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Research Article

The (G'/G)-Expansion Method and Its Application for Higher-Order Equations of KdV (III)

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New exact traveling wave solutions of a higher-order KdV equation type are studied by the (G'/G)-expansion method, where $G = G(\xi)$ satisfies a second-order linear differential equation. The traveling wave solutions are expressed by the hyperbolic functions, the trigonometric functions, and the rational functions. The property of this method is that it is quite simple and understandable.

1. Introduction

Nonlinear evolution equations (NLEEs) are widely used as models to describe the complex physical phenomena and a troublesome and tedious but very important problem is to find exact solutions of NLEEs. In recent years, more and more researchers investigated exact traveling wave solutions of NLEEs and lots of effective methods have been proposed, such as the inverse scattering method [1], the Backlund transform method [2, 3], the Darboux transform method [4], the Hirota bilinear transformation method [5], the Exp-function method [6, 7], the tanh-function method [8, 9], the Weierstrass elliptic function method [10], and the Jacobi elliptic function expansion method [11].

Recently, the (G'/G)-expansion method, firstly presented by Wang et al. [12], has been widely used to search for various kinds of exact solutions of NLEEs. For instance, Malik et al. [13] applied the (G'/G)-expansion method in getting traveling wave solutions of some nonlinear partial differential equations. Bekir [14] was concerned with this method to study nonlinear evolution equations for constructing wave solutions. Zayed [15] investigated the higher-dimensional nonlinear evolution equations by using the same method to get solutions. In [16], Naher et al. implemented the method for constructing abundant traveling wave solutions of the Caudrey-Dodd-Gibbon equation. Lately, the further developed methods named the generalized (G'/G)-expansion method [17], the modified (G'/G)-expansion method [18], the extended (G'/G)-expansion method [19], the improved (G'/G)-expansion method [20], the generalized and improved (G'/G)-expansion method [21], and the $(G'/G, 1/G)$-expansion method [22] have been proposed for constructing exact solutions to NLEEs.

The main purpose of this paper is to use (G'/G)-expansion method to find the exact solutions of higher-order equations of KdV type (III):

$$u_t + u_x + \alpha u u_x + \beta u_{xxx} + \alpha^2 \rho_1 u^2 u_x + \alpha\beta \left(\rho_2 u u_{xxx} + \rho_3 u_x u_{xx}\right) = 0, \quad (1)$$

where ρ_i $(i = 1, 2, 3)$ are free parameters and α and β are positive real constants, which characterize the long wavelength and short amplitude of the waves, respectively. This equation, arising as models in theory of water wave which were first proposed by Fokas [23], is a water wave equation of KdV type which is more physically and practically meaningful, and some interesting results related to (1) have been obtained by many authors. For example, in [24], Li et al. obtained some exact explicit parametric representations of solitary wave, kink and antikink wave solutions, and breaking wave solutions of (1) under special parametric conditions. Some peculiar exact traveling wave solutions including solitary wave, cusp wave, and loop solution with singular or nonsingular character of (1) were discussed under some particular parameters in [25]. By using the bifurcation theory of planar dynamical system and elliptic function integral method, the explicit and implicit solutions of periodic wave and solitary cusp wave of (1) were obtained in [26].

In this paper, we seek the exact solutions of (1) with the parameters ρ_i ($i = 1, 2, 3$) being arbitrary constants and obtain new solutions. The rest of this paper is organized as follows. In Section 2, the (G'/G)-expansion method is introduced briefly. Section 3 is devoted to applying this method to the shallow water wave model of generalized KdV equation. The last section is a short summary and discussion.

2. Description of the (G'/G)-Expansion Method

We consider a nonlinear partial differential equation (PDE) in two independent variables x and t, which is given by

$$F\left(u, u_x, u_t, u_{xx}, u_{xt}, \ldots\right) = 0, \quad (2)$$

where $u = u(x, t)$ is an unknown function and F is a polynomial with respect to u and its partial derivatives which involve the highest order derivatives and the nonlinear terms. We give the main steps of the (G'/G)-expansion method in the following.

Step 1. We use a traveling wave variable

$$u(x, t) = U(\xi), \quad \xi = x - ct, \quad (3)$$

where c is the wave speed and ξ is the combination of two independent variables x and t. Then we can rewrite (2) as the following nonlinear ordinary differential equation (ODE):

$$F\left(U, U', U'', \ldots\right) = 0, \quad (4)$$

where the primes denote differentiation with respect to ξ.

If possible, integrating (4) term by term for one or more times yields constant(s) of integration. For simplicity, the integration constants may be zero.

Step 2. Suppose that the solution of ODE (4) can be written as follows:

$$U(\xi) = \sum_{i=0}^{n} a_i \left(\frac{G'}{G}\right)^i, \quad (5)$$

where a_i ($i = 0, 1, 2, \ldots, n$) are constants to be determined later, n is a positive integer, and $G = G(\xi)$ satisfies the following second-order linear ordinary differential equation:

$$G'' + \lambda G' + \mu G = 0, \quad (6)$$

where λ and μ are real constants. The general solutions of (6) can be listed as follows.

When $\Delta = \lambda^2 - 4\mu > 0$, we obtain the hyperbolic function solution of (6):

$$G(\xi) = e^{-(\lambda/2)\xi} \left(A_1 \cosh\left(\frac{\sqrt{\Delta}}{2}\xi\right) + A_2 \sinh\left(\frac{\sqrt{\Delta}}{2}\xi\right)\right). \quad (7)$$

When $\Delta = \lambda^2 - 4\mu < 0$, we obtain the trigonometric function solution of (6):

$$G(\xi) = e^{-(\lambda/2)\xi} \left(A_1 \cos\left(\frac{\sqrt{-\Delta}}{2}\xi\right) + A_2 \sin\left(\frac{\sqrt{-\Delta}}{2}\xi\right)\right). \quad (8)$$

When $\Delta = \lambda^2 - 4\mu = 0$, we obtain the solution of (6):

$$G(\xi) = e^{-(\lambda/2)\xi} \left(A_1 + A_2 \xi\right), \quad (9)$$

where A_1 and A_2 are arbitrary constants.

Step 3. Determine the positive integer n by balancing the highest order derivatives and nonlinear terms in (4).

Step 4. Substituting (5) along with (6) into (4) and then setting all the coefficients of $(G'/G)^k$ ($k = 1, 2, \ldots$) of the resulting system's numerator to zero yield a set of overdetermined nonlinear algebraic equations for c and a_i ($i = 0, 1, \ldots$).

Step 5. Assuming that the constants c and a_i can be obtained by solving the algebraic equations in Step 4, then substituting these constants and the known general solutions of (6) into (5) we can obtain the exact solutions of (1) immediately.

3. Exact Solutions of (1)

In this section, we apply the (G'/G)-expansion method to construct the traveling wave solutions of the higher-order KdV type equation (1).

By using the traveling wave variable (4), (1) is converted into the following ODE for $U(\xi)$:

$$\begin{aligned} &(1 - c) U' + \alpha U U' + \beta U''' + \alpha^2 \rho_1 U^2 U' \\ &\quad + \alpha\beta \left(\rho_2 U U''' + \rho_3 U' U''\right) = 0. \end{aligned} \quad (10)$$

By integrating (10) with respect to the variable ξ and assuming a zero constant of integration, we have

$$\begin{aligned} &(1 - c) U + \frac{\alpha}{2} U^2 + \beta U'' + \frac{\alpha^2 \rho_1}{3} U^3 \\ &\quad + \alpha\beta \left(\rho_2 U U'' + \frac{1}{2}\left(\rho_3 - \rho_2\right) U'^2\right) = 0. \end{aligned} \quad (11)$$

The homogeneous balance between U^3 and $(U')^2$ in (11) implies $n = 2$. Suppose that the solution of ODE (11) is of the following form:

$$U(\xi) = a_2 \left(\frac{G'}{G}\right)^2 + a_1 \left(\frac{G'}{G}\right) + a_0. \quad (12)$$

Substituting (12) along with (6) into (11) and then setting all the coefficients of $(G'/G)^k$ ($k = 0, 1, \ldots$) of the resulting system's numerator to zero, we obtain a set of overdetermined nonlinear algebraic equations about a_0, a_1, a_2, and c as follows:

$$\begin{aligned}
&\frac{\alpha^2\rho_1 a_2^3}{3} + 2\alpha\beta\rho_3 a_2^2 + 4\alpha\beta\rho_2 a_2^2 = 0,\\
&2\alpha\beta\rho_3 a_1 a_2 + 4\alpha\beta\rho_3 a_2^2\lambda + \alpha^2\rho_1 a_1 a_2^2\\
&\quad + 6\alpha\beta\rho_2 a_1 a_2 + 6\alpha\beta\rho_2 a_2^2\lambda = 0,\\
&6\beta a_2 + \frac{3\alpha\beta\rho_2 a_1^2}{2} + \alpha^2\rho_1 a_1^2 a_2 + 6\alpha\beta\rho_2 a_0 a_2\\
&\quad + \frac{\alpha\beta\rho_3 a_1^2}{2} + 4\alpha\beta\rho_3 a_1 a_2\lambda + 9\alpha\beta\rho_2 a_1 a_2\lambda\\
&\quad + 2\alpha\beta\rho_3 a_2^2\lambda^2 + 4\alpha\beta\rho_2 a_2^2\mu + 2\alpha\beta\rho_2 a_2^2\lambda^2\\
&\quad + 4\alpha\beta\rho_3 a_2^2\mu + \alpha^2\rho_1 a_0 a_2^2 + \frac{\alpha a_2^2}{2} = 0,\\
&2\alpha\beta\rho_2 a_2^2\mu\lambda + 2\alpha^2\rho_1 a_0 a_1 a_2 + 2a_1\beta + 2\alpha\beta\rho_2 a_0 a_1\\
&\quad + 2\alpha\beta\rho_2 a_1^2\lambda + \frac{\alpha^2\rho_1 a_1^3}{3} + 3\alpha\beta\rho_2 a_1 a_2\lambda^2\\
&\quad + 6\alpha\beta\rho_2 a_1 a_2\mu + 10a_2\beta\lambda + \alpha a_1 a_2\\
&\quad + 10\alpha\beta\rho_2 a_0 a_2\lambda + \alpha\beta\rho_3 a_1^2\lambda + 4\alpha\beta\rho_3 a_2^2\lambda\mu\\
&\quad + 2\alpha\beta\rho_3 a_1 a_2\lambda^2 + 4\alpha\beta\rho_3 a_1 a_2\mu = 0,\\
&2\alpha\beta\rho_3 a_2^2\mu^2 + \alpha\beta\rho_2 a_1^2\mu + 8\alpha\beta\rho_2 a_0 a_2\mu\\
&\quad + 4\beta a_2\lambda^2 + 8a_2\beta\mu + 3\alpha\beta\rho_2 a_1 a_2\lambda\mu + 3a_1\beta\lambda\\
&\quad + \frac{a_1^2\alpha}{2} - ca_2 + 4\alpha\beta\rho_3 a_1 a_2\lambda\mu + \frac{\alpha\beta\rho_3 a_1^2\lambda^2}{2}\\
&\quad + \alpha\beta\rho_3 a_1^2\mu + 3\alpha\beta\rho_2 a_0 a_1\lambda + \alpha a_0 a_2\\
&\quad + \frac{\alpha\beta\rho_2 a_1^2\lambda^2}{2} + \alpha^2\rho_1 a_0 a_1^2 + 4\alpha\beta\rho_2 a_0 a_2\lambda^2\\
&\quad + a_2 + \alpha^2\rho_1 a_0^2 a_2 = 0,\\
&a_1 + 6\alpha\beta\rho_2 a_0 a_2\lambda\mu + 2\alpha\beta\rho_2 a_0 a_1\mu + 2\alpha\beta\rho_3 a_1 a_2\mu^2\\
&\quad - ca_1 + 2\beta a_1\mu + \alpha^2\rho_1 a_0^2 a_1\\
&\quad + a_1\beta\lambda^2 + \alpha\beta\rho_3 a_1^2\lambda\mu + 6\beta a_2\lambda\mu\\
&\quad + \alpha a_0 a_1 + \alpha\beta\rho_2 a_0 a_1\lambda^2 = 0,\\
&a_0 + a_1\beta\lambda\mu + 2\alpha\beta\rho_2 a_0 a_2\mu^2 + 2a_2\beta\mu^2\\
&\quad + \alpha\beta\rho_2 a_0 a_1\lambda\mu - ca_0\\
&\quad + \frac{\alpha^2\rho_1 a_0^3}{3} + \frac{\alpha\beta\rho_3 a_1^2\mu^2}{2} + \frac{\alpha a_0^2}{2} - \frac{\alpha\beta\rho_2 a_1^2\mu^2}{2} = 0.
\end{aligned} \tag{13}$$

Solving the system of algebraic equations with the aid of Maple, we obtain the following two different sets of solutions.

Case 1. We have

$$\begin{aligned}
a_0 &= -\frac{3\beta\lambda^2(\rho_3+2\rho_2)}{2\alpha\rho_1} - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)},\\
a_1 &= -\frac{6\beta\lambda(\rho_3+2\rho_2)}{\alpha\rho_1},\\
a_2 &= -\frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1}, \qquad c = 1 - \frac{\rho_3-2\rho_1+2\rho_2}{(\rho_3+2\rho_2)(\rho_3+\rho_2)},\\
\mu &= \frac{\lambda^2}{4} + \frac{\rho_3-2\rho_1+2\rho_2}{4\beta(\rho_3+2\rho_2)(\rho_3+\rho_2)},
\end{aligned} \tag{14}$$

where λ is an arbitrary constant.

Case 2. We have

$$\begin{aligned}
a_0 &= -\frac{W}{\alpha\rho_1}, \qquad a_1 = -\frac{6\beta\lambda(\rho_3+2\rho_2)}{\alpha\rho_1},\\
a_2 &= -\frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1},\\
c &= \frac{1}{16}2\rho_3(\rho_3+4\rho_1-\rho_2)(\rho_3+\rho_2)W\\
&\quad + 3\beta\rho_3(\rho_3+4\rho_1-\rho_2)(\rho_3+\rho_2)(\rho_3+2\rho_2)\lambda^2\\
&\quad - 16\rho_3^3\rho_1 + 16\rho_2^3\rho_1 - 6\rho_2^2\rho_3 + 3\rho_3^3 - 16\rho_2^2\rho_1\\
&\quad + 6\rho_2\rho_1\rho_3 + 16\rho_2^2\rho_1\rho_3 - 2\rho_3^2\rho_1\\
&\quad - 16\rho_2\rho_1\rho_3^2 + 16\rho_2\rho_1^2 + 3\rho_2\rho_3^2 - 8\rho_3\rho_1^2,\\
\mu &= -\frac{W}{4\beta(\rho_3+2\rho_2)} + \frac{\lambda^2}{8} + \frac{1}{8\beta(\rho_3+\rho_2)}\\
&\quad - \frac{\rho_1}{4\beta(\rho_3+\rho_2)(\rho_3+2\rho_2)},
\end{aligned} \tag{15}$$

where W is a root of the equation $9\beta(\rho_3+2\rho_2)^2\lambda^2(\beta(\rho_2+\rho_3)(\rho_2-\rho_3)\lambda^2+1)+6\rho_1(\rho_3+2\rho_2)(6\beta\rho_2\lambda^2+1)-12\rho_1^2+[6(\rho_2-\rho_3)(\rho_3+2\rho_2)(2\beta\lambda^2(\rho_2+\rho_3)+1)-24\rho_1\rho_2]Z-4(\rho_2+\rho_3)(\rho_2-\rho_3)Z^2=0$ with respect to Z and λ is an arbitrary constant.

For Case 1, substituting (14) into (12), we obtain the following traveling wave solution:

$$\begin{aligned}
U(\xi) &= -\frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1}\left(\frac{G'}{G}\right)^2 - \frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1}\lambda\left(\frac{G'}{G}\right)\\
&\quad - \frac{3\beta\lambda^2(\rho_3+2\rho_2)}{2\alpha\rho_1} - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)}.
\end{aligned} \tag{16}$$

Substituting the general solutions of (6) into (16), we can get three types of traveling wave solutions of (1) as follows.

When $\Delta = \lambda^2 - 4\mu > 0$, the hyperbolic function solution of (1) is of the following form:

$$u(x,t) = -\frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1} \times \left[\frac{\lambda^2}{4} - \frac{\lambda\sqrt{\lambda^2-4\mu}}{2}\frac{A_1\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}{A_1\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)} + \frac{\lambda^2-4\mu}{4}\left(\frac{A_1\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}{A_1\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}\right)^2\right]$$
$$- \frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1}\lambda\left[-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{A_1\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}{A_1\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}\right]$$
$$- \frac{3\beta\lambda^2(\rho_3+2\rho_2)}{2\alpha\rho_1} - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)} = -\frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1}\frac{\lambda^2-4\mu}{4} \times \left(\frac{A_1\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}{A_1\cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)+A_2\sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}\right)^2 - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)}, \tag{17}$$

where $\xi = x - ct, c = 1 - (\rho_3 - 2\rho_1 + 2\rho_2)/(\rho_3+2\rho_2)(\rho_3+\rho_2)$, and A_1 and A_2 are arbitrary constants. Obviously, the various known hyperbolic function solutions can be rewritten if A_1, A_2, and λ are taken as special values, as follows:

(1) if $A_1 = 0$ and $A_2 \neq 0$, we have

$$u(\xi,t) = -\frac{3\beta(\rho_3+2\rho_2)}{2\alpha\rho_1}(\lambda^2-4\mu) \times \coth^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)}, \tag{18}$$

(2) if $A_2 = 0$ and $A_1 \neq 0$, we have

$$u(\xi,t) = -\frac{3\beta(\rho_3+2\rho_2)}{2\alpha\rho_1}(\lambda^2-4\mu) \times \tanh^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi\right) - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)}, \tag{19}$$

(3) if $A_1 \neq 0$ and $A_1^2 > A_2^2$, we have

$$u(\xi,t) = -\frac{3\beta(\rho_3+2\rho_2)}{2\alpha\rho_1}(\lambda^2-4\mu) \times \operatorname{sech}^2\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+\xi_0\right) - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)}, \tag{20}$$

where $\xi = x - ct, c = 1 - (\rho_3 - 2\rho_1 + 2\rho_2)/(\rho_3+2\rho_2)(\rho_3+\rho_2)$, and $\xi_0 = \tanh^{-1}(A_2/A_1)$.

When $\Delta = \lambda^2 - 4\mu < 0$, we obtain the following trigonometric function solutions of (1):

$$u(x,t) = -\frac{6\beta(\rho_3+2\rho_2)}{\alpha\rho_1}\frac{4\mu-\lambda^2}{4} \times \left(\frac{A_1\sin\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right)+A_2\cos\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right)}{A_1\cos\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right)+A_2\sin\left(\left(\sqrt{4\mu-\lambda^2}/2\right)\xi\right)}\right)^2 - \frac{3(\rho_3-2\rho_1+2\rho_2)}{2\alpha\rho_1(\rho_2+\rho_3)}, \tag{21}$$

where $\xi = x - ct$, $c = 1 - (\rho_3 - 2\rho_1 + 2\rho_2)/(\rho_3 + 2\rho_2)(\rho_3 + \rho_2)$, and A_1 and A_2 are arbitrary constants. Obviously, the various known trigonometric function solutions can be rewritten if A_1, A_2, and λ are taken as special values, as follows:

(1) if $A_1 = 0$ and $A_2 \neq 0$, we have

$$u(\xi, t) = -\frac{3\beta(\rho_3 + 2\rho_2)}{2\alpha\rho_1}\left(4\mu - \lambda^2\right) \times \cot^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) - \frac{3(\rho_3 - 2\rho_1 + 2\rho_2)}{2\alpha\rho_1(\rho_2 + \rho_3)}, \tag{22}$$

(2) if $A_2 = 0$ and $A_1 \neq 0$, we have

$$u(\xi, t) = -\frac{3\beta(\rho_3 + 2\rho_2)}{2\alpha\rho_1}\left(4\mu - \lambda^2\right) \times \tan^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi\right) - \frac{3(\rho_3 - 2\rho_1 + 2\rho_2)}{2\alpha\rho_1(\rho_2 + \rho_3)}, \tag{23}$$

(3) if $A_1 \neq 0$ and $A_1^2 > A_2^2$, we have

$$u(\xi, t) = -\frac{3\beta(\rho_3 + 2\rho_2)}{2\alpha\rho_1}\left(4\mu - \lambda^2\right) \times \sec^2\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi + \xi_0\right) - \frac{3(\rho_3 - 2\rho_1 + 2\rho_2)}{2\alpha\rho_1(\rho_2 + \rho_3)}, \tag{24}$$

where $\xi = x - ct$, $c = 1 - (\rho_3 - 2\rho_1 + 2\rho_2)/(\rho_3 + 2\rho_2)(\rho_3 + \rho_2)$, and $\xi_0 = \tan^{-1}(A_2/A_1)$.

When $\Delta = \lambda^2 - 4\mu = 0$, we obtain the following rational function solutions of (1):

$$u(x, t) = -\frac{6\beta(\rho_3 + 2\rho_2)}{\alpha\rho_1}\left(\frac{A_2}{A_1 + A_2\xi}\right)^2 - \frac{3(\rho_3 - 2\rho_1 + 2\rho_2)}{2\alpha\rho_1(\rho_2 + \rho_3)}, \tag{25}$$

where $\xi = x - ct$, $c = 1 - (\rho_3 - 2\rho_1 + 2\rho_2)/(\rho_3 + 2\rho_2)(\rho_3 + \rho_2)$, and A_1 and A_2 are arbitrary constants. Obviously, the various known trigonometric function solutions can be rewritten if A_1, A_2, and λ are taken as special values, Case 2 can be discussed similarly to Case 1, and we omit the details for brevity.

Remark 1. The solutions of (1) we obtained in this work involving arbitrary parameters ρ_i ($i = 1, 2, 3$) have not been given in other literatures.

Remark 2. The solutions (18), (19), and (20) are solitary wave solutions which can explain the relationship between the waveform, speed of the traveling water waves, and the amplitude; that is, most of the smooth water waves propagate in the form of solitary waves; when the traveling speed of the water wave increases, the amplitude of solitary waves will become higher and the width of the waveform will become narrower. On the contrary, once the speed slows down, the amplitude of solitary waves will become lower and the width of the waveform will become wider.

Remark 3. The physical meaning of other solutions for this equation is still unclear. For instance, the relationship between the solutions (21)–(25) and the movement of water waves and the dynamic behaviors that these solutions can demonstrate are both unknown to us. However, the rest of these questions is still worthy of further observation and investigation by the researchers in the territory of experimental physics, and we also hope that more researchers should pay attention to the investigation of this area.

4. Conclusions

The higher-order KdV type equation was investigated by using (G'/G)-expansion method and we successfully obtained some exact solutions expressed by hyperbolic functions, trigonometric functions, and rational functions. As far as we know, there is no previous work about the solutions of (1) with arbitrary parameters. The obtained results are verified by putting them back into the original equation with the aid of Maple. The applicability of this algorithm to other NLEEs in mathematical physics illustrates its effectiveness and powerfulness.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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