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Research Article

Measurement of Congestion in the Simultaneous Presence of Desirable and Undesirable Outputs

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The concept of congestion, which is mainly applied in economics, refers to a situation where inputs are overinvested. Many studies have focused on congestion measurement by means of data envelopment analysis (DEA). However, most of the previous investigations only considered the framework of desirable outputs. In fact, firms in the real world unavoidably generate undesirable outputs (such as pollutants or wastes) along with desirable outputs. Therefore, a new scheme is required for measuring congestion in the simultaneous presence of both desirable and undesirable outputs. This paper develops a nonradial efficiency measure for including undesirable outputs into the environmental performance. Based on the proposed model, a new definition and a new approach are presented to deal with congestion in the simultaneous presence of desirable and undesirable outputs. Then, this paper uses the presented method to study the pollutants (waste gas emission and waste discharge) of 31 administrative regions of China. The finding indicates that 7 industries pay attention to the reduction of their pollutants accompanying improvement of their commercial targets. Consequently, they do not show congestion in any input.

1. Introduction

The concept of congestion, which is mainly applied in economics, refers to a situation where inputs are overinvested [1]. A typical example of congestion is the case where too many men in an underground coal mine may reduce the output of coal.

The topic of congestion was initially defined and extended by the essay of Fare and Svensson [2] in 1980. Afterwards, it was examined by Fare and Grosskopf [3] within the data envelopment analysis framework. They imposed the assumptions of weak and strong disposability on the production possibility set to identify the evidence of congestion. Besides, their approach suffers from some weaknesses in treating congestion [4]. In addition, a slacks-based measure was presented by Cooper et al. [5] that has some strong points compared to the previous method and can identify the congested inputs and the amount of congestion in each input. Moreover, Jahanshahloo and Khodabakhshi [6] developed an input relaxation model for improving outputs and, accordingly, calculated the input congestion based on the proposed model. Wei and Yan [7] estimated congestion by the ratio of technical efficiency to pure technical efficiency. However, their approach shows only existence or nonexistence of congestion and cannot provide a value for measuring the amount of congestion in each input.

Indeed, the investigations into congestion within the DEA framework have received considerable attention in the last few decades. Some of the other investigations in this field include Asgharian et al. [8], Flegg and Allen [9], Khodabakhshi [10], Sueyoshi and Sekitani [11], and Tone and Sahoo [12], to name a few.

According to Cooper et al. [1], the common understanding of congestion is that a decrease (increase) in one or more inputs results in an increase (decrease) in one or more outputs. From this viewpoint, all outputs are expected to rise. Outputs of this kind are called desirable outputs. However, an important issue is that, in the real world, undesirable outputs (such as pollutants or wastes) are unavoidably generated along with desirable outputs. Therefore, a new scheme is required for measuring congestion in the simultaneous presence of both desirable and undesirable outputs.

In recent years, many researchers have been trying to model undesirable outputs within the DEA framework.

A common treatment of undesirable outputs is to regard them as inputs and apply the traditional DEA models. Some of the works in this area include Dyson et al. [13], Hailu and Veeman [14], Dyckhoff and Allen [15], and Sueyoshi and Goto [16]. Nevertheless, this routine causes two problems. First, the free disposability principle between inputs and bad outputs implies that a finite amount of inputs can produce an infinite amount of undesirable outputs, while this is physically impossible [17]. Second, the free disposability principle does not recognize the relation between desirable and undesirable outputs [18].

Seiford and Zhu [19] employed an alternative approach that first multiplies each undesirable output by −1 and then adds a big enough positive scalar to them in order to let all negative undesirable outputs be positive. Nevertheless, this approach is only valid under the variable returns to scale condition, and, furthermore, it does not reflect a rational production possibility set.

Scheel [20] inverted the undesirable output values and treated them as desirable. This nonlinear transformation may change the efficiency frontiers, though, and hence result in erroneous efficiency scores.

However, this subject was first methodically dealt with by Fare et al.'s publications [17, 21]. They thought of undesirable outputs as outputs and tried to incorporate them into production possibility set under a new axiom. Thus, they employed the weak disposability assumption, which had been introduced by Shephard [22], between good and bad outputs. Later, the study was extended in this direction by scholars such as Fare et al. [23], Zhou et al. [24], Kuosmanen [25], and Kousmanen and Podinovski [18].

Although there are many papers in the DEA literature which discuss the theory and applications of congestion (see, e.g., [1, 4, 7–12, 26, 27] among others), studying this subject in the presence of undesirable outputs is very young. In existing paper only, Wu et al. [28] have studied congestion measurement considering both desirable and undesirable outputs. However, they chose the method of Seiford and Zhu [19] to address undesirable outputs, and, moreover, they applied Wei and Yan approach [7] and determined only existence or nonexistence of congestion.

In this paper, we choose the Kuosmanen [25] technology to address the undesirable outputs. The reasons for choosing this technology have been clarified in Section 2. Employing the Kuosmanen technology, we present a nonradial efficiency measure that incorporates in both good and bad outputs. Then, following Cooper's idea for measuring congestion, a new definition and a new approach are presented to deal with congestion in the simultaneous presence of desirable and undesirable outputs.

The rest of this paper is arranged as follows. In Section 2, we review the two-model approach of Cooper et al. [5] for congestion measurement and the Kuosmanen [25] technology for addressing undesirable outputs. Section 3 provides an efficiency measure that incorporates in both good and bad outputs. Thereafter, we focus on congestion in the simultaneous presence of both desirable and undesirable outputs and offer a plain definition. Furthermore, an approach is proposed in order to deal with this type of congestion. In Section 4, the results of the presented models are supplied and interpreted, regarding an empirical application corresponding to 31 administrative regions of China. The summary of the study and directions for future researches are provided in Section 5.

2. Preliminaries

The assumption in this paper is that there are n observed and comparable Decision Making Units (DMUs). The jth DMU, $j \in \{1, \dots, n\}$, is determined by the vector (x_j, g_j, b_j), where $x_j = (x_{1j}, x_{2j}, \dots, x_{mj}) \in R^m$, $x_j \geq 0$, $x_j \neq 0$, is the vector of inputs, $g_j = (g_{1j}, g_{2j}, \dots, g_{sj}) \in R^s$, $g_j \geq 0$, $g_j \neq 0$, is the vector of desirable (good) outputs, and $b_j = (b_{1j}, b_{2j}, \dots, b_{hj}) \in R^h$, $b_j \geq 0$, $b_j \neq 0$, is the vector of undesirable (bad) outputs. Next, Cooper's method for congestion evaluation [5] and the Kuosmanen technology [25] for addressing undesirable outputs are delineated.

2.1. Cooper's Method. Here, we review Cooper's approach for congestion evaluation, which considers only desirable outputs. This technique was first published by Cooper et al. [5] and was soon thereafter examined on real data and refined by Brockett et al. [26]. This approach, which is a slacks-based method, progresses in two stages. The first stage is directed to find a target point for each DMU via the output-oriented version of the BCC model. The output-oriented version has received more attention in congestion assessment because the input-oriented version may lead to wrong results in this area [1]. In the second stage, the outputs are fixed to those of the target point, and, afterwards, the maximum amount that can be added to the target's inputs is computed.

In Cooper's method, at first, the following well-known BCC model is solved:

$$
\begin{aligned}
\max \quad & \varphi + \varepsilon \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right) \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ik} \quad (i = 1, \dots, m), \\
& \sum_{j=1}^{n} \lambda_j g_{rj} - s_r^+ = \varphi g_{rk} \quad (r = 1, \dots, s), \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0 \quad (j = 1, \dots, n), \\
& s_i^- \geq 0 \quad (i = 1, \dots, m), \\
& s_r^+ \geq 0 \quad (r = 1, \dots, s).
\end{aligned}
\tag{1}
$$

In Model (1), λ_j $(j = 1, \dots, n)$ is the jth structural variable associated with the jth DMU, and the k index refers to the DMU under evaluation. Also, s_i^- and s_r^+ are, respectively, the slack variables for the decrease in the ith input and increase in the rth output. In addition, ε is a non-Archimedean element and applies only in theory in order to avoid rewriting the constraints of the model. Actually, the two-phase procedure is utilized to handle this element.

Let an optimal solution of Model (1) be $(\lambda^*, \varphi^*, s^{-*}, s^{+*})$. Thus, the DMU_k is called strongly efficient if and only if $\varphi^* = 1$, $s^{-*} = 0$, and $s^{+*} = 0$ hold for every optimal solution of the model. Otherwise, the DMU_k can be projected on a strongly efficient target point $(\widehat{x}_k, \widehat{g}_k)$ by the following formulations:

$$\widehat{x}_k = \sum_{j=1}^{n} \lambda_j^* x_j = x_k - s^{-*},$$
$$\widehat{g}_k = \sum_{j=1}^{n} \lambda_j^* g_j = \varphi^* g_k + s^{+*}. \tag{2}$$

It is widely known that inefficiency is a necessary condition for the existence of congestion [1]. After recognizing whether DMU_k is inefficient, Brockett et al. [26] employed the target point (2) on the right-hand side of Model (3) as follows:

$$\begin{aligned}
\max \quad & \sum_{i=1}^{m} \delta_i^+ \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} - \delta_i^+ = \widehat{x}_{ik} \quad (i = 1, \ldots, m), \\
& \sum_{j=1}^{n} \lambda_j g_{rj} = \widehat{g}_{rk} \quad (r = 1, \ldots, s), \\
& \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0 \quad (j = 1, \ldots, n), \\
& 0 \leq \delta_i^+ \leq s_i^{-*} \quad (i = 1, \ldots, m).
\end{aligned} \tag{3}$$

This model calculates the maximum amount that can be added to the ith input of the target point in order to remain in T_{NEW}, which is obtained by assuming weak input disposability. In this way, the nonzero slack cannot be associated with any input. T_{NEW} is shown as follows:

$$T_{\mathrm{NEW}} = \left\{ (x, g) \mid x = \sum_{j=1}^{n} \lambda_j x_j, g \leq \sum_{j=1}^{n} \lambda_j g_j, \right.$$
$$\left. \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0; j = 1, \ldots, n \right\}. \tag{4}$$

Eventually, Cooper's measure of the ith input congestion, which is denoted here by s_i^c, is defined as

$$s_i^c = s_i^{-*} - \delta_i^{+*} \quad (i = 1, \ldots, m). \tag{5}$$

2.2. The Kuosmanen Technology. The production technology is characterized by the set $T = \{(x, g, b) \mid x$ can produce $(g, b)\}$. Consider the following principles which have been introduced in the DEA literature [17, 25] for incorporating undesirable factors into production technology.

(A1) Strong (free) disposability of inputs and good outputs. If $(x, g, b) \in T$, $0 \leq g' \leq g$, and $x' \geq x$, then $(x', g', b) \in T$.

(A2) Weak disposability of good and bad outputs. If $(x, g, b) \in T$, $0 \leq \theta \leq 1$, then $(x, \theta g, \theta b) \in T$.

(A3) T is convex.

Axiom (A2) recognizes the relation between good and bad outputs because the pollutants and wastes can be reduced in proportion to the reduction of good outputs. The multiplier θ used in this axiom is pointed out as the abatement factor [25].

Kousmanen [25] applied n different abatement factors to deal with axiom (A2). He argued that the correct minimum extrapolation technology necessitates n distinctive abatement factors. Therefore, he employed distinctive abatement factors θ_j corresponding to each observed firm $j = 1, \ldots, n$ and developed the following technology (T_K):

$$T_K = \left\{ (x, g, b) \mid \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \theta_j \lambda_j g_j \geq g, \right.$$
$$\sum_{j=1}^{n} \theta_j \lambda_j b_j = b, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \tag{6}$$
$$\left. (j = 1, \ldots, n), \ 0 \leq \theta_j \leq 1, \ (j = 1, \ldots, n) \right\}.$$

Subsequently, Kuosmanen and Podinovski [18] proved that T_K is indeed the only technology that contains all the observed firms and satisfies the minimum extrapolation principle of DEA under the mentioned axioms of (A1), (A2), and (A3).

In addition, it should be noted that T_K is nonlinear, since θ_j is multiplied by λ_j. Nevertheless, Kuosmanen [25] stated that T_K can be linearized as follows:

$$\lambda_j = \underbrace{\theta_j \lambda_j}_{\eta_j} + \underbrace{\left(1 - \theta_j\right) \lambda_j}_{\mu_j} \quad (j = 1, \ldots, n),$$

$$T_K = \left\{ (x, g, b) \mid \sum_{j=1}^{n} \left(\eta_j + \mu_j\right) x_j \leq x, \sum_{j=1}^{n} \eta_j g_j \geq g, \right.$$
$$\sum_{j=1}^{n} \eta_j b_j = b, \sum_{j=1}^{n} \left(\eta_j + \mu_j\right) = 1, \tag{7}$$
$$\left. \eta_j \geq 0, \mu_j \geq 0, \ (j = 1, \ldots, n) \right\}.$$

3. The Proposed Models

3.1. An Efficiency Measure. This section attempts to provide an efficiency measure that incorporates in both desirable and undesirable outputs. For this purpose, the Russell efficiency measure [29] of DEA is employed. This measure includes all the inefficiencies that the model can identify, as described by Pastor et al. [29]. Here, we have used the Kuosmanen technology to address the undesirable outputs. The proposed measure, which is denoted by RM_U, is as follows:

$$\begin{aligned}
RM_U &= \min \frac{(1/(m+h))\left(\sum_{i=1}^{m}\theta_i+\sum_{f=1}^{h}\gamma_f\right)}{(1/s)\sum_{r=1}^{s}\varphi_r} \\
\text{s.t.}\quad & (\theta_1 x_{1k},\dots,\theta_m x_{mk},\varphi_1 g_{1k},\dots,\varphi_s g_{sk}, \\
& \gamma_1 b_{1k},\dots,\gamma_h b_{hk})^t \in T_K \\
& 0 \le \theta_i \le 1 \quad (i=1,\dots,m), \\
& \varphi_r \ge 1 \quad (r=1,\dots,s), \\
& 0 \le \gamma_f \le 1 \quad (f=1,\dots,h).
\end{aligned} \tag{8}$$

In Model (8), the constraints $\theta_i \le 1$, $\varphi_r \ge 1$, and $\gamma_f \le 1$ are included in the model in order to see whether a DMU can be found to dominate DMU_k. If this is not possible, that is, $RM_U = 1$, DMU_k is environmentally efficient; otherwise, it is inefficient. Here, if $x_{ik} = 0$ ($b_{fk} = 0$), then the term θ_i (γ_f) is deleted from the objective function. Moreover, if $g_{rk} = 0$, then it is replaced with a very small positive number which serves as a penalty. Using T_K, the outcome model is

$$\begin{aligned}
RM_U &= \min \frac{(1/(m+h))\left(\sum_{i=1}^{m}\theta_i+\sum_{f=1}^{h}\gamma_f\right)}{(1/s)\sum_{r=1}^{s}\varphi_r} \\
\text{s.t.}\quad & \sum_{j=1}^{n}\left(\eta_j+\mu_j\right)x_{ij} \le \theta_i x_{ik} \quad (i=1,\dots,m), \\
& \sum_{j=1}^{n}\eta_j g_{rj} \ge \varphi_r g_{rk} \quad (r=1,\dots,s), \\
& \sum_{j=1}^{n}\eta_j b_{fj} = \gamma_f b_{fk} \quad (f=1,\dots,h), \\
& \sum_{j=1}^{n}\left(\eta_j+\mu_j\right) = 1, \quad \eta_j \ge 0, \\
& \mu_j \ge 0 \quad (j=1,\dots,n), \\
& 0 \le \theta_i \le 1 \quad (i=1,\dots,m), \\
& \varphi_r \ge 1 \quad (r=1,\dots,s), \\
& 0 \le \gamma_f \le 1 \quad (f=1,\dots,h).
\end{aligned} \tag{9}$$

The RM_U measure incorporates all of the inefficiencies that the model can identify. Simply, it can be verified that all of the constraints of the first, second, and third groups are binding on the optimality of Model (9). In the following theorem, we demonstrate that RM_U lies between zero and unity.

Theorem 1. *Consider the following:*

$$0 < RM_U \le 1.$$

Proof. Since $\theta_i = 1$ $(\forall i)$, $\varphi_r = 1$ $(\forall r)$, $\gamma_f = 1$ $(\forall f)$ $\mu_j = 0$, $(\forall j)$, $\eta_j = 0$, $(\forall j \ne k)$, and $\eta_k = 1$ is a feasible solution to Model (9) with objective function value 1, $RM_U \le 1$. Moreover, $\theta_i \ge 0$ $(\forall i)$, $\varphi_r \ge 1$ $(\forall r)$, and $\gamma_f \ge 0$ $(\forall f)$, and, therefore, $RM_U \ge 0$. Now, we only need to prove that $RM_U \ne 0$. Suppose that $RM_U = 0$. This implies that $\theta_i = 0$ $(\forall i)$ and $\gamma_f = 0$ $(\forall f)$. Consequently, the undesirable constraints yield $\eta_j = 0$ $(\forall j)$, and hence the desirable constraints lead to $\varphi_r g_{rk} \le 0$ $(\forall r)$. At last, it is inferred that $g_{rk} = 0$ $(\forall r)$, while this is a contradiction. Consequently, $RM_U \ne 0$. □

Note that Model (9) is a fractional programming problem; however, it can be transformed into an equivalent linear programming problem by using the Charnes-Cooper transformation. Holding $(1/s)\sum_{r=1}^{s}\varphi_r = 1/t$ and multiplying each constraint by t, we thereafter let $t\eta_j = \eta'_j$, $t\mu_j = \mu'_j$, $t\theta_i = \theta'_i$, $t\varphi_r = \varphi'_r$, and $t\gamma_f = \gamma'_f$, and thus the following linear problem is achieved:

$$\begin{aligned}
RM_U &= \min \frac{1}{m+h}\left(\sum_{i=1}^{m}\theta'_i+\sum_{f=1}^{h}\gamma'_f\right) \\
\text{s.t.}\quad & \sum_{j=1}^{n}\left(\eta'_j+\mu'_j\right)x_{ij} \le \theta'_i x_{ik} \quad (i=1,\dots,m), \\
& \sum_{j=1}^{n}\eta'_j g_{rj} \ge \varphi'_r g_{rk} \quad (r=1,\dots,s), \\
& \sum_{j=1}^{n}\eta'_j b_{fj} = \gamma'_f b_{fk} \quad (f=1,\dots,h), \\
& \sum_{j=1}^{n}\left(\eta'_j+\mu'_j\right) = t, \\
& \eta'_j \ge 0,\ \mu'_j \ge 0 \quad (j=1,\dots,n), \\
& 0 \le \theta'_i \le t \quad (i=1,\dots,m), \\
& \varphi'_r \ge t \quad (r=1,\dots,s), \\
& 0 \le \gamma'_f \le t \quad (f=1,\dots,h), \\
& \sum_{r=1}^{s}\varphi'_r = s, \quad t \ge 0.
\end{aligned} \tag{10}$$

As a result, we can easily obtain RM_U via a linear problem for assessing the environmental performance of DMU_k. Note that since $t > 0$, the transformation is reversible. Let $(\eta'^*, \mu'^*, \theta'^*, \varphi'^*, \gamma'^*, t^*)$ be an optimal solution of Model (10); consequently, $(\eta^* = \eta'^*/t^*, \mu^* = \mu'^*/t^*, \theta^* = \theta'^*/t^*, \varphi^* = \varphi'^*/t^*, \gamma^* = \gamma'^*/t^*)$ is an optimal solution of Model (9).

The output-oriented version of the proposed measure can be defined by ignoring the reduction of inputs, that is, by omitting θ_i, in the objective function of Model (9). Subsequently, the efficiency score $RM_U(O)$ can be achieved as follows:

$$\begin{aligned}
&RM_U(O)\\
&\quad= \min \frac{(1/h)\sum_{f=1}^{h}\gamma_f}{(1/s)\sum_{r=1}^{s}\varphi_r}\\
&\quad\quad \text{s.t.} \quad \sum_{j=1}^{n}\left(\eta_j+\mu_j\right)x_{ij} \le x_{ik} \quad (i=1,\ldots,m),\\
&\quad\quad\quad \sum_{j=1}^{n}\eta_j g_{rj} \ge \varphi_r g_{rk} \quad (r=1,\ldots,s),\\
&\quad\quad\quad \sum_{j=1}^{n}\eta_j b_{fj} = \gamma_f b_{fk} \quad (f=1,\ldots,h),\\
&\quad\quad\quad \sum_{j=1}^{n}\left(\eta_j+\mu_j\right) = 1,\\
&\quad\quad\quad \eta_j \ge 0,\ \mu_j \ge 0 \quad (j=1,\ldots,n),\\
&\quad\quad\quad \varphi_r \ge 1 \quad (r=1,\ldots,s),\\
&\quad\quad\quad 0 \le \gamma_f \le 1 \quad (f=1,\ldots,h).
\end{aligned} \tag{11}$$

In a similar way, as described above, the equivalent linear program of the output-oriented Model (11) can be obtained as follows:

$$\begin{aligned}
&RM_U(O)\\
&\quad= \min \frac{1}{h}\sum_{f=1}^{h}\gamma'_f\\
&\quad\quad \text{s.t.} \quad \sum_{j=1}^{n}\left(\eta'_j+\mu'_j\right)x_{ij} \le t x_{ik} \quad (i=1,\ldots,m),\\
&\quad\quad\quad \sum_{j=1}^{n}\eta'_j g_{rj} \ge \varphi'_r g_{rk} \quad (r=1,\ldots,s),\\
&\quad\quad\quad \sum_{j=1}^{n}\eta'_j b_{fj} = \gamma'_f b_{fk} \quad (f=1,\ldots,h),\\
&\quad\quad\quad \sum_{j=1}^{n}\left(\eta'_j+\mu'_j\right) = t,\\
&\quad\quad\quad \eta'_j \ge 0,\ \mu'_j \ge 0 \quad (j=1,\ldots,n),\\
&\quad\quad\quad \varphi'_r \ge t \quad (r=1,\ldots,s),\\
&\quad\quad\quad 0 \le \gamma'_f \le t \quad (f=1,\ldots,h),\\
&\quad\quad\quad \sum_{r=1}^{s}\varphi'_r = s, \quad t \ge 0.
\end{aligned} \tag{12}$$

3.2. Congestion in the Simultaneous Presence of Desirable and Undesirable Outputs. As noted earlier, in traditional congestion, only desirable outputs are considered. Nevertheless, in the real world, undesirable outputs (such as pollutants or wastes) are unavoidably generated along with desirable outputs. In this subsection, we attempt to develop a new scheme for measuring congestion in the simultaneous presence of both desirable and undesirable outputs. The definition we present is as follows.

Definition 2. Congestion occurs in the performance of $\mathrm{DMU}_k = (x_k, g_k, b_k)$ whenever a reduction in one or more inputs can increase one or more desirable outputs and decrease one or more undesirable outputs without worsening any other input, desirable output, or undesirable output. In contrast, whenever an increase in one or more inputs can decrease one or more desirable outputs and increase one or more undesirable outputs without improving any other input, desirable output, or undesirable output.

Now we have tried to identify this type of congestion and determine its sources and amounts. For this purpose, first, the output-oriented version of the proposed measure is employed. This model determines the maximally possible amount of desirable outputs and the minimally possible amount of undesirable outputs that can be acquired by DMU_k.

In fact, Model (11) recognizes whether DMU_k could achieve more desirable outputs and fewer undesirable outputs. If this is not possible, that is, if $RM_U(O) = 1$, then congestion does not prevail at DMU_k. Otherwise, congestion may be present in the performance of DMU_k. Therefore, to capture the input congestion and identify its sources and amounts, Model (13) is employed as follows:

$$\begin{aligned}
&\min \quad \sum_{i=1}^{m} s_i^c\\
&\text{s.t.} \quad \sum_{j=1}^{n}\left(\eta_j+\mu_j\right)x_{ij} = x_{ik} - s_i^c \quad (i=1,\ldots,m),\\
&\quad\quad \sum_{j=1}^{n}\eta_j g_{rj} \ge \varphi_r^* g_{rk} \quad (r=1,\ldots,s),\\
&\quad\quad \sum_{j=1}^{n}\eta_j b_{fj} = \gamma_f^* b_{fk} \quad (f=1,\ldots,h),
\end{aligned}$$

Table 1: Data set of industry of China in 2010.

District	DMU	Input1 TIFA	Input2 EC	Desirable output GIOV	Undesirable out1 TWGE	Undesirable out2 TWWD
Anhui	D1	9,121.829	1077.91	18,732	17,849	70,971
Beijing	D2	4,554.356	809.9	13,699.84	4,750	8,198
Chongqing	D3	5,049.258	626.44	9,143.55	10,943	45,180
Fujian	D4	6,534.803	1315.09	21,901.23	13,507	124,168
Gansu	D5	2,274.305	804.43	4,882.68	6,252	15,352
Guangdong	D6	11,903.36	4060.13	85,824.64	24,092	187,031
Guangxi	D7	5,166.135	993.24	9,644.13	14,520	165,211
Guizhou	D8	2,483.012	835.38	4,206.37	10,192	14,130
Hainan	D9	903.8264	159.02	1,381.25	1,360	5,782
Hebei	D10	11,737.07	2691.52	31,143.29	56,324	114,232
Heilongjiang	D11	5,019.085	747.84	9,535.15	10,111	38,921
Henan	D12	12,868.24	2353.96	34,995.53	22,709	150,406
Hubei	D13	7,276.638	1330.44	21,623.12	13,865	94,593
Hunan	D14	7,374.157	1171.91	19,008.83	14,673	95,605
Inner Mongolia	D15	6,831.416	1536.83	13,406.11	27,488	39,536
Jiangsu	D16	18,977.92	3864.37	92,056.48	31,213	263,760
Jiangxi	D17	6,696.149	700.51	13,883.06	9,812	72,526
Jilin	D18	6,313.748	576.98	13,098.35	8.240	38,656
Liaoning	D19	12,480.94	1715.26	36,219.42	26,955	71,521
Ningxia	D20	1,193.702	546.77	1,924.39	16,324	21,977
Qinghai	D21	789.5051	465.18	1,481.99	3,952	9,031
Shaanxi	D22	5,462.784	859.22	11,199.84	13,510	45,487
Shandong	D23	17,664.34	3298.46	83,851.4	43,837	208,257
Shanghai	D24	4,252.32	1295.87	30,114.41	12,969	36,696
Shanxi	D25	4,702.091	1460	12,471.33	35,190	49,881
Sichuan	D26	9,790.274	1549.03	23,147.38	20,107	93,444
Tianjin	D27	4,571.888	645.74	16,751.82	7,686	19,680
Tibet	D28	306.567	20.41	62.22	16	736
Xinjiang	D29	2,749.838	661.96	5,341.9	9,310	25,413
Yunnan	D30	4,024.972	1004.07	6,464.63	10,978	30,926
Zhejiang	D31	10,246.41	2820.93	51,394.2	20,434	217,426

$$\sum_{j=1}^{n}\left(\eta_j+\mu_j\right)=1,$$
$$\eta_j \geq 0, \quad \mu_j \geq 0 \quad \left(j=1,\ldots,n\right),$$
$$s_i^c \geq 0 \quad \left(i=1,\ldots,m\right). \tag{13}$$

Fixing the outputs on the obtained optimal amounts, Model (13) computes the minimum amount that can be reduced from the ith input of DMU_k in order to acquire these optimal amounts of outputs and also reach $T_{K_{\text{NEW}}}$.

$T_{K_{\text{NEW}}}$, which is gained by changing the input inequalities of T_K to input equalities, is the NEW technology corresponding to Kuosmanen technology and is as follows:

$$T_{K_{\text{NEW}}}=\left\{(x,g,b)\mid\sum_{j=1}^{n}\left(\eta_j+\mu_j\right)x_j=x,\sum_{j=1}^{n}\eta_j g_j\geq g,\right.$$
$$\sum_{j=1}^{n}\eta_j b_j=b,\sum_{j=1}^{n}\left(\eta_j+\mu_j\right)=1, \tag{14}$$
$$\left.\eta_j\geq 0,\ \mu_j\geq 0,\ \left(j=1,\ldots,n\right)\right\}.$$

After solving Model (13), if $s_i^c > 0$, then s_i^c is the amount of congestion in the ith input of DMU_k. Otherwise, if $s_i^c = 0$, congestion does not occur in the ith input of DMU_k.

4. Numerical Example

In this section, we apply the proposed method for assessing the congestion in 31 administrative regions of China. See the data set in Table 1 which is adopted form Wu et al.'s paper [28]. These data have two inputs: the total investment in fixed assets of industry (TIFA) and the electricity consumption by industry (EC), one desirable output: the gross industrial output value (GIOV), and two undesirable outputs: the total volume of industrial waste gas emission (TWGE) and the total volume of waste water discharge (TWWD). For ease of comparison, we have named the industries D1 to D31, which are depicted in the second column of Table 1.

We utilized the general algebraic modeling system (GAMS) software for the calculations. Table 2 displays the results of the presented models. The first column shows the results of the proposed measure which is computed by Model (10). D2, D6, D16, D23, D24, D27, and D28 are identified as the environmentally efficient industries and attain $RM_U = 1$. Thus, we can conclude that these 7 industries pay attention to

Table 2: Results of the proposed models.

District	DMU	RM_U	$RM_U(O)$	s_1^c	s_2^c
Anhui	D1	0.455	0.317	4054.732	41.241
Beijing	D2	1	1	0	0
Chongqing	D3	0.380	0.205	1481.994	0
Fujian	D4	0.526	0.372	1144.784	135.603
Gansu	D5	0.331	0.231	0	418.298
Guangdong	D6	1	1	0	0
Guangxi	D7	0.262	0.133	611.779	183.340
Guizhou	D8	0.257	0.161	0	410.458
Hainan	D9	0.375	0.248	0	27.604
Hebei	D10	0.364	0.290	7343.447	1344.598
Heilongjiang	D11	0.386	0.237	798.638	0
Hainan	D12	0.495	0.433	6144.005	584.392
Hubei	D13	0.517	0.395	1914.956	163.486
Hunan	D14	0.467	0.322	2278.853	122.766
Inner Mongolia	D15	0.319	0.186	2277.060	726.930
Jiangsu	D16	1	1	0	0
Jiangxi	D17	0.505	0.360	2412.607	0
Jilin	D18	0.604	0.516	2487.404	0
Liaoning	D19	0.652	0.635	7390.180	116.468
Ningxia	D20	0.150	0.047	0	361.478
Qinghai	D21	0.229	0.114	0	355.012
Shaanxi	D22	0.381	0.217	908.428	49.320
Shandong	D23	1	1	0	0
Shanghai	D24	1	1	0	0
Shanxi	D25	0.300	0.136	147.735	650.100
Sichuan	D26	0.444	0.350	4273.281	313.386
Tianjin	D27	1	1	0	0
Tibet	D28	1	1	0	0
Xinjiang	D29	0.287	0.162	0	187.446
Yunnan	D30	0.255	0.165	0	292.561
Zhejiang	D31	0.697	0.597	1851.267	312.372

the reduction of their pollutants accompanying improvement of their commercial targets.

The second column depicts the results of the output-oriented measure which is computed by Model (12). It should be noticed that some industries (such as D7, D8, D15, D20, D21, D25, D29, and D30) obtain a very low $RM_U(O)$. As a result, these industries should be seriously concerned about their outputs. Especially, they should pay more attention to the reduction of their pollutants and wastes.

The amount of congestion in each input, which is calculated by Model (13), is presented in the two last columns of Table 2. The 7 above-mentioned industries, which performed efficiently, do not show congestion in any input. The other industries evidence congestion in one or both of their inputs. For instance, congestion exists in the second input of D5. Therefore, D5 can reduce its electricity consumption in amounts of $s_2^c = 418.298$ and, accordingly, its desirable output increases and its undesirable outputs decrease. Or D25 shows congestion in both of its inputs. Therefore, it can reduce its investment in amounts of $s_1^c = 147.735$ and its electricity consumption in amounts of $s_2^c = 650.100$ and, accordingly, its desirable output increases and its undesirable outputs decrease.

5. Summary and Conclusion

The concept of congestion, which is mainly applied in the economics, points out a situation where inputs are overinvested. Many researchers have studied this subject using data envelopment analysis (DEA). However, most of the previous investigations only considered the framework of desirable outputs (see, e.g., [6–8, 10–12]). In fact, firms in the real world unavoidably generate undesirable outputs (like pollutants or wastes) along with desirable outputs. Therefore, a new scheme is required for measuring congestion in the simultaneous presence of both desirable and undesirable outputs.

In this paper, we briefly introduced the Kuosmanen [25] technology which is available in the DEA literature for modeling environmental performance under the weak disposability assumption of desirable and undesirable outputs.

Then, we attempted to present a nonradial efficiency measure that incorporates in both desirable and undesirable outputs. Based on the proposed model, a new definition and a new approach are presented to deal with congestion in the simultaneous presence of desirable and undesirable outputs.

Afterwards, the presented method is applied to study the pollutants (waste gas emission and waste discharge) of 31 administrative regions of China. The finding of this paper shows that 7 industries pay attention to the reduction of their pollutants accompanying improvement of their commercial targets. Consequently, they do not show congestion in any input. The other industries evidence congestion in one or both of their inputs. Reducing the amount of congestion in each input, these industries can enhance their desirable output and decrease their undesirable outputs.

Possible future research is to study ranking of DMUs with undesirable outputs. Ranking efficient DMU is an important issue in DEA literature and many researchers have worked in this domain; see, for example, [30–38]. However, the research on this subject in the presence of undesirable outputs can be a future research agenda.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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