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Research Article

Solving Fokker-Planck Equations on Cantor Sets Using Local Fractional Decomposition Method

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The local fractional decomposition method is applied to approximate the solutions for Fokker-Planck equations on Cantor sets with local fractional derivative. The obtained results give the present method that is very effective and simple for solving the differential equations on Cantor set.

1. Introduction

The Fokker-Planck equation [1–16] plays an important role in describing the system dynamics. For example, the Langevin approach for microscopic dynamics [1], the dynamics of energy cascade in turbulence [2], the population dynamics [3], the chaotic universe dynamics [4], the fatigue crack growth dynamics [5], the fission dynamics [6], the dynamics of distributions of heavy quarks [7] and financial returns [8], the spin relaxation dynamics [9], the electron dynamics of plasmas and semiconductors [10], the critical dynamics [11], and the fiber dynamics [12] had been investigated by using the Fokker-Planck equation (see [13, 14] and the cited references therein) and its 3 solution was presented by the different methods. There are some methods for solving the differential equations, such as Adomian decomposition, Homotopy perturbation, Variational iteration, Metropolis Monte Carlo, multiscale finite element and finite difference methods (see, e.g., [15–20]), and others [21–25].

In recent years, fractional calculus was applied to model many fractal dynamical systems [26–30]. The fractional Fokker-Planck equation [26–38], as one of dynamical equations, has interested many researchers. Its solution was also investigated in [39–41]. However, the above fractional Fokker-Planck equation did not describe the nondifferentiable behaviors of dynamical systems because of the limit of the fractional operators. In order to overcome the above problems, the local fractional calculus was developed and applied to the fractal phenomenon in science and engineering [42–53]. Local fractional Fokker-Planck equation [45], which was an analog of a diffusion equation with local fractional derivative, was suggested as follows:

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x,t) = \frac{\Gamma(1+\alpha)}{4} \chi_C(t) \frac{\partial^2}{\partial x^2} u(x,t), \tag{1}$$

where the local fractional operator was the Kolwankar-Gangal local fractional differential operator. In [46], the Fokker-Planck equation on a Cantor set with local fractional derivative was presented as follows:

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x,t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x,t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x,t), \tag{2}$$

where the local fractional partial differential operator of order α $(0 < \alpha \le 1)$ was defined as [42–44]

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x_0,t) = \frac{\Delta^\alpha \left(u(x_0,t) - u(x_0,t_0)\right)}{(t-t_0)^\alpha} \tag{3}$$

with

$$\Delta^{\alpha}\left(u\left(x_{0},t\right)-u\left(x_{0},t_{0}\right)\right)\cong\Gamma\left(1+\alpha\right)\left[u\left(x_{0},t\right)-u\left(x_{0},t_{0}\right)\right]. \tag{4}$$

Analytical and approximate solutions for local fractional differential equations were presented by different researchers (see [43, 46–53] and the cited references therein). Applications of local fractional decomposition method were presented (see [51–53] and the cited references therein). Our main purpose of the paper is to apply the local fractional decomposition method to solve the Fokker-Planck equations on a Cantor set.

In this paper, Section 2 gives the recent results for local fractional integral operator. The local fractional decomposition method is analyzed in Section 3. The approximate solutions are presented in Section 4. Finally, conclusions are given in Section 5.

2. The Local Fractional Integral Operator

In this section, we introduce the local fractional integral operator and its recent results.

Definition 1 (see [42, 46–53]). Let the function $f(x) \in C_{\alpha}(a,b)$, if it is valid for

$$\left|f\left(x\right)-f\left(x_{0}\right)\right|<\varepsilon^{\alpha}, \tag{5}$$

where $|x-x_0|<\delta$, for $\varepsilon>0$ and $\varepsilon \in R$.

Definition 2 (see [42, 43, 46–53]). Let $f(x) \in C_{\alpha}[a,b]$. The local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is defined as

$$\begin{aligned}{}_{a}I_{b}^{(\alpha)}f\left(x\right)&=\frac{1}{\Gamma\left(1+\alpha\right)}\int_{a}^{b}f\left(t\right)\left(dt\right)^{\alpha}\\&=\frac{1}{\Gamma\left(1+\alpha\right)}\lim_{\Delta t\to 0}\sum_{j=0}^{j=N-1}f\left(t_{j}\right)\left(\Delta t_{j}\right)^{\alpha},\end{aligned} \tag{6}$$

where the partitions of the interval $[a,b]$ are denoted by (t_j,t_{j+1}), $j=0,\ldots,N-1$, $t_0=a$ and $t_N=b$ with $\Delta t_j=t_{j+1}-t_j$ and $\Delta t=\max\{\Delta t_0,\Delta t_1,\Delta t_j,\ldots\}$.

Some properties of local fractional integrals operator used in the paper are listed as follows [42, 51]:

$$\begin{aligned}{}_{a}I_{x}^{(\alpha)}f\left(x\right)g^{(\alpha)}\left(x\right)&=\left[f\left(x\right)g\left(x\right)\right]\Big|_{a}^{x}-{}_{a}I_{x}^{(\alpha)}f^{(\alpha)}\left(x\right)g\left(x\right),\\{}_{a}I_{b}^{(\alpha)}\,{}_{c}I_{d}^{(\alpha)}\psi\left(x,y\right)&={}_{c}I_{d}^{(\alpha)}\,{}_{a}I_{b}^{(\alpha)}\psi\left(x,y\right),\\{}_{a}I_{x}^{(\alpha)}\,{}_{a}I_{\tau}^{(\alpha)}f\left(t\right)&={}_{a}I_{x}^{(\alpha)}\left[\frac{\left(x-t\right)^{\alpha}}{\Gamma\left(1+\alpha\right)}f\left(t\right)\right].\end{aligned} \tag{7}$$

The formulas of local fractional integrals operator used in the paper are listed as follows [42]:

$$\begin{aligned}{}_{0}I_{x}^{(\alpha)}E_{\alpha}\left(x^{\alpha}\right)&=E_{\alpha}\left(x^{\alpha}\right)-1,\\{}_{0}I_{x}^{(\alpha)}\frac{x^{n\alpha}}{\Gamma\left(1+n\alpha\right)}&=\frac{x^{(n+1)\alpha}}{\Gamma\left(1+\left(n+1\right)\alpha\right)}.\end{aligned} \tag{8}$$

Definition 3 (see [42, 43, 46–53]). Let $f(x) \in C_{\alpha}[a,b]$. The local fractional derivative of $f(x)$ of order α in the interval $[a,b]$ is defined as

$$\frac{d^{\alpha}f\left(x_{0}\right)}{dx^{\alpha}}=\frac{\Delta^{\alpha}\left(f\left(x\right)-f\left(x_{0}\right)\right)}{\left(x-x_{0}\right)^{\alpha}}, \tag{9}$$

where

$$\Delta^{\alpha}\left(f\left(x\right)-f\left(x_{0}\right)\right)\cong\Gamma\left(1+\alpha\right)\left[f\left(x\right)-f\left(x_{0}\right)\right]. \tag{10}$$

The formulas of local fractional differential operator used in the paper are listed as follows [42]:

$$\begin{aligned}\frac{d^{\alpha}}{dx^{\alpha}}E_{\alpha}\left(x^{\alpha}\right)&=E_{\alpha}\left(x^{\alpha}\right),\\\frac{d^{\alpha}}{dx^{\alpha}}E_{\alpha}\left(-x^{\alpha}\right)&=-E_{\alpha}\left(x^{\alpha}\right),\\\frac{d^{\alpha}}{dx^{\alpha}}\frac{x^{3\alpha}}{\Gamma\left(1+3\alpha\right)}&=\frac{x^{2\alpha}}{\Gamma\left(1+2\alpha\right)}.\end{aligned} \tag{11}$$

3. Analysis of the Method

In this section, the local fractional decomposition method for a class of differential equations defined on Cantor is given.

We now write (2) in the following form:

$$L_{t}^{(\alpha)}u\left(x,t\right)=-L_{x}^{(\alpha)}u\left(x,t\right)+L_{xx}^{(2\alpha)}u\left(x,t\right), \tag{12}$$

where $L_{xx}^{(2\alpha)}=\partial^{2\alpha}/\partial x^{2\alpha}$ is a 2αth local fractional differential operator with respect to x, $L_{t}^{(\alpha)}=\partial^{\alpha}/\partial t^{\alpha}$ is a αth local fractional differential operator with respect to t, and $L_{x}^{(\alpha)}=\partial^{\alpha}/\partial x^{\alpha}$ *is* αth local fractional differential operator with respect to x.

The initial condition reads as follows:

$$u\left(x,0\right)=f\left(x\right)\quad\left(0\le x\le l\right). \tag{13}$$

We now define the αth-fold local fractional integral operator in the form

$$L_{t}^{(-\alpha)}m\left(t\right)={}_{0}I_{t}^{(\alpha)}m\left(s\right). \tag{14}$$

In view of (13), we structure

$$L_{t}^{(-\alpha)}L_{t}^{(\alpha)}u\left(x,t\right)=L_{t}^{(-\alpha)}\left\{-L_{x}^{(\alpha)}u\left(x,t\right)+L_{xx}^{(2\alpha)}u\left(x,t\right)\right\}. \tag{15}$$

Hence, from (15), we have

$$u\left(x,t\right)=r\left(x\right)-L_{t}^{(-\alpha)}L_{x}^{(\alpha)}u\left(x,t\right)+L_{t}^{(-\alpha)}L_{xx}^{(2\alpha)}u\left(x,t\right), \tag{16}$$

where the nondifferentiable term $r(x)$ is obtained from the initial condition.

Making use of (16), for $n\ge 0$, we give the recurrence relationship in the following form:

$$u_{n+1}\left(x,t\right)=-L_{t}^{(-\alpha)}L_{x}^{(\alpha)}u_{n}\left(x,t\right)+L_{t}^{(-\alpha)}L_{xx}^{(2\alpha)}u_{n}\left(x,t\right), \tag{17}$$

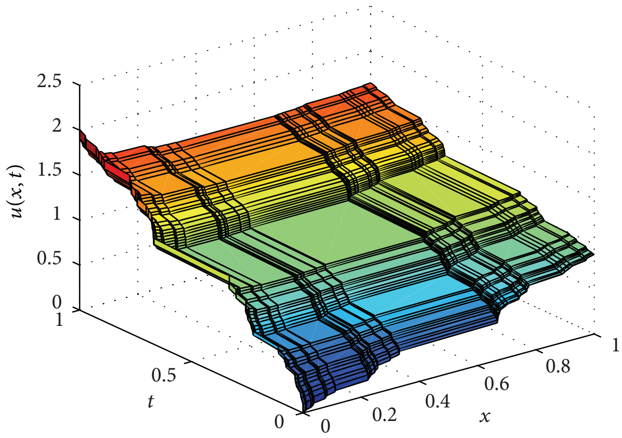

FIGURE 1: Graph of $u(x,t)$ with the parameter $\alpha = \ln 2/\ln 3$.

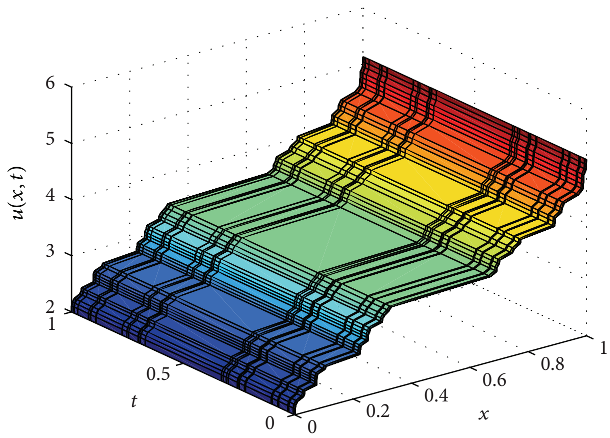

FIGURE 2: Graph of $u(x,t)$ with the parameter $\alpha = \ln 2/\ln 3$.

subject to the initial value

$$u_0(x,t) = r(x). \tag{18}$$

Finally, the approximation form of the solution reads as

$$u(x,t) = \lim_{n\to\infty} \phi_n(x,t) = \lim_{n\to\infty} \sum_{i=0}^{\infty} u_i(x,t). \tag{19}$$

4. The Approximate Solutions

In this section, we investigate the approximate solutions for Fokker-Planck equations on Cantor sets with local fractional derivative by using the local fractional decomposition method.

Example 1. Let us consider the following Fokker-Planck equation on a Cantor set with local fractional derivative in the form

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x,t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x,t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x,t), \tag{20}$$

subject to the initial value

$$u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{21}$$

In view of (17), we have the recurrence formulas in the form

$$u_{n+1}(x,t) = -L_t^{(-\alpha)} L_x^{(\alpha)} u_n(x,t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u_n(x,t), \tag{22}$$

$$u_0(x,t) = u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{23}$$

From (23), we obtain the following approximate formulas:

$$\begin{aligned} u_1(x,t) &= -\frac{x^\alpha}{\Gamma(1+\alpha)} \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^\alpha}{\Gamma(1+\alpha)}, \\ u_2(x,t) &= \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}, \\ u_3(x,t) &= 0, \\ &\vdots \\ u_n(x,t) &= 0. \end{aligned} \tag{24}$$

Hence, the nondifferentiable solution for (20) with the initial value (21) is

$$\begin{aligned} u(x,t) = &\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \\ &- \frac{x^\alpha}{\Gamma(1+\alpha)} \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^\alpha}{\Gamma(1+\alpha)}, \end{aligned} \tag{25}$$

and its graph is shown in Figure 1.

Example 2. We consider the Fokker-Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha}{\partial t^\alpha} u(x,t) = -\frac{\partial^\alpha}{\partial x^\alpha} u(x,t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}} u(x,t), \tag{26}$$

together with initial condition

$$u(x,0) = E_\alpha(x^\alpha). \tag{27}$$

From (17), we get the recurrence formulas in the form

$$u_{n+1}(x,t) = -L_t^{(-\alpha)} L_x^{(\alpha)} u_n(x,t) + L_t^{(-\alpha)} L_{xx}^{(2\alpha)} u_n(x,t), \tag{28}$$

$$u_0(x,t) = u(x,0) = E_\alpha(x^\alpha). \tag{29}$$

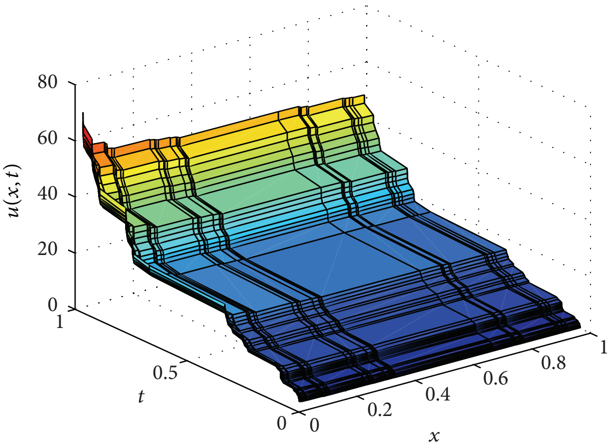

FIGURE 3: Graph of $u(x,t)$ with the parameter $\alpha = \ln 2/\ln 3$.

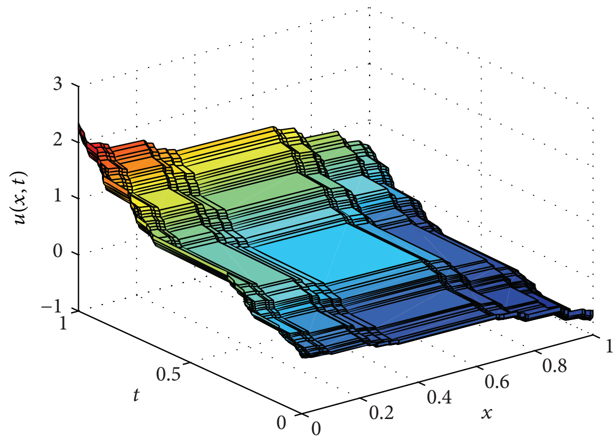

FIGURE 4: Graph of $u(x,t)$ with the parameter $\alpha = \ln 2/\ln 3$.

Making use of (29), we reach the following formulas:

$$u_1(x,t) = 0,\quad u_2(x,t) = 0,\quad \vdots \quad u_n(x,t) = 0. \tag{30}$$

Therefore, the nondifferentiable solution of (26) with the initial value (27) reads as follows:

$$u(x,t) = E_\alpha(x^\alpha) \tag{31}$$

with plot shown in Figure 2.

Example 3. We present the Fokker-Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha}{\partial t^\alpha}u(x,t) = -\frac{\partial^\alpha}{\partial x^\alpha}u(x,t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}}u(x,t) \tag{32}$$

and suggest the initial condition

$$u(x,0) = E_\alpha(-x^\alpha). \tag{33}$$

In view of (17), the recurrence formulas can be written as

$$u_{n+1}(x,t) = -L_t^{(-\alpha)}L_x^{(\alpha)}u_n(x,t) + L_t^{(-\alpha)}L_{xx}^{(2\alpha)}u_n(x,t), \tag{34}$$

$$u_0(x,0) = u(x,0) = E_\alpha(-x^\alpha). \tag{35}$$

From (35), we get the following approximate equalities:

$$\begin{aligned} u_1(x,0) &= \frac{2t^\alpha}{\Gamma(1+\alpha)}E_\alpha(-x^\alpha), \\ u_2(x,0) &= \frac{4t^{2\alpha}}{\Gamma(1+2\alpha)}E_\alpha(-x^\alpha), \\ u_3(x,0) &= \frac{8t^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(-x^\alpha) \\ &\vdots \\ u_n(x,t) &= \frac{2^n t^{n\alpha}}{\Gamma(1+n\alpha)}E_\alpha(-x^\alpha). \end{aligned} \tag{36}$$

Therefore, the nondifferentiable solution of (32) with the initial value (33) reads as follows:

$$u(x,t) = E_\alpha(2t^\alpha)E_\alpha(-x^\alpha), \tag{37}$$

together with plot shown in Figure 3.

Example 4. We suggest the Fokker-Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha}{\partial t^\alpha}u(x,t) = -\frac{\partial^\alpha}{\partial x^\alpha}u(x,t) + \frac{\partial^{2\alpha}}{\partial x^{2\alpha}}u(x,t), \tag{38}$$

and the initial condition is

$$u(x,0) = -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}. \tag{39}$$

From (17), the recurrence formulas read as follows:

$$u_{n+1}(x,t) = -L_t^{(-\alpha)}L_x^{(\alpha)}u_n(x,t) + L_t^{(-\alpha)}L_{xx}^{(2\alpha)}u_n(x,t), \tag{40}$$

$$u_0(x,t) = u(x,0) = -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}. \tag{41}$$

Hereby, from (41), we have the following formulas:

$$u_1(x,t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{t^{\alpha}}{\Gamma(1+\alpha)}\frac{x^{\alpha}}{\Gamma(1+\alpha)},$$
$$u_2(x,t) = -\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\frac{x^{\alpha}}{\Gamma(1+\alpha)} + \frac{2t^{2\alpha}}{\Gamma(1+2\alpha)},$$
$$u_3(x,t) = \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}, \tag{42}$$
$$u_3(x,t) = 0,$$
$$\vdots$$
$$u_n(x,t) = 0.$$

So the nondifferentiable solution of (32) with the initial value (33) is

$$u(x,t) = \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{t^{\alpha}}{\Gamma(1+\alpha)}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{2t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{x^{3\alpha}}{\Gamma(1+3\alpha)} - \frac{t^{\alpha}}{\Gamma(1+\alpha)}\frac{x^{\alpha}}{\Gamma(1+\alpha)} - \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\frac{x^{\alpha}}{\Gamma(1+\alpha)}, \tag{43}$$

together with plot illustrated in Figure 4.

5. Conclusions

In this work, we had used the local fractional decomposition method to solve the Fokker-Planck equations on Cantor sets which were described by the local fractional differential operator. The nondifferentiable solutions were obtained. The obtained results show that the present method is a very effective and powerful mathematical tool for finding the nondifferentiable solutions for the local fractional differential equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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