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Research Article

New Results on Passivity Analysis for Uncertain Neural Networks with Time-Varying Delay

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The paper investigates the stability and passivity analysis problems for a class of uncertain neural networks with time-delay via delta operator approach. Both the parameter uncertainty and the generalized activation functions are considered in this paper. By constructing an appropriate Lyapunov-Krasovskii functional, some new stability and passivity conditions are obtained in terms of linear matrix inequalities (LMIs). The main characteristic of this paper is to obtain novel stability and passivity analysis criteria for uncertain neural networks with time-delay in the delta operator system framework. A numerical example is presented to demonstrate the effectiveness of the proposed results.

1. Introduction

Recently, neural networks have attracted considerable attention due to their applications in wide areas such as associative memory [1, 2], pattern recognition [1, 3], and optimization problems [4–7]. Recently, some stability conditions [8–12] and passivity analysis for neural networks [13–15] have been reported in the literature. The effect of time-delays [16–25] cannot be ignored in the real systems due to the facts that the delays can lead to instability [26–29], oscillation, or chaos. Recently, the stability results of time-delay neural networks have been presented in [30–36]. In addition, many results on passivity of neural networks with time-delay have been proposed [37–41].

It is well known that the discrete systems are often used for computer realization and continuous systems are frequently applied to theoretical analysis, respectively. Sampling continuous systems can lead to considerable discrete systems. When the sampling is fast using the traditional shift operator, the poles are located in the stable boundary. Then, the discrete systems will lose stability in finite word length computer. Goodwin proposed delta operator approach in [42] which is used to replace the aforementioned operator with sample continuous systems, which can unify some previous related results of the continuous and discrete systems into the framework of the delta operator systems. The delta operator is defined by

$$\delta x(t) = \begin{cases} \dfrac{dx(t)}{dt}, & T = 0; \\ \dfrac{x(t+T) - x(t)}{T}, & T \neq 0, \end{cases} \quad (1)$$

where T is a sampling period. More recently, much attention has been focused on the stability and stabilization problems for some delta operator systems [43–47]. However, it should be mentioned that there are few achievements about passivity analysis for uncertain discrete neural networks with time-varying delay via delta operator approach; instability exists in the applications of neural networks when the sampling rate is high, which motivates this research.

In this paper, the stability and passivity problems are investigated for uncertain neural networks with time-varying delay via delta operator approach. Both the parameter uncertainty and the generalized activation functions are considered in this paper. By choosing a new type of Lyapunov functional in delta domain and employing some novel methods to handle the delays, some stability and passivity criteria are proposed. The proposed conditions are expressed in terms of linear matrix inequalities (LMIs), which are dependent on the sampling period. The main characteristic of this paper is to obtain some stability and passivity analysis criteria for uncertain neural networks with time-varying delay in the delta operator system framework. Finally, a numerical example is given to demonstrate the effectiveness of the developed results.

Notation. Throughout this paper, for the sake of convenience, we use t_k to denote kT, where T is the sampling period. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The superscript T stands for matrix transposition. The notation diag$\{\cdots\}$ denotes a block-diagonal matrix. For real symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the $X - Y$ is positive semidefinite (resp., positive-definite). I is the identity matrix with appropriate dimensions. The symbol "$*$" stands for the symmetric term in a matrix. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions.

2. Problem Formulation

Consider the following uncertain neural network with time-varying delay:

$$\begin{aligned}\delta x(t_k) = & -(W_1 + \Delta W_1(t_k))\, x(t_k) \\ & + (W_2 + \Delta W_2(t_k))\, f(x(t_k)) \\ & + (W_3 + \Delta W_3(t_k))\, f(x(t_k - d_k)) \\ & + u(t_k),\end{aligned} \quad (2)$$

$$y(t_k) = f(x(t_k)), \quad (3)$$

where $x(t_k) \in \mathbb{R}^n$ stands for the network state at time t_k; $f(x(t_k)) \in \mathbb{R}^n$ denotes the activation at time t_k; $u(t_k) \in \mathbb{R}^n$ is the external input at time t_k; $y(t_k)$ is the output vector; W_1 is a positive diagonal matrix; $\Delta W_1(t_k)$, $\Delta W_2(t_k)$, and $\Delta W_3(t_k)$ are unknown matrices; d_k is a time-varying delay $0 \leq d_m \leq d_k \leq d_M$, with $d_m = n_m T$ and $d_M = n_M T$; n_m and n_M are two known positive and finite integers; T is the sampling period. The time-varying parameter uncertainties $\Delta W_1(t_k)$, $\Delta W_2(t_k)$, and $\Delta W_3(t_k)$ are assumed to be in the following form:

$$\begin{aligned}&[\Delta W_1(t_k) \;\; \Delta W_2(t_k) \;\; \Delta W_3(t_k)] \\ &\quad = HF(t_k)[E_1 \;\; E_2 \;\; E_3],\end{aligned} \quad (4)$$

where H, E_1, E_2, and E_3 are known constant matrices; $F(t_k)$ is an unknown time-varying matrix satisfying

$$F^T(t_k) F(t_k) \leq I. \quad (5)$$

The activation function $f(x(t_k))$ satisfies the following condition:

$$K_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq K_i^+, \quad i = 1, 2, \ldots, n, \quad (6)$$

with $\alpha_1 \neq \alpha_2$,

$$\begin{aligned}K^- &= \operatorname{diag}[K_1^- \;\; K_2^- \;\; \cdots \;\; K_n^-], \\ K^+ &= \operatorname{diag}[K_1^+ \;\; K_2^+ \;\; \cdots \;\; K_n^+].\end{aligned} \quad (7)$$

Before ending this section, some preliminaries are recalled which are used to prove the main results in the next section.

Definition 1 (see [48]). A delta operator system is asymptotically stable, if the following conditions hold:

(i) $V(x(t)) \geq 0$ with equality if and only if $x(t) = 0$;

(ii) $\delta V(x(t)) = [V(x(t+T)) - V(x(t))]/T < 0$,

where $V(x(t))$ is a Lyapunov function in the delta domain.

Definition 2. The neural network (2) and (3) is called passive if there exists a scalar $\gamma \geq 0$ such that

$$\sum_{k=0}^{\infty} \left[-\gamma u^T(t_k) u(t_k) - 2y^T(t_k) u(t_k)\right] \leq 0. \quad (8)$$

Lemma 3 (see [49]). *The property of delta operator for any time function $x(t)$ and $y(t)$ can be represented as*

$$\delta(x(t) y(t)) = \delta(x(t)) y(t) + x(t) \delta y(t) + T\delta(x(t)) \delta y(t), \quad (9)$$

where T is a sampling period.

Lemma 4 (see [50]). *For any constant positive semidefinite symmetric matrix W, two positive integers r and r_0 satisfy $r \geq r_0 \geq 1$; the following inequality holds:*

$$\begin{aligned}&\left(\sum_{i=r_0}^{r} x(i)\right)^T W \left(\sum_{i=r_0}^{r} x(i)\right) \\ &\quad \leq (r - r_0 + 1) \sum_{i=r_0}^{r} x^T(i) W x(i).\end{aligned} \quad (10)$$

Lemma 5 (see [51]). *Let U, V, W, and M be real matrices of appropriate dimensions with M satisfying $M = M^T$; then $M + UVW + W^T V^T U^T < 0$ for all $V^T V \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that $M + \varepsilon^{-1} UU^T + \varepsilon W^T W < 0$.*

Lemma 6 (see [52] Schur complement). *Given constant matrices Ω_1, Ω_2, and Ω_3 with appropriate dimensions, where $\Omega_1^T = \Omega_1$, and $\Omega_2^T = \Omega_2 > 0$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$, if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0. \quad (11)$$

3. Main Results

In this section, the stability and the passivity results for discrete-time uncertain neural network with time-varying delay via delta operator are given. Firstly, the stability conditions are given in the following part.

3.1. Stability Analysis. In order to consider the stability condition for uncertain neural networks with time-delay (2), we define $\Delta W_i(t_k) = 0$ $(i = 1, 2, 3)$ and $u(t_k) = 0$ in (2). Then, we can have the following neural networks with time-delay:

$$\delta x(t_k) = -W_1 x(t_k) + W_2 f(x(t_k)) + W_3 f(x(t_k - d_k)). \quad (12)$$

For neural networks with time-delay in (12), the stability criterion is obtained in the following theorem.

Theorem 7. *For given scalars $0 \le d_m \le d_k \le d_M$, neural network (12) with (6) is asymptotically stable, if there exist $P > 0$, $Q_i > 0$ $(i = 1, 2, 3)$, $R > 0$, and $S > 0$, positive definite diagonal matrices L_1, L_2, X_1, and X_2, such that the following LMI holds:*

$$\Xi = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0, \quad (13)$$

where

$$\Theta_{11} = \begin{bmatrix} \Psi_{11} & -SW_1 & 0 \\ * & \Psi_{22} & 0 \\ * & * & -Q_2 \end{bmatrix},$$

$$\Theta_{12} = \begin{bmatrix} 0 & SW_2 & SW_3 \\ \frac{1}{n_M T} R & PW_2 & \Psi_{25} \\ 0 & 0 & X_2 K^+ \end{bmatrix},$$

$$\Theta_{22} = \begin{bmatrix} -Q_1 - \frac{1}{n_M T} R & 0 & 0 \\ * & \Psi_{55} & 0 \\ * & * & \Psi_{66} \end{bmatrix},$$

$$\begin{aligned} \Psi_{11} &= TP + Tn_M R - S - S^T, \\ \Psi_{22} &= -2L_1 K^- + 2L_2 K^+ + Q_1 + Q_2 \\ &\quad + T(d+1) Q_2 - \frac{1}{n_M T} R \\ &\quad - PW_1 - W_1^T P, \\ \Psi_{25} &= L_1 - L_2 + X_1 K^+ + PW_2, \\ \Psi_{55} &= -2X_1 + Q_3, \\ \Psi_{66} &= -2X_2 - Q_3, \\ L_1 &= \text{diag}\{l_{11} \ \ l_{12} \ \cdots \ l_{1n}\}, \\ L_2 &= \text{diag}\{l_{21} \ \ l_{22} \ \cdots \ l_{2n}\}, \\ X_1 &= \text{diag}\{\lambda_{11} \ \ \lambda_{12} \ \cdots \ \lambda_{1n}\}, \\ X_2 &= \text{diag}\{\lambda_{21} \ \ \lambda_{22} \ \cdots \ \lambda_{2n}\}. \end{aligned} \quad (14)$$

Proof. Choose a Lyapunov-Krasovskii functional in delta domain as follows:

$$V(t_k) = \sum_{i=1}^{5} V_i(t_k), \quad (15)$$

with

$$\begin{aligned} V_1(t_k) &= x^T(t_k) P x(t_k), \\ V_2(t_k) &= 2T \sum_{j=1}^{n} l_{1j} \int_0^{x_j(t_k - T)} \left(f_j(s) - K_j^- s\right) ds \\ &\quad + 2T \sum_{j=1}^{n} l_{2j} \int_0^{x_j(t_k - T)} \left(K_j^+ s - f_j(s)\right) ds, \\ V_3(t_k) &= T \sum_{i=1}^{n_M} x^T(t_k - iT) Q_1 x(t_k - iT) \\ &\quad + T \sum_{i=1}^{n_k} x^T(t_k - iT) Q_2 x(t_k - iT) \\ &\quad + T \sum_{i=1}^{n_k} f^T(x(t_k - iT)) Q_3 f(x(t_k - iT)), \\ V_4(t_k) &= T^2 \sum_{i=n_m}^{n_M} \sum_{j=1}^{i} x^T(t_k - jT) Q_3 x(t_k - jT), \\ V_5(t_k) &= \sum_{i=1}^{n_M} \sum_{j=1}^{i} e^T(t_k - jT) \text{Re}(t_k - jT), \end{aligned} \quad (16)$$

where

$$e(t_k) = x(t_k) - x(t_k + T) = -T \delta x(t_k). \quad (17)$$

Applying Lemma 3, the delta-domain form of $V_1(t_k)$ can be obtained as

$$\begin{aligned} \delta V_1(t_k) &= \delta^T x(t_k) P x(t_k) + x^T(t_k) P \delta x(t_k) \\ &\quad + T \delta^T x(t_k) P \delta x(t_k) \\ &= -x^T(t_k) W_1^T P x(t_k) + x^T(t_k) W_2^T P f(x(t_k)) \\ &\quad + x^T(t_k) W_3^T P f(x(t_k - d_k)) \\ &\quad - x^T(t_k) P W_1 x(t_k) + x^T(t_k) P W_2 f(x(t_k)) \\ &\quad + x^T(t_k) P W_3 f(x(t_k - d_k)) \\ &\quad + T \delta^T x(t_k) P \delta x(t_k). \end{aligned} \quad (18)$$

From (6), for the scalars $\lambda_{1i} \ge 0, \lambda_{2i} \ge 0$, one can have

$$\begin{aligned} &2 \sum_{i=1}^{n} \lambda_{1i} f_i(x_i(t_k)) \left[f_i(x_i(t_k)) - K_i^+ x_i(t_k) \le 0\right], \\ &2 \sum_{i=1}^{n} \lambda_{2i} f_i x_i(t_k - n_k T) \\ &\quad \times \left[f_i(x_i(t_k - n_k T)) - K_i^+ x_i(t_k - n_k T)\right] \le 0, \end{aligned} \quad (19)$$

which can be equivalently denoted as

$$\begin{aligned}&2f^T(x(t_k))X_1f(x(t_k))\\&\qquad-2f^T(x(t_k))K^+X_1x(t_k)\le 0,\\&2f^T(x(t_k-n_kT))X_2f(x(t_k-n_kT))\\&\qquad-2f^T(x(t_k-n_kT))K^+X_2x(t_k-n_kT)\le 0.\end{aligned}\tag{20}$$

Then, we obtain the following inequality:

$$\begin{aligned}\delta V_2(t_k)&=2\sum_{j=1}^{n}l_{1j}\left[\int_0^{x_j(t_k)}\left(f_j(s)-K_j^-s\right)ds-\int_0^{x_j(t_k-T)}\left(f_j(s)-K_j^-s\right)ds\right]\\&\quad+2\sum_{j=1}^{n}l_{2j}\left[\int_0^{x_j(t_k)}\left(K_j^+s-f_j(s)\right)ds-\int_0^{x_j(t_k-T)}\left(K_j^+s-f_j(s)\right)ds\right]\\&\le 2\sum_{j=1}^{n}l_{1j}x_j(t_k)\left[f_j\left(x_j(t_k)\right)-K_j^-x(t_k)\right]\\&\quad+2\sum_{j=1}^{n}l_{2j}x_j(t_k)\left[K_j^+x(t_k)-f_j\left(x_j(t_k)\right)\right]\\&\le 2x^T(t_k)L_1\left[f(x(t_k))-K^-x(t_k)\right]\\&\quad+2x^T(t_k)L_2\left[K^+x(t_k)-f(x(t_k))\right]\\&\quad-2f^T(x(t_k))X_1f(x(t_k))\\&\quad+2f^T(x(t_k))K^+X_1x(t_k)\\&\quad-2f^T(x(t_k-n_kT))X_2f(x(t_k-n_kT))\\&\quad+2f^T(x(t_k-n_kT))K^+X_2x(t_k-n_kT).\end{aligned}\tag{21}$$

Taking the delta operator manipulations of $V_3(t_k)$, we can obtain that

$$\begin{aligned}&\delta V_3(t_k)\\&=\frac{1}{T}\times T\left[\sum_{i=1}^{n_M}x^T(t_k-iT+T)Q_1x(t_k-iT+T)-\sum_{i=1}^{n_M}x^T(t_k-iT)Q_1x(t_k-iT)\right]\\&\quad+\frac{1}{T}\times T\left[\sum_{i=1}^{n_k}x^T(t_k-iT+T)Q_2x(t_k-iT+T)-\sum_{i=1}^{n_k}x^T(t_k-iT)Q_2x(t_k-iT)\right]\\&\quad+\frac{1}{T}\times T\left[\sum_{i=1}^{n_k}f^T(x(t_k-iT+T))\times Q_3f(x(t_k-iT+T))-\sum_{i=1}^{n_k}f^T(x(t_k-iT))Q_3f(x(t_k-iT))\right]\\&\le x^T(t_k)Q_1x(t_k)-x^T(t_k-n_MT)Q_1x(t_k-n_MT)\\&\quad+x^T(t_k)Q_2x(t_k)-x^T(t_k-n_kT)Q_2x(t_k-n_kT)\\&\quad+T\sum_{i=n_m}^{n_M}x^T(t_k-iT)Q_2x(t_k-iT)\\&\quad+f^T(x(t_k))Q_3f(x(t_k))-f^T(x(t_k-n_kT))\\&\quad\times Q_3f(x(t_k-n_kT)).\end{aligned}\tag{22}$$

Taking the delta operator manipulations of $V_4(t_k)$, the following results can be obtained:

$$\begin{aligned}&\delta V_4(t_k)\\&=\frac{1}{T}\times T^2\left[\sum_{i=n_m}^{n_M}\sum_{j=1}^{i}x^T(t_k-jT+T)\times Q_3x(t_k-jT+T)-\sum_{i=n_m}^{n_M}\sum_{j=1}^{i}x^T(t_k-jT)Q_3x(t_k-jT)\right]\end{aligned}\tag{23}$$

$$\begin{aligned}&=T\left[\sum_{i=n_m}^{n_M}x^T(t_k)Q_3x(t_k)\times\sum_{i=n_m}^{n_M}-x^T(t_k-iT)Q_3x(t_k-iT)\right]\\&=T(d+1)x^T(t_k)Q_3x(t_k)\\&\quad-T\sum_{i=n_m}^{n_M}x^T(t_k-iT)Q_3x(t_k-iT),\end{aligned}\tag{24}$$

where $d=n_M-n_m$.

Taking the delta operator manipulations of $V_5(t_k)$ and using Lemma 4, it can be found that

$$\begin{aligned}&\delta V_5(t_k)\\&=\frac{1}{T}\left[\sum_{i=1}^{n_M}\sum_{j=1}^{i}e^T(t_k-jT+T)\times \mathrm{Re}(t_k-jT+T)-\sum_{i=1}^{n_M}\sum_{j=1}^{i}e^T(t_k-jT)\mathrm{Re}(t_k-jT)\right]\\&=\frac{n_M}{T}e^T(t_k)\mathrm{Re}(t_k)\\&\quad-\frac{1}{T}\sum_{i=1}^{n_M}e^T(t_k-iT)\mathrm{Re}(t_k-iT)\end{aligned}$$

$$\begin{aligned} &\leq T n_M \delta^T x(t_k) R \delta x(t_k) \\ &\quad - \frac{1}{n_M T} [x(t_k - n_M T) - x(t_k)]^T \\ &\quad \times R [x(t_k - n_M T) - x(t_k)]. \end{aligned} \tag{25}$$

For a given positive definite matrix P_1 with appropriately dimensions, one has that

$$\begin{aligned} r_1(t_k) = 2\delta^T x(t_k) S [&-W_1 x(t_k) + W_2 f(x(t_k)) \\ &+ W_3 f(x(t_k - d_k)) - \delta x(t_k)] = 0. \end{aligned} \tag{26}$$

Combining (18) and (21)–(26), the following inequality holds:

$$\begin{aligned} \delta V(t_k) &= \sum_{i=1}^{5} \delta V_i(t_k) \\ &\leq \eta^T(x(t_k)) \Xi \eta(x(t_k)), \end{aligned} \tag{27}$$

where

$$\begin{aligned} \eta^T(x(t_k)) = \big[&\delta^T x(t_k), x^T(t_k), x^T(t_k - n_k T), \\ &x^T(t_k - n_M T), f^T(x(t_k)), \\ &f^T(x(t_k - n_k T))\big]. \end{aligned} \tag{28}$$

It can be seen from Theorem 7 that $\Xi < 0$, which means that $\delta V(t_k) < 0$. Then, based on Definition 1, neural network (12) is asymptotically stable. The proof is completed. □

In the subsection, we continue to consider the robust stability problem of neural network (2) without inputs $u(t_k)$; equivalently, we have

$$\begin{aligned} \delta x(t_k) = &-(W_1 + \Delta W_1(t_k)) x(t_k) \\ &+ (W_2 + \Delta W_2(t_k)) f(x(t_k)) \\ &+ (W_3 + \Delta W_3(t_k)) f(x(t_k - d_k)). \end{aligned} \tag{29}$$

For neural network (29), the robust stability criterion is given in the following theorem

Theorem 8. *For given scalars $0 \leq d_m \leq d_k \leq d_M$, neural network (29) under the conditions (4)–(6) is robustly asymptotically stable, if there exist $P > 0$, $Q_i > 0$ $(i = 1, 2, 3)$, $R > 0$, $S > 0$ positive definite diagonal matrices L_1, $L_2 X_1$, and X_2, such that the following LMI holds:*

$$\widehat{\Xi} = \begin{bmatrix} \widehat{\Theta}_{11} & \widehat{\Theta}_{12} \\ * & \widehat{\Theta}_{22} \end{bmatrix} < 0, \tag{30}$$

where

$$\begin{aligned} \widehat{\Theta}_{11} &= \begin{bmatrix} \Psi_{11} & SW_1 & 0 & 0 \\ * & \Psi_{22} & 0 & \frac{1}{n_M T} R \\ * & * & -Q_2 & 0 \\ * & * & * & -Q_1 - \frac{1}{n_M T} R \end{bmatrix}, \\ \widehat{\Theta}_{12} &= \begin{bmatrix} SW_2 & SW_3 & SH & 0 \\ \Psi_{25} & PW_3 & PH & -\varepsilon E_1^T \\ 0 & X_2 K^+ & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ \widehat{\Theta}_{22} &= \begin{bmatrix} \Psi_{55} & 0 & 0 & \varepsilon E_2^T \\ * & \Psi_{66} & 0 & \varepsilon E_3^T \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix}, \end{aligned} \tag{31}$$

with Ψ_{11}, Ψ_{22}, Ψ_{25}, Ψ_{55}, Ψ_{66}, L_1, L_2, X_1, and X_2 defined in Theorem 7.

Proof. We choose the same Lyapunov-Krasovskii functional as Theorem 7. According to neural network (2), we have the following equation:

$$\begin{aligned} r_2(t_k) = 2\delta^T x(t_k) \\ \times S [&-(W_1 + \Delta W_1(t_k)) x(t_k) \\ &+ (W_2 + \Delta W_2(t_k)) f(x(t_k)) \\ &+ (W_3 + \Delta W_3(t_k)) f(x(t_k - d_k)) \\ &- \delta x(t_k)] = 0. \end{aligned} \tag{32}$$

Similar to the proof of Theorem 7, we can have that

$$\begin{aligned} \delta V(t_k) &= \sum_{i=1}^{5} \delta V_i(t_k) \\ &= \sum_{i=1}^{5} \delta V_i(t_k) + r_2(t_k) \\ &\leq \eta^T(x(t_k)) \breve{\Xi} \eta(x(t_k)) \\ &= \eta^T(x(t_k)) \\ &\quad \times \left(\Xi + \varphi F(t_k) \psi + \psi^T F^T(t_k) \varphi^T\right) \eta(x(t_k)), \end{aligned} \tag{33}$$

where

$$\breve{\Xi} = \begin{bmatrix} \breve{\Theta}_{11} & \breve{\Theta}_{12} \\ * & \breve{\Theta}_{22} \end{bmatrix},$$

$$\breve{\Theta}_{11} = \begin{bmatrix} \Psi_{11} & -SW_{1k}(t_k) & 0 \\ * & \breve{\Psi}_{22} & 0 \\ * & * & -Q_2 \end{bmatrix},$$

$$\breve{\Theta}_{12} = \begin{bmatrix} 0 & SW_{2k}(t_k) & SW_{3k}(t_k) \\ \frac{1}{n_M T}R_2 & PW_{2k}(t_k) & \breve{\Psi}_{25} \\ 0 & 0 & X_2K^+ \end{bmatrix},$$

$$\breve{\Theta}_{22} = \begin{bmatrix} -Q_1 - \frac{1}{n_M T}R & 0 & 0 \\ * & \Psi_{55} & 0 \\ * & * & \Psi_{66} \end{bmatrix}, \quad (34)$$

$$\varphi^T = \begin{bmatrix} H^TS^T & H^TP^T & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\psi = \begin{bmatrix} 0 & -E_1 & 0 & 0 & E_2 & E_3 \end{bmatrix},$$

$$\breve{\Psi}_{22} = -2L_1K^- + 2L_2K^+ + Q_1 + Q_2 + T(d+1)Q_2 - \frac{1}{n_M T}R - PW_{1k}(t_k) - W_{1k}^T(t_k)P,$$

$$\breve{\Psi}_{25} = L_1 - L_2 + X_1K^+ + PW_{2k}(t_k),$$

with $W_{1k}(t_k) = W_1 + \Delta W_1(t_k)$, $W_{2k}(t_k) = W_2 + \Delta W_2(t_k)$, $W_{3k}(t_k) = W_3 + \Delta W_3(t_k)$, and Ξ, Ψ_{11}, Ψ_{55}, Ψ_{66}, L_1, L_2, X_1, and X_2 have been defined in Theorem 7.

Applying Schur complement to (30), we have

$$\Xi + \varepsilon^{-1}\varphi\varphi^T + \varepsilon\psi^T\psi < 0. \quad (35)$$

By Lemma 5, from the inequality (35), we can easily obtain

$$\Xi + \varphi F(t_k)\psi + \psi^T F^T(t_k)\varphi^T < 0. \quad (36)$$

Consequently, $\delta V(t_k) < 0$; from Definition 1, neural network in (29) is robustly asymptotically stable. The proof is completed. □

3.2. Passivity Analysis. In this subsection, the passivity analysis results are given in the following part. We first consider (2) and (3) without the parameter uncertainties $\Delta W_i(t_k) = 0$, $(i = 1, 2, 3)$. Then, the following neural network can be obtained:

$$\begin{aligned} \delta x(t_k) &= -W_1 x(t_k) + W_2 f(x(t_k)) \\ &\quad + W_3 f(x(t_k - d_k)) + u(t_k), \\ y(t_k) &= f(x(t_k)). \end{aligned} \quad (37)$$

Theorem 9. *For given scalars $0 \le d_m \le d_k \le d_M$, neural network is passive in (37), if there exist $P > 0$, $Q_i > 0$ $(i = 1, 2, 3)$, $R > 0$, $S > 0$ positive definite diagonal matrices L_1, L_2, X_1, and X_2, such that the following LMI holds:*

$$\widetilde{\Xi} < 0, \quad (38)$$

where

$$\widetilde{\Xi} = \begin{bmatrix} \widetilde{\Theta}_{11} & \widetilde{\Theta}_{12} \\ * & \widetilde{\Theta}_{22} \end{bmatrix},$$

$$\widetilde{\Theta}_{12} = \begin{bmatrix} 0 & SW_2 & SW_3 & S \\ \frac{1}{n_M T}R & \Psi_{25} & PW_3 & P \\ 0 & 0 & X_2K^+ & 0 \end{bmatrix}, \quad (39)$$

$$\widetilde{\Theta}_{22} = \begin{bmatrix} -Q_1 - \frac{1}{n_M T}R & 0 & 0 & 0 \\ * & \Psi_{55} & 0 & -I \\ * & * & \Psi_{66} & 0 \\ * & * & * & -\gamma I \end{bmatrix},$$

and $\widetilde{\Theta}_{11} = \Theta_{11}$, Ψ_{25}, Ψ_{55}, and Ψ_{66} have been defined in Theorem 7.

Proof. In order to present the passivity condition for neural network (37), we choose the same Lyapunov-Krasovskii functional as Theorem 7. By following the same line of proof of Theorem 7 and considering the following inequality:

$$\begin{aligned} &\sum_{k=0}^{\infty}\left[-\gamma u^T(t_k)u(t_k) - 2y^T(t_k)u(t_k)\right] \\ &\quad = \sum_{k=0}^{\infty}\left[-\gamma u^T(t_k)u(t_k) - 2y^T(t_k)u(t_k) + \delta V(t_k)\right] \\ &\qquad - V(\infty) + V(0) \\ &\quad \le \sum_{k=0}^{\infty}\left[-\gamma u^T(t_k)u(t_k) - 2y^T(t_k)u(t_k) + \delta V(t_k)\right], \end{aligned} \quad (40)$$

it can be seen from the LMI condition (40) that

$$\sum_{k=0}^{\infty}\left[-\gamma u^T(t_k)u(t_k) - 2y^T(t_k)u(t_k)\right] \le 0, \quad (41)$$

which means that neural network in (37) is passive. This finishes the proof. □

In the following theorem, the passivity condition for uncertain neural network with time-varying delay in (2) and (3) is presented.

Theorem 10. *For given scalars $0 \le d_m \le d_k \le d_M$, neural network in (2) and (3) is passive, if there exist $P > 0$, $Q_i > 0$ $(i = 1, 2, 3)$, $R > 0$, $S > 0$ positive definite diagonal matrices L_1, L_2, X_1, and X_2, such that the following LMI holds:*

$$\begin{bmatrix} \overline{\Theta}_{11} & \overline{\Theta}_{12} \\ * & \overline{\Theta}_{22} \end{bmatrix} < 0, \quad (42)$$

where

$$\overline{\Theta}_{12} = \begin{bmatrix} SW_2 & SW_3 & S & SH & 0 \\ \Psi_{25} & PW_3 & P & PH & -\varepsilon E_1^T \\ 0 & 0 & 0 & 0 & 0 \\ 0 & X_2K^+ & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{\Theta}_{22} = \begin{bmatrix} \Psi_{55} & 0 & -I & 0 & \varepsilon E_2^T \\ * & \Psi_{66} & 0 & 0 & \varepsilon E_3^T \\ * & * & -\gamma I & 0 & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix}. \quad (43)$$

and $\overline{\Theta}_{11} = \widehat{\Theta}_{11}$, Ψ_{25}, Ψ_{55}, *and* Ψ_{66} *have been defined in Theorem 8.*

Proof. In order to present the passivity condition for neural network in (2) and (3), we choose the same Lyapunov-Krasovskii functional as Theorem 7. By following the same line of proof of Theorems 8 and 9, Theorem 10 can be proved. □

4. A Numerical Example

In this section, the following numerical example is presented to demonstrate the effectiveness of the proposed results.

Example 1. Consider discrete-time neural networks (2) and (3) with the following parameters:

$$W_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix},$$
$$W_3 = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix},$$
$$H = 0.1 \times I, \quad E_1 = 0.1 \times I, \quad (44)$$
$$E_2 = 0.2 \times I, \quad E_3 = 0.3 \times I,$$
$$K^- = \operatorname{diag}\{-1, -1\},$$
$$K^+ = \operatorname{diag}\{0.9, 0.9\}.$$

In order to illustrate the effectiveness of the obtained results, we choose $T = 0.05$, $n_m = 10$, and $n_M = 20$. Then, using the Matlab LMI toolbox to solve the LMI in (42), we obtain a solution as follows:

$$P = \begin{bmatrix} 5.2080 & 2.0984 \\ 2.0984 & 9.7576 \end{bmatrix},$$
$$S = \begin{bmatrix} 1.2475 & 0.1776 \\ 0.1776 & 2.9572 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 0.2133 & -0.0219 \\ -0.0219 & 1.2999 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 0.4684 & 0.2752 \\ 0.2752 & 1.7084 \end{bmatrix},$$
$$Q_3 = \begin{bmatrix} 7.3290 & 6.0654 \\ 6.0654 & 12.4586 \end{bmatrix},$$
$$R = \begin{bmatrix} 0.6211 & 0.0621 \\ 0.0621 & 1.5979 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 0.0313 & 0 \\ 0 & 0.2507 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 2.7406 & 0 \\ 0 & 0.7734 \end{bmatrix},$$
$$X_1 = \begin{bmatrix} 9.4819 & 0 \\ 0 & 31.8737 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 0.6547 & 0 \\ 0 & 3.2283 \end{bmatrix},$$
$$\gamma = 48.7332, \quad \varepsilon = 3.1900. \quad (45)$$

5. Conclusions

In this paper, the problems of stability and passivity analysis for discrete-time neural networks with time-varying delay have been studied via delta operator approach. This paper has considered the parameter uncertainty and the generalized activation functions. By constructing appropriate Lyapunov-Krasovskii functional, some novel stability and passivity criteria have been proposed in the delta operator system framework. The obtained conditions have been expressed in terms of LMI, which can be easily solved by standard software. A numerical example has been given to illustrate the effectiveness of the proposed results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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