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Research Article

On-Line Booking Policies and Competitive Analysis of Medical Examination in Hospital

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From the on-line point, we consider the hospital's medical examination appointment problem with hierarchical machines. This approach eliminates the need for both demand forecasts and a risk-neutrality assumption. Due to different unit revenue, uncertain demand, and arrival of patients, we design on-line booking policies for two kinds of different situations from the perspective of on-line policy and competitive analysis. After that, we prove the optimal competitive ratios. Through numerical examples, we compare advantages and disadvantages between on-line policies and traditional policies, finding that there is different superiority for these two policies under different arrival sequences.

1. Introduction

In this paper, we investigate an on-line hierarchical scheduling problem on medical equipment, such as CT scanning. In hospital, patients arrive one by one randomly and each may be scheduled only on a certain machine. When a patient is assigned to a machine, it is not permitted to be man-modified. Namely, each patient has a set of available devices and a corresponding hierarchy. Each device has a patients set and a corresponding hierarchy. Patients cannot be scheduled on a device with lower hierarchy and the devices can only examine patients with no higher hierarchy. This constraint reflects different service capabilities of the devices, while it is very common in the practice (such as the customer level in services and the server level in communications). These issues can be summarized as hierarchical scheduling problem.

Suppose the service capacity of each device is N in a given cycle, the service times of each patient are the same but the revenue is different. The goal of this study is to examine as many patients as possible to maximize hospitals' social revenue with successful appointments. Because it is unknown in advance how many patients will be present and which hierarchies they belong to, this social revenue-maximization problem is an on-line fashion.

This problem is motivated by practical cases in the Radiology Department of the West China Hospital. In this department, the CT scanning divides into two types depending on the complexity of the examinations, enhanced CT scanning and regular CT scanning. There are two CTs; CT1 is used for doing the regular scanning while CT2 is usually for enhanced scanning. The examination time of regular scanning is 1-2 minutes (excluding the preparation time and adjustment time of examination, such as the time for position adjustment and breath adjustment). The enhanced scanning needs to inject contrast agent, test allergy, and enhancer. It is worth noting that enhancer is unique to timeliness, so the patients should complete the examination as soon as enhancer is injected; otherwise the efficacy will expire.

In the actual appointment, the regular scanning patients can book CT1 or CT2, while the enhanced scanning patients can only book CT2. Now the appointment mode of the West China Hospital is patients making appointments in the integrated service station during a period of half day. Patients arrive at the service station one by one (inpatients usually arrive in batch) to make an appointment; then physicians judge people as regular scanning patients or enhanced scanning patients by synthesizing all kinds of factors. After that, nurses arrange the regular scanning patients to CT1 and the enhanced scanning patients to CT2. When the capacity of CT1 is lacking, nurses will arrange patients on the first-come-first-served basis and transfer surplus patients to CT2. The obvious disadvantage of this mode is that it does not consider the different unit revenue from two types of patients. In general, the unit revenue from regular scanning patients is lower than the revenue from enhanced scanning patients. Certainly we cannot measure patients' value from the perspective of revenue, and that is why the hospitals choose the first-come-first-served as the basic rule. But from the social return, serving an enhanced scanning patient is also more important than serving a regular scanning patient. For CT2, if it accepts a regular scanning patient, it may reject a potential enhanced scanning patient; if it refuses the present regular scanning patient ("refuse" means patients have to be delayed to another day), it may be idle and loses this revenue. Therefore the service station should determine the number of regular scanning patients in the CT2 to generate better returns. Our study considers this problem from the view of on-line booking and competitive analysis.

Currently, there is a lot of literature on hierarchical on-line scheduling, such as literature [1–6]. However, few scholars considered such social revenue maximization problem in hospital. Compared with the traditional literature, there are two differences. First, the capacity of the device is not limited in the traditional literature; all jobs must be machined. Second, the termination condition of traditional scheduling algorithm is that all jobs are completed, and the goal is usually to complete the processing of all jobs as soon as possible. The common optimization objectives are minimizing the total completion time ($\sum C_j$) [7–9] or the maximum completion time ($C_{\max}$) [10, 11], while our work is to maximize the social return.

In this paper, we assume that the service capacity of each device in a certain period is N and the service time of each patient is the same, but the unit revenue is different. Thus, the goal of this paper is to maximize the total social revenue from all accepted patients in certain period. Further, we assume that the patients and the devices are hierarchical. A patient can make a reservation of a device whose grade is not lower than the level of his own.

Next, considering the different unit revenue from different types of patients, the paper studies two-hierarchy on-line scheduling problem in two situations.

2. On-Line Booking and Competitive Analysis

Suppose there are two devices: M_1 and M_2. The grade of M_1 is 1 and that of M_2 is 2. The same capacity of each device is N. Two types of patients are involved: type 1 and type 2. The level of type 1 is 1 and that of type 2 is 2. Namely, patients of type 1 can book M_1 and M_2, while patients of type 2 can only book M_2. Assume the service time of each patient is the same and the unit revenue is different. The unit revenue of examining each type 1 patient with M_1 is r_1 while it is r_1' with M_2, and the unit revenue of examining each type 2 patient with M_2 is r_2. Obviously, $r_1 \geq r_1'$; because the cost of examining the patients of type 1 with M_2 is higher than examining them with M_1, there will be lower social income. So it is impossible for the hospital manager to arrange patients of type 1 to M_2 when M_1 has reservation capability. As for r_1' and r_2, which is higher is uncertain, so we discuss the on-line problem from two perspectives.

2.1. $r_1' \leq r_2$. In this situation, the unit revenue of using M_2 to examine type 2 patients is higher than to examine type 1 patients. The manager should decide whether to arrange type 1 patients to M_2 or not. On the one hand, the service capacity of M_1 is insufficient, and the number of type 1 patients is much larger than that of type 2 patients; it is better to transfer some patients of type 1 to M_2. On the other hand, if there are plenty of type 2 patients, only few type 1 patients can be arranged to M_2 while enough capacity should be kept for patients of type 2.

This paper designs an optimal on-line policy ST1; moreover, the competitive ratio is $c = r_2(r_1+r_1')/(2r_1'r_2+r_1r_2-r_1'^2)$.

Suppose n_1 is the number of type 1 patients who are arranged to M_1, n_2 is the number of type 1 and type 2 patients who use M_2, and n_2^1 is the number of type 1 patients with M_2. The on-line policy is as follow.

ST1 Policy. (1) Suppose $\tilde{n}_1$ is the total number of being accepted type 1 patients by M_1 before the next arriving type 1 patient, $\tilde{n}_2$ is the total number of being accepted type 1 and type 2 patients by M_2 before the next arriving patient, $\tilde{n}_2^1$ is the total number of being accepted type 1 patients by M_2 before the next arriving type 1 patient. At the beginning of the appointment time, $\tilde{n}_1 = 0$, $\tilde{n}_2 = 0$, and $\tilde{n}_2^1 = 0$.

(2) For the next arriving patient, consider the following.

(A_1) The patient type is type 1. If $\tilde{n}_1 < N$, arrange the patient to M_1. If $\tilde{n}_1 = N$, $\tilde{n}_2 < N$, and $\tilde{n}_2^1 < y_1 N$ (where $y_1 = r_1'(r_1+r_2)/(2r_1'r_2+r_1r_2-r_1'^2)$), arrange the patient to M_2. Otherwise, refuse.

(A_2) The patient type is type 2. If $\tilde{n}_2 < N$, arrange the patient to M_2. Otherwise, refuse.

Theorem 1. *The competitive ratio of ST1 policy is $c_1 = r_2(r_1 + r_1')/(2r_1'r_2 + r_1r_2 - r_1'^2)$.*

Proof. For any input sequence I, $V'(I)$ is the total social revenue of ST1 policy and $V^*(I)$ is the total social revenue of the optimal off-line policy. Consider the following four circumstances.

(1) $n_1 < N$. In this case, for any input sequence I, on-line algorithm ST1 is the same as the optimal off-line algorithm; both of them will arrange all patients of type 1 to M_1 and all patients of type 2 to M_2. So, $V'(I) = V^*(I)$; namely, $V'(I)/V^*(I) = 1$.

(2) $n_1 = N$, $n_2 = N$. At the moment, the total social revenue of ST1 policy is

$$\begin{aligned} V'(I) &= r_1 N + r_1' n_2^1 + r_2\left(N - n_2^1\right) \\ &\geq r_1 N + r_1' y_1 N + r_2\left(N - y_1 N\right) \\ &= r_1 N + r_1' y_1 N + r_2\left(1 - y_1\right) N. \end{aligned} \quad (1)$$

The optimal off-line revenue is $V^*(I) \leq r_1 N + r_2 N$; then

$$\begin{aligned} \frac{V'(I)}{V^*(I)} &\geq \frac{r_1 N + r_1' n_2^1 + r_2\left(N - n_2^1\right)}{r_1 N + r_2 N} \\ &\geq \frac{r_1 N + r_1' y_1 N + r_2\left(1 - y_1\right) N}{r_1 N + r_2 N} \\ &= \frac{r_1 + r_1' y_1 + r_2\left(1 - y_1\right)}{r_1 + r_2} = \frac{r_2\left(r_1 + r_1'\right)}{2r_1' r_2 + r_1 r_2 - r_1'^2}. \end{aligned} \quad (2)$$

(3) $n_1 = N$, $n_2 < N$, and before the policy stops, there is always $n_2^1 < y_1 N$. By this time, all arrived demands can be accepted. So $V'(I) = V^*(I)$; namely, $V'(I)/V^*(I) = 1$.

(4) $n_1 = N$, $n_2 < N$, and at some period before the policy stops, $n_2^1 = y_1 N$. In this case, off-line adversary will design a special input sequence: after $n_2^1 = y_1 N$, only type 1 patients will come. Under this kind of bad sequence, the total social revenue of ST1 policy is

$$V'(I) = r_1 N + r_1' y_1 N + r_2\left(n_2 - y_1 N\right). \quad (3)$$

The total social revenue of the optimal off-line policy is

$$V^*(I) = r_1 N + r_1'\left(N - \left(n_2 - y_1 N\right)\right) + r_2\left(n_2 - y_1 N\right). \quad (4)$$

Then

$$\begin{aligned} \frac{V'(I)}{V^*(I)} &= \frac{r_1 N + r_1' y_1 N + r_2\left(n_2 - y_1 N\right)}{r_1 N + r_1'\left(N - \left(n_2 - y_1 N\right)\right) + r_2\left(n_2 - y_1 N\right)} \\ &\geq \frac{r_1 N + r_1' y_1 N + r_2\left(n_2 - y_1 N\right)}{r_1 N + r_1' N + r_2\left(n_2 - y_1 N\right)} \\ &\geq \frac{r_1 N + r_1' y_1 N}{r_1 N + r_1' N} = \frac{r_1 + r_1' y_1}{r_1 + r_1'} \\ &= \frac{r_2\left(r_1 + r_1'\right)}{2r_1' r_2 + r_1 r_2 - r_1'^2}. \end{aligned} \quad (5)$$

To sum up, for any input sequence I, there is always

$$V(I) \geq \frac{r_2\left(r_1 + r_1'\right)}{2r_1' r_2 + r_1 r_2 - r_1'^2} V^*(I). \quad (6)$$

Then the competitive ratio of ST1 policy is $c_1 = r_2(r_1 + r_1')/(2r_1' r_2 + r_1 r_2 - r_1'^2)$. □

Next, we will prove that ST1 algorithm is the optimal on-line scheduling algorithm.

Theorem 2. *ST1 algorithm is the optimal on-line scheduling algorithm.*

Proof. In order to prove that ST1 algorithm is the optimal on-line scheduling algorithm, we only need to prove that, for any other on-line scheduling algorithm, its competitive ratio is not more than $r_2(r_1 + r_1')/(2r_1' r_2 + r_1 r_2 - r_1'^2)$.

Consider the following two special arrival sequences I_1 and I_2. In I_1, there first come $2N$ type 1 patients, and then come N type 2 patients. In I_2, there first come $2N$ type 1 patients, and then have no type 2 patients to come. At the moment, the total revenue of optimal off-line policy is

$$V^*\left(I_1\right) = r_1 N + r_2 N, \qquad V^*\left(I_2\right) = r_1 N + r_1' N. \quad (7)$$

For any on-line scheduling algorithm A, let xN be the total number of type 1 patients that can be accepted by M_2. Obviously, $0 \leq x \leq 1$. Accordingly,

$$\begin{aligned} V\left(I_1\right) &= r_1 N + r_1' xN + r_2\left(N - xN\right), \\ V\left(I_2\right) &= r_1 N + r_1' xN. \end{aligned} \quad (8)$$

By calculating, we find the following.

When

$$\begin{aligned} x \geq y_1 &= \frac{\left(r_1 + r_2\right) r_1'}{2r_1' r_2 - r_1'^2 + r_1 r_2}, \\ V\left(I_1\right) &\leq \frac{r_2\left(r_1 + r_1'\right)}{2r_1' r_2 + r_1 r_2 - r_1'^2} V^*\left(I_1\right). \end{aligned} \quad (9)$$

When

$$\begin{aligned} x \leq y_1 &= \frac{\left(r_1 + r_2\right) r_1'}{2r_1' r_2 - r_1'^2 + r_1 r_2}, \\ V\left(I_2\right) &\leq \frac{r_2\left(r_1 + r_1'\right)}{2r_1' r_2 + r_1 r_2 - r_1'^2} V^*\left(I_2\right). \end{aligned} \quad (10)$$

Then, for any on-line scheduling algorithm A, there exists some arrival sequence, whose total on-line revenue is not more than $r_2(r_1 + r_1')/(2r_1' r_2 + r_1 r_2 - r_1'^2)$ times for the total optimal off-line revenue. Namely, for any on-line scheduling algorithm A, its competitive ratio is not more than $r_2(r_1 + r_1')/(2r_1' r_2 + r_1 r_2 - r_1'^2)$.

Combined with Theorem 1, we can see that ST1 algorithm is the optimal on-line scheduling algorithm; its competitive ratio is $r_2(r_1 + r_1')/(2r_1' r_2 + r_1 r_2 - r_1'^2)$. □

2.2. $r_1' > r_2$. In this condition, the unit revenue of using M_2 to examine type 1 patients is higher than to examine type 2 patients. The manager should decide whether to accept the request from type 2 patients. If the service capacity of M_1 is sufficient and the amount of type 2 patients far exceeds that

Table 1: Comparison of two strategies in Case 1.

		Patient number									
		1	2	3	4	5	6	7	8	9	10
	Patient type	1	1	1	1	1	1	1	1	1	2
Device	ST1 policy (total revenue: 4850)	1	1	1	1	1	1	1	1	1	2
	Traditional policy (total revenue: 5450)	1	1	1	1	1	1	1	1	1	2
		Patient number									
		11	12	13	14	15	16	17	18	19	20
	Patient type	1	1	1	1	1	1	2	2	1	1
Device	ST1 policy (total revenue: 4850)	1	1	1	1	1	1	2	2	1	1
	Traditional policy (total revenue: 5450)	1	1	1	1	1	1	2	2	1	1
		Patient number									
		21	22	23	24	25	26	27	28	29	30
	Patient type	1	1	1	1	1	1	1	1	1	1
Device	ST1 policy (total revenue: 4850)	1	1	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
	Traditional policy (total revenue: 5450)	1	1	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
		Patient number									
		31	32	33	34	35	36	37	38	39	40
	Patient type	1	1	1	1	1	1	1	1	1	1
Device	ST1 policy (total revenue: 4850)	**2**	**2**	**2**	**2**	0	0	0	0	0	0
	Traditional policy (total revenue: 5450)	**2**	**2**	**2**	**2**	*2*	*2*	*2*	*2*	*2*	*2*

of type 1 patients, it is better to accept a part of requests from type 2 patients. On the other hand, if there will be lots of type 1 patients, the manager cannot respond too much requests from type 2 patients and keep enough capacity of M_2 for subsequent patients of type 1.

Next, we design an optimal on-line policy ST2; moreover, the competitive ratio is $c = r_1'(r_1 + r_2)/(2r_1'r_2 + r_1r_1' - r_2^2)$.

Suppose n_1 is the number of patients of type 1 who are arranged to M_1, n_2 is the number of type 1 and type 2 patients who use M_2, and n_2^2 is the number of type 2 patients with M_2. The on-line policy is as follow.

ST2 Policy. (1) Suppose $\tilde{n}_1$ is the total number of being accepted type 1 patients by M_1 before the next arriving type 1 patient, $\tilde{n}_2$ is the total number of being accepted type 1 and type 2 patients by M_2 before the next arriving patient, $\tilde{n}_2^2$ is the total number of being accepted type 2 patients by M_2 before the next arriving type 2 patient. At the beginning of the appointment time, $\tilde{n}_1 = 0$, $\tilde{n}_2 = 0$, and $\tilde{n}_2^2 = 0$.

(2) For the next arriving patient, consider the following.

(B_1) The patient type is type 1. If $\tilde{n}_1 < N$, arrange the patient to M_1. If $\tilde{n}_1 = N$ and $\tilde{n}_2 < N$, arrange the patient to M_2. Otherwise, refuse.

(B_2) The patient type is type 2. If $\tilde{n}_2 < N$ and $\tilde{n}_2^2 < y_2N$ (where $y_2 = r_2(r_1 + r_1')/(2r_1'r_2 + r_1r_1' - r_2^2)$), arrange the patient to M_2. Otherwise, refuse.

Theorem 3. *ST2 algorithm is the optimal on-line scheduling algorithm; its competitive ratio is* $c_2 = r_1'(r_1+r_2)/(2r_1'r_2+r_1r_1' - r_2^2)$.

The proof of Theorem 3 is similar to the proof of Theorems 1 and 2, so it is not explained here.

Through the conclusions of three theorems above, we can find the difference between on-line policy and off-line policy when the service capacity of M_1 is insufficient. If the service capacity of M_1 is enough, all patients of type 1 will be arranged to M_1, not to M_2. When the service capacity of M_1 is not enough, the subsequent patients of type 1 could be arranged to M_2; there are two different on-line policies according to different unit revenue. For ST1 policy, considering the unit revenue of using M_2 to examine type 2 patients is higher than to examine type 1 patients, when M_2 has reservation capability, the manager should control the number of type 1 patients on M_2 and be sure the request of type 2 patients can be met as far as possible. For ST2 policy, considering the unit revenue of using M_2 to examine type 1 patients is higher than to examine type 2 patients, the manager should control the number of patients of type 2 on M_2 and make sure patients of type 1 can be serviced.

3. Contrast with Traditional Reservation Policy

For $r_1' < r_2$, by calculating, we have $y_1 = r_1'(r_1 + r_2)/(2r_1'r_2 + r_1r_2 - r_1'^2) < c_1 = r_2(r_1 + r_1')/(2r_1'r_2 + r_1r_2 - r_1'^2)$. Obviously, y_1 is

Table 2: Comparison of two strategies in Case 2.

		Patient number									
		1	2	3	4	5	6	7	8	9	10
	Patient type	1	1	1	2	1	1	1	1	2	1
Device	ST1 policy (total revenue: 6350)	1	1	1	2	1	1	1	1	2	1
	Traditional policy (total revenue: 5450)	1	1	1	2	1	1	1	1	2	1
		Patient number									
		11	12	13	14	15	16	17	18	19	20
	Patient type	1	1	1	1	1	1	1	1	1	1
Device	ST1 policy (total revenue: 6350)	1	1	1	1	1	1	1	1	1	1
	Traditional policy (total revenue: 5450)	1	1	1	1	1	1	1	1	1	1
		Patient number									
		21	22	23	24	25	26	27	28	29	30
	Patient type	1	1	1	1	1	1	1	1	1	1
Device	ST1 policy (total revenue: 6350)	1	1	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
	Traditional policy (total revenue: 5450)	1	1	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
		Patient number									
		31	32	33	34	35	36	37	38	39	40
	Patient type	1	1	1	1	1	1	1	1	1	2
Device	ST1 policy (total revenue: 6350)	**2**	**2**	**2**	***0***	***0***	***0***	***0***	***0***	***0***	2
	Traditional policy (total revenue: 5450)	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	2
		Patient number									
		41	42	43	44	45	46	47	48	49	50
	Patient type	2	2	2	2	2	2	2	2	2	2
Device	ST1 policy (total revenue: 6350)	2	2	2	2	2	2	0	0	0	0
	Traditional policy (total revenue: 5450)	0	0	0	0	0	0	0	0	0	0

non-increasing in r_1, r_2 and is non-decreasing in r_1'. c_1 is not decreasing with r_1 and is not increasing with r_2. However, it is not sure of $c_1 = r_2(r_1+r_1')/(2r_1'r_2+r_1r_2-r_1'^2)$'s monotonicity about r_1', because

$$\begin{aligned}\frac{\partial c_1}{\partial r_1'} &= \frac{r_2\left(2r_1'r_2+r_1r_2-r_1'^2\right)-r_2\left(r_1+r_1'\right)\left(2r_2-2r_1'\right)}{\left(2r_1'r_2+r_1r_2-r_1'^2\right)^2} \\ &= \frac{r_2\left(r_1'^2-r_1r_2+2r_1r_1'\right)}{\left(2r_1'r_2+r_1r_2-r_1'^2\right)^2}.\end{aligned} \tag{11}$$

When $r_1' \in (0, -r_1+\sqrt{r_1^2+r_1r_2})$, $\partial c_1/\partial r_1' < 0$ and $c_1 = r_2(r_1+r_1')/(2r_1'r_2+r_1r_2-r_1'^2)$ is non-increasing in r_1'.

When $r_1' \in [-r_1+\sqrt{r_1^2+r_1r_2}, r_2)$, $\partial c_1/\partial r_1' \geq 0$ and $c_1 = r_2(r_1+r_1')/(2r_1'r_2+r_1r_2-r_1'^2)$ is non-decreasing in r_1'.

In order to compare on-line booking in this paper with the traditional reservation (first-come-first-appointed), we consider the following two extreme cases.

Case 1. From the certain moment when the service capacity of M_1 is insufficient, the subsequent people are all type 1 patients.

Case 2. From the certain moment when the service capacity of M_1 is insufficient, some patients of type 2 arrive after patients of type 1.

Consistent with the description above, we define y_1N as the upper limit of type 1 patients on M_2. Suppose $N = 20$, $r_1 = 150$, $r_1' = 100$, and $r_2 = 250$. In order to simplify the comparison process, suppose $\tilde{n}_2^1 < y_1N$, where $\tilde{n}_2^1$ is the number of type 1 patients accepted by M_2. The analysis process is similar when $\tilde{n}_2^1 = y_1N$. The revenue of on-line and traditional policy is shown in Tables 1 and 2.

In Table 1, ST1 policy controls the accepted number of type 1 patients on M_2, and traditional reservation policy accepts appointments one by one according to the first-come-first-appointed policy until the service capacity of M_2 runs out. So the revenue from traditional policy is higher than on-line policy.

In Table 2, ST1 policy refuses some patients of type 1 and keeps the capacity of M_2 for subsequent patients of type 2 who bring more yields. Traditional policy accepts the type 1 patients who come first so that the more valuable patients of type 2 are rejected by M_2.

The above comparison shows that two strategies exhibit different advantages according to different demand sequences. On-line policy in this paper is based on a worst case analysis and the actual arrival sequence is not always worst, so it is necessary to improve on-line revenue management policy design on the actual distribution of demand.

4. Conclusion

The medical technology appointment is core problem of hospital management. The nurses in service station face unknown demand sequences and should make a decision immediately when each patient arrives. On-line policy and competitive analysis are exactly suitable for decision problem with unknown demand sequences. This paper studies the revenue management problem of hospital medical examination according to the on-line policy. Based on the literature [12], we analyze the medical examination appointment model under different yield scenarios and design the optimal on-line booking policy. Finally, we compare the on-line policy with traditional reservation policy by numerical examples.

In the future, it is necessary to dynamically adjust the existing reservation policy based on demand information, because the demand sequence generally follows certain rules in practice. According to these laws, designing on-line policy with the latest achievements in competitive analysis will be a more realistic theoretical issue.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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