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Research Article

Solvability for a Coupled System of Fractional Integrodifferential Equations with m-Point Boundary Conditions on the Half-Line

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The aim of this paper is to study the solvability for a coupled system of fractional integrodifferential equations with multipoint fractional boundary value problems on the half-line. An example is given to demonstrate the validity of our assumptions.

1. Introduction

The theory of derivatives and integrals of fractional order has undergone rapid development over the years and played a very important role in modern applied mathematical models of real processes arising in phenomena studied in physics, mechanics, engineering, and so on [1–3]. Recently, the existence of solutions for coupled systems involving fractional differential equations is one of the theoretical fields investigated by many authors [4–13].

Very recently, Wang et al. [10] studied the existence of solutions for the following coupled system of nonlinear fractional differential equations by using Schauder's fixed point theorem:

$$\begin{gathered} D^{p}u(t)+f(t,v(t))=0,\quad 2<p<3,\ t\in J:=[0,\infty),\\ D^{q}v(t)+g(t,u(t))=0,\quad 2<q<3,\ t\in J:=[0,\infty),\\ u(0)=u'(0)=0,\qquad D^{p-1}u(\infty)=\sum_{i=1}^{m-2}\beta_i u(\xi_i),\qquad (1)\\ v(0)=v'(0)=0,\qquad D^{q-1}v(\infty)=\sum_{i=1}^{m-2}\gamma_i v(\xi_i), \end{gathered}$$

where $f,g\in C(J\times\mathbb{R},\mathbb{R})$, $0<\xi_1<\xi_2<\cdots<\xi_{m-2}<\infty$, and D^p and D^q denote Riemann-Liouville fractional derivatives of order p and order q, respectively; also $\beta_i>0$, $\gamma_i>0$ are such that $0<\sum_{i=1}^{m-2}\beta_i\xi_i^{p-1}<\Gamma(p)$ and $0<\sum_{i=1}^{m-2}\gamma_i\xi_i^{q-1}<\Gamma(q)$.

Motivated by [10], in this paper, we consider a coupled system of nonlinear fractional integrodifferential equations on an unbounded domain and more general boundary conditions:

$$\begin{gathered} D^{\alpha}u(t)+f\left(t,v(t),I^{\alpha}v(t)\right)=0,\quad t\in J=[0,\infty),\\ D^{\beta}v(t)+g\left(t,u(t),I^{\beta}u(t)\right)=0,\quad t\in J=[0,\infty),\\ u(0)=u'(0)=0,\qquad v(0)=v'(0)=0,\\ D^{\alpha-1}u(\infty)=\sum_{i=1}^{m-2}a_i u(\xi_i)+\sum_{i=1}^{m-2}b_i D^{\alpha-1}u(\xi_i),\\ D^{\beta-1}v(\infty)=\sum_{i=1}^{m-2}c_i v(\xi_i)+\sum_{i=1}^{m-2}d_i D^{\beta-1}v(\xi_i), \end{gathered}\qquad (2)$$

where $2<\alpha,\beta\le 3$, $0<\xi_1<\xi_2<\cdots<\xi_{m-2}<\infty$, $a_i,b_i,c_i,d_i\ge 0$ are real numbers, $f,g\in C(J\times\mathbb{R}\times\mathbb{R},\mathbb{R})$, and Ddenotes Riemann-Liouville fractional derivative. It is clear that boundary value problem (2) includes problem (1) as special case.

Integrodifferential equations have become important in recent years as mathematical models of phenomena in both the physical and social sciences. In particular, some physical phenomena involving certain type of memory effects are represented by integrodifferential equations [14–18].

However, to the best of our knowledge, no work has been reported on the existence results for coupled system of nonlinear fractional integrodifferential equations on an unbounded domain.

The paper is organized as follows. In Section 2, we recall some basic definitions, notations, and preliminary facts. Section 3 is devoted to the existence results for system of nonlinear fractional integrodifferential equations on an unbounded domain. In Section 4, an example is given to demonstrate the applicability of our results.

2. Preliminaries

In this section, we will first recall some basic definitions and lemmas which are used in what follows and can be found in [2, 19].

Definition 1. The Riemann-Liouville fractional integral of order $\delta > 0$ of a function $f : (0,\infty) \to \mathbb{R}$ is given by

$$I^{\delta} f(t) = \frac{1}{\Gamma(\delta)} \int_0^t (t-s)^{\delta-1} f(s)\,ds, \tag{3}$$

provided that the right-hand side is pointwise defined.

Definition 2. The Riemann-Liouville fractional derivative of order $\delta > 0$ of a continuous function $f : (0,\infty) \to \mathbb{R}$ is given by

$$D^{\delta} f(t) = \frac{1}{\Gamma(n-\delta)} \left(\frac{d}{dt}\right)^n \int_0^t (t-s)^{n-\delta-1} f(s)\,ds, \tag{4}$$

where $n = [\alpha] + 1$, provided that the right-hand side is pointwise defined.

Remark 3. The following properties are well known:

$$\begin{gathered} D^{\delta} I^{\delta} f(t) = f(t), \quad \delta > 0,\ f(t) \in L^1(0,\infty), \\ D^{\alpha} I^{\delta} f(t) = I^{\delta-\alpha} f(t), \quad \delta > \alpha > 0,\ f(t) \in L^1(0,\infty), \\ D^{\delta} t^{\lambda} = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\delta+1)} t^{\lambda-\delta}, \quad \lambda > -1,\ t > 0. \end{gathered} \tag{5}$$

Lemma 4. *For $\delta > 0$, the equation $D^{\delta}u(t) = 0$ is valid if and only if*

$$\begin{gathered} u(t) = c_1 t^{\delta-1} + c_2 t^{\delta-2} + \cdots + c_n t^{\delta-n}, \\ c_i \in \mathbb{R}, \quad j = 1, 2, \ldots, n, \end{gathered} \tag{6}$$

where n is the smallest integer greater than or equal to δ.

Lemma 5. *Assume that $D^{\delta}u(t) \in L^1(0,\infty)$; then,*

$$\begin{gathered} I^{\delta} D^{\delta} u(t) = u(t) + c_1 t^{\delta-1} + c_2 t^{\delta-2} + \cdots + c_n t^{\delta-n}, \\ c_i \in \mathbb{R}, \quad j = 1, 2, \ldots, n, \end{gathered} \tag{7}$$

where n is the smallest integer greater than or equal to δ.

For any $\delta > 1$ we can define the space

$$X_{\delta} = \left\{ u \in C[0,\infty) : \sup_{t\in J} \frac{|u(t)|}{1+t^{\delta-1}} < \infty \right\}, \tag{8}$$

equipped with the norm

$$\|u\|_{\delta} = \sup_{t\in J} \frac{|u(t)|}{1+t^{\delta-1}}. \tag{9}$$

Clearly, $(X_{\delta}, \|u\|_{\delta})$ is a Banach space [19]. For $(u,v) \in X_{\alpha} \times X_{\beta}$ we define

$$\|(u,v)\|_{\alpha,\beta} = \max\left\{\|u\|_{\alpha}, \|v\|_{\beta}\right\}; \tag{10}$$

then, $(X_{\alpha} \times X_{\beta}, \|\cdot\|_{\alpha,\beta})$ is a Banach space.

3. Main Results

In this section, we prove the existence results for the boundary value problem (2). For convenience we use the following notation:

$$\Delta_{\alpha} = \Gamma(\alpha) - \sum_{i=1}^{m-2} a_i \xi_i^{\alpha-1} - \Gamma(\alpha) \sum_{i=1}^{m-2} b_i. \tag{11}$$

By replacing α, a_i, b_i with β, c_i, d_i, respectively, we can define Δ_{β}.

Lemma 6. *Let $h \in C[0,\infty)$ and $\Delta_{\alpha} > 0$; then, the unique solution of*

$$D^{\alpha} u(t) + h(t) = 0, \quad 2 < \alpha < 3,\ t \in J = [0,\infty), \tag{12}$$

$$u(0) = u'(0) = 0, \tag{13}$$

$$D^{\alpha-1} u(\infty) = \sum_{i=1}^{m-2} a_i u(\xi_i) + \sum_{i=1}^{m-2} b_i D^{\alpha-1} u(\xi_i) \tag{14}$$

is given by

$$u(t) = \int_0^{\infty} G_{\alpha}(t,s) h(s)\,ds, \tag{15}$$

where $G_{\alpha}(t,s)$ is Green's function given by

$$\begin{aligned} G_{\alpha}(t,s) = {} & K_{\alpha}(t,s) + \frac{t^{\alpha-1}}{\Delta_{\alpha}} \sum_{i=1}^{m-2} a_i K_{\alpha}(\xi_i, s) \\ & + \frac{t^{\alpha-1}}{\Delta_{\alpha}} \sum_{i=1}^{m-2} b_i H(\xi_i, s), \end{aligned} \tag{16}$$

with

$$K_{\alpha}(t,s) = \frac{1}{\Gamma(\alpha)} \begin{cases} t^{\alpha-1} - (t-s)^{\alpha-1}, & 0 \le s \le t < \infty, \\ t^{\alpha-1}, & 0 \le t \le s < \infty, \end{cases} \tag{17}$$

$$H(t,s) = \begin{cases} 0, & 0 \le s \le t < \infty, \\ 1, & 0 \le t \le s < \infty. \end{cases} \tag{18}$$

Proof. By Lemma 5, the solution of (12) can be writen as

$$u(t) = c_1 t^{\alpha-1} + c_2 t^{\alpha-2} + c_3 t^{\alpha-3} - I^{\alpha} h(t). \quad (19)$$

Using the boundary conditions (13), we find that $c_2 = c_3 = 0$ and

$$D^{\alpha-1} u(t) = c_1 \Gamma(\alpha) - I^1 h(t). \quad (20)$$

Now considering the second boundary condition, we have

$$\begin{aligned} c_1 = \frac{1}{\Delta_\alpha} \Bigg(\int_0^\infty h(s)\,ds & - \frac{\sum_{i=1}^{m-2} a_i}{\Gamma(\alpha)} \int_0^{\xi_i} (\xi_i - s)^{\alpha-1} h(s)\,ds \\ & - \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(s)\,ds \Bigg). \end{aligned} \quad (21)$$

Therefore, the unique solution of the boundary value problem (12)–(14) is

$$\begin{aligned} u(t) &= \frac{t^{\alpha-1}}{\Delta_\alpha} \Bigg(\int_0^\infty h(s)\,ds - \frac{\sum_{i=1}^{m-2} a_i}{\Gamma(\alpha)} \int_0^{\xi_i} (\xi_i - s)^{\alpha-1} h(s)\,ds - \sum_{i=1}^{m-2} b_i \int_0^{\xi_i} h(s)\,ds \Bigg) \\ &\quad - \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} h(s)\,ds \\ &= \int_0^\infty K_\alpha(t,s) h(s)\,ds + \frac{\sum_{i=1}^{m-2} a_i t^{\alpha-1}}{\Delta_\alpha} \int_0^\infty \frac{\xi_i^{\alpha-1}}{\Gamma(\alpha)} h(s)\,ds \\ &\quad + \frac{\Gamma(\alpha) \sum_{i=1}^{m-2} b_i t^{\alpha-1}}{\Delta_\alpha} \int_0^\infty \frac{h(s)}{\Gamma(\alpha)}\,ds - \frac{\sum_{i=1}^{m-2} a_i t^{\alpha-1}}{\Delta_\alpha} \int_0^{\xi_i} \frac{(\xi_i - s)^{\alpha-1}}{\Gamma(\alpha)} h(s)\,ds \\ &\quad - \frac{\Gamma(\alpha) \sum_{i=1}^{m-2} b_i t^{\alpha-1}}{\Delta_\alpha} \int_0^{\xi_i} \frac{h(s)}{\Gamma(\alpha)}\,ds \\ &= \int_0^\infty K_\alpha(t,s) h(s)\,ds + \frac{\sum_{i=1}^{m-2} a_i t^{\alpha-1}}{\Delta_\alpha} \int_0^\infty K_\alpha(\xi_i, s) h(s)\,ds \\ &\quad + \frac{\sum_{i=1}^{m-2} b_i t^{\alpha-1}}{\Delta_\alpha} \int_0^\infty H(\xi_i, s) h(s)\,ds \\ &= \int_0^\infty G_\alpha(t,s) h(s)\,ds, \end{aligned} \quad (22)$$

where $G_\alpha(t,s)$, $K_\alpha(t,s)$, and $H(t,s)$ are defined by (16), (17), and (18), respectively. The proof is complete. □

Now, we introduce the following function:

$$\begin{aligned} G_\beta(t,s) = K_\beta(t,s) &+ \frac{t^{\beta-1}}{\Delta_\beta} \sum_{i=1}^{m-2} c_i K_\beta(\xi_i, s) \\ &+ \frac{t^{\beta-1}}{\Delta_\beta} \sum_{i=1}^{m-2} d_i H(\xi_i, s), \end{aligned} \quad (23)$$

where

$$K_\beta(t,s) = \frac{1}{\Gamma(\beta)} \begin{cases} t^{\beta-1} - (t-s)^{\beta-1}, & 0 \le s \le t < \infty, \\ t^{\beta-1}, & 0 \le t \le s < \infty. \end{cases} \quad (24)$$

Remark 7. From the definition of $G_\alpha(t,s)$ and $G_\beta(t,s)$, for any $(s,t) \in [0,\infty) \times [0,\infty)$, we have

$$\frac{G_\alpha(t,s)}{1 + t^{\alpha-1}} \le Q, \qquad \frac{G_\beta(t,s)}{1 + t^{\beta-1}} \le Q, \quad (25)$$

where

$$\begin{aligned} Q = \max \Bigg\{ & \frac{1}{\Gamma(\alpha)} + \frac{\sum_{i=1}^{m-2} a_i \xi_{m-2}^{\alpha-1}}{\Gamma(\alpha) \Delta_\alpha} + \frac{\sum_{i=1}^{m-2} b_i}{\Delta_\alpha}, \\ & \frac{1}{\Gamma(\beta)} + \frac{\sum_{i=1}^{m-2} c_i \xi_{m-2}^{\beta-1}}{\Gamma(\beta) \Delta_\beta} + \frac{\sum_{i=1}^{m-2} d_i}{\Delta_\beta} \Bigg\}. \end{aligned} \quad (26)$$

Let an operator $T : X_\alpha \times X_\beta \to X_\alpha \times X_\beta$ be defined by

$$\begin{aligned} T(u,v) &= (T_1(v), T_2(u)) \\ &= \Bigg(\int_0^\infty G_\alpha(t,s) f(s, v(s), I^\alpha v(s))\,ds, \\ &\qquad \int_0^\infty G_\beta(t,s) g\left(s, u(s), I^\beta u(s)\right) ds \Bigg). \end{aligned} \quad (27)$$

From the definition of operator T, the problem (2) has a solution if and only if the operator T has a fixed point.

Theorem 8. *Assume the following.*

(H_1) *There exist nonnegative functions $a(t), b(t), \phi(t) \in C[0,\infty)$ such that*

$$|f(t,x,y)| \le a(t)|x| + b(t)|y| + \phi(t),$$

$$\int_0^\infty \phi(t)\,dt < \infty, \qquad \Delta_\alpha > 0,$$

$$\begin{aligned}&\int_0^\infty \left(1+t^{\alpha-1}\right) a(t)\,dt\\ &\quad+\frac{1}{\Gamma(\alpha)}\int_0^\infty \left[\frac{t^\alpha}{\alpha}+B(\alpha,\alpha)\,t^{2\alpha-1}\right] b(t)\,dt\\ &\quad<\frac{1}{Q},\end{aligned} \tag{28}$$

where $B(\alpha,\alpha)$ is the beta-function.

(H_2) *There exist nonnegative functions $c(t), d(t), \varphi(t) \in C[0,\infty)$ such that*

$$\begin{gathered}\left|g(t,x,y)\right| \le c(t)\,|x| + d(t)\,|y| + \varphi(t),\\ \int_0^\infty \varphi(t)\,dt<\infty, \qquad \Delta_\beta>0,\end{gathered}$$

$$\begin{aligned}&\int_0^\infty \left(1+t^{\beta-1}\right) c(t)\,dt\\ &\quad+\frac{1}{\Gamma(\beta)}\int_0^\infty \left[\frac{t^\beta}{\beta}+B(\beta,\beta)\,t^{2\beta-1}\right] d(t)\,dt\\ &\quad<\frac{1}{Q},\end{aligned} \tag{29}$$

where $B(\beta,\beta)$ is the beta-function.

Then, the system (2) *has a solution.*

Proof. Take

$$\begin{aligned}R>\max\Bigg\{&\left(Q\int_0^\infty \phi(t)\,dt\right)\\ &\times\left(1-Q\int_0^\infty \left(1+t^{\alpha-1}\right)a(t)\,dt-\frac{Q}{\Gamma(\alpha)}\right.\\ &\qquad\left.\times\int_0^\infty\left[\frac{t^\alpha}{\alpha}+B(\alpha,\alpha)\,t^{2\alpha-1}\right]b(t)dt\right)^{-1},\\ &\left(Q\int_0^\infty \varphi(t)\,dt\right)\\ &\times\left(1-Q\int_0^\infty \left(1+t^{\beta-1}\right)c(t)\,dt-\frac{Q}{\Gamma(\beta)}\right.\\ &\qquad\left.\times\int_0^\infty\left[\frac{t^\beta}{\beta}+B(\beta,\beta)\,t^{2\beta-1}\right]d(t)\,dt\right)^{-1}\Bigg\},\end{aligned} \tag{30}$$

and define a ball

$$B_R=\left\{(u,v)\in X_\alpha\times X_\beta : \|(u,v)\|_{\alpha,\beta}\le R\right\}. \tag{31}$$

At the first step, we prove that the operator T transforms the ball B_R into itself. For any $(u,v)\in B_R$ we have

$$\begin{aligned}&\|T_1(v)\|_\alpha\\ &=\sup_{t\in J}\frac{1}{1+t^{\alpha-1}}\left|\int_0^\infty G_\alpha(t,s)\,f\left(s,v(s),I^\alpha v(s)\right)ds\right|\\ &\le\sup_{t\in J}\frac{1}{1+t^{\alpha-1}}\\ &\quad\times\int_0^\infty G_\alpha(t,s)\left[a(s)\,|v(s)|+b(s)\left|I^\alpha v(s)\right|+\phi(s)\right]ds\\ &\le Q\int_0^\infty a(s)\,|v(s)|\,ds+\frac{Q}{\Gamma(\alpha)}\\ &\quad\times\int_0^\infty b(s)\left[\int_0^s (s-\tau)^{\alpha-1}v(\tau)\,d\tau\right]ds\\ &\quad+Q\int_0^\infty \phi(s)\,ds\\ &\le Q\|v\|_\alpha\int_0^\infty\left(1+s^{\alpha-1}\right)a(s)\,ds\\ &\quad+\frac{Q\|v\|_\alpha}{\Gamma(\alpha)}\int_0^\infty b(s)\left[\int_0^s\left(1+\tau^{\alpha-1}\right)(s-\tau)^{\alpha-1}d\tau\right]ds\\ &\quad+Q\int_0^\infty \phi(s)\,ds\\ &\le Q\|v\|_\alpha\int_0^\infty\left(1+s^{\alpha-1}\right)a(s)\,ds\\ &\quad+\frac{Q\|v\|_\alpha}{\Gamma(\alpha)}\int_0^\infty b(s)\left[s^\alpha\int_0^1(1-\theta)^{\alpha-1}+s^{2\alpha-1}\right.\\ &\qquad\qquad\left.\times\int_0^1\theta^{\alpha-1}(1-\theta)^{\alpha-1}d\theta\right]ds\\ &\quad+Q\int_0^\infty \phi(s)\,ds\\ &\le Q\|v\|_\alpha\int_0^\infty\left(1+s^{\alpha-1}\right)a(s)\,ds\\ &\quad+\frac{Q\|v\|_\alpha}{\Gamma(\alpha)}\int_0^\infty b(s)\left[\frac{s^\alpha}{\alpha}+B(\alpha,\alpha)\,s^{2\alpha-1}\right]ds\\ &\quad+Q\int_0^\infty \phi(s)\,ds<R.\end{aligned} \tag{32}$$

In a similar way, we can get

$$\begin{aligned}&\|T_2(u)\|_\beta\\ &\quad\le Q\|u\|_\beta\int_0^\infty\left(1+s^{\beta-1}\right)c(s)\,ds\\ &\qquad+\frac{Q\|u\|_\beta}{\Gamma(\beta)}\int_0^\infty d(s)\left[\frac{s^\beta}{\beta}+B(\beta,\beta)\,s^{2\beta-1}\right]ds\\ &\qquad+Q\int_0^\infty \varphi(t)\,ds<R.\end{aligned} \tag{33}$$

Hence, $\|T(u,v)\|_{\alpha,\beta}\le R$ and this shows that $TB_R\subset B_R$.

Next, we show that $T : B_R \to B_R$ is completely continuous. First, Let $(u_n, v_n) \to (u, v)$ as $n \to \infty$ in B_R. From (32) we have

$$\int_0^\infty \left|f\left(s, v(s), I^\alpha v(s)\right)\right| ds < \frac{R}{Q}. \tag{34}$$

Then, by the Lebsegue dominated convergence theorem and continuity of f, we obtain

$$\begin{aligned}\int_0^\infty f\left(s, v_n(s), I^\alpha v_n(s)\right) ds \\ \longrightarrow \int_0^\infty f\left(s, v(s), I^\alpha v(s)\right) ds,\end{aligned} \tag{35}$$

as $n \to \infty$. Therefore, by Remark 7, we have

$$\begin{aligned}&\left\|T_1(v_n) - T_1(v)\right\|_\alpha \\ &\quad \le Q\left|\int_0^\infty f\left(s, v_n(s), I^\alpha v_n(s)\right) ds \right. \\ &\qquad \left. - \int_0^\infty f\left(s, v(s), I^\alpha v(s)\right) ds\right| \longrightarrow 0,\end{aligned} \tag{36}$$

as $n \to \infty$. Similar process can be repeated for T_2; thus, operator T is continuous.

Now, we show that $T : B_R \to B_R$ is equicontinuous operator. Let $L > 0$ and $t_1, t_2 \in [L, \infty)$; without loss of generality, we may assume that $t_1 < t_2$. Since $K_\alpha(\xi_i, s) \le \xi_{m-2}^{\alpha-1}/\Gamma(\alpha)$ and $H(\xi_i, s) \le 1$, for any $s > 0$ and $i = 1, 2, \ldots, m-2$, we have

$$\begin{aligned}&\left|\frac{T_1(u)(t_2)}{1+t_2^{\alpha-1}} - \frac{T_1(u)(t_1)}{1+t_1^{\alpha-1}}\right| \\ &\quad \le \int_0^\infty \left|\left(\frac{G_\alpha(t_2,s)}{1+t_2^{\alpha-1}} - \frac{G_\alpha(t_1,s)}{1+t_1^{\alpha-1}}\right)\right. \\ &\qquad \left. \times f\left(s, v(s), I^\alpha v(s)\right)\right| ds \\ &\quad \le \int_0^\infty \left|\left(\frac{K_\alpha(t_2,s)}{1+t_2^{\alpha-1}} - \frac{K_\alpha(t_1,s)}{1+t_1^{\alpha-1}}\right)\right. \\ &\qquad \left. \times f\left(s, v(s), I^\alpha v(s)\right)\right| ds \\ &\qquad + \left[\frac{\sum_{i=1}^{m-2} a_i \xi_{m-2}^{\alpha-1}}{\Gamma(\alpha)\Delta_\alpha} + \frac{\sum_{i=1}^{m-2} b_i}{\Delta_\alpha}\right] \\ &\qquad \times \left|\frac{t_2^{\alpha-1}}{1+t_2^{\alpha-1}} - \frac{t_1^{\alpha-1}}{1+t_1^{\alpha-1}}\right| \\ &\qquad \times \int_0^\infty \left|f\left(s, v(s), I^\alpha v(s)\right)\right| ds.\end{aligned} \tag{37}$$

In view of (37), by the similar process used in [20], we can easily prove that operator T_1 is equicontinuous. Similar process can be repeated for T_2; thus, T is equicontinuous. On the other hand, TB_R is uniformly bounded as $TB_R \subset B_R$. Therefore, T is completely continuous operator. Hence, by Schauder fixed point theorem the boundary value problem (2) has at least one solution in B_R. □

4. An Example

Consider the following boundary value problem on unbounded domain:

$$\begin{aligned}D^{2.25}u(t) &= \frac{\cos t \sin(|v(t)|)}{(1+t^{1.25})(10+t)^2} \\ &\quad + \frac{\left|\int_0^\infty (t-s)^{1.25} e^{-5t} v(s)\, ds\right|}{\Gamma(2.25)\left(t^{2.25} + 0.248 t^{3.5}\right)} + \frac{t^2 e^{-t}}{1+t^2}, \\ D^{2.5}v(t) &= \frac{\sin t \arctan(|u(t)|)}{(1+t^{1.5})(8+t)^2} \\ &\quad + \frac{\left|\int_0^\infty (t-s)^{1.5} e^{-4t} u(s)\, ds\right|}{\Gamma(2.5)\left(t^{2.5} + 0.183 t^4\right)} + \frac{t^4 e^{-t}}{1+t^4}, \\ &u(0) = u'(0) = 0, \qquad v(0) = v'(0) = 0, \\ D^{1.25}u(\infty) &= \frac{1}{2}u\left(\frac{1}{4}\right) + \frac{1}{4}u(1) + \frac{1}{10}D^{1.25}u\left(\frac{1}{4}\right) \\ &\quad + \frac{3}{10}D^{1.25}u(1), \\ D^{1.5}v(\infty) &= \frac{1}{5}v\left(\frac{1}{4}\right) + \frac{1}{10}v(1) + \frac{3}{11}D^{1.5}v\left(\frac{1}{4}\right) \\ &\quad + \frac{1}{7}D^{1.5}v(1).\end{aligned} \tag{38}$$

Here $t \in [0, \infty)$, $\alpha = 2.25$, $\beta = 2.5$, $\xi_1 = 1/4$, $\xi_2 = 1$, $a_1 = 1/2$, $a_2 = 1/4$, $b_1 = 1/10$, $b_2 = 3/10$, $c_1 = 1/5$, $c_2 = 1/10$, $d_1 = 3/11$, and $d_2 = 1/7$. We have

$$\begin{aligned}f(t,x,y) &= \frac{\cos t \sin(|x|)}{(1+t^{1.25})(10+t)^2} \\ &\quad + \frac{e^{-5t}|y|}{\left(t^{2.25} + 0.248t^{3.5}\right)} + \frac{t^2 e^{-t}}{1+t^2}, \\ g(t,x,y) &= \frac{\sin t \arctan(|x|)}{(1+t^{1.5})(8+t)^2} \\ &\quad + \frac{e^{-4t}|y|}{\left(t^{2.5} + 0.183t^4\right)} + \frac{t^4 e^{-t}}{1+t^4}.\end{aligned} \tag{39}$$

For

$$\begin{aligned}a(t) &= \frac{1}{(1+t^{1.25})(10+t)^2}, \\ b(t) &= \frac{e^{-5t}}{\left(t^{2.25} + 0.248t^{3.5}\right)},\end{aligned}$$

$$\begin{aligned}\phi(t) &= \varphi(t) = e^{-t},\\ c(t) &= \frac{1}{\left(1+t^{1.5}\right)(8+t)^2},\\ d(t) &= \frac{e^{-4t}}{\left(t^{2.5}+0.183t^4\right)},\end{aligned} \tag{40}$$

by direct calculation we obtain $\Delta_\alpha = 0.342$, $\Delta_\beta = 0.645$, $Q = 3.98$, and

$$\begin{aligned}&\int_0^\infty \left(1+t^{1.25}\right) a(t)\,dt + \frac{1}{\Gamma(2.25)}\\ &\quad\times \int_0^\infty \left[\frac{t^{2.25}}{2.25} + B(2.25, 2.25)\, t^{3.5}\right] b(t)\,dt\\ &= 0.17480 < \frac{1}{Q} = 0.25125,\\ &\int_0^\infty \left(1+t^{1.5}\right) c(t)\,dt + \frac{1}{\Gamma(2.5)}\\ &\quad\times \int_0^\infty \left[\frac{t^{2.5}}{2.5} + B(2.5, 2.5)\, t^{4}\right] d(t)\,dt\\ &= 0.20000 < \frac{1}{Q} = 0.25125.\end{aligned} \tag{41}$$

Thus all the conditions of Theorem 8 are satisfied and the problem (38) has at least one solution.

5. Conclusion

In the current paper, we have studied the existence results for a coupled system of nonlinear fractional integrodifferential equations with m-point fractional boundary conditions on an unbounded domain. The result obtained in this paper is based on Schauder's fixed point theorem. In order to show the validity of the assumptions made in our result, we also include an illustrative example.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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