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Research Article

Constrained Weak Nash-Type Equilibrium Problems

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A constrained weak Nash-type equilibrium problem with multivalued payoff functions is introduced. By virtue of a nonlinear scalarization function, some existence results are established. The results extend the corresponding one of Fu (2003). In particular, if the payoff functions are singlevalued, our existence theorem extends the main results of Fu (2003) by relaxing the assumption of convexity.

1. Introduction

For a long time, real valued functions have played a central role in game theory. More recently, motivated by applications to real-world situations, many authors have studied the existence of solutions of Pareto equilibria of multiobjective game with vector payoff functions; for example, see [1–4] and the references therein. Notice that most payoffs may be one collection of things from many collections of things in the real world; reference [5] studied the constrained Nash-type equilibrium problem with multivalued payoff functions and proved the existence results.

In the paper, let I be an index set, Z_i a real topological vector space, and X_i $(i \in I)$ a Hausdorff topological space. Let $X = \prod_{i\in I} X_i$ and $X^i = \prod_{j\in I, j\neq i} X_j$. For each $x \in X$, let x_i and x^i denote the ith coordinate of x and the projection of x on X^i, respectively. In the sequel, we may write $x = (x_i)_{i\in I} = (x_i, x^i)$. For all $i \in I$, let C_i be a convex, closed, and pointed cone of Z_i, with apex at the origin and with nonempty interior; let $F_i : X_i \times X^i \to 2^{Z_i}$ and $S_i : X \to 2^{X_i}$. We consider a class of constrained weak Nash-type equilibrium problems with multivalued payoff functions.

(*CWNEP*) Finding an $\overline{x} = (\overline{x})_{i\in I} \in X$ such that, for each $i \in I$, $u_i \in S_i(\overline{x})$, and $\overline{z}_i \in F_i(\overline{x}_i, \overline{x}^i)$, there exists $z_i \in F_i(u_i, \overline{x}^i)$ satisfying

$$z_i - \overline{z}_i \notin -\operatorname{int} C_i. \qquad (1)$$

Then, $\overline{x}$ is a solution of (*CWNEP*).

The following problems are special cases of (*CWNEP*).

(i) If, for each $i \in I$, F_i is a singlevalued function, $Z_i = R$, and $S_i(X) = X_i$, (*CWNEP*) reduces to the Nash equilibrium problem [6].

(ii) Let X, Y, and Z be real Hausdorff topological vector spaces, and let C and D be two nonempty subsets of X and Y, respectively. Let $P \subset Z$ be a closed convex and pointed cone with $\operatorname{int} P \neq \emptyset$, let $S : C \times D \to 2^C$ and $T : C \times D \to 2^D$ be two set-valued mappings, and let $f, g : C \times D \to Z$ be two vector-valued mappings. The problem (*CWNEP*) reduces to a class of symmetric vector quasiequilibrium problems (for short, SVQEP) that consists in finding $(\overline{x}, \overline{y}) \in C \times D$ such that $\overline{x} \in S(\overline{x}, \overline{y})$, $\overline{y} \in T(\overline{x}, \overline{y})$, and

$$\begin{aligned} f(x, \overline{y}) - f(\overline{x}, \overline{y}) &\notin -\operatorname{int} P, \quad \forall x \in S(\overline{x}, \overline{y}), \\ g(\overline{x}, y) - g(\overline{x}, \overline{y}) &\notin -\operatorname{int} P, \quad \forall y \in T(\overline{x}, \overline{y}), \end{aligned} \qquad (2)$$

which was considered by Fu [7].

In this paper, we obtain the existence result for (*CWNEP*). Our existence theorem extends the main result of [6] from singlevalued case to multivalued case. In particular, if the payoff functions are singlevalued, our existence theorem extends the corresponding result in [7] by relaxing the assumption of convexity.

The rest of the paper is organized as follows. In Section 2, we state some notations and preliminary results for multivalued mappings. We recall the nonlinear scalarization function and its properties. In Section 3, we show existence result for (*CWNEP*).

2. Preliminaries

Let us first recall some definitions of continuity for set-valued mappings. Let X and Y be two topological spaces. $T : X \to 2^Y$ is a set-valued mapping. T is said to be upper semicontinuous at $x_0 \in X$ if, for each open set V containing $T(x_0)$, there is an open set U containing x_0 such that, for each $t \in U$, $T(t) \subseteq V$. It is said to be upper semicontinuous if it is upper semicontinuous at every point $x \in X$. T is said to be lower semicontinuous at $x_0 \in X$ if, for each open set V with $T(x_0) \cap V \neq \emptyset$, there is an open set U containing x_0 such that, for each $t \in U$, $T(t) \cap V \neq \emptyset$. It is said to be lower semicontinuous on X if it is lower semicontinuous at every point $x \in X$. T is said to be continuous at x_0 if it is both upper semicontinuous and lower semicontinuous at x_0. It is said to be continuous on X if it is continuous at every point $x \in X$.

From [7, Lemma 2], T is l.s.c. at $x \in X$ if and only if, for any $y \in T(x)$ and any net $\{x_n\}$, $x_n \to x$, there is a net $\{y_n\}$ such that $y_n \in T(x_n)$ and $y_n \to y$. T is closed if and only if, for any net $\{x_n\}$, $x_n \to x$, and any net $\{y_n\}$, $y_n \in T(x_n)$, $y_n \to y$, one has $y \in T(x)$.

Definition 1. Assume that X is a Hausdorff topological space and Z is a real topological vector space. Let E be a nonempty convex subset of X, let $H : E \to 2^Z$ be a set-valued mapping, and let $P \subset Z$ be a closed convex and pointed cone with $\operatorname{int} P \neq \emptyset$. H is said to be generalized Luc's quasi-P-convex on E if, for every $x_1, x_2 \in E$, $\lambda \in [0,1]$, and $y \in H(\lambda x_1 + (1-\lambda)x_2)$, there exist $z_1 \in H(x_1)$ and $z_2 \in H(x_2)$ such that

$$y \in z - C, \quad z \in C(z_1, z_2), \tag{3}$$

where $C(z_1, z_2)$ is the set of all upper bounds of z_1 and z_2; that is,

$$C(z_1, z_2) = \{z \in Z \mid z_1 \in z - P, z_2 \in z - P\}. \tag{4}$$

Remark 2. Definition 1 is a generalization of the concept of Luc's quasi-P-convexity in [8].

Now we recall the definition of the nonlinear scalarization function [9, 10] as follows.

Definition 3. Let Z be a real topological vector space, and let $P \subset Z$ be a closed convex and pointed cone with $e \in \operatorname{int} P$. The nonlinear scalarization function $\xi_e : Z \to R$ is defined by

$$\xi_e(y) = \min\{t \in R \mid y \in te - P\}. \tag{5}$$

Lemma 4 (see [9]). *The nonlinear scalarization function has the following main properties:*

(i) $\xi_e(\cdot)$ *is continuous and convex;*

(ii) $\xi_e(\cdot)$ *is subadditive; that is,* $\xi_e(y_1+y_2) \le \xi_e(y_1)+\xi_e(y_2)$;

(iii) $\xi_e(\cdot)$ *is strictly monotone; that is, if* $y_1 - y_2 \in \operatorname{int} P$, *then* $\xi_e(y_1) > \xi_e(y_2)$.

3. Existence for the Solution of (CWNEP)

Throughout this section, let E_i ($i \in I$) be a locally convex Hausdorff topological vector space, and let Z_i be a real Hausdorff topological vector space. Let X_i be a nonempty, compact convex subset of Z_i, respectively. Let $C_i \subset Z_i$ be a closed convex and pointed cone with $e_i \in \operatorname{int} C_i$. Suppose that $S_i : X \to 2^{X_i}$ is a continuous set-valued mapping with compact convex values and $F_i : X_i \times X^i \to 2^{Z_i}$ is a continuous set-valued mapping with compact values. For every $i \in I$, set $\xi_{e_i}(F_i(x,y)) = \bigcup_{u_i \in F_i(x,y)} \xi_{e_i}(u_i)$.

Lemma 5 (see [11]). *Let E be a nonempty compact convex subset of a locally convex Hausdorff topological space X. If $G : E \to 2^E$ is upper semicontinuous and, for each $x \in E$, $G(x)$ is a nonempty, closed, and convex subset, then there exists an $\overline{x} \in E$ such that $\overline{x} \in G(\overline{x})$.*

Theorem 6. *Suppose that the following conditions hold:*

(i) $S_i : X \to 2^{X_i}$ *is continuous with compact convex values;*

(ii) $F_i : X_i \times X^i \to 2^{Z_i}$ *are continuous with compact values;*

(iii) *for each fixed* $x_i \in X_i$, $F_i(\cdot, x^i)$ *is generalized Luc's quasi-C_i-convex.*

Then, there exists an $\overline{x} \in X_i \times X^i$ such that, for each $i \in I$, $u_i \in S_i(\overline{x})$, and $\overline{z}_i \in F_i(\overline{x}_i, \overline{x}^i)$, there exists $z_i \in F_i(u_i, \overline{x}^i)$ satisfying

$$z_i - \overline{z}_i \notin -\operatorname{int} C_i. \tag{6}$$

Proof. We define a set-valued mapping $A_i : X \to 2^{X_i}$ by

$$\begin{aligned} A_i(x) = \Big\{ & u_i \in S_i(x) \mid \max \xi_{e_i}\left(F_i\left(u_i, x^i\right)\right) \\ & = \min \bigcup_{x_i \in S_i(x)} \max \xi_{e_i}\left(F_i\left(x_i, x^i\right)\right) \Big\}. \end{aligned} \tag{7}$$

It follows from [12, pages 110–119, Propositions 6 and 21] that $\max \xi_{e_i}(F_i(\cdot, x^i))$ is upper semicontinuous for each fixed $x^i \in X^i$. By [12, page 112, Proposition 11], the set

$$\bigcup_{\theta \in S(x)} \max \xi_{e_i}(F(\theta, y)) \tag{8}$$

is compact. Therefore, $A_i(x)$ is nonempty for every $x \in X$.

Let

$$\begin{gathered} \{x_n\} \in X, \quad x_n \longrightarrow x_0, \\ u_{i,n} \in A_i(x_n), \quad u_{i,n} \longrightarrow u_{i,0}. \end{gathered} \tag{9}$$

We must show that $u_{i,0} \in A_i(x_0)$. First, note that $u_{i,n} \in A_i(x_n)$ and then $u_{i,n} \in S_i(x_n)$. As $S_i(\cdot)$ is upper semicontinuous and the set $S_i(x_0)$ is compact, it follows that $u_{i,0} \in S_i(x_0)$. Suppose that $u_{i,0} \notin A_i(x_0)$. Then, there exists a vector $w_{i,0} \in S_i(x_0)$ satisfying

$$\max \xi_{e_i}\left(F_i\left(w_{i,0}, x_0^i\right)\right) < \max \xi_{e_i}\left(F_i\left(u_{i,0}, x_0^i\right)\right). \quad (10)$$

As $S_i(\cdot)$ is lower semicontinuous, there exists $w_{i,n} \in S_i(x_n)$, such that $w_{i,n} \to w_{i,0}$. It follows from compactness of $F_i(w_{i,n}, x_n^i)$ that there exists $z_{i,n} \in F_i(w_{i,n}, x_n^i)$ such that

$$\xi_{e_i}\left(z_{i,n}\right) = \max \xi_{e_i}\left(F_i\left(w_{i,n}, x_n^i\right)\right). \quad (11)$$

It follows from the upper semicontinuity of $F_i(\cdot,\cdot)$ and the compactness of $X^i \times X_i$ that $F_i(x^i, x_i)$ is compact. Hence, for the net $\{z_{i,n}\}$, there exists a subnet of $\{z_{i,n}\}$ converging to $z_{i,0}$. Without loss of generality, assume $z_{i,n} \to z_{i,0}$. Now we prove that

$$\xi_{e_i}\left(z_{i,0}\right) = \max \xi_{e_i}\left(F_i\left(w_{i,0}, x_0^i\right)\right). \quad (12)$$

Since the mapping $F_i(\cdot,\cdot)$ is upper semicontinuous and the set $F_i(w_{i,0}, x_0^i)$ is compact, we have $\xi_{e_i}(z_{i,0}) \in \xi_{e_i}(F_i(w_{i,0}, x_0^i))$.

Now, suppose that $\xi_{e_i}(z_{i,0}) \neq \max \xi_{e_i}(F_i(w_{i,0}, x_0^i))$. Namely, there exists $v_{i,0} \in F_i(w_{i,0}, x_0^i)$ such that $\xi_{e_i}(v_{i,0}) > \xi_{e_i}(z_{i,0})$. As $F_i(\cdot,\cdot)$ is lower semicontinuous, there exists $v_{i,n} \in F_i(w_{i,n}, x_n^i)$ such that $v_{i,n} \to v_{i,0}$. Since $\xi_{e_i}(\cdot)$ is continuous, for n large enough,

$$\xi_{e_i}\left(v_{i,n}\right) > \xi_{e_i}\left(z_{i,n}\right), \quad (13)$$

which is a contradiction to (11).

From the compactness of $F_i(u_{i,n}, x_n^i)$, we take $\tilde{z}_{i,n} \in F(u_{i,n}, x_n^i)$ such that

$$\xi_{e_i}\left(\tilde{z}_{i,n}\right) = \max \xi_{e_i}\left(F_i\left(u_{i,n}, x_n^i\right)\right). \quad (14)$$

By the compactness of $F_i(x_i, x^i)$, we can choose a converging subnet of $\{\tilde{z}_{i,n}\}$, which is denoted without loss of generality by the original net $\{\tilde{z}_{i,n}\}$. Assume $\tilde{z}_{i,n} \to \tilde{z}_{i,0}$. Similar to the preceding proof, we have

$$\xi_{e_i}\left(\tilde{z}_{i,0}\right) = \max \xi_{e_i}\left(F\left(u_{i,0}, x_0^i\right)\right). \quad (15)$$

Then, by (10), $\xi_{e_i}(z_{i,0}) < \xi_{e_i}(\tilde{z}_{i,0})$.

It follows from the continuity of $\xi_{e_i}(\cdot)$ that $\xi_{e_i}(z_{i,n}) \to \xi_{e_i}(z_{i,0})$ and $\xi_{e_i}(\tilde{z}_{i,n}) \to \xi_{e_i}(\tilde{z}_{i,0})$. Therefore, $\xi_{e_i}(z_{i,n}) < \xi_{e_i}(\tilde{z}_{i,n})$, when n is large enough. It is said that

$$\max \xi_{e_i}\left(F_i\left(w_{i,n}, x_n^i\right)\right) < \max \xi_{e_i}\left(F_i\left(u_{i,n}, x_n^i\right)\right). \quad (16)$$

By the definition of $A_i(\cdot)$ and $u_{i,n} \in A_i(x_n)$, we have

$$\max \xi_{e_i}\left(F_i\left(u_{i,n}, x_n^i\right)\right) = \min \bigcup_{x_{i,n} \in S_i(x_n)} \max \xi_{e_i}\left(F_i\left(x_{i,n}, x_n^i\right)\right). \quad (17)$$

This, however, contradicts the fact $u_{i,n} \in A_i(x_n)$. Therefore, the mapping $A_i(\cdot)$ is closed.

Let $u_{i,1}, u_{i,2} \in A_i(x)$, $\lambda \in (0,1)$, and

$$\alpha_0 = \min \bigcup_{\theta_i \in S_i(x)} \max \xi_{e_i}\left(F_i\left(\theta_i, x^i\right)\right). \quad (18)$$

From the definition of $A_i(\cdot)$, we have $u_{i,1}, u_{i,2} \in S_i(x)$ and

$$\max \xi_{e_i}\left(F\left(u_{i,1}, x^i\right)\right) = \max \xi_{e_i}\left(F\left(u_{i,2}, x^i\right)\right) = \alpha_0. \quad (19)$$

As $S_i(x)$ is convex-valued, $\lambda u_{i,1} + (1-\lambda)u_{i,2} \in S_i(x)$.

According to the generalized Luc's quasi-C_i-convexity of $F_i(\cdot, x^i)$, we get that, for all $z_i' \in F_i(\lambda u_{i,1} + (1-\lambda)u_{i,2}, x^i)$, there exist $z_{i,1} \in F_i(u_{i,1}, x^i)$ and $z_{i,2} \in F_i(u_{i,2}, x^i)$ such that

$$z_i' \in z_i - C_i, \quad \forall z_i \in C\left(z_{i,1}, z_{i,2}\right). \quad (20)$$

Without loss of generality, suppose $l_1 = \xi_{e_i}(z_{i,1})$ and $l_2 = \xi_{e_i}(z_{i,2})$, $l_1 \geq l_2$; we have $z_{i,1} \in l_1 e_i - C_i$ and $z_{i,2} \in l_2 e_i - C_i \subset l_1 e_i - C_i$. From (20), $z_i' \in l_1 e_i - C_i$. By the monotonicity of $\xi_{e_i}(\cdot)$,

$$\xi_{e_i}\left(z_i'\right) \leq \xi_{e_i}\left(l_1 e_i\right) = l_1. \quad (21)$$

As

$$l_1 \leq \max\left(\max \xi_{e_i}\left(F\left(u_{i,1}, x^i\right)\right), \max \xi_{e_i}\left(F\left(u_{i,2}, x^i\right)\right)\right) = \alpha_0. \quad (22)$$

therefore, $\xi_{e_i}(z_i') \leq \alpha_0$.

Since

$$z_i' \in F_i\left(\lambda u_{i,1} + (1-\lambda) u_{i,2}, x^i\right) \quad (23)$$

is arbitrary, we have

$$\max \xi_{e_i}\left(F_i\left(\lambda u_{i,1} + (1-\lambda) u_{i,2}, x^i\right)\right) \leq \alpha_0. \quad (24)$$

By the fact that $F_i(\lambda u_{i,1} + (1-\lambda)u_{i,2}, x^i)$ is compact and $\xi_{e_i}(\cdot)$ is continuous, there exists

$$\tilde{z}_i \in F_i\left(\lambda u_{i,1} + (1-\lambda) u_{i,1}, x^i\right) \quad (25)$$

such that

$$\xi_{e_i}\left(\tilde{z}_i\right) = \max \xi_{e_i}\left(F_i\left(\lambda u_{i,1} + (1-\lambda) u_{i,2}, x^i\right)\right). \quad (26)$$

Thus, $\xi_{e_i}(\tilde{z}_i) \leq \alpha_0$. It follows from the definition of α_0 that

$$\max \xi_{e_i}\left(F_i\left(\lambda u_{i,1} + (1-\lambda) u_{i,2}, x^i\right)\right) = \alpha_0. \quad (27)$$

Thus, $\lambda u_{i,1} + (1-\lambda)u_{i,2} \in A_i(x)$; namely, $A_i(x)$ is a convex set.

Define $A : X \to 2^X$ by $A(x) = \Pi_{i \in I} A_i(x), \forall x \in X$. Therefore, $A(x)$ is a nonempty, convex, and closed subset of X for each $x \in X$. Since $A_i(\cdot)$ is closed, so is $A(\cdot)$, and since $A(x) \subseteq X$, X is compact, by [12, page 111, Corollary 9], $A(\cdot)$ is upper semicontinuous. By Lemma 5, there exists a point $\bar{x} \in X$ such that $\bar{x} \in A(\bar{x})$.

By the definition of $A(\cdot)$, we have

$$\overline{x}_i \in S_i(\overline{x}),$$
$$\max \xi_{e_i}\left(F_i\left(x_i, \overline{x}^i\right)\right) \geq \max \xi_{e_i}\left(F\left(\overline{x}_i, \overline{x}^i\right)\right) \quad (28)$$
$$\forall x_i \in S_i(\overline{x}), \quad i \in I.$$

From (28), $\forall \overline{z}_i \in F_i(\overline{x}_i, \overline{x}^i)$,

$$\max \xi_{e_i}\left(F_i\left(x_i, \overline{x}^i\right)\right) \geq \xi_{e_i}\left(\overline{z}_i\right). \quad (29)$$

By the compactness of $F_i(x_i, \overline{x}^i)$ and the continuity of $\xi_{e_i}(\cdot)$, there exists $z_i \in F_i(x_i, \overline{x}^i)$, such that $\xi_{e_i}(z_i) = \max \xi_{e_i}(F_i(x_i, \overline{x}^i))$. Thus, for all $\overline{z}_i \in F(\overline{x}_i, \overline{x}^i)$, there exists $z_i \in F_i(x_i, \overline{x}^i)$ such that $\xi_{e_i}(\overline{z}_i) \leq \xi_{e_i}(z_i)$. Then, it follows from the subadditivity of $\xi_{e_i}(\cdot)$ that

$$\xi_{e_i}\left(z_i - \overline{z}_i\right) \geq 0. \quad (30)$$

By Lemma 4, we get

$$z_i - \overline{z}_i \notin -\operatorname{int} P. \quad (31)$$

So $\overline{x}$ is a solution of (CWNEP) and this completes the proof. □

Let X, Y, and Z be real Hausdorff topological vector spaces, and let C and D be two compact subsets of X and Y, respectively.

Corollary 7. *Let X, Y, and Z be real Hausdorff topological vector spaces, and let C and D be two nonempty subsets of X and Y, respectively. Let $P \subset Z$ be a closed convex and pointed cone with* $\operatorname{int} P \neq \emptyset$. *Assume that*

(1) $S : C \times D \to 2^C$ *and* $T : C \times D \to 2^D$ *are continuous and compact, and for each* $(x, y) \in C \times D$, $S(x, y)$ *and* $T(x, y)$ *are nonempty, closed convex subsets;*

(2) $f, g : C \times D \to Z$ *are continuous;*

(3) *for any fixed* $y \in D$, $f(\cdot, y)$ *is Luc's quasi-P-convex; for any fixed* $x \in C$, $g(x, \cdot)$ *is Luc's quasi-P-convex.*

Then there exists $(\overline{x}, \overline{y}) \in C \times D$ *such that* $\overline{x} \in S(\overline{x}, \overline{y})$, $\overline{y} \in T(\overline{x}, \overline{y})$, *and*

$$f(x, \overline{y}) - f(\overline{x}, \overline{y}) \notin -\operatorname{int} P, \quad \forall x \in S(\overline{x}, \overline{y}),$$
$$g(\overline{x}, y) - g(\overline{x}, \overline{y}) \notin -\operatorname{int} P, \quad \forall y \in T(\overline{x}, \overline{y}). \quad (32)$$

Remark 8. Since both the class of properly quasi-P-convex functions and the class of P-convex functions (see [7]) are larger than the class of Luc's quasi-P-convex functions, Corollary 7 improves [7, Theorem].

Example 9. Suppose that $X = Y = R$, $C = D = [0, 1]$, and $P = R^3_+$ and let $S : C \times D \to 2^C$ and $T : C \times D \to 2^D$ be defined as $S(x, y) = C$ and $T(x, y) = D$, respectively. For all $(x, y) \in R^2$, let

$$f(x, y) = \left(x^2, 1 - x^2, y\right),$$
$$g(x, y) = \left(x, y^2, 1 - y^2\right). \quad (33)$$

It is clear that the mappings f and g are not properly quasi-P-convex (see [7]), but all the conditions of Corollary 7 hold. It is easy to see from [7] that both the class of properly quasi-P-convex functions and the class of P-convex functions (see [7]) are larger than the class of Luc's quasi-P-convex functions, and then Corollary 7 improves [7, Theorem].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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