering, whether a demand point is provided by a service facility depends on the distance between this point and the service facilities. If the distance between the two is less than or equal to a predetermined distance, the facility can provide service. This is called covered distance or coverage radius [12–15]. Determining the coverage radius will need apriority or practical experience and many of the studies mentioned above usually give directly coverage radius to be used to solve the covering problem, but do not discuss in detail how to reasonably determine the coverage radius.

Significant difference between water supply engineering system and general transport logistics system lies in its carrier medium. Layout of pipeline and diameter of pipes selection directly affect construction investment, while construction investment has a direct impact on the economic efficiency of the project. Common logistics model generally assumed that delivery route from the facility or distribution center to demand point is radial [16, 17], but for water supply project, a special logistics system, it should be consistent with the views of integrated logistics management system; that is, for the arrangement of the water purification plant and water demand point, water distribution, there exists relationship of interdependence.

Whether a water supply project can achieve the expected economic benefits is closely related with its subsidiary pipeline system optimized layout. Combining the path arrangement with facility location to research has attracted more and more attention of scholars. Sun and Gao [18] fully take account of the interests of logistics planning department and the customer and also the site selection impact on route arrangement, thus establishing a bilevel programming

model for solving the logistics distribution center location problem. Zare Mehrjerdi and Nadizadeh [19] have established the capacitated location-routing problem with fuzzy demands (CLRP-FD), combining the facility location problem with multipath optimization problem together. However, in this particular village water supply project logistics system, research combining the pipe network optimization, and purification plant sitting are rare.

In this paper, covering model for the water purification plant location problem is studied. In determining the coverage radius, the impact of network optimization problems on the site selection is considered, and meanwhile economic and engineering factors based on planned investment returns during the period are taken into consideration. Combined with the water supply network system optimization design theory, the distribution network optimization and best water supply radius are connected to achieve nesting between water pipe network optimization model and location selection model, so that the coverage radius has more theoretical and practical significance and also is in line with the integrated logistics management system. Due to the characteristics of the set covering model, the results are a number of possible solutions rather than single optimal solution. Therefore, this paper introduces the model of minimum present value, in order to optimize the site selection secondly to get the best location results. Finally, through a practical calculation example, the feasibility and practicality of the nested site selection model is demonstrated.

## 2. Confirmation of Economic Water Supply Radius

This part firstly analyzes the investment benefits of the water treatment plant based on the income of the water treatment, the construction expenses of the water treatment plant, investment in the infrastructure of the pipe network, and the energy expenses of the pump stations and establishes the present value model of project benefits during the planning period. It makes the optimization design for the affiliated pipe network of the water treatment plant when considering the investment in the infrastructure of the pipe network, which achieved the combination of the engineering factor and economy analysis. It gets the hydraulic loss along the pipe network and the functional relation between the investment in the infrastructure and the water supply radius through fitting, so as to get the functional expression between the present earning value and the water supply radius during the planning period of the water treatment plant and confirms the best water supply radius. This paper expects to provide a basis for the establishment of quadratic optimization model of the site selection by using NPV maximum water supply radius or the water supply radius with the minimum average total cost.

*2.1. The Established Present Value Model during the Planning Period.* In order to save the power expenses in rural water

supply projects, we usually adopt the water supply mode which takes the plant as the center and radiates in a circular way [20]. The investment in the water supply project generally consists of the investment from the water treatment plant, the construction of pipe network, and maintenance and management expenses. The management expense mainly needs to consider the energy costs required in the process of lifting the water by the pump station. We do not consider the expenses of the pump stations, water tanks, and water towers when considering the project economy due to their low radios in the total expenses of the project and take the economization of the pipe network plans as the main factor [14]. The income of the water supply project mainly consists of the income from water charge, which is related to the water requirement and price in the range of water supply areas.

Suppose that the population within the water supply area of a water treatment plant is averagely distributed and that adopted water supply mode is taking the plant as the center and radiating in a circular way; besides, suppose the water supply duration is 15 years. The model for present earning value of water treatment plants within planning periods may be expressed as follows:

$$\begin{aligned} NPV = & \sum_{t=2}^{15} \frac{3.15 \times 10^4 c \pi R^2 P q}{(1+r)^t} - \sum_{t=2}^{15} \frac{K (H_0 + \sum_{i \in LM} h_i) Q_p}{(1+r)^t} \\ & - \beta (\pi R^2 P q)^\theta - \sum_{i=1}^P (a + b d_i^\alpha) l_i, \end{aligned} \quad (1)$$

where,  $Q$  indicates water requirement in L/s;  $R$  indicates water supply radius in km;  $c$  indicates water price in RMB Yuan/m<sup>3</sup>;  $P$  indicates average population density with the area in person/km<sup>2</sup>;  $q$  indicates water demand quota for households in L/(person·d);  $\beta$  indicates investment coefficient;  $\theta$  indicates investment scale index;  $d_i$  and  $l_i$  indicate the diameter and length of pipe section  $i$ ;  $a + b d_i^\alpha$  indicates pipe network construction cost (including pipe materials and accessories cost as well as pipe laying cost);  $H_0 = Z_m + H_a - Z_p$  indicates static head of water pumps;  $H_p = H_0 + \sum h_i$  indicates water pump lift (m) and  $\sum h_i$  indicates the total frictional head loss of all pipe sections;  $K$  indicates economical indexes related to water pump operating, namely, annual power rate incurred for lifting 1 m<sup>3</sup>/s of hydraulic pressure by 1 m H<sub>2</sub>O  $K = (24 \times 365 \times 10000 \gamma \sigma) / 102 \eta = 86000 (\gamma \sigma / \eta)$  [7];  $\sigma$  indicates electricity price, RMB Yuan/kw·h;  $\gamma$  indicates nonuniform coefficient of water supply energy with the planned years, without water-distribution ducts of water towers or the pipe network of water towers in front of the network;  $\gamma = 0.1 \sim 0.4$ , indicates pipe network of water towers in front of the network,  $\gamma = 0.5 \sim 0.75$ ; LM indicates pipe section collection of any one pipeline from the pump station to control point  $m$  and  $p$  is the total of pipe sections.  $Q_p$  indicates the total flow entering into pipe network or water outflow (m<sup>3</sup>/s).

For any one pipe section, in the case that  $q_i$  and  $d_i$  are certain, head loss can be calculated; namely,  $h_i = k (q_i^m l_i / d_i^n)$ ; coefficient  $k$  and indexes  $m$  and  $n$  are calculated according

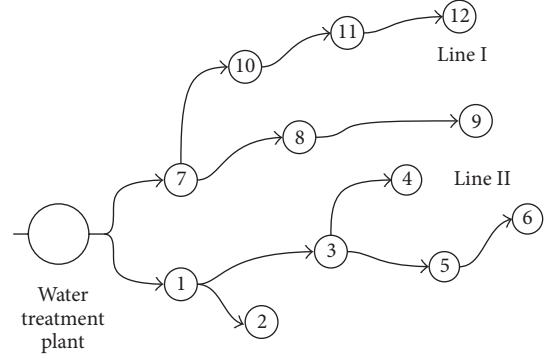


FIGURE 1: The schematic diagram for the minimal spanning trees between each spot in need of water and the water plant within one area.

to head loss computation formula. If  $d_i$  in Formula (1) is expressed in  $q_i$  and  $h_i$ , we may obtain

$$\begin{aligned} NPV = & \sum_{t=2}^{15} \frac{3.15 \times 10^4 c \pi R^2 P q}{(1+r)^t} - \sum_{t=2}^{15} \frac{k (H_0 + \sum_{i \in LM} h_i) Q_p}{(1+r)^t} \\ & - \beta (\pi R^2 P q)^\theta - \sum_{t=1}^P [a + b k^{\alpha/m} (q_i^{2\alpha/m} h_i^{-\alpha/m} l_i^{\alpha/m})] l_i. \end{aligned} \quad (2)$$

## 2.2. Optimal Design for the Affiliated Pipe Network System.

As the flow distribution of each pipe section in tree-like pipe network for single-water source is unique, the formula above may be utilized to determine the relationship between water supply radius and net present value of water pipe network within one area, thereby, determining the most economical water supply radius. In rural areas, the users of water supply projects are dispersed in many spots and wide areas, and layout modes for water pipe network are various, so that it is impossible to accurately calculate the relationship between water a variety of supply pipe network and relevant hydraulic parameters [21]. Since a model is closely related to layout modes of pipe network and flow distribution when it is used to calculate net present value of water supply projects within one area, before the economical water supply radius for a water plant is determined, we should first optimize the design for water supply network in the area to determine the optimal layout mode, the economical pipe diameter, and the head loss of each pipe section.

In determining a network layout mode, we adopt the method of minimal spanning tree [22] to connect the minimal spanning tree between each spot in need of water as the layout mode for the main pipe of water supply network, then take the total investment in infrastructure construction of main pipe, and present value of network operating cost (electric charge of water pumps) as the minimum objective function, next utilize Lagrange's undetermined multiplier to determine each length of each main pipe, the economical pipe diameter, and the flow and head loss for each pipe section in water supply pipe network when water supply radiuses differ.

In Figure 1, Position 0 indicates a water treatment plant; Positions 1 to 12 indicate the spots in need of water; then we

connect the minimal spanning trees between each spot in need of water and the plant to obtain a result.

After flow of each main pipe is determined according to node flow balance condition, we may utilize investment in infrastructure construction and operating cost (electric charge of water pumps) of pipe network as objective function to determine the economical pipe diameters of each main pipe; namely, before the optimal water supply radius is determined, we should take into account the configuration and optimization of affiliated pipe network in advance. Due to various service radiuses of water treatment plants, they also have various layout modes and scale of affiliated pipe network; when water supply radiuses differ, the economical pipe diameters and head loss of pipe sections in the same position are correspondingly different. Let us take the single-water source tree-like network in Figure 1 as an example; suppose the water supply radius of pipe network with this scale is  $R$  and the total flow entering into pipe network  $Q$  and the flow  $Q_i$  of each node are given; besides, the hydraulic-pressure elevation  $H_2, H_6, H_4, H_9, H_{12}$  for Nodes 2, 6, 4, 9, and 12 are also given, and the hydraulic-pressure elevation  $H_p$  of starting point 0 is undetermined; in addition, head loss  $h_{ij}$  of 11 pipe sections and hydraulic-pressure elevation  $H_0$  of starting point are also unknown. In order to achieve economical  $H_p$  and  $h_{ij}$  value, the following 6 constraint conditions must be met:

$$\begin{aligned} H_p - (h_{0-1} + h_{1-3} + h_{3-5} + h_{5-6}) &= H_6, \\ H_p - (h_{0-1} + h_{1-2}) &= H_2, \\ H_p - (h_{0-1} + h_{1-3} + h_{3-4}) &= H_4, \\ H_p - (h_{0-7} + h_{7-8} + h_{8-9}) &= H_9, \\ H_p - (h_{0-7} + h_{7-10} + h_{10-11} + h_{11-12}) &= H_{12}. \end{aligned} \quad (3)$$

The function expression of the sum between the investment in infrastructure construction and operating cost of pipe network is as follows:

$$\begin{aligned} F(h_{ij}) &= W + \lambda_2 (H_2 + h_{0-1} + h_{1-2} - H_0) \\ &+ \lambda_3 (H_4 + h_{0-1} + h_{3-4} - H_0) \\ &+ \lambda_4 (H_9 + h_{0-7} + h_{7-8} + h_{8-9} - H_0) \\ &+ \lambda_5 (H_{12} + h_{0-7} + h_{7-10} - h_{10-11} - H_0), \end{aligned} \quad (4)$$

where,  $W = \sum_1^P [a + bk^{a/m} q_{ij}^{2a/m} h_{ij}^{-a/m} l_{ij}^{a/m}] l_{ij} + \sum_{t=2}^{15} (KQ_p H_p / (1+r)^t)$ ,

$$H_p = H_0 = h_{0-1} + h_{1-3} + h_{3-5} + h_{5-6} + H_6. \quad (5)$$

To obtain the partial derivative of function  $F$  to  $h_{ij}$ , we suppose it to be zero and eliminate undetermined multiplier  $\lambda$ ; then the result is as follows:

$$\begin{aligned} A_{0-1} &= A_{1-2} + A_{3-4} + \sum_{t=2}^{15} \frac{KQ}{(1+r)^t}, \\ A_{1-3} &= A_{3-4} + \sum_{t=2}^{15} \frac{KQ}{(1+r)^t}, \\ A_{3-5} &= A_{5-6} = \sum_{t=2}^{15} \frac{KQ}{(1+r)^t}, \\ A_{7-8} &= A_{8-9}, \\ A_{0-7} &= A_{7-8} + A_{7-10}, \\ A_{7-10} &= A_{10-11} = A_{11-12}, \end{aligned} \quad (6)$$

where,  $A_{i-j} = (\alpha/m)bk^{a/m} q_{i-j}^{2\alpha/m} l_{i-j}^{-(\alpha+m)/m} h_{i-j}^{-(\alpha+m)/m}$ .

Using the formula above and the above-mentioned constraint equation, we may successively obtain the values of  $h_{3-5}, h_{5-6}, h_{3-4}, h_{1-3}, h_{1-2}, h_{0-1}, H_0$ , namely, the head loss  $h_{ij}$  of all pipe sections in Pipeline II. Suppose in the function formula  $H_p = H_0 = h_{0-7} + h_{7-10} + h_{10-11} + h_{11-12} + H_{12}$ ; then we may obtain the head loss of all pipe sections in Pipeline I; in a similar way, we may obtain the pipe diameters and head loss of all pipe sections with various water supply radiuses.

The economical pipe diameter  $d_{ij}$  of pipe sections may be obtained by using head loss formula; in this paper, Hayden William formula is selected [23]. Consider

$$h = \frac{10.67q^{1.852l}}{C^{1.852}D^{4.87}}, \quad (7)$$

where  $l$  is length of pipe section (m),  $D$  is pipe diameter (m),  $q$  is flow of pipe section ( $m^3/s$ ), and  $C$  is a coefficient, which is related to types of pipe materials. For instance, PVC pipes are used in water supply projects; then  $C = 150$ .

**2.3. Set up the Functional Relation.** Use MATLAB to fit the obtained results, respectively, after getting the hydraulic loss along the pipe network and the functional relation between the investment in the infrastructure and the water supply radius; the functional expression between the present earning value and water supply radius during the planning period of the water treatment plant can also be gained according to the above mentioned present value model. Then we could obtain the best water supply radius with the maximum NPV. Similarly, according to the functional relation between the total cost of the average water production and water treatment volume, we could work out the best water production volume and the best water supply radius when the total cost of unit water production is minimum. The water supply radius under the maximal condition of NPV and the water supply radius under the minimum condition of the unit water production cost have practical significance of their own. If the maximum present earnings of the project shall be realized in the planning period, the water supply scale can be confirmed by the water supply radius under the maximum condition of NPV. If the actual project scale cannot reach



this construction scale for the actual massive investment, excessive loan interest, and complexity of the construction conditions, the water supply scale can be confirmed through the water supply radius under the minimum condition of the unit water supply cost. At this moment the total unit water production reaches the minimum; the input-output ratio reaches the maximum and the economic efficiency of the water treatment plant reaches the maximum.

### 3. Nested Site Selection Model for Waterworks Based on the Most Economical Water Supply Radius

Applying the most economical method of water supply radius identified above, engineering factor (actual pipe network layout) and economic analysis (analysis of investment returns) can be combined reasonably to get the best water radius in sense of the practical works. Use set covering model to determine waterworks site choice of site means to use as little waterworks as possible to cover water demand points. This reduces the water treatment plant construction cost and operating cost. This model nests network optimization model with set covering secondary optimization model and the water purification plant site is built on the basis of network optimization, considering the effect of pipeline distribution layout on water plant construction and investment returns in order to make site model closer to practical engineering.

Assume that within a water area, there are  $m$  water demand points, set covering model is as follows:

$$\min \sum_{k \in M} W_k. \tag{8}$$

Constraint equation is as follows:

$$\sum_{k \in B(i)} U_{ik} = 1 \quad (i = 1, 2, \dots, m), \tag{9}$$

$$\sum_{i \in A(k)} X_i \cdot U_{ik} \leq C_k \cdot W_k \quad (i = 1, 2, \dots, m; k \in B(i)), \tag{10}$$

$$X_i, C_k \geq 0, \quad (i = 1, 2, \dots, m; k \in B(i)), \tag{11}$$

$$W_k \in \{0, 1\}, \tag{12}$$

$$U_{ik} \in \{0, 1\}, \tag{13}$$

$$W_k = U_{ik} \quad \text{if } i = k. \tag{14}$$

In the formula,  $M = \{1, 2, \dots, m\}$  indicates the need of  $m$  water points within a region;  $C_k$  represents water supply capacity of water purification plant  $k$ ;  $X_i$  represents water demand of the  $i$ th point;  $A(k)$  represents the collection of water demand points of water purification plant  $k$  to be covered;  $B(i)$  represents the  $i$ th water demand point water purification plant; Generally,  $A(k)$  and  $B(i)$  is consistent, but if some other restrictions are considered, differences may occur;  $W_k$  indicates whether  $k$ th waterworks is to be used;  $U_{ik}$  is whether the  $i$ th point is required to be covered by the  $k$ th waterworks.

The objective function is to select preferably site which can cover  $m$  waterworks points in the minimum quantity

from the existing  $m$  water demand points; constraint equation (1) represents every water demand point can be covered with water purification plant and each point can only be supplied by one water purification plant, and this constraint is taken into account in the actual rural water supply projects. Building two water supply pipes to one water supply objective is very uneconomical and is also not in line with the actual situation of the rural water supply; constraint equation (2) is a water supply capacity to meet the requirements of the plant, that is, its water needs covering water demand point range is not greater than the water capacity of the water treatment plant; constraint equation (3) means water demand in water demand points and water production capacity of the water purification plant are non-negative; constraint equation (4) represents whether the decision variables  $k$  of waterworks are enabled; constraint equation (6) means decision variables whether the  $i$ th point of water demand is provided by the  $k$ th plant supply; constraint equation (7) means that when plant  $k$  is selected, plant  $k$  is to meet the water demand of local residents.

To solve the above set covering model with constraints extremes, branch and bound method can be used for accurate calculation [24–27]. Due to the above set covering model NP-hard problem, with the expansion of the scale of the problem, the time which is used to obtain the optimal solution by virtue of exact solution will be in exponential growth. So many studies are seeking to use heuristic algorithms to solve set covering problem with large-scale, numerous variables [28, 29]. In this paper, LINGO software programming is used to solve the problem. Considering the LINGO can only obtain a feasible solution once but cannot find all of the feasible solutions, one feasible solution can be set as the boundary condition, thus obtaining all of the feasible solutions using MATLAB.

When solving scale becomes larger, the collection of site selection of plant solved by set covering model is nonunique. After the position and the number of water purification plant initially identified, use the minimum model of present value to make secondary optimization of water purification plant sitting point and coverage. With this model, a number of possible solutions can be further optimized, resulting in the rank of present value of the water site collection, where we can choose the present value of the minimum cost as the optimal solution. When calculating the length of pipeline lying, this model is unlike the general logistics distribution center location model which assumes that distribution from the facility or distribution center to demand points (customer sites) is radiation-like, otherwise, it is based on the minimum spanning tree model taking optimization of water distribution pipelines into consideration.

The minimum cost model of present value is as follows:

$$\begin{aligned} \min R = & \sum_{i \in A(k)} \sum_{k=1}^p \sum_{t=2}^{15} \frac{L_{ik} \cdot C_{ik}}{(1+r)^t} \cdot (365X_{ik}) \cdot U_{ik} \\ & + \sum_{j=1}^n \sum_{k=1}^p \sum_{t=2}^{15} \frac{S_{kj} \cdot D_{kj}}{(1+r)^t} \cdot (365Y_{kj}) \cdot V_{kj} \end{aligned}$$

$$\begin{aligned}
& + \sum_{k=1}^P F_k \cdot W_k + \sum_{i \in A(k)} \sum_{k=1}^P L_{ik} \cdot P_{ik1} \cdot U_{ik} \\
& + \sum_{j=1}^n \sum_{k=1}^P S_{kj} \cdot P_{ik2} \cdot V_{kj}.
\end{aligned} \tag{15}$$

Constraint equation is as follows:

$$\begin{aligned}
L_{ik} &= \min \sum_{(i,j) \in A(k)} d_{ij} x_{ij}, \\
\sum_j^{j \in N} x_{ij} &\geq 1 \quad \forall i \in N, \quad j \neq i, \\
\sum_{j=1}^N \sum_{i=1}^N x_{ij} &= N - 1, \quad j \neq i, \\
F_k &= F \left( \sum_{j=1}^n Y_{kj} \cdot W_k \right) \\
&(j = 1, 2, \dots, n, \quad k = 1, 2, \dots, p), \\
\left( \sum_{k=1}^m X_{ik} \cdot U_{ik} \right) &= \left( \sum_{j=1}^n Y_{kj} \cdot V_{kj} \right) \\
&(i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n, \quad k = 1, 2, \dots, p), \\
\sum_{k=1}^p U_{ik} &= 1 \quad (i = 1, 2, \dots, m, \quad k = 1, 2, \dots, p), \\
U_{ik} &\leq W_k \quad (i = 1, 2, \dots, m, \quad k = 1, 2, \dots, p), \\
Q_{\min} &\leq \sum_{i=1}^m X_{ik} U_{ik} \leq Q_{\max} \\
&(i = 1, 2, \dots, m, \quad k = 1, 2, \dots, p), \\
X_{ik}, Y_{kj} &\geq 0 \\
&(i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n, \quad k = 1, 2, \dots, p) \\
W_k &\in \{0, 1\}, \\
U_{ik} &\in \{0, 1\}, \\
V_{kj} &\in \{0, 1\}, \\
x_{ij} &\in \{0, 1\},
\end{aligned} \tag{16}$$

where  $T$  is planned life and  $t_0$ -year is construction period;  $r$  is the discount rate;  $C_{ik}$  is the expense of water supply unit cost per unit distance (Yuan/m<sup>3</sup>/km) where  $k$ th waterworks provides water demand for the  $i$ th point;  $X_{ik}$  is the daily quantity of water (m<sup>3</sup>/d)  $k$ th waterworks provides for the  $i$ th point of water;  $L_{ik}$  is the pipeline length between the  $k$ th

plant and the  $i$ th water demand between point (km);  $D_{kj}$  is the expense of unit cost where water purification plant intake water from the source  $j$  (RMB/m<sup>3</sup>/km);  $Y_{kj}$  means daily water consumption (m<sup>3</sup>/d) which plant  $k$  intake water from the water source  $j$ ;  $S_{kj}$  is the length of pipeline between the  $k$ th plant and source  $j$  (km);  $F_k$  is the fixed investment during planned construction period (RMB);  $Q_{\min}$  is the minimum size of the water purification plant control building (m<sup>3</sup>/d);  $Q_{\max}$  is the maximum size of the water purification plant control building (m<sup>3</sup>/d);  $p_{ik1}$  is the average comprehensive unit price of UPVC pipeline construction (yuan/m);  $p_{ik2}$  is the average integrated unit for ductile iron pipe construction (yuan/m).

The objective function is the minimum model of present value in planned useful life, covering the present value of water costs in rural water supply and distribution system (including the cost of water demand point of water distribution from water purification plants and the present value of taking water from water source), the present value of the investment and present value of construction investment of water distribution network infrastructure. Constraint (1) indicates setting the minimum spanning tree water of the water plant  $k$  water demand point range in the area as a dry pipe installation route. Constraint (2) is a function of fixed investment between waterworks system and the actual amount of water; constraint (3) is the balance between supply and demand of water supply and water demand; constraint equation (4) represents a water demand point which is provided by one water plant; the relationship between the two is "many-to-one"; constraint (6) means when there is no demand for water, water purification plant is not enabled, but as long as water demand is present, water purification plants must be enabled; constraint (7) means the water plant size control; constraint (8) the amount of water supply is nonnegative. Constraint (9) is the decision variables whether the water purification plant is enabled; constraint (10) is decision variables whether the demand for water is transported to the water purification plant; constraint (11) is decision variables whether a waterworks takes water from one source; constraint (12) indicates within the water demand points of plant  $k$ , whether to arrange pipes between water demand points  $i$  and  $j$ .

Set the feasible solution achieved by set covering model as minimum cost optimization model, the best result of purification plant siting can be obtained. The following example demonstrates the feasibility and practicality on basis of secondary optimization model of the most economical water supply radius.

#### 4. Case Study

A county plans to build water supply system within geographic scope and needs to establish some water treatment plants within water supply region. According to population distribution within the region, permanent residents of the county are clustered as per township into Jinjiang town (1), Chang'an town (9), Taiping town (17), Shankou town (16), Meiting town (18), Qiaotou town (8), Fushan town

(7), Laocheng town (4), Bailian town (12), Macun town (10), Yongfa town (3), Xinwu town (11), Ruixi town (2), Wenru town (13), Shifu town (15), Zhongxing town (5), Renxing town (14), and Jiale town (6). A total of 18 large water demand points are included. The numbers of large water demand points are shown in parentheses. Assume waterworks candidate point set and water demand point set are the same. According to large data amount of distance matrix between the various water demand points which does not list here and the distribution of water resources in the county, select the A, B, C, D, E, and F, six water source points, among which E and F are reservoir water savings, and the remaining water sources are river water.

Firstly, determine the most economical water supply radius of the county's water supply project. Assume that county distribution within the water supply range and the water demand situation are known. By calculating hydraulic parameters for each pipe segment in different water radius, we get loss value on water conservancy project and cost of the pipe network of radius of 4 km, 6 km, 8 km, and 10 km. The functional relationship between loss value on water conservancy project and investment in infrastructure and water supply radius is calculated as follows:

$$\sum_{i \in LM} h_i = 0.0198R^3 - 1.1068R^2 + 17.4301R + 0.2697, \tag{17}$$

$$\sum_{i=1}^P (a + bd_i^\alpha) l_i = 2.6105R^2 + 11.4909R - 40.01.$$

Set  $c = 1.4 \text{ Yuan/m}^3$ ,  $p = 533 \text{ person/km}^2$ ,  $\gamma = 0.2$ ,  $\sigma = 0.6 \text{ Yuan/kwh}$ ,  $\eta = 0.7$ ,  $H_0 = 20 \text{ m}$ , and  $r = 11\%$ . According to *Technical code for water supply engineering of town and village* (SL310-2004), in the fifth regions, the water supply condition is full day water supply. When there is wash trough and other health facilities, the maximum residential water usage quantity is 90~140 L/person-d [30]. In this paper, set  $q_c = 110 \text{ L/person-d}$ . About the relationship between construction investment and water scale, this paper refers to the result of [31],  $W = 0.206Q^3 - 108.727Q^2 + 44340.48Q + 904200$ , in which the unit of  $Q$  is L/s and  $W$  is Yuan.

Figure 2 shows that when the best water amount is  $0.78 \text{ m}^3/\text{s}$ , NPV reaches its maximum, the corresponding water radius is about 20 km, and the present value of the project revenue is approximately  $1.2 \times 10^4$  million. As water radius continues to increase, NPV decreases. When  $R$  is 26 km, the present value of the gain is zero. To achieve minimum unit cost of water supply project, the best water scale is  $0.3 \text{ m}^3/\text{s}$ , and the corresponding water radius is 12.4 km.

Taking into account the initial investment to maximize present value of earnings in the realization of the project within planning period is huge, in order to adapt to the water demand size of the county, water supply radius of 12.4 km achieves smallest total cost of the unit water supply as set covering model coverage radius. In the above set covering model, screened plant  $k$  covering water demand points  $A(k)$  and plant  $i$  covering water demand point  $B(i)$  can be determined. Water supply capacity of plant  $k$  is

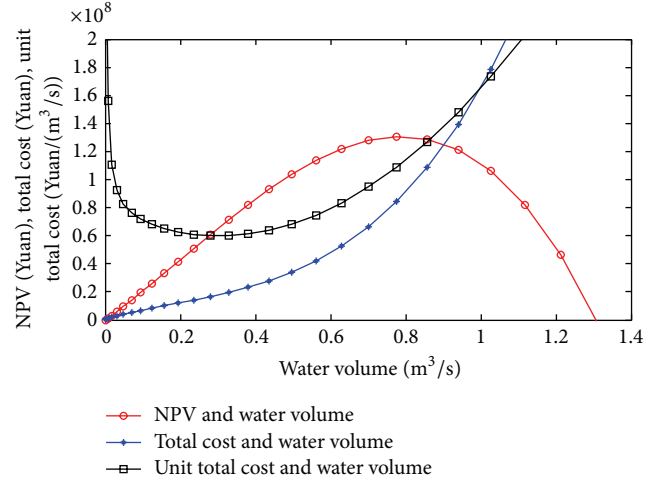


FIGURE 2: Functional relationships between water volume and NPV, total cost, and unit total cost.

$C_k = 0.3 \text{ m}^3/\text{s}$ . Take the average integrated unit price of UPVC pipeline construction is 130 yuan/m and the average integrated unit price of ductile iron pipeline construction is 950 yuan/m,  $C_{ik} = D_{ik} = 2.6 \text{ yuan/m}^3/\text{km}$ . Write programs using MATLAB to solve optimization. The result shows that the total cost of water supply system in planned life (15 years) within the county  $8.6 \times 10^7$  yuan, a total of six waterworks, 1, 2, 5, 6, 7, and 10. Specific location of water purification plant and its services range is shown in Figure 3.

### 5. Conclusion

In this paper, set covering model represents water purification plant location problem. In determining its coverage radius, impact of pipeline layout on the site selection is taken into consideration, and investment returns and the pipe network optimization theory are combined to analyze the best radius of the water purification plant, which is set as the coverage radius. In rural water supply project, as a kind of special logistics system, determination of the coverage radius is not based on transcendental experience, which is more theoretical and practical. When determining the radius of coverage, we should fully reflect the views of integrated logistics management system and combine pipelines optimization and location. Taking into account of multiple solutions of set covering model, this paper introduces secondary optimization of minimum cost model of the present value to obtain the best waterworks site. Finally, this paper combines with a county water supply project example to validate the model and the experimental results show that the proposed model is feasible and effective. So due to the advantages, decision authorities can introduce the specific model to assist them to arrange the county-level water supply infrastructure, which is more scientific and precise than depending just on experience. This model assumes that there is no other water plant in the region; that is, this water plant is the first entrant to the market in the region without other competitors. Actually, competitors are prevalent and, therefore, the existing water treatment plant

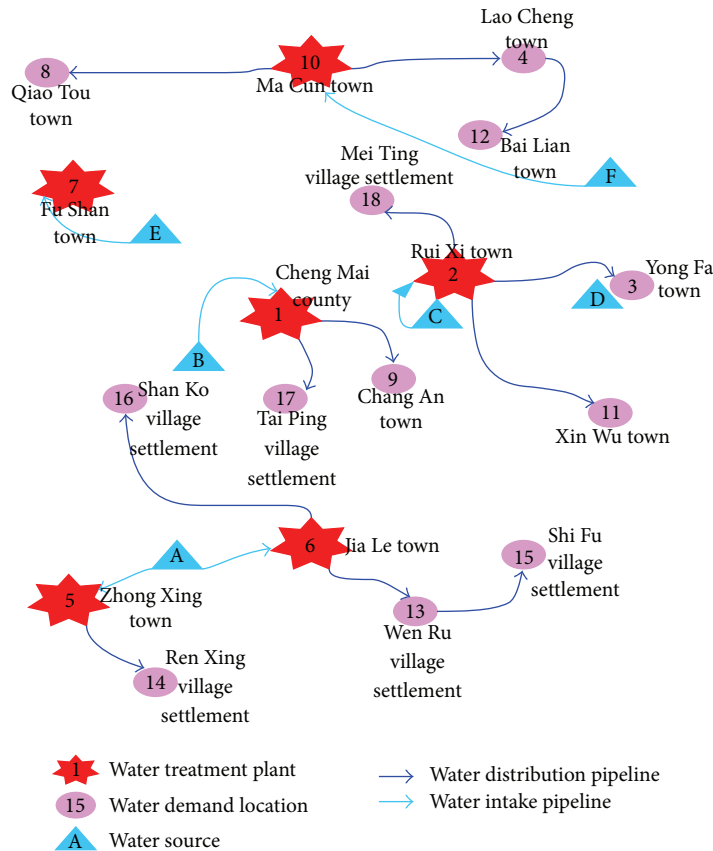


FIGURE 3: Specific location of water purification plant and its services range.

competitors need to be taken into account. Thus, the model needs to be furthestmost developed to accommodate to a wider range of circumstances.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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