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Research Article

Stabilization of a Class of Stochastic Systems with Time Delays

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The problem of exponential stability is investigated for a class of stochastic time-delay systems. By using the decomposition technique and Lyapunov stability theory, two improved exponential stability criteria are derived. Finally, a numerical example is given to illustrate the effectiveness and the benefit of the proposed method.

1. Introduction

In fact, time delay constantly occurs in the real world, which results in instability of systems. Thus, the stability problem for time-delay systems has been studied for many years [1–8]. On the other hand, stochastic modelling has come to play an important role in many fields of science or industry. Stability analysis for stochastic systems has become increasingly meaningful. A number of results have appeared in the literature [9–19]. For instance, in [11], the author provided the criteria for the stability of a class of stochastic systems by Lyapunov theory.

It's worth noting that, the grey systems can be established when parameters are evaluated by grey numbers (see [20]). Until now, there have been a few papers tackling the stability of the systems; some important and innovative results are obtained [20–23]. In [23], the authors provided the delay-dependent criteria for exponential robust stability in the forms of nonlinear matrix inequalities and linear matrix inequalities.

In this paper, we deal with the exponential stability for the time-delay grey stochastic systems. By using the method of [21–23] and Lyapunov stability theory, two improved criteria of mean-square exponential stability are proposed. At last, a numerical example is given to verify the criteria.

Notations. R^n denotes the n dimensional Euclidean space, the superscript "T" denotes matrix transposition, and the notation $X \geq Y$, where X and Y are symmetric matrices, means that $X-Y$ is positive semidefinite. Let $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and F_0 contains all P-null sets). Let $w(t) = (w_1(t), \ldots, w_m(t))^T$ be an m-dimensional Brownian motion defined on the probability space. Let $\tau > 0$ and $C([-\tau, 0]; R^n)$ be the family of continuous functions φ from $[-\tau, 0]$ to R^n. Let $L^2_{F_0}([-\tau, 0]; R^n)$ be the family of all F_0-measurable $C([-\tau, 0]; R^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0} E|\xi(\theta)|^2 < \infty$.

2. Problem Formulation

Consider

$$\begin{aligned} dx(t) &= [Ax(t) + Bx(t-\tau) + Du(t)]\,dt \\ &\quad + \sigma(x(t), x(t-\tau), t)\,dW(t), \quad t \geq 0, \\ x_0 &= \xi, \quad \xi \in L^2_{F_0}([-\tau, 0]; R^n), \ -\tau \leq t \leq 0. \end{aligned} \tag{1}$$

Definition 1. If there is at least a grey matrix among matrices A, B, and D of system (1), then (1) is called grey stochastic time-delay system.

Hence

$$\begin{aligned} dx(t) &= [A(\otimes)x(t) + B(\otimes)x(t-\tau) + Du(t)]\,dt \\ &\quad + \sigma(x(t), x(t-\tau), t)\,dW(t), \quad t \geq 0, \\ x_0 &= \xi, \quad \xi \in L^2_{F_0}([-\tau, 0]; R^n), \ -\tau \leq t \leq 0, \end{aligned} \tag{2}$$

where $A(\otimes)$ and $B(\otimes)$ are grey matrices and $A(\otimes) = (\otimes_{ij}^{a})$, $B(\otimes) = (\otimes_{ij}^{b})$.

Clearly, if matrices $A(\otimes)$ and $B(\otimes)$ are replaced by the deterministic matrices A and B, the grey system (2) becomes system (1). The equations

$$[L_a, U_a] = \left\{A(\hat{\otimes}) = (a_{ij}) : \underline{a_{ij}} \le a_{ij} \le \overline{a_{ij}},\ i, j = 1, 2, \ldots, n\right\},$$
$$[L_b, U_b] = \left\{B(\hat{\otimes}) = (b_{ij}) : \underline{b_{ij}} \le b_{ij} \le \overline{b_{ij}},\ i, j = 1, 2, \ldots, n\right\} \tag{3}$$

are said to be the continuous matrix-covered sets of $A(\otimes)$ and $B(\otimes)$.

Definition 2 (see [20]). System (2) is said to be robustly exponentially stable in the mean square, if, for all $\xi \in L^2_{F_0}([-\tau, 0]; R^n)$ and arbitrary matrices $A(\hat{\otimes}) \in [L_a, U_a]$, $B(\hat{\otimes}) \in [L_b, U_b]$, there exist scalars $r > 0$ and $C > 0$, such that

$$E|x(t,\xi)|^2 \le Ce^{-rt} \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2, \quad t \ge 0. \tag{4}$$

In addition, the following assumptions are made on the system (2).

(H1) $H : R^n \times R^n \times R_+ \rightarrow R^{n\times n}$.

(H2) Supposing there exist scalars $\alpha \ge 0$, $\beta \ge 0$, such that, for $(x, y, t) \in H : R^n \times R^n \times R_+$, the inequality holds, $\text{Trace}[\sigma^T(x, y, t)\sigma(x, y, t)] \le \alpha|x|^2 + \beta|y|^2$.

Before giving the main results, we first present Lemmas 3 and 4, which are important for the proof of main theorems.

Lemma 3 (see [20]). *For arbitrary whitened matrix $A(\hat{\otimes}) \in [L_a, U_a]$, it follows that*

(i) $A(\hat{\otimes}) = L_a + \Delta A$

(ii) $0 \le \Delta A \le U_a - L_a$

(iii) $\|A(\hat{\otimes})\| = \|L_a\| + \|U_a - L_a\|$,

where $L_a = (\underline{a_{ij}})_{m\times n}$, $U_a = (\overline{a_{ij}})_{m\times n}$, $\Delta A = (\delta_{ij}\hat{r}_{ij})_{m\times n}$, $\delta_{ij} = \overline{a_{ij}} - \underline{a_{ij}} \ge 0$.

Lemma 4 (see [18]). *Let N be a real matrix of appropriate dimensions; for any vectors $x, y \in R^n$, one has $2x^TNy \le \varepsilon x^Tx + \varepsilon^{-1}y^TN^TNy$.*

3. Proof of the Main Theorem

In this section, we discuss the exponential stability for system (2); two improved criteria for robust exponential stability in mean square are proposed.

Theorem 5. *System* (2) *is exponentially robustly stable in mean square. If there exist positive scalars ε_1, ε_2, and ε_3, such that*

$$\lambda_{\max}(L_a + L_a^T) + k_1 + k_2 < 0 \tag{5}$$

here

$$k_1 = 1 + \varepsilon_1 + \varepsilon_1^{-1}\|U_a - L_a\|^2 + \varepsilon_2 + \alpha,$$
$$k_2 = \varepsilon_2^{-1}(1 + \varepsilon_3)\lambda_{\max}(L_b^TL_b) + \varepsilon_2^{-1}(1 + \varepsilon_3^{-1})\|U_b - L_b\|^2 + \beta. \tag{6}$$

Then, for all $\xi \in L^2_{F_0}([-\tau, 0]; R^n)$, the following inequality holds:

$$E|x(t,\xi)|^2 \le (1 + k_2\tau\, e^{r\tau}) e^{-rt} \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2, \quad t \ge 0, \tag{7}$$

where r is the unique positive solution of the following equation:

$$r + \lambda_{\max}(L_a + L_a^T) + k_1 + k_2e^{r\tau} = 0. \tag{8}$$

Proof. First, define Lyapunov-Krasovskii functional as follows:

$$V(x(t), t) = e^{rt}x^T(t)x(t) + \int_{-\tau}^{0} e^{r(t+\theta)}x^T(t+\theta)x(t+\theta)\,d\theta. \tag{9}$$

Then, we have

$$\begin{aligned} LV(x(t), t) = {} & (r + 1)e^{rt}x^T(t)x(t) \\ & - e^{r(t-\tau)}x^T(t-\tau)x(t-\tau) \\ & + e^{rt}\left\{2x^T(t)A(\hat{\otimes})x(t) \right. \\ & + 2x^T(t)B(\hat{\otimes})x(t-\tau) \\ & + \text{Trace}\left[\sigma^T(x(t), x(t-\tau), t) \right. \\ & \left.\left. \times \sigma(x(t), x(t-\tau), t)\right]\right\}. \end{aligned} \tag{10}$$

Using Lemmas 3 and 4, we derive

$$\begin{aligned} & 2x^T(t)A(\hat{\otimes})x(t) \\ & \le \lambda_{\max}(L_a + L_a^T)x^T(t)x(t) \\ & \quad + \varepsilon_1x^T(t)x(t) + \varepsilon_1^{-1}\|U_a - L_a\|^2x^T(t)x(t), \\ & 2x^T(t)B(\hat{\otimes})x(t-\tau) \\ & \le \varepsilon_2x^T(t)x(t) + \varepsilon_2^{-1}(1 + \varepsilon_3)\lambda_{\max} \\ & \quad \times (L_b^TL_b)x^T(t-\tau)x(t-\tau) \\ & \quad + \varepsilon_2^{-1}(1 + \varepsilon_3^{-1})\|U_b - L_b\|^2x^T(t-\tau)x(t-\tau). \end{aligned} \tag{11}$$

By assumption (H2), we can obtain

$$\begin{aligned} & \text{Trace}\left[\sigma^T(x(t), x(t-\tau), t)\sigma(x(t), x(t-\tau), t)\right] \\ & \le \alpha x^T(t)x(t) + \beta x^T(t-\tau)x(t-\tau). \end{aligned} \tag{12}$$

Substituting (11)–(12) into (10), we see that

$$\begin{aligned}
&LV(x(t),t)\\
&\leq\left[1+r+\lambda_{\max}\left(L_a+L_a^T\right)+\varepsilon_1\right.\\
&\quad\left.+\varepsilon_1^{-1}\|U_a-L_a\|^2+\varepsilon_2+\alpha\right]e^{rt}x^T(t)\,x(t)\\
&\quad+\left[\varepsilon_2^{-1}(1+\varepsilon_3)\,\lambda_{\max}\left(L_b^TL_b\right)\right.\\
&\quad\left.+\varepsilon_2^{-1}\left(1+\varepsilon_3^{-1}\right)\|U_b-L_b\|^2+\beta\right]\\
&\quad\times e^{rt}x^T(t-\tau)\,x(t-\tau).
\end{aligned}\tag{13}$$

Using Itô's differential formula and integrating both sides, we obtain

$$\begin{aligned}
EV(x(t),t)\leq&\sup_{-\tau\leq\theta\leq0}E|\xi(\theta)|^2\\
&+\left[1+r+\lambda_{\max}\left(L_a+L_a^T\right)\right.\\
&\left.+\varepsilon_1+\varepsilon_1^{-1}\|U_a-L_a\|^2+\varepsilon_2+\alpha\right]\\
&\times\int_0^t e^{rs}E|x(s)|^2ds\\
&+\left[\varepsilon_2^{-1}(1+\varepsilon_3)\,\lambda_{\max}\left(L_b^TL_b\right)\right.\\
&\left.+\varepsilon_2^{-1}\left(1+\varepsilon_3^{-1}\right)\|U_b-L_b\|^2+\beta\right]\\
&\times\int_0^t e^{rs}E|x(s-\tau)|^2ds.
\end{aligned}\tag{14}$$

Moreover, we have

$$\begin{aligned}
\int_0^t e^{rs}E|x(s-\tau)|^2ds\leq{}&e^{r\tau}\int_0^t e^{ru}E|x(u)|^2du\\
&+\tau e^{r\tau}\sup_{-\tau\leq\theta\leq0}E|\xi(\theta)|^2.
\end{aligned}\tag{15}$$

Combining (14) with (15) and noting the definitions of k_1, k_2, we see that

$$\begin{aligned}
&EV(x(t),t)\\
&\leq\left(1+k_2\tau e^{r\tau}\right)\sup_{-\tau\leq\theta\leq0}E|\xi(\theta)|^2\\
&\quad+\left[r+\lambda_{\max}\left(L_a+L_a^T\right)+k_1+k_2e^{r\tau}\right]\\
&\quad\times\int_0^t e^{rs}E|x(s)|^2ds.
\end{aligned}\tag{16}$$

Furthermore,

$$\begin{aligned}
&f(r)=r+\lambda_{\max}\left(L_a+L_a^T\right)+k_1+k_2e^{r\tau},\\
&\text{then }f'(r)=1+k_2\tau e^{r\tau}.
\end{aligned}\tag{17}$$

Since $f'(r)>0$, $f(0)=\lambda_{\max}(L_a+L_a^T)+k_1+k_2$ and $f(+\infty)=+\infty$.

When (5) holds, (8) must have a unique solution $r>0$. Hence, we have

$$E|x(t,\xi)|^2\leq\left(1+k_2\tau e^{r\tau}\right)e^{-rt}\sup_{-\tau\leq\theta\leq0}E|\xi(\theta)|^2,\quad t\geq0.\tag{18}$$

This completes the proof of Theorem 5. □

By a similar approach in [22], whitened system of (2) can be written as

$$\begin{aligned}
dx(t)={}&\left[\left(A(\hat{\otimes})+L_b\right)x(t)-L_b\left(x(t)-x(t-\tau)\right)\right.\\
&\left.+\Delta Bx(t-\tau)+Du(t)\right]dt\\
&+\sigma(x(t),x(t-\tau),t)\,dW(t).
\end{aligned}\tag{19}$$

Since

$$\begin{aligned}
&x(t)-x(t-\tau)\\
&=\int_{t+\theta}^t\left[A(\hat{\otimes})x(s)+B(\hat{\otimes})x(s-\tau)+Du(s)\right]ds\\
&\quad+\int_{t+\theta}^t\sigma(x(s),x(s-\tau),s)\,dW(s),\quad t\geq\tau,
\end{aligned}\tag{20}$$

then, we can introduce

$$\begin{aligned}
&H(x,t)\\
&=\begin{cases}\displaystyle\int_{t+\theta}^t\left[A(\hat{\otimes})x(s)+B(\hat{\otimes})x(s-\tau)\right]ds\\
\displaystyle\quad+\int_{t+\theta}^t\sigma(x(s),x(s-\tau),s)\,dW(s), & t\geq\tau\\
x(t)-x(t+\theta), & 0\leq t\leq\tau.\end{cases}
\end{aligned}\tag{21}$$

Combining (19) and (21) together, whitened system of (2) can be rewritten as

$$\begin{aligned}
dx(t)={}&\left[\left(A(\hat{\otimes})+L_b\right)x(t)-L_bH(x,t)\right.\\
&\left.+\Delta Bx(t-\tau)+Du(t)\right]dt\\
&+\sigma(x(t),x(t-\tau),t)\,dW(t),\quad t\geq0,\\
&x_0=\xi,\quad\xi\in L_{F_0}^2\left([-\tau,0];R^n\right),\ -\tau\leq t\leq0.
\end{aligned}\tag{22}$$

Lemma 6. *For all $\xi\in L_{F_0}^2([-\tau,0];R^n)$ and $t>0$, the following inequality holds:*

$$\begin{aligned}
&\int_0^t e^{rs}E|H(x(s),s)|^2ds\\
&\leq\left(k_1\tau e^{r\tau}+k_2\tau e^{2r\tau}\right)\int_0^t e^{rs}E|x(s)|^2ds\\
&\quad+k_2\tau^2e^{2r\tau}\sup_{-\tau\leq\theta\leq0}E|\xi(\theta)|^2+\sup_{-\tau\leq\theta\leq0}\psi(\theta),
\end{aligned}\tag{23}$$

where

$$l_1 = 4\tau(\|L_a\| + \|U_a - L_a\|)^2 + 2\alpha,$$
$$l_2 = 4\tau(\|L_b\| + \|U_b - L_b\|)^2 + 2\beta, \quad (24)$$
$$\psi(\theta) = \int_0^\tau e^{rs} E|x(s) - x(s-\tau)|^2 ds.$$

Proof. First, by the definition of $H(x,t)$, we can derive

$$\begin{aligned} E|H(x(s),s)|^2 &\le 2E\left|\int_{t+\theta}^t [A(\widehat{\otimes})x(s) + B(\widehat{\otimes})x(s-\tau)]\,ds\right|^2 \\ &+ 2E\left|\int_{t+\theta}^t \sigma(x(s), x(s-\tau), s)\,dW(s)\right|^2. \end{aligned} \quad (25)$$

Clearly,

$$\begin{aligned} E&\left|\int_{t+\theta}^t [A(\widehat{\otimes})x(s) + B(\widehat{\otimes})x(s-\tau)]\,ds\right|^2 \\ &\le 2\tau(\|L_a\| + \|U_a - L_a\|)^2 \int_{t+\theta}^t E|x(s)|^2 ds \\ &+ 2\tau(\|L_b\| + \|U_b - L_b\|)^2 \int_{t+\theta}^t E|x(s-\tau)|^2 ds. \end{aligned} \quad (26)$$

By assumption (H2), we see that

$$\begin{aligned} E&\left|\int_{t+\theta}^t \sigma(x(s), x(s-\tau), s)\,dW(s)\right|^2 \\ &\le \alpha\int_{t+\theta}^t E|x(s)|^2 ds + \beta\int_{t+\theta}^t E|x(s-\tau)|^2 ds. \end{aligned} \quad (27)$$

By (25)–(27) and noting the definitions of k_1, k_2, we can obtain

$$\begin{aligned} E|H(t,x(t))|^2 &\le k_1\int_{t+\theta}^t E|x(s)|^2 ds \\ &+ k_2\int_{t+\theta}^t E|x(s-\tau)|^2 ds, \quad t \ge \tau. \end{aligned} \quad (28)$$

By (28) and integrating both sides, we have

$$\begin{aligned} \int_0^t & e^{rs} E|H(x(s),s)|^2 ds \\ &\le \sup_{-\tau\le\theta\le 0}\psi(\theta) + k_1\int_\tau^t e^{rs}\int_{s+\theta}^s E|x(u)|^2 du\,ds \\ &+ k_2\int_\tau^t e^{rs}\int_{s+\theta}^s E|x(u-\tau)|^2 du\,ds. \end{aligned} \quad (29)$$

Moreover, we can obtain

$$\begin{aligned} \int_\tau^t & e^{rs}\int_{s+\theta}^s E|x(u-\tau)|^2 du\,ds \\ &\le \tau e^{2r\tau}\int_0^t e^{rs} E|x(s)|^2 ds \\ &+ \tau^2 e^{2r\tau}\sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2, \end{aligned} \quad (30)$$
$$\int_\tau^t e^{rs}\int_{s+\theta}^s E|x(u)|^2 du\,ds \le \tau e^{r\tau}\int_0^t e^{rs} E|x(s)|^2 ds.$$

Substituting (30) into (29) and noting the definitions of l_1, l_2, and $\psi(\theta)$, (23) holds. The proof of Lemma 6 is completed. □

By (22) and Lemma 6, another criterion for system (2) will be given.

Theorem 7. *System (2) is exponentially robustly stable in mean square. If there exist positive scalars ε_1, ε_2, and ε_3, such that*

$$\lambda_{\max}(L_a + L_a^T + L_b + L_b^T) + m_1 + m_2 + m_3 < 0, \quad (31)$$

here

$$m_1 = 1 + \varepsilon_1 + \varepsilon_1^{-1}\|U_a - L_a\|^2 + \varepsilon_2 + \varepsilon_3 + \alpha,$$
$$m_2 = \varepsilon_3^{-1}\|U_b - L_b\|^2 + \beta, \quad (32)$$
$$m_3 = \varepsilon_2^{-1}(l_1 + l_2)\tau\,\lambda_{\max}(L_b^T L_b).$$

Then, for all $\xi \in L_{F_0}^2([-\tau,0];R^n)$, the following inequality holds:

$$\begin{aligned} E|x(t,\xi)|^2 \le \Big[&\left(1 + m_2\tau e^{r\tau} + \varepsilon_2^{-1} l_2\tau^2 e^{2r\tau}\lambda_{\max}(L_b^T L_b)\right) \\ &\times \sup_{-\tau\le\theta\le 0} E|\xi(\theta)|^2 \\ &+ \varepsilon_2^{-1}\lambda_{\max}(L_b^T L_b)\sup_{-\tau\le\theta\le 0}\psi(\theta)\Big] e^{-rt}, \end{aligned} \quad (33)$$

where r is the unique positive solution of the following equation:

$$\begin{aligned} r + \lambda_{\max}&(L_a + L_a^T + L_b + L_b^T) + m_1 + m_2 e^{r\tau} \\ &+ \varepsilon_2^{-1}(l_1\tau e^{r\tau} + l_2\tau e^{2r\tau})\lambda_{\max}(L_b^T L_b) = 0. \end{aligned} \quad (34)$$

Proof. Similar to the proof process of Theorem 5, we can derive

$$\begin{aligned} LV(x(t),t) \le &\left[r + \lambda_{\max}(L_a + L_a^T + L_b + L_b^T) + m_1\right] \\ &\times e^{rt}x^T(t)x(t) \\ &+ m_2 e^{rt}x^T(t-\tau)x(t-\tau) \\ &+ \varepsilon_2^{-1}\lambda_{\max}(L_b^T L_b)e^{rt}|H(x(t),t)|^2. \end{aligned} \quad (35)$$

Using Itô's differential formula and integrating and by the definition of $H(x,t)$, we obtain

$$\begin{aligned} EV(x(t),t) \leq & \left[r + \lambda_{\max}\left(L_a + L_a^T + L_b + L_b^T\right) + m_1 \right. \\ & + m_2 e^{rt} + \varepsilon_2^{-1}\left(l_1 \tau e^{r\tau} + l_2 \tau e^{2r\tau}\right) \\ & \left. \times \lambda_{\max}\left(L_b^T L_b\right)\right] \\ & \times \int_0^t e^{rs} E|x(s)|^2 ds \\ & + \left[1 + m_2 \tau e^{r\tau} \right. \\ & \left. + \varepsilon_2^{-1} l_2 \tau^2 e^{2r\tau} \lambda_{\max}\left(L_b^T L_b\right)\right] \sup_{-\tau\leq\theta\leq 0} E|\xi(\theta)|^2 \\ & + \varepsilon_2^{-1}\lambda_{\max}\left(L_b^T L_b\right) \sup_{-\tau\leq\theta\leq 0} \psi(\theta). \end{aligned} \tag{36}$$

If $g(r) = r + \lambda_{\max}(L_a + L_a^T + L_b + L_b^T) + m_1 + m_2 e^{r\tau} + \varepsilon_2^{-1}(l_1 \tau e^{r\tau} + l_2 \tau e^{2r\tau})\lambda_{\max}(L_b^T L_b)$, then $g'(r) = 1 + m_2 \tau e^{r\tau} + \varepsilon_2^{-1}(l_1 \tau^2 e^{r\tau} + 2 l_2 \tau^2 e^{2r\tau})\lambda_{\max}(L_b^T L_b)$.

Because $g'(r) > 0$, $g(0) = \lambda_{\max}(L_a + L_a^T + L_b + L_b^T) + m_1 + m_2 + m_3$ and $g(+\infty) = +\infty$.

If (31) holds, (34) must have a unique solution $r > 0$.

Therefore, we can easily get

$$\begin{aligned} EV(x(t),t) \leq & \left[1 + m_2 \tau e^{r\tau} + \varepsilon_2^{-1} l_2 \tau^2 e^{2r\tau} \lambda_{\max}\left(L_b^T L_b\right)\right] \\ & \times \sup_{-\tau\leq\theta\leq 0} E|\xi(\theta)|^2 \\ & + \varepsilon_2^{-1}\lambda_{\max}\left(L_b^T L_b\right) \sup_{-\tau\leq\theta\leq 0} \psi(\theta), \end{aligned} \tag{37}$$

or equivalently (33) holds; the proof of Theorem 7 is completed. □

4. Examples

Consider a stochastic time-delay system

$$\begin{aligned} dx(t) = & [A(\otimes)x(t) + B(\otimes)x(t-0.5)]\,dt \\ & + \sigma(x(t), x(t-0.5), t)\,dW(t), \\ x_0 = \xi, \quad & \xi \in L_{F_0}^2\left([-0.5, 0]; R^2\right), \ -0.5 \leq t \leq 0, \end{aligned} \tag{38}$$

where

$$\begin{aligned} L_a &= \begin{bmatrix} -3.35 & 0.22 \\ 0.23 & -3.34 \end{bmatrix}; \qquad U_a = \begin{bmatrix} -3.15 & 0.32 \\ 0.31 & -3.45 \end{bmatrix} \\ L_b &= \begin{bmatrix} -1.15 & 0.20 \\ 0.23 & -1.16 \end{bmatrix}; \qquad U_b = \begin{bmatrix} -1.12 & 0.22 \\ 0.31 & -1.09 \end{bmatrix}. \end{aligned} \tag{39}$$

Respectively, L_a, U_a, L_b, and U_b are the lower bound and upper bound matrices of $A(\otimes)$ and $B(\otimes)$.

In addition,

$$\sigma(x(t), x(t-0.5), t) = \begin{bmatrix} \frac{1}{2} x_1(t)\sin\left(x_2(t-0.5)\right) \\ \frac{1}{2} x_2(t)\sin\left(x_1(t-0.5)\right) \end{bmatrix}. \tag{40}$$

Clearly,

$$\begin{aligned} & \operatorname{Trace}\left[\sigma^T(x(t), x(t-0.5), t)\,\sigma(x(t), x(t-0.5), t)\right] \\ & \quad \leq 0.25 x^2(t). \end{aligned} \tag{41}$$

By using the method of [21], we can obtain that $r = 1.7513$ or $r = 1.8162$, which indicates that the system (38) is exponentially stable in mean square.

5. Conclusion

In this paper, we have investigated a class of grey stochastic systems with time delay; by constructing a suitable Lyapunov-Krasovskii functional combined with Itô's differential formula, two improved exponential stability criteria are derived. The criteria obtained in this paper are so conveniently verified that the results in this paper should be proved to be very useful in applications.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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