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Research Article

Distributionally Robust Self-Scheduling Optimization with CO_2 Emissions Constraints under Uncertainty of Prices

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As a major energy-saving industry, power industry has implemented energy-saving generation dispatching. Apart from security and economy, low carbon will be the most important target in power dispatch mechanisms. In this paper, considering a power system with many thermal power generators which use different petrochemical fuels (such as coal, petroleum, and natural gas) to produce electricity, respectively, we establish a self-scheduling model based on the forecasted locational marginal prices, particularly taking into account CO_2 emission constraint, CO_2 emission cost, and unit heat value of fuels. Then, we propose a distributionally robust self-scheduling optimization model under uncertainty in both the distribution form and moments of the locational marginal prices, where the knowledge of the prices is solely derived from historical data. We prove that the proposed robust self-scheduling model can be solved to any precision in polynomial time. These arguments are confirmed in a practical example on the IEEE 30 bus test system.

1. Introduction

Generation self-scheduling in a pool-based electricity market has been recently studied in the power systems literature [1–3]. The self-schedules are required in developing successful bidding strategies and constructing hourly bidding curves for consideration by the independent system operator. In order to obtain successful generation bids, the generation companies have to self-schedule their unit by maximizing the expected profit based on the forecasted location marginal prices and accounting for the network security constraints. For a price-taker generation and a single time period, the generation schedule x is obtained from the following deterministic self-scheduling problem:

$$\max_{x\in X} \xi^T x - \sum_{i=1}^{m} C_i(x_i), \tag{1}$$

where x is an $(m\times 1)$ column vector containing the generation schedule, ξ is an $(m\times 1)$ column vector of locational marginal prices (LMPs), $X \subset R^m$ is the feasible region which is a convex and compact set, and $C_i(x_i)$ is the generation quadratic cost function:

$$C_i(x_i) = a_i + b_i x_i + c_i x_i^2 \quad i = 1, \ldots, m. \tag{2}$$

The issue of interest for this work is that, since the electricity prices are of stochastic nature, the generation company cannot be certain about the revenue. Measuring the underlying risk due to this uncertainty is crucial not only for assessing profitability but also for generation self-scheduling. Stochastic programming can effectively describe self-scheduling problems in uncertain environments. Unfortunately, although the self-scheduling problem is a convex optimization problem, to solve it one must often resort to Monte Carlo approximations, which can be computationally challenging. A more challenging difficulty that arises in practice is the need to commit to a distribution given only limited information about the stochastic parameters [4].

In an effort to address these issues, robust formulations for self-scheduling problems were proposed; see [2, 3]. Jabr [2] considers a generation self-scheduling model based on a worst-case conditional robust profit with partial information on the probability distribution of prices. It is assumed that the nominal distribution and a set of possible distributions were given. Uncertainty of prices is represented by box and ellipsoidal uncertainty sets. However, in practice, true probability distribution of prices cannot be known exactly. Their solution can be misleading when there is ambiguity in the choice of a distribution for the random prices. Recently, Delage and Ye [4] proposed a distributionally robust optimization model that describes uncertainty in both the distribution form and moments (mean and covariance matrix). By deriving a new form of confidence region for the mean and the covariance matrix of a random vector, it was showed how the proposed distribution set can be well justified when addressing data-driven problems (i.e., problems where the knowledge of the random parameters is solely derived from historical data).

On the other hand, as a major energy-saving industry, power industry has implemented energy-saving generation dispatching. Apart from security and economy, low carbon will be the most important target in power dispatch mechanisms. It becomes a common target for the global power industry to build a more safe, economic, and low-carbon power system [5]. So more attention should be paid to the reduction of CO_2 emission in power system operation [5, 6].

The main contribution of this paper is twofold. First, by considering a power system with many thermal power generators which use different petrochemical fuels (such as coal, petroleum, and natural gas) to produce electricity, respectively, we establish a self-scheduling model based on the forecasted locational marginal prices, particularly taking into account CO_2 emission constraint, CO_2 emission cost, and unit heat value of fuels. This problem is important and timely as world leaders and international organizations discuss the roles and responsibilities of each country and sector of economic activity in the path towards a sustainable future. Second, motivated by the work of Delage and Ye [4], we propose a distributionally robust self-scheduling optimization model under uncertainty in both the distribution form and moments of the locational marginal prices, where the knowledge of the prices is solely derived from historical data. Then we prove that the proposed robust self-scheduling model can be solved to any precision in polynomial time. These arguments are confirmed in a practical example on the IEEE 30 bus test system.

2. Robust Self-Scheduling Problem with Moment Uncertainty and CO_2 Emission Constraint

2.1. Power Dispatch with CO_2 Emission Constraint. In this section, we describe the power scheduling problem with CO_2 emission constraint.

Given is a thermal power system with M thermal power generators $i \in M = \{1, \ldots, m\}$, which use petrochemical fuels as their fuels, such as coal, petroleum, and natural gas. The quantity of CO_2 emission of each power generator i can be represented as

$$E_i = F_i h_i, \tag{3}$$

where F_i is the quantity of the fuel which is consumed to produce electricity and $h_i (\geq 0)$ is the amount of CO_2 emissions by per unit of fuel complete combustion. The generating capacity of this power generator is

$$x_i = F_i p_i \varphi_i, \tag{4}$$

where $p_i (> 0)$ is the unit heat value of the fuel and $\varphi_i (> 0)$ is the energy conversion efficiency.

By (3) and (4), we get the carbon characteristic function of power generator i:

$$E_i = \frac{h_i}{p_i} \frac{1}{\varphi_i} x_i. \tag{5}$$

The objective is the maximization of the expected profit based on the forecasted locational marginal prices ξ, particularly taking into account CO_2 emission constraints and CO_2 emission cost. The cost consists of the variable cost of electricity production of the thermal generators and the cost of CO_2 emission. So the power scheduling problem with CO_2 emission constraints can be modeled as the following deterministic self-scheduling problem:

$$\begin{aligned} \max_{x \in X} \quad & E\left[\xi^T x - \sum_{i=1}^{m} C_i(x_i) - \sum_{i=1}^{m} \delta \cdot \frac{h_i}{p_i} \frac{1}{\varphi_i} x_i\right] \\ \text{s.t.} \quad & \sum_{i=1}^{m} \frac{h_i}{p_i} \frac{1}{\varphi_i} x_i \leq \overline{E}, \end{aligned} \tag{6}$$

where x is an $(m\times 1)$ column vector containing the generation schedule, ξ is an $(m \times 1)$ column vector of forecasted locational marginal prices (LMPs), X is the feasible region, δ is the CO_2 emission price with the unit £/ton, $\overline{E}$ is the maximum allowable CO_2 emissions, and $C_i(x_i)$ is the generator quadratic cost function:

$$C_i(x_i) = a_i + b_i x_i + c_i x_i^2, \quad i = 1, \ldots, m, \tag{7}$$

where $c_i > 0$ for $i = 1, \ldots, m$.

Letting $D_i(x_i) \equiv \delta(h_i/p_i)(1/\varphi_i)x_i$, by (5), problem (6) can be rewritten as follows:

$$\begin{aligned} \max_{x \in X} \quad & E\left[\xi^T x - \sum_{i=1}^{m} C_i(x_i) - \sum_{i=1}^{m} D_i(x_i)\right] \\ \text{s.t.} \quad & \sum_{i=1}^{m} \frac{D_i(x_i)}{\delta} \leq \overline{E}. \end{aligned} \tag{8}$$

2.2. Robust Self-Scheduling Problem with Moment Uncertainty and CO_2 Emission Constraints. In practice, It is often the case that one has limited information about the locational marginal prices ξ driving the uncertain parameters that are involved in the decision making process. The data samples may be not available, or the data samples may be unstable. In such situations, it might instead be safer to rely on estimates of the mean μ_0 and covariance matrix Σ_0 of the random vector ξ, for example, using empirical estimates. However, we believe that, in such problems, it is also rarely the case that one is entirely confident in these estimates. For this reason, motivated by Delage and Ye [4], we propose representing the uncertainty using two constraints parameterized by $\gamma_1 \geq 0$ and $\gamma_2 \geq 1$:

$$(\mathbb{E}[\xi]-\mu_0)^T\Sigma_0^{-1}(\mathbb{E}[\xi]-\mu_0) \leq \gamma_1, \tag{9}$$

$$\mathbb{E}\left[(\xi-\mu_0)(\xi-\mu_0)^T\right] \preceq \gamma_2\Sigma_0, \tag{10}$$

where constraint (9) assumes that the mean of price ξ lies in an ellipsoid of size γ_1 centered at the estimate μ_0 and constraint (10) forces the centered second-moment matrix of ξ to lie in a positive semidefinite cone defined with a matrix inequality. In other words, it describes how likely ξ is to be close to μ_0 in terms of the correlations expressed in Σ_0. Finally, the parameters γ_1 and γ_2 provide natural means of quantifying one's confidence in μ_0 and Σ_0, respectively.

Denote the distributional set as

$$\begin{aligned}\mathscr{D}_1&(\mathscr{F},\mu_0,\Sigma_0,\gamma_1,\gamma_2)\\ &=\Big\{F\in\mathscr{U} \mid \mathbb{P}(\xi\in\mathscr{F})=1,\\ &\quad(\mathbb{E}[\xi]-\mu_0)^T\Sigma_0^{-1}(\mathbb{E}[\xi]-\mu_0)\leq\gamma_1,\\ &\quad\mathbb{E}\left[(\xi-\mu_0)(\xi-\mu_0)^T\right]\preceq\gamma_2\Sigma_0\Big\},\end{aligned} \tag{11}$$

where $\mathscr{U}$ is the set of all probability measures on the measurable space $(\mathbb{R}^m,\mathscr{B})$, with $\mathscr{B}$ being the Borel σ-algebra on $\mathbb{R}^m$, and $\mathscr{F} \in \mathbb{R}^m$ is any closed convex set known to contain the support of F. The set $\mathscr{D}_1(\mathscr{F},\mu_0,\Sigma_0,\gamma_1,\gamma_2)$, which will also be referred to in shorthand notation as $\mathscr{D}_1$, can be seen as a generalization of many previously proposed sets.

Next we will study worst-case expected results over the choice of a distribution in the distributional set $\mathscr{D}_1$. This leads to solving the distributionally robust self-scheduling optimization with moment uncertainty of prices (DRSSO):

$$\begin{aligned}v=\max_{x\in X}&\left(\min_{F\in\mathscr{D}_1}\mathbb{E}_F\left[\xi^Tx-\sum_{i=1}^m C_i(x_i)-\sum_{i=1}^m D_i(x_i)\right]\right)\\ \text{s.t.}&\quad\sum_{i=1}^m\frac{D_i(x_i)}{\delta_i}\leq\overline{E},\end{aligned} \tag{12}$$

where $\mathbb{E}_F[\cdot]$ is the expectation taken with respect to the random vector ξ given that it follows the probability distribution $F\in\mathscr{D}_1$. It is easy to see that DRSSO problem (12) is equivalent to the following problem:

$$\begin{aligned}-v=\min_{x\in X}&\left(\max_{F\in\mathscr{D}_1}\mathbb{E}_F\left[-\xi^Tx+\sum_{i=1}^m C_i(x_i)+\sum_{i=1}^m D_i(x_i)\right]\right)\\ \text{s.t.}&\quad\sum_{i=1}^m\frac{D_i(x_i)}{\delta_i}\leq\overline{E}.\end{aligned} \tag{13}$$

We start by considering the question of solving the inner maximization problem of the problem (13) that uses the set $\mathscr{D}_1$.

Definition 1. Given any fixed $x \in X$, let $\Phi(x;\gamma_1,\gamma_2)$ be the optimal value of the moment problem:

$$\max_{F\in\mathscr{D}_1}\mathbb{E}_F\left[-\xi^Tx+\sum_{i=1}^m C_i(x_i)+\sum_{i=1}^m D_i(x_i)\right]. \tag{14}$$

Applying the duality theory and robust optimization methods [7–10], by [4, Lemma 1], we can circumvent the difficulty of finding the optimal value of the problem (14).

Lemma 2. *For a fixed $x\in\mathbb{R}^n$, suppose that $\gamma_1\geq 0$, $\gamma_2\geq 1$, $\Sigma_0 \succ 0$. Then $\Phi(x;\gamma_1,\gamma_2)$ must be equal to the optimal value of the problem* (15):

$$\begin{aligned}\min_{Q,q,r,t}\quad & r+t\\ \text{s.t.}\quad & t\geq\left(\gamma_2\Sigma_0+\mu_0\mu_0^T\right)\cdot Q+\mu_0^Tq\\ &\quad+\sqrt{\gamma_1}\left\|\Sigma_0^{1/2}(q+2Q\mu_0)\right\|\\ & r\geq-\xi^Tx+\sum_{i=1}^m C_i(x_i)\\ &\quad+\sum_{i=1}^m D_i(x_i)-\xi^TQ\xi-\xi^Tq,\quad\forall\xi\in\mathscr{F}\\ & Q\succeq 0,\end{aligned} \tag{15}$$

where $(A\cdot B)$ refers to the Frobenius inner product between matrices, $Q\in\mathbb{R}^{m\times m}$ is a symmetric matrix, the vector $q\in\mathbb{R}^m$, and $r,t\in\mathbb{R}$. In addition, if $\Phi(x;\gamma_1,\gamma_2)$ is finite, then the set of optimal solutions to problem (15) *must be nonempty.*

Proof. Let $h(x,\xi) = -\xi^Tx+\sum_{i=1}^m C_i(x_i)+\sum_{i=1}^m D_i(x_i)$. Then $h(x,\xi)$ is F-integrable for all $F\in\mathscr{D}_1$ and the conclusions are followed by [4, Lemma 1]. This completes the proof. □

The following result shows that the DRSSO problem (13) is a tractable problem.

Theorem 3. *Assume that the set X is convex and compact. Then DRSSO problem* (13) *is equivalent to the following convex optimization problem:*

$$\begin{aligned}-v=\min_{x,Q,q,r,t,\eta,y,\omega}\quad & r+t\\ \text{s.t.}\quad & t\geq\left(\gamma_2\Sigma_0+\mu_0\mu_0^T\right)\cdot Q+\mu_0^Tq\\ &\quad+\sqrt{\gamma_1}\left\|\Sigma_0^{1/2}(q+2Q\mu_0)\right\|\\ &\begin{bmatrix}Q & \dfrac{q}{2}+\dfrac{x}{2}\\ \dfrac{q^T}{2}+\dfrac{x^T}{2} & r-\eta\end{bmatrix}\succeq 0\end{aligned}$$

$$\begin{aligned}
&Q \succeq 0\\
&\eta \geq \sum_{i=1}^{m}(a_i + b_i x_i) + \sum_{i=1}^{m}\delta_i \frac{h_i}{p_i}\frac{1}{\varphi_i}x_i + y,\\
&y \geq \sum_{i=1}^{m}\omega_i^2\\
&\omega_i = \sqrt{c_i}x_i\\
&\sum_{i=1}^{m}\frac{h_i}{p_i}\frac{1}{\varphi_i}x_i \leq \overline{E}\\
&x \in X.
\end{aligned} \tag{16}$$

In addition, the DRSSO problem (13) *can be solved to any precision* ϵ *in time polynomial in* $\log(1/\epsilon)$ *and the size of* x *and* ξ.

Proof. By Lemma 2, the DRSO problem (13) must be equal to the following problem:

$$\begin{aligned}
-\nu = \min_{x,Q,q,r,t} \quad & r + t\\
\text{s.t.} \quad & t \geq \left(\gamma_2\Sigma_0 + \mu_0\mu_0^T\right)\cdot Q + \mu_0^T q\\
&\quad + \sqrt{\gamma_1}\left\|\Sigma_0^{1/2}\left(q + 2Q\mu_0\right)\right\|,\\
& r \geq -\xi^T x + \sum_{i=1}^{m}C_i\left(x_i\right)\\
&\quad + \sum_{i=1}^{m}D_i\left(x_i\right) - \xi^T Q\xi - \xi^T q, \quad \forall\xi \in \mathcal{F},\\
& C_i\left(x_i\right) = a_i + b_i x_i + c_i x_i^2, \quad i = 1,\ldots,m\\
& D_i\left(x_i\right) = \delta_i\frac{h_i}{p_i}\frac{1}{\varphi_i}x_i, \quad i = 1,\ldots,m,\\
& Q \succeq 0,\\
& \sum_{i=1}^{m}\frac{h_i}{p_i}\frac{1}{\varphi_i}x_i \leq \overline{E},\\
& x \in X.
\end{aligned} \tag{17}$$

Let

$$\begin{aligned}
&\eta \geq \sum_{i=1}^{m}(a_i + b_i x_i) + \sum_{i=1}^{m}\delta_i \frac{h_i}{p_i}\frac{1}{\varphi_i}x_i + y,\\
&y \geq \sum_{i=1}^{m}\omega_i^2,\\
&\omega_i = \sqrt{c_i}x_i.
\end{aligned} \tag{18}$$

Then $\eta \geq \sum_{i=1}^{m}C_i(x_i) + \sum_{i=1}^{m}D_i(x_i)$, and the inequality (17) can be replaced by

$$\xi^T Q\xi + \xi^T q + \xi^T x + r - \eta \geq 0, \quad \forall\xi \in \mathcal{F}. \tag{19}$$

Note that the constraint in (19) can be written as the following LMI:

$$\begin{gathered}
\begin{bmatrix}\xi\\1\end{bmatrix}^T \begin{bmatrix} Q & \dfrac{q + x}{2}\\ \dfrac{q^T + x^T}{2} & r - \eta\end{bmatrix}\begin{bmatrix}\xi\\1\end{bmatrix} \geq 0, \quad \forall\xi \in \mathcal{F}\\
\Longleftrightarrow \begin{bmatrix} Q & \dfrac{q}{2} + \dfrac{x}{2}\\ \dfrac{q^T}{2} + \dfrac{x^T}{2} & r - \eta\end{bmatrix} \succeq 0.
\end{gathered} \tag{20}$$

So DRSSO problem (13) is equivalent to the convex optimization problem (16).

Since $f(x,\xi) = -\xi^T x + \sum_{i=1}^{m}C_i(x_i) + \sum_{i=1}^{m}D_i(x_i)$ is convex in x and concave in ξ and X is convex and compact, the assumptions in [4] are satisfied. So a straightforward application of [4, Proposition 2] shows that DRSSO problem (13) can be solved in polynomial time. This completes the proof. □

If X is a convex polyhedron, then the convex optimization problem (16) is a semidefinite program that can be solved by SeDuMi conic optimizer.

3. Numerical Example

We present our simulation results on the IEEE 30 bus test system and get the results by using the SeDuMi conic optimizer running on an *Intel Core* i3-2350 M (2.30 GHz) PC with 2 GB RAM. There are 6 power generations with coal as their fuel in this system. Reference [11] gives the network and load data for this system. The generator data is listed in Table 1. A historical data set of 100 samples of prices vector ξ is shown in [2, Table 2]. We assume that the generating units in Table 1 belong to the same generation company.

In implementing our method, the distributional set is formulated as $\mathcal{D}_1(\mathbb{R}^6, \mu_0, \Sigma_0, 0.2, 2.3)$, where μ_0 and Σ_0 are the empirical estimates of the mean and covariance matrix of prices vector ξ shown in [2, Table 2].

We choose the parameters γ_1 and γ_2 based on some simple statistical analysis of a lot of experiments. We convert the calorific value of different kinds of coals to standard unit of coal and let the unit heat value of fuel p equal 8.13 (kW·h)/kg standard unit of coal, the amount of CO_2 emissions by per unit of fuel complete combustion h equal 2.62 kg CO_2/kg standard unit of coal in uniform, and the maximum allowable CO_2 emissions $\overline{E}$ equal 150 ton.

If we fix $\gamma_2 = 1.2$ and let γ_1 vary from 0 to 2, it can be shown that the profit decreases when γ_1 increases from 0 to 1.2, and the profit is almost invariable after $\gamma_1 > 1.2$. Table 2 illustrates the generation self-scheduling result for γ_1 from 0 to 1 and $\gamma_2 = 1.2$.

The value of γ_1 reflects the stability of the electricity price. The electricity price is more instable when the value of γ_1

Table 1: Generator data.

Bus number	$x_{\min}$, MW	$x_{\max}$, MW	Cost coefficients		
			a, £/h	b, £/MWh	c, £/MW^{2}h
1	50	200	0	2.00	0.00375
2	20	80	0	1.75	0.01750
5	15	50	0	1.00	0.06250
8	10	35	0	3.25	0.00834
11	10	30	0	3.00	0.02500
13	12	40	0	3.00	0.02500

Table 2: Generation self-scheduling result for γ_1 from 0 to 1 ($\gamma_2 = 1.2$).

γ_1	Bus 1, MW	Bus 2, MW	Bus 5, MW	Bus 8, MW	Bus 11, MW	Bus 13, MW	Profit, £
0	135.00	35.37	16.45	35.00	30.00	38.67	229.52
0.2	103.36	33.44	16.20	35.00	30.00	36.77	188.78
0.4	91.42	32.45	16.08	35.00	30.00	35.94	174.14
0.6	82.87	31.62	15.98	35.00	30.00	35.29	163.71
0.8	76.13	30.88	15.90	35.00	30.00	34.73	155.41
1	70.55	30.20	15.82	35.00	30.00	34.23	148.44

is higher. If generations produce too much electricity, they would face more risks. The decision makers would produce less electricity when the value of γ_1 is high. And due to the basic power needs, the generating capacity would tend to be stable if the value of γ_1 is too high. Figure 1 shows the result intuitively.

However, if we fix γ_1 and let γ_2 vary from 1 to 10, numerical results show that the profit is almost invariable. This implies that γ_2 is not sensitive to the model.

Now we study three kinds of fuels with different unit heat values. The unit heat value p of natural gas, coal, and oil is shown in [12, Table 1]. And we can get h, the quantity of CO_2 released by the unit fuel burnt fully, by some simple calculations. The relevant data is listed in Table 3. From this table, we can know that the unit heat value of natural gas is higher than that of coking coal and the quantity of CO_2 released by the unit natural gas is lower than that of it. Let $\delta = 0.2$ £/ton, $\overline{E} = 135.0$ ton. We assume the 6 buses all use one fuel as their power fuel; the three results are shown in Table 4.

The results show that using natural gas gets more economic benefits. This is because the unit natural gas can produce more quantity of heat and release fewer CO_2. And the result of using oil is between natural gas and coking coal. In addition, we can find that the quantity of CO_2 emission of coking coal and the quantity of CO_2 emission of oil have reached the maximum allowable value, respectively.

For different power generators, we let their efficiency increase in turns. Numerical results show that the profits increase as their efficiency increases.

We consider the carbon price's effect in the following. Figure 2 illustrates the effect of carbon price δ on the generation self-scheduling. The upper part of the figure shows that the profit and CO_2 emission decrease when the carbon price δ increases with the limit of $\overline{E}$. And the lower part of the figure shows the result without the limit of $\overline{E}$. When δ is very

Table 3: Relevant data of fuels.

Power fuel	h_i, kg(CO_2)/kg	p_i, Kwh/kg
Coking coal	3.04	7.90
Natural gas	2.18	10.81
Oil	3.06	11.61

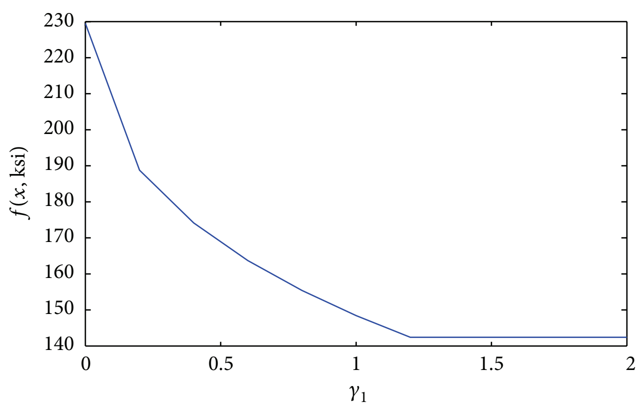


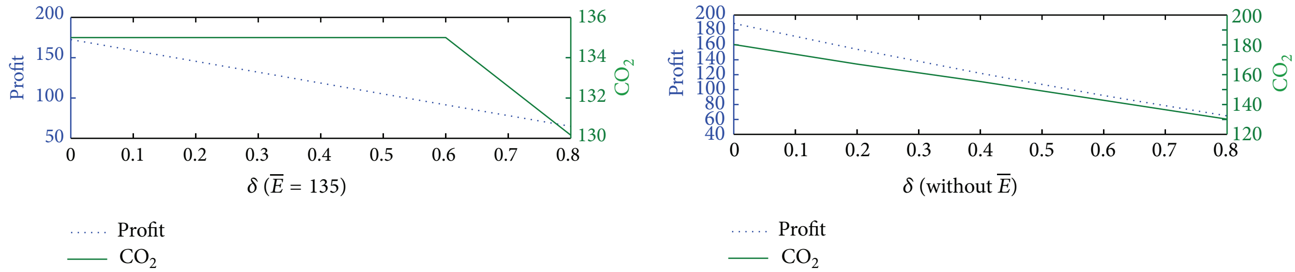
Figure 1: Relation between profit and γ_1.

small, the CO_2 emission is so large because there is no limit of $\overline{E}$. That is, $\overline{E}$ makes a difference on reducing carbon emissions. When δ increases, the profit decreases. So government should consider the effect of carbon price when they set the carbon price.

We let the carbon price δ equal 0.2 £/ton and 0.6 £/ton, respectively, and $\overline{E}$ range from 120 ton to 180 ton. Figure 3 illustrates this result. Due to the emission restriction decreases, the object f and CO_2 emission go up. And they tend to be stable along with the value of $\overline{E}$ becoming larger. By the comparison between the upper and the lower part

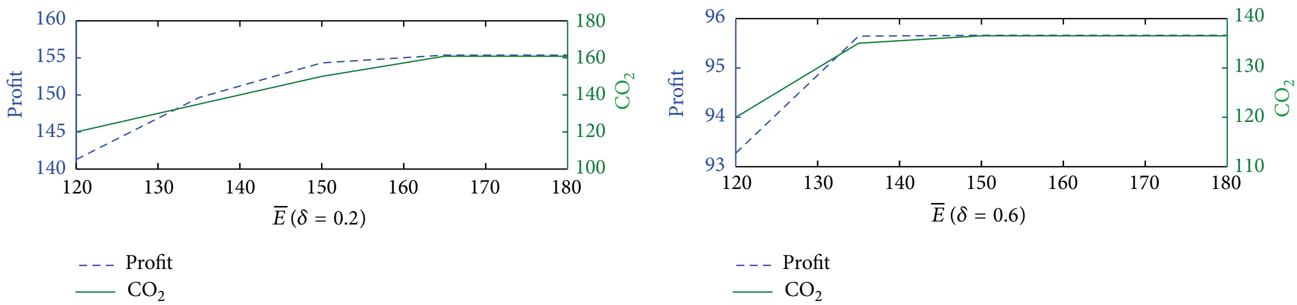
Table 4: The result of using one kind of fuel ($\delta = 0.2$ £/ton, $\overline{E} = 135.0$ ton).

Bus number	1, MW	2, MW	5, MW	8, MW	11, MW	13, MW	Profit, £	CO_2, t
Natural gas	99.24	31.18	15.47	35.00	30.00	35.31	174.12	119.1
Oil	68.42	25.36	15.00	35.00	30.00	31.10	154.89	135.0
Coking coal	50.00	20.00	15.00	24.88	15.55	14.90	113.84	135.0



(a) (b)

Figure 2: Profit and CO_2 emission with increasing carbon price δ.



(a) (b)

Figure 3: Profit and CO_2 emission with increasing $\overline{E}$.

of Figure 3, the quality of CO_2 emission tends to reach an invariable value earlier as $\delta = 0.6$ £/ton. This shows that a suitable carbon tax is a good means to reduce carbon dioxide emissions.

Figure 4 demonstrates the change of generation self-schedule when $\overline{E}$ increases. At first when $\overline{E}$ increases, some generation schedule x increases. But when the $\overline{E}$ achieves a point, the effect of restriction disappears and the generation schedules are invariable.

4. Conclusion

This paper studies worst-case profit self-schedules of price-taker generators with the constraints of CO_2 emission in pool-based electricity markets. A distributionally robust self-scheduling optimization model describes uncertainty of prices in both distribution form and moments (mean and covariance matrix), where the knowledge of the prices is solely derived from historical data. It is proved that the proposed robust self-scheduling model can be solved to any precision in polynomial time. These arguments are confirmed in a practical example on the IEEE 30 bus test system. Numerical results show that parameter γ_1 of mean is sensitive to the solution and parameter γ_2 of covariance

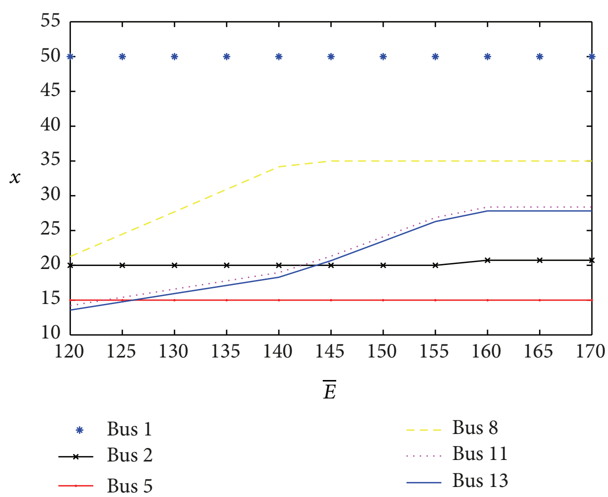


Figure 4: Generation self-schedule x with increasing $\overline{E}$.

is not sensitive to the solution. Through the comparison between the different fuels and energy conversion efficiencies, we find that the power stations should choose the fuels with lower h and higher p as their power fuels. And the energy conversion efficiency is also very important to the power stations; we should try to improve it. Through the comparison between the carbon prices δ and $\overline{E}$, it is showed that the maximum allowable CO_2 emissions makes a difference in reducing carbon emissions and a reasonable carbon tax is a good means to reduce carbon dioxide emissions.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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