Research Article

New Uncertainty Measure of Rough Fuzzy Sets and Entropy Weight Method for Fuzzy-Target Decision-Making Tables

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In the rough fuzzy set theory, the rough degree is used to characterize the uncertainty of a fuzzy set, and the rough entropy of a knowledge is used to depict the roughness of a rough classification. Both of them are effective, but they are not accurate enough. In this paper, we propose a new rough entropy of a rough fuzzy set combining the rough degree with the rough entropy of a knowledge. Theoretical studies and examples show that the new rough entropy of a rough fuzzy set is suitable. As an application, we introduce it into a fuzzy-target decision-making table and establish a new method for evaluating the entropy weight of attributes.

1. Introduction

The rough set theory, introduced by Pawlak [1, 2], is a useful and important tool to deal with uncertain information systems. It has been studied and applied in many fields, such as artificial intelligence, making decision, machine learning, and pattern recognition. In the traditional rough set theory, targets are often ordinary. However, in actual problems, we often encounter some cases in which targets are fuzzy [3]. In order to describe this kind of fuzzy targets, Dubois and Prade [4, 5] proposed the concept of rough fuzzy set, which is a good combination of rough sets and fuzzy sets. As an important issue in the rough fuzzy set theory, measuring uncertainty has been widely studied (see [6–9]). There are two primary reasons which cause the uncertainty. One reason is that the boundary region of the target's approximation space could produce roughness. The larger the boundary region is, the larger the roughness of the target is. The kind of roughness is classified into system uncertainty and measured by the rough degree [3, 10] from the view of algebra. Another reason is the roughness produced by a knowledge which is a division and divided by the binary relation on universe. The more objects the equivalent classes have, the rougher the knowledge is. This kind of uncertainty is classified into knowledge's uncertainty and measured by the rough entropy [11, 12] of a knowledge from the view of information theory.

However, there exist some deficiencies about these uncertainty measures of rough fuzzy sets mentioned above. On the one hand, the rough degree is not strictly monotonic with finer knowledge. That implies that the rough degree is not accurate enough. On the other hand, the rough entropy of knowledge does not reflect the uncertainty produced by the boundary region of approximation space. In order to overcome these limitations, we propose a new rough entropy about rough fuzzy sets, which considers not only the rough degree but also the rough entropy of a knowledge. This is the first aim in this paper (see Section 3).

In multiattribute decision-making problems, the research of evaluating attribute weights is always a hot area. There are some traditional methods to determine attribute weights, such as expert grading, binary comparing, fuzzy statistics, and grey relational analysis. Nevertheless these methods depend largely on decision maker's experience. Now, the rough set theory has been introduced into research of determining weights. Usually, there are two kinds of methods. From the view of algebraic theory [13, 14], the rough degree is used to determine weights of attribute. This method is effective but sometimes leads to an impractical case in which some weights may be zero. From the view of information theory [15, 16], information entropy is used to determine weights. This method can avoid the case in which the weight is equal to zero. But it may lead to an awkward case in which

weights of some redundant attributes are bigger than that of nonredundant attributes weights. In this paper, we will introduce the new rough entropy of rough fuzzy sets into fuzzy-target decision-making tables. As an application, we give a new method to evaluate entropy weights. That is the second aim in this paper (see Section 4).

2. Basic Concepts of Rough Fuzzy Sets

In order to deal with complicated fuzzy information systems integrating the rough set theory, Dubois and Prade in [4] introduced equivalent relations into fuzzy sets and proposed the concept of rough fuzzy sets.

Let U be universe, and let $R \subseteq U \times U$ be an equivalent relation. Then (U, R) constitutes a Pawlak approximation space. The set of equivalent classes $U/R = \{X_1, X_2, \ldots, X_m\}$ generated by R on U is called a knowledge. $[x]_R$ denotes the equivalent class of the element x. $F(U)$ denotes the set of all fuzzy sets on U.

Definition 1 (see [3, 10]). Let (U, R) be a Pawlak approximation space, and let A be a fuzzy set on U. Then the lower and upper approximation sets of A on (U, R) are defined by $\underline{R}(A)$ and $\overline{R}(A)$, respectively, where

$$\underline{R}(A)(x) = \min\{A(y) \mid y \in [x]_R\}, \quad \forall x \in U,$$
$$\overline{R}(A)(x) = \max\{A(y) \mid y \in [x]_R\}, \quad \forall x \in U. \tag{1}$$

$\underline{R}$ and $\overline{R}$ are called the lower and upper approximation operators. The ordered pair $(\underline{R}(A), \overline{R}(A))$ is called rough fuzzy set. If $\underline{R}(A) = \overline{R}(A)$, A is said to be definable. Otherwise, A is said to be rough.

In Definition 1, $\underline{R}(A)(x)$ can be understood as the membership that the element x affiliated at lowest level to A and $\overline{R}(A)(x)$ as the membership that the element x affiliated possibly to A. If A is degenerated to an ordinary set, $\underline{R}(A)$ and $\overline{R}(A)$ are degenerated accordingly to the lower and upper approximation set of traditional Pawlak rough set, respectively.

Let U be a universe and $A \in F(U)$. Then the cut sets of $\underline{R}(A)$ and $\overline{R}(A)$ are given, respectively, as follows:

$$(\underline{R}(A))_\alpha = \{x \in U : \underline{R}(A)(x) \geq \alpha\},$$
$$(\overline{R}(A))_\beta = \{x \in U : \overline{R}(A)(x) \geq \beta\}. \tag{2}$$

Definition 2 (see [12]). Let R and S be equivalent relations, $U/R = \{X_1, X_2, \ldots, X_m\}$, and $U/S = \{Y_1, Y_2, \ldots, Y_n\}$. If there is $Y_j \in U/S$ for any $X_i \in U/R$ such that $X_i \subseteq Y_j$, then it is said that the knowledge U/R is finer than the knowledge U/S and denotes it by $R \preceq S$.

If U/R is finer than U/S and $U/R \neq U/S$, then it is said that U/R is strictly finer than U/S and denotes it by $R \prec S$.

3. Uncertainty Measures of Rough Fuzzy Sets

Two kinds of methods to measure uncertainty of rough fuzzy sets are often used. One is the rough degree, which is defined

from an algebra point of view. Another is the rough entropy, which is defined from an information theory point of view.

3.1. Rough Degree of a Rough Fuzzy Set. Since a fuzzy target is described by use of the lower and upper approximation sets, the noncoincidence of the lower and upper approximation sets results in roughness. The larger the boundary region formed by the lower and upper approximation sets is, the more the roughness is. In order to quantify the roughness, rough degree was introduced.

Definition 3 (see [3, 10]). Let (U, R) be a Pawlak space and $A \in F(U)$. Then for any α, β ($0 < \beta \leq \alpha \leq 1$), the accuracy degree $\eta_A^R(\alpha, \beta)$ and rough degree $\rho_A^R(\alpha, \beta)$ of A under R are defined, respectively, as follows:

$$\eta_A^R(\alpha, \beta) = \frac{\left|(\underline{R}(A))_\alpha\right|}{\left|(\overline{R}(A))_\beta\right|},$$

$$\rho_A^R(\alpha, \beta) = 1 - \eta_A^R(\alpha, \beta) = 1 - \frac{\left|(\underline{R}(A))_\alpha\right|}{\left|(\overline{R}(A))_\beta\right|}, \tag{3}$$

where $|\cdot|$ denotes the number of elements belonging to a finite set. If $\overline{R}(A)_\beta = \Phi$, then it is stipulated that $\rho_A^R(\alpha, \beta) = 0$.

In Definition 3, rough degree characterizes the roughness using the lower and upper approximation sets to approach a fuzzy target and thresholds α and β make the representation of rough degree more flexible.

Theorem 4 (see [3]). *Let (U, R) be a Pawlak space and $A \in F(U)$. For any α, β ($0 < \beta \leq \alpha \leq 1$), the rough degree in Definition 3 satisfies the following properties:*

(i) $0 \leq \rho_A^R(\alpha, \beta) \leq 1$;

(ii) $\rho_A^R(\alpha, \beta)$ *does not decrease with α and does not increase with β;*

(iii) *If $\max_{1 \leq i \leq m} \min_{x \in X_i} A(x) < \alpha$, then $\rho_A^R(\alpha, \beta) = 1$;*

(iv) *If $\alpha = \beta$ and $A(x) = c_i$ (c_i is a constant) for any $x \in X_i$ ($i \leq r$), then $\rho_A^R(\alpha, \beta) = 1$.*

Theorem 5 (see [3]). *Let (U, R) and (U, S) be two Pawlak spaces, and $R \preceq S$. Then, $\rho_A^R(\alpha, \beta) \leq \rho_A^S(\alpha, \beta)$.*

Theorem 5 describes that the rough degree is a monotone decrease with finer knowledge. However, this result is not sufficiently accurate; see an illustrative example.

Example 6. Let (U, R) be a Pawlak approximation space, where $U = \{a_1, a_2, a_3, a_4, b_1, b_2, b_3, c_1, c_2\}$, $R = \{r_1, r_2, r_3\}$, and $A \in F(U)$. Given a fuzzy-target decision-making decision table, see Table 1.

TABLE 1: Fuzzy-target decision-making table.

U	r_1	r_2	r_3	A
a_1	1	1	1	0.6
a_2	1	1	2	0.7
a_3	1	2	1	0.7
a_4	1	2	2	0.9
b_1	2	3	3	0.5
b_2	2	3	3	0.4
b_3	2	3	3	0.7
c_1	3	4	4	0.8
c_2	3	4	4	0.7

Let $R_1 = \{r_1\}$, $R_2 = \{r_1, r_2\}$, and $R_3 = \{r_1, r_2, r_3\}$. Then

$$\frac{U}{R_1} = \{\{a_1, a_2, a_3, a_4\}, \{b_1, b_2, b_3\}, \{c_1, c_2\}\},$$

$$\frac{U}{R_2} = \{\{a_1, a_2\}, \{a_3, a_4\}, \{b_1, b_2, b_3\}, \{c_1, c_2\}\}, \quad (4)$$

$$\frac{U}{R_3} = \{\{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{b_1, b_2, b_3\}, \{c_1, c_2\}\}.$$

We may see that

$$R_3 \prec R_2 \prec R_1. \quad (5)$$

By Definition 1, we have

$$\underline{R_1}(A) = \left\{\frac{0.6}{a_1}, \frac{0.6}{a_2}, \frac{0.6}{a_3}, \frac{0.6}{a_4}, \frac{0.4}{b_1}, \frac{0.4}{b_2}, \frac{0.4}{b_3}, \frac{0.7}{c_1}, \frac{0.7}{c_2}\right\},$$

$$\overline{R_1}(A) = \left\{\frac{0.9}{a_1}, \frac{0.9}{a_2}, \frac{0.9}{a_3}, \frac{0.9}{a_4}, \frac{0.7}{b_1}, \frac{0.7}{b_2}, \frac{0.7}{b_3}, \frac{0.8}{c_1}, \frac{0.8}{c_2}\right\},$$

$$\underline{R_2}(A) = \left\{\frac{0.6}{a_1}, \frac{0.6}{a_2}, \frac{0.7}{a_3}, \frac{0.7}{a_4}, \frac{0.4}{b_1}, \frac{0.4}{b_2}, \frac{0.4}{b_3}, \frac{0.7}{c_1}, \frac{0.7}{c_2}\right\},$$

$$\overline{R_2}(A) = \left\{\frac{0.7}{a_1}, \frac{0.7}{a_2}, \frac{0.9}{a_3}, \frac{0.9}{a_4}, \frac{0.7}{b_1}, \frac{0.7}{b_2}, \frac{0.7}{b_3}, \frac{0.8}{c_1}, \frac{0.8}{c_2}\right\}, \quad (6)$$

$$\underline{R_3}(A) = \left\{\frac{0.6}{a_1}, \frac{0.7}{a_2}, \frac{0.7}{a_3}, \frac{0.9}{a_4}, \frac{0.4}{b_1}, \frac{0.4}{b_2}, \frac{0.4}{b_3}, \frac{0.7}{c_1}, \frac{0.7}{c_2}\right\},$$

$$\overline{R_3}(A) = \left\{\frac{0.6}{a_1}, \frac{0.7}{a_2}, \frac{0.7}{a_3}, \frac{0.9}{a_4}, \frac{0.7}{b_1}, \frac{0.7}{b_2}, \frac{0.7}{b_3}, \frac{0.8}{c_1}, \frac{0.8}{c_2}\right\}.$$

Let $\alpha = \beta = 0.6$. Then

$$\underline{R_i}(A)_{0.6} = \{a_1, a_2, a_3, a_4, c_1, c_2\}, \quad i = 1, 2, 3,$$

$$\overline{R_i}(A)_{0.6} = \{a_1, a_2, a_3, a_4, b_1, b_2, b_3, c_1, c_2\}, \quad i = 1, 2, 3. \quad (7)$$

We have

$$\rho_A^{R_i}(0.6, 0.6) = \frac{1}{3}, \quad i = 1, 2, 3. \quad (8)$$

It is possible that the values of rough degree are invariant under the finer knowledge from Example 6. This reveals that rough degree is not equipped with strict monotone with the finer knowledge. It implies the limitations of rough degree in Definition 3. Hence, it is necessary to find a more accurate measure to characterize the roughness of rough fuzzy sets.

3.2. Rough Entropy of a Knowledge.

Entropy is an important notion measuring uncertainty of information systems in the information theory; see [17]. A knowledge in rough set theory is regarded as a division of universe. This implies that knowledge itself is granular and uncertain. The bigger the knowledge granules are, the rougher the division is, and, accordingly, the bigger the uncertainty of knowledge is. In order to quantify the uncertainty that resulted in knowledge granules, Liang and Shi in [11] introduced a definition of rough entropy for a knowledge.

Definition 7 (see [11]). Let (U, R) be a Pawlaw space and $U/R = \{X_1, X_2, \ldots, X_m\}$. Then rough entropy of the knowledge U/R is defined as follows:

$$E(R) = -\sum_{i=1}^{m} \frac{|X_i|}{|U|} \log_2 \frac{1}{|X_i|}. \quad (9)$$

Theorem 8 (see [11]). *Let (U, R) and (U, S) be two Pawlak spaces. Then $R \prec S$ implies that $E(R) < E(S)$.*

From Theorem 8, one can see that the finer the division of U under R is, the smaller the rough entropy of a knowledge is.

3.3. Rough Entropy of a Rough Fuzzy Set.

The uncertainty of a rough fuzzy set not only depends on the roughness of the fuzzy target itself, but also depends on the uncertainty of a knowledge. From the discussion in Section 3.1, we know that although rough degree $\rho_A^R(\alpha, \beta)$ can reflect the roughness of a fuzzy target which is characterized approximately by lower and upper approximation sets, $\rho_A^R(\alpha, \beta)$ is not strictly monotone and cannot reflect the influence with the changes of knowledge granules. From the discussion in Section 3.2, we know that the rough entropy $E(R)$ of a knowledge can reflect the roughness of the knowledge and is strictly monotone, but it cannot reflect the roughness of a fuzzy target itself. Based on these deficiencies, we propose a new rough entropy of a rough fuzzy set.

Definition 9. Let (U, R) be a Pawlak approximation space, $A \in F(U)$, and $U/R = \{X_1, X_2, \ldots, X_m\}$. Rough entropy of A under R is defined as follows:

$$E_{\text{rough}}^R(A) = \rho_A^R(\alpha, \beta) E(R). \quad (10)$$

Theorem 10. *Rough entropy of A defined in Definition 9 has the following properties.*

(i) *For any given $\alpha, \beta(\min_{x \in U}\{A(x)\} < \beta \leq \alpha < \max_{x \in U}\{A(x)\})$, $E_{\text{rough}}^R(A)$ reaches the maximum $\log_2|U|$ when $U/R = \{U\}$.*

(ii) *$E_{\text{rough}}^R(A)$ reaches the minimum 0 when $U/R = \{\{x_1\}, \{x_2\}, \ldots, \{x_n\}\}$.*

Proof. (i) Let $U/R = \{U\}$. Then for any $x \in U$, we have

$$\underline{R}(A)(x) = \min\{A(y) \mid y \in [x]_R\} = \min\{A(y) \mid y \in U\};$$

$$\overline{R}(A)(x) = \max\{A(y) \mid y \in [x]_R\} = \max\{A(y) \mid y \in U\}. \quad (11)$$

Obviously, $\underline{R}(A)(x) < \alpha$, $\overline{R}(A)(x) > \beta$. Then $\underline{R}(A) = \Phi$ and $\overline{R}(A)_\beta = U$. By Definition 3, we have

$$\rho_A^R (\alpha, \beta) = 1 - \frac{\left|\underline{R}(A)_\alpha\right|}{\left|\overline{R}(A)_\beta\right|} = 1. \tag{12}$$

In addition, by Definition 7, we have

$$E (R) = -\frac{|U|}{|U|} \log_2 \frac{1}{|U|} = \log_2 |U|. \tag{13}$$

Hence, we obtain

$$E_{\text{rough}}^R (A) = \rho_A^R (\alpha, \beta) E (R) = \log_2 |U|. \tag{14}$$

(ii) If $U/R = \{\{x_1\}, \{x_2\}, \ldots, \{x_n\}\}$, we have

$$E (R) = -\sum_{i=1}^n \frac{|\{x_i\}|}{|U|} \log_2 \frac{1}{|\{x_i\}|} = 0. \tag{15}$$

Hence, we obtain $E_{\text{rough}}^R(A) = 0$. $\qquad\square$

Theorem 11. *Let (U, R) and (U, S) be Pawlak approximation spaces, and $A \in F(U)$. If $R \prec S$, one has*

$$E_{rough}^R (A) < E_{rough}^S (A). \tag{16}$$

Proof. Assuming that $R \prec S$, we have $\rho_A^R(\alpha, \beta) \le \rho_A^S(\alpha, \beta)$ by Theorem 5 and $E(R) < E(S)$ by Theorem 8. Then we obtain $E_{\text{rough}}^R(A) < E_{\text{rough}}^S(A)$. $\qquad\square$

Theorem 11 implies that the rough entropy of a rough fuzzy set in Definition 9 is strictly monotone.

Example 12. For the fuzzy set A given in Example 6, we compute the rough entropy as follows:

$$E_{\text{rough}}^{R_1} (A)$$

$$= \rho_A^{R_1} (\alpha, \beta) E (R_1)$$

$$= \frac{1}{3} \left[-\left(\frac{4}{9} \times \log_2 \frac{1}{4} + \frac{3}{9} \log_2 \frac{1}{3} + \frac{2}{9} \log_2 \frac{1}{2} \right) \right]$$

$$= \frac{10}{27} + \frac{2}{9} \log_2 3,$$

$$E_{\text{rough}}^{R_2} (A)$$

$$= \rho_A^{R_2} (\alpha, \beta) E (R_2)$$

$$= \frac{1}{3} \left[-\left(\frac{2}{9} \times 2 \times \log_2 \frac{1}{2} + \frac{3}{9} \log_2 \frac{1}{3} + \frac{2}{9} \log_2 \frac{1}{2} \right) \right]$$

$$= \frac{6}{27} + \frac{2}{9} \log_2 3,$$

$$E_{\text{rough}}^{R_3} (A)$$

$$= \rho_A^{R_3} (\alpha, \beta) E (R_3)$$

$$= \frac{1}{3} \left[-\left(\frac{1}{9} \times 4 \times \log_2 1 + \frac{3}{9} \log_2 \frac{1}{3} + \frac{2}{9} \log_2 \frac{1}{2} \right) \right]$$

$$= \frac{2}{27} + \frac{2}{9} \log_2 3. \tag{17}$$

Then we obtain $E_{\text{rough}}^{R_3}(A) < E_{\text{rough}}^{R_2} < E_{\text{rough}}^{R_1}(A)$ when $R_3 \prec R_2 \prec R_1$.

This example validates that the rough entropy in Definition 9 is more accurate than the rough degree in Definition 3, since it is equipped with strict monotonicity.

4. Entropy Weight for a Fuzzy-Concept Decision-Making Table

4.1. Weight of an Attribute. Given a fuzzy-target decision-making table $S = (U, R, A, V, W)$, where U is an object set, R is a conditional attribute set whose value set V is a crisp set, and A is a fuzzy target whose value set W is a fuzzy set. Using the rough fuzzy set theory in Section 3, the fuzzy target A can be described approximately by the lower and upper approximation sets $\underline{R}(A)$, $\overline{R}(A)$ under the attribute set R, and the rough entropy of the fuzzy target A under the attribute set R is

$$E_{\text{rough}}^R (A) = \rho_A^R (\alpha, \beta) E (R). \tag{18}$$

The significance of an attribute is obtained by the change value of rough degree when the attribute is removed from the attribute set. The larger the change is, the more the significance of the attribute is. The significance $\text{sig}(r_i, R, A)$ of the attribute r_i is

$$\text{sig} (r_i, R, A) = \rho_A^{R-\{r_i\}} (\alpha, \beta) - \rho_A^R (\alpha, \beta). \tag{19}$$

Then the weight of the condition attribute $r_i \in R$ is

$$w (r_i, R, A) = \frac{\text{sig} (r_i, R, A)}{\sum_{j=1}^n \text{sig} (r_i, R, A)}. \tag{20}$$

We can see that $\text{sig}(r_i, R, A)$ and $w(r_i, R, A)$ may equal zero since $\rho_A^R(\alpha, \beta)$ is not strictly monotone with finer knowledge R. Nevertheless, for a certain decision table, each attribute is significant in a way [18]. That is to say, each attribute's weight may be tiny but could not be equivalent to zero. In order to avoid the deficiency of $\text{sig}(r_i, R, A)$ and $w(r_i, R, A)$ above, we propose a new method to calculate the weight.

Definition 13. Let $S = (U, R, A, V, W)$ be a fuzzy-target decision-making table and $R = \{r_1, r_2, \ldots, r_p\}$. The significance of r_i $(i = 1, 2, \ldots, p)$ is defined by

$$\text{sig}^* (r_i, R, A) = E_{\text{rough}}^{R-\{r_i\}} (A) - E_{\text{rough}}^R (A); \tag{21}$$

TABLE 2: Fuzzy-target decision-making table.

U	Outlook (r_1)	Temperature (r_2)	Humidity (r_3)	Windy (r_4)	Decision (A)
x_1	Sunny	Hot	High	False	0.4
x_2	Sunny	Hot	High	True	0.5
x_3	Overcast	Hot	High	False	0.6
x_4	Rain	Mild	High	False	0.8
x_5	Rain	Cool	Normal	False	0.7
x_6	Rain	Cool	Normal	True	0.7
x_7	Overcast	Cool	Normal	True	0.5
x_8	Sunny	Mild	High	False	0.6
x_9	Sunny	Cool	Normal	False	0.6
x_{10}	Rain	Mild	High	False	0.9
x_{11}	Sunny	Mild	Normal	True	0.5
x_{12}	Overcast	Mild	High	True	0.7
x_{13}	Overcast	Hot	Normal	False	0.6
x_{14}	Rain	Mild	High	True	0.8
x_{15}	Sunny	Hot	High	True	0.6

and weight of r_i is defined by

$$w^* (r_i, R, A) = \frac{\text{sig}^* (r_i, R, A)}{\sum_{j=1}^n \text{sig}^* (r_i, R, A)}. \tag{22}$$

Theorem 14. *Let $S = (U, R, A, V, W)$ be a fuzzy-target decision-making table and $R = \{r_1, r_2, \ldots, r_p\}$. Then for any integers i ($1 \le i \le p$), one has $w^*(r_i, R, A) > 0$.*

Proof. Since $R \prec R - \{r_i\}$ holds, by Theorem 11, we have

$$E_{\text{rough}}^{R-\{r_i\}} (A) > E_{\text{rough}}^R (A). \tag{23}$$

Then

$$\text{sig}^* (r_i, R, A) > 0. \tag{24}$$

Hence, we obtain

$$w^* (r_i, R, A) = \frac{\text{sig}^* (r_i, R, A)}{\sum_{j=1}^n \text{sig}^* (r_i, R, A)} > 0. \tag{25}$$

□

Theorem 14 shows that the weight of attribute defined in Definition 13 cannot be equivalent to zero. This is what we expect.

4.2. Numerical Example.

Example 15. Given a fuzzy-target decision-making table for people's trip in an area influenced by weather conditions, see Table 2. Using two different methods, we analyze the weight of every weather condition which affects people's decision for a trip.

Let $U = \{x_1, x_2, \ldots, x_{15}\}$ and $R = $ Outlook (r_1), Temperature (r_2), Humidity (r_3), Windy (r_4). Then we have

$$
\begin{aligned}
\frac{U}{R} &= \{\{x_1\}, \{x_2, x_{15}\}, \{x_3\}, \{x_4, x_{10}\}, \\
&\quad \{x_5\}, \{x_6\}, \{x_7\}, \\
&\quad \{x_8\}, \{x_9\}, \{x_{11}\}, \{x_{12}\}, \{x_{13}\}, \{x_{14}\}\}, \\
\frac{U}{(R - \{r_1\})} &= \{\{x_1, x_3\}, \{x_2, x_{15}\}, \{x_4, x_8, x_{10}\}, \{x_5, x_9\}, \\
&\quad \{x_6, x_7\}, \{x_{11}\}, \{x_{12}, x_{14}\}, \{x_{13}\}\}, \\
\frac{U}{(R - \{r_2\})} &= \{\{x_1, x_8\}, \{x_2, x_{15}\}, \{x_3\}, \{x_4, x_{10}\}, \{x_5\}, \\
&\quad \{x_6\}, \{x_7\}, \{x_9\}, \{x_{11}\}, \{x_{12}\}, \{x_{13}\} \{x_{14}\}\}, \\
\frac{U}{(R - \{r_3\})} &= \{\{x_1\}, \{x_2, x_{15}\}, \{x_3, x_{13}\}, \{x_4, x_{10}\}, \{x_5\}, \\
&\quad \{x_6\}, \{x_7\}, \{x_8\}, \{x_9\}, \{x_{11}\}, \{x_{12}\}, \{x_{14}\}\}, \\
\frac{U}{(R - \{r_4\})} &= \{\{x_1, x_2, x_{15}\}, \{x_3\}, \{x_4, x_{10}, x_{14}\}, \{x_5, x_6\}, \\
&\quad \{x_7\}, \{x_8\}, \{x_9\}, \{x_{11}\}, \{x_{12}\}, \{x_{13}\}\}.
\end{aligned}
\tag{26}
$$

Let $\alpha = 0.7$, $\beta = 0.6$; we obtain

$$
\begin{aligned}
\rho_A^R (0.7, 0.6) &= 0.5, \\
\rho_A^{R-\{r_1\}} (0.7, 0.6) &= 0.857, \\
\rho_A^{R-\{r_2\}} (0.7, 0.6) &= 0.538, \\
\rho_A^{R-\{r_3\}} (0.7, 0.6) &= 0.5, \\
\rho_A^{R-\{r_4\}} (0.7, 0.6) &= 0.538.
\end{aligned}
\tag{27}
$$

Then, by computation using rough degree, the significance $\mathrm{sig}(r_i, R, A)$ of attribute r_i $(i = 1, 2, 3, 4)$ is, respectively,

$$\mathrm{sig}\,(r_1, R, A) = \rho_A^{R-\{r_1\}}\,(0.7, 0.6) - \rho_A^R\,(0.7, 0.6) = 0.357, \tag{28}$$

$$\mathrm{sig}\,(r_2, R, A) = \rho_A^{R-\{r_2\}}\,(0.7, 0.6) - \rho_A^R\,(0.7, 0.6) = 0.038, \tag{29}$$

$$\mathrm{sig}\,(r_3, R, A) = \rho_A^{R-\{r_3\}}\,(0.7, 0.6) - \rho_A^R\,(0.7, 0.6) = 0, \tag{30}$$

$$\mathrm{sig}\,(r_4, R, A) = \rho_A^{R-\{r_4\}}\,(0.7, 0.6) - \rho_A^R\,(0.7, 0.6) = 0.038, \tag{31}$$

and the weight of r_i $(i = 1, 2, 3, 4)$ is, respectively,

$$w\,(r_1, R, A) = 0.824, \tag{32}$$

$$w\,(r_2, R, A) = 0.088, \tag{33}$$

$$w\,(r_3, R, A) = 0, \tag{34}$$

$$w\,(r_4, R, A) = 0.088. \tag{35}$$

On the other hand, we compute the modified significance $\mathrm{sig}^*(r_i, R, A)$ and weight $w^*(r_i, R, A)$ given in Definition 13. According to Definition 7, we compute

$$\begin{aligned} E\,(R) &= 0.267, \\ E\,(R - \{r_1\}) &= 0.984, \\ E\,(R - \{r_2\}) &= 0.400, \\ E\,(R - \{r_3\}) &= 0.400, \\ E\,(R - \{r_4\}) &= 0.767, \end{aligned} \tag{36}$$

and according to Definition 9, we have

$$\begin{aligned} E_{\mathrm{rough}}^R &= 0.133, \\ E_{\mathrm{rough}}^{R-\{r_1\}}\,(A) &= 0.843, \\ E_{\mathrm{rough}}^{R-\{r_2\}}\,(A) &= 0.215, \\ E_{\mathrm{rough}}^{R-\{r_3\}}\,(A) &= 0.200, \\ E_{\mathrm{rough}}^{R-\{r_4\}}\,(A) &= 0.413. \end{aligned} \tag{37}$$

Hence, according to Definition 13, we obtain

$$\begin{aligned} \mathrm{sig}^*\,(r_1, R, A) &= E_{\mathrm{rough}}^{R-\{r_1\}}\,(A) - E_{\mathrm{rough}}^R\,(A) = 0.610, \\ \mathrm{sig}^*\,(r_2, R, A) &= E_{\mathrm{rough}}^{R-\{r_2\}}\,(A) - E_{\mathrm{rough}}^R\,(A) = 0.082, \\ \mathrm{sig}^*\,(r_3, R, A) &= E_{\mathrm{rough}}^{R-\{r_3\}}\,(A) - E_{\mathrm{rough}}^R\,(A) = 0.067, \\ \mathrm{sig}^*\,(r_4, R, A) &= E_{\mathrm{rough}}^{R-\{r_4\}}\,(A) - E_{\mathrm{rough}}^R\,(A) = 0.280, \end{aligned} \tag{38}$$

and the weight $w^*(r_i, R, A)$ of attribute r_i $(i = 1, 2, 3, 4)$ is, respectively,

$$w^*\,(r_1, R, A) = 0.587, \tag{39}$$

$$w^*\,(r_2, R, A) = 0.080, \tag{40}$$

$$w^*\,(r_3, R, A) = 0.064, \tag{41}$$

$$w^*\,(r_4, R, A) = 0.269. \tag{42}$$

By contrasting two kinds of weights above, we find that the latter method involving the rough entropy of a fuzzy set given in Definition 13 keeps basically identical trend with the former one only using the rough degree. That means the latter method is effective. However, it is unpractical that the weight of attribute r_3 is zero in the former case. The latter method avoids this case. This implies that the latter method is better than the former one. Moreover, the weights of attributes r_2 and r_4 are equivalent in the former result. It does not reflect the influence of knowledge's roughness. The latter method makes up for this deficiency. According to these results, we can conclude that our method involving new rough entropy of a fuzzy set is superior to the method only using rough degree.

5. Conclusion

A new rough entropy for measuring uncertainty in the rough fuzzy set theory has been proposed. Using it, we can measure not only the rough degree induced by the boundary region of a fuzzy target in an approximation space, but also the roughness from the classification of a knowledge. In a fuzzy-target decision-making table, we have established a new method based on the new rough entropy to evaluate attribute weights.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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