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Research Article

Stochastic Finite-Time H_∞ Performance Analysis of Continuous-Time Systems with Random Abrupt Changes

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The problem of H_∞ control performance analysis of continuous-time systems with random abrupt changes is concerned in this paper. By employing an augmented multiple mode-dependent Lyapunov-Krasovskii functional and using some integral inequalities, new sufficient conditions are obtained relating to finite-time bounded and an H_∞ performance index. The finite-time H_∞ control performance problem is solved and desired controller is given to ensure the system trajectory stays within a prescribed bound during a given time interval. At last, two numerical examples are provided to show that our results are less conservative than the existing ones.

1. Introduction

It is well known that Markovian jump systems were introduced when the physical models are always subject to random changes, which can be also regarded as a special class of hybrid systems because of the structures are subject to random abrupt changes [1]. In the recent years, there are a lot of people towards to Markovian jump systems for its widely applications, for example, target tracking, robotics, manufacturing systems, aircraft control, and power systems [2–4]. Markovian jump systems are regarded as a special class of stochastic systems which switches from one to another at different time in the finite operation modes. Many important topics have been studied for Markovian jumping systems such as stability, control synthesis, stabilization, and filter design [5–7].

On the other hand, time delay is very common in practical dynamical systems, for example, networked control systems, chemical processes, communication systems, and so on [8–20]. Therefore, during the past two decades, various research topics have been considered for Markovian jump systems with time-varying delays [8–14]. It worth pointing out that when time delay is small enough in linear Markovian jump systems, the delay-dependent criteria are always less conservative than delay-independent ones. Over the past few years, for Markovian jump systems, many important topics related to delay-dependent have been extensively studied [14, 15].

Generally speaking, finite-time stability is investigated to address these transient performances of control systems in finite-time interval. Up to now, the concept of finite-time stability has been revisited with different systems, and many important results are obtained for finite-time stability and finite-time boundedness [21–26]. However, to the best of authors' knowledge, the stochastic finite-time H_∞ control for Markovian jump systems has not been fully studied. There is some room for next investigation due to the fact that analysis methods in existing references seem still conservative.

The major contribution of this paper is that we introduce a newly Lyapunov-Krasovskii functional for Markovian jump system. Some sufficient conditions are obtained to ensure the finite-time stability and bounded of the closed-loop Markovian jump systems. Compared with traditional methods of MJSs, it is shown the less conservative results can be obtained and the desired H_∞ control performance is obtained by employing mode-dependent Lyapunov functional instead of mode-independent Lyapunov functional. The finite-time bounded criterion can be dealt with in the terms of LMIs. Finally, the effectiveness of the developed techniques is also illustrated by two numerical examples.

2. Preliminaries

Given the probability space (Ω, F, P), where Ω, F, and P represent the sample space, the algebra of events, and the probability measure defined on F, respectively, the following Markovian jump systems over the probability space (Ω, F, P) are considered:

$$\begin{gathered}\dot{x}(t)=A_{r_t}x(t)+A_{\tau r_t}x\left(t-\tau_{r_t}(t)\right)+B_{r_t}u(t)+D_{r_t}\omega(t),\\ z(t)=C_{r_t}x(t)+C_{\tau r_t}x\left(t-\tau_{r_t}(t)\right)+F_{r_t}\omega(t),\\ x(t)=\varphi(t),\quad t=[-h,0],\end{gathered}\tag{1}$$

where $x(t) \in \mathscr{R}^n$ represents the state vector of Markovian jump system, $u(t) \in \mathscr{R}^m$ is the control input, $z(t) \in \mathscr{R}^q$ denotes the controlled output and $\varphi(t), t = [-h, 0]$, where $r_0 \in \mathscr{N}$ is initial condition. $\omega(k) \in \mathscr{R}^q$ denotes the disturbance input which satisfies

$$\int_0^T \omega^{\mathrm{T}}(t)\,\omega(t)\,dt \le d. \tag{2}$$

Firstly, taking value on the finite set $\mathscr{N} = \{1, 2, \ldots, N\}$, let the random form process $\{r_t, t \ge 0\}$ be the stochastic process with transition rate matrix $\Omega = \{\pi_{ij}\}$, $i, j \in \mathscr{N}$ and let the transition probabilities also be denoted as follows:

$$\Pr\left(r_{t+\Delta}=j \mid r_t=i\right)=\rho_{ij}+\pi_{ij}\Delta+o(\Delta), \tag{3}$$

where

$$\varrho_{ij}=\begin{cases}0, & \text{if } i\neq j\\ 1 & \text{if } i=j,\end{cases}\tag{4}$$

and $\Delta > 0$, $\pi_{ij} \ge 0$, for $i \ne j$, denotes the mode i in time t to time $t + \Delta$ with mode j,

$$-\pi_{ii}=\sum_{j=1, j\neq i}^{N}\pi_{ij}\tag{5}$$

for each mode $i \in \mathscr{N}$, $\lim_{\Delta\to 0_+}(o(\Delta)/\Delta) = 0$. $\tau_i(t)$ denotes the time-varying delay, which satisfies

$$\begin{gathered}0<\tau_i(t)\le h_i<\infty,\\ \dot{\tau}_i(t)\le\mu_i,\end{gathered}\tag{6}$$

where $h = \max\{h_i, i \in \mathscr{N}\}$ is the given upper bound of time-varying delays $\tau_i(t)$ and $\mu = \max\{\mu_i, i \in \mathscr{N}\}$ is the given upper bound of $\dot{\tau}_i(t)$. All the matrices are known matrices with the appropriate dimension.

In this paper, the objective is to design a state feedback controller as follows:

$$u(t)=K_i x(t),\tag{7}$$

where K_i is the controller gains to be designed.

Definition 1. System (1) is said to be finite-time bounded with respect to (c_1, c_2, T, R, d), if condition (2) and the following inequality hold:

$$\begin{gathered}\sup_{-h\le v\le 0}\mathbb{E}\left\{x^{\mathrm{T}}(v)Rx(v),\dot{x}^{\mathrm{T}}(v)R\dot{x}(v)\right\}\le c_1\\ \Longrightarrow\mathbb{E}\left\{x^{\mathrm{T}}(t)Rx(t)\right\}<c_2,\quad\forall t\in[0,T],\end{gathered}\tag{8}$$

where $c_2 > c_1 \ge 0$ and $R > 0$.

Definition 2 (see [8]). Considering system (1) with the stochastic Lyapunov function $V(x_t, r_t)$, we get the weak infinitesimal operator as follows:

$$\begin{aligned}\pounds V\left(x_t,r_t,t\right)&=\lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[\mathbb{E}\left\{V\left(x_{t+\Delta t},r_{t+\Delta t},t+\Delta t\right)\right\}\right.\\&\qquad\left.-V\left(x_t,i,t\right)\right]\\&=\frac{\partial}{\partial t}V\left(x_t,i,t\right)+\frac{\partial}{\partial x}V\left(x_t,i,t\right)\dot{x}(t,i)\\&\quad+\sum_{j=1}^{N}\pi_{ij}V\left(x_t,j,t\right).\end{aligned}\tag{9}$$

Definition 3. Given a constant scalar $T > 0$ and for all admissible $\omega(t)$ given in condition (2), if the Markovian jump system (1) is finite-time stochastic bounded and controller outputs satisfy condition (7) with attenuation $\gamma > 0$,

$$\mathbb{E}\left\{\int_0^T z^{\mathrm{T}}(t)\,z(t)\,dt\right\}\le\gamma^2 e^{\eta T}\mathbb{E}\left\{\int_0^T\omega^{\mathrm{T}}(t)\,\omega(t)\,dt\right\}.\tag{10}$$

The Markovian jump system (1) is called the finite-time stochastic bounded with a disturbance attenuation γ.

Lemma 4 (see [27]). *Let f_i: $\mathscr{R}^m \to \mathscr{R}(i = 1, 2, \ldots, N)$ have positive values in an open subset $\mathscr{D}$ of $\mathscr{R}^m$. Then, the reciprocally convex combination of f_i over $\mathscr{D}$ satisfies*

$$\begin{gathered}\min_{\{\beta_i\mid\beta_i>0,\sum_i\beta_i=1\}}\sum_i\frac{1}{\beta_i}f_i(t)=\sum_i f_i(t)+\max_{g_{i,j}(t)}\sum_{i\neq j}g_{i,j}(t)\\ \textit{subject to}\ \left\{g_{i,j}:\mathscr{R}^m\longrightarrow\mathscr{R},g_{j,i}(t)=g_{i,j}(t),\right.\\ \left.\begin{bmatrix}f_i(t) & g_{i,j}(t)\\ g_{i,j}(t) & f_j(t)\end{bmatrix}\ge 0\right\}.\end{gathered}\tag{11}$$

Lemma 5. *For any constant matrix $M \in \mathscr{R}^{m\times m}$ with $M > 0$, scalars $a < b \le 0$, vector function $x : [a,b] \to \mathscr{R}^m$, such that the integrals in the following are well-defined; then,*

$$-\frac{a^2-b^2}{2}\int_a^b\int_{t+s}^t x^{\mathsf{T}}(s)\,Mx(s)\,ds\,d\theta \le -\left[\int_a^b\int_{t+s}^t x(s)\,ds\,d\theta\right]^{\mathsf{T}} M\left[\int_a^b\int_{t+s}^t x(s)\,ds\,d\theta\right]. \tag{12}$$

3. Finite-Time H_∞ Performance Analysis

The issue of stability analysis of Markovian jump system (1) subject to $u(t) = 0$ is given firstly. Therefore, the finite-time stability is obtained in this section.

Theorem 6. *System* (1) *is called the finite-time bounded with respect to (c_1, c_2, d, R, T), if there exist matrices*

$$\begin{gathered} P_i > 0, \quad Q_{li} > 0 \quad (l = 1,2), \quad Q > 0, \\ \mathscr{X}_i = \begin{bmatrix} X_{1i} & X_{2i} \\ X_{3i} & X_{4i}\end{bmatrix} > 0, \quad \mathscr{X} = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4\end{bmatrix} > 0, \\ Y_s > 0 \quad (s = 1,2), \quad Z_i > 0, \\ Z > 0, \quad H > 0, \end{gathered} \tag{13}$$

scalars $c_1 < c_2$, $T > 0$, $\lambda_s > 0$, $(s = 1,2,\ldots,12)$, $\eta > 0$, and $\Lambda > 0$, such that $\forall i, j \in \mathscr{N}$ and the inequalities hold as follows:

$$e^{\delta h}\sum_{j=1}^N \pi_{ij}Q_{1j} + e^{\delta h}\sum_{j=1, j\ne i}^N \pi_{ij}Q_{2j} < Q, \tag{14}$$

$$\sum_{j=1}^N \pi_{ij}\mathscr{X}_j - \mathscr{X} < 0, \tag{15}$$

$$\sum_{j=1}^N \pi_{ij}Z_j - Z < 0, \tag{16}$$

$$\begin{bmatrix} \dfrac{\mathscr{X}_i}{h} & \mathscr{S}_i \\ * & \dfrac{\mathscr{X}_i}{h}\end{bmatrix} > 0, \tag{17}$$

$$\begin{bmatrix} Y_1 & W_1 \\ * & Y_2\end{bmatrix} > 0, \tag{18}$$

$$\begin{bmatrix} Y_1 & W_2 \\ * & Y_2\end{bmatrix} > 0, \tag{19}$$

$$\Xi_i = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & S_{4i} & \Xi_{14i} & \Xi_{15i} & -S_{3i} + \dfrac{Z_i + Z_i^{\mathsf{T}}}{h} & P_iD_i \\ * & \Xi_{22i} & -S_{4i} + \dfrac{X_{4i}}{h} & 0 & \dfrac{X_{3i}}{2h} + \dfrac{X_{2i}^{\mathsf{T}}}{2h} & S_{3i} - S_{2i}^{\mathsf{T}} - \dfrac{X_{3i}}{2h} - \dfrac{X_{2i}^{\mathsf{T}}}{2h} & 0 \\ * & * & -Q_{1i} - \dfrac{X_{4i}}{h} - W_2 & 0 & 0 & S_{2i}^{\mathsf{T}} + \dfrac{X_{3i}}{2h} + \dfrac{X_{2i}^{\mathsf{T}}}{2h} & 0 \\ * & * & * & \Xi_{44i} & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55i} & -S_{1i} - \dfrac{Z_i}{h^2} - \dfrac{Z_i^{\mathsf{T}}}{h^2} & 0 \\ * & * & * & * & * & -\dfrac{X_{1i}}{h} - \dfrac{Z_i}{h^2} - \dfrac{Z_i^{\mathsf{T}}}{h^2} & 0 \\ * & * & * & * & * & * & -\delta H \end{bmatrix} < 0, \tag{20}$$

$$c_1\Lambda + d\delta\lambda_{12}\frac{1}{\eta}\left(1 - e^{-\eta T}\right) < \lambda_1 e^{-\eta T}c_2, \tag{21}$$

where

$$\begin{aligned} \Xi_{11i} = {} & \sum_{j=1}^N \pi_{ij}P_j + \delta P_i + P_iA_i + A_i^{\mathsf{T}}P_i + e^{\delta h}(Q_{1i} + Q_{2i}) \\ & + hQ + \frac{e^{\delta h - 1}}{\delta}X_{1i} \\ & + \frac{e^{\delta h} - \delta h e^{\delta h} - 1}{\delta^2}X_1 - \frac{X_{4i}}{h} + hY_1 + W_1 - Z_i - Z_i^{\mathsf{T}}, \end{aligned}$$

$$\Xi_{12i} = P_iA_{\tau i} + \frac{X_{4i}}{h} - S_{4i},$$

$$\Xi_{14i} = \frac{e^{\delta h} - 1}{2\delta}\left(X_{2i} + X_{3i}^{\mathsf{T}}\right) + \frac{e^{\delta h} - \delta h e^{\delta h} - 1}{2\delta^2}\left(X_2 + X_3^{\mathsf{T}}\right),$$

$$\Xi_{15i} = -\frac{X_{3i}}{2h} - \frac{X_{2i}^{\mathsf{T}}}{2h} + \frac{Z_i}{h^2} + \frac{Z_i^{\mathsf{T}}}{h^2},$$

$$\Xi_{22i} = -\left(1 - \mu_i\right)Q_{2i} - \frac{2X_{4i}}{h} + S_{4i} + S_{4i}^{\mathsf{T}} - W_1 + W_2,$$

$$\Xi_{44i} = \frac{e^{\delta h}-1}{\delta}X_{4i} + \frac{e^{\delta h}-\delta h e^{\delta h}-1}{\delta^2}X_{4i} + hY_2 + \frac{e^{\delta h}-\delta h-1}{\delta^2}Z_i + \frac{h\delta^2 e^{\delta h}+e^{\delta h}+\delta h+1}{\delta^3}Z,$$

$$\Xi_{55i} = -\frac{X_{4i}}{h} - \frac{Z_i}{h^2} - \frac{Z_i^{\top}}{h^2},$$

$$\Lambda = \lambda_2 + he^{\delta h}(\lambda_3+\lambda_4) + h^2 e^{\delta h}\lambda_5 + h^2 e^{\delta h}\lambda_6 + \frac{1}{2}h^3 e^{\delta h}\lambda_7 + h^2 e^{\delta h}(\lambda_8+\lambda_9) + \frac{1}{2}h^3 e^{\delta h}\lambda_{10} + \frac{1}{6}h^4 e^{\delta h}\lambda_{11},$$

$$\lambda_1 = \max_{i\in\mathcal{N}}\lambda_{\max}(P_i), \qquad \lambda_2 = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{P}_i),$$

$$\lambda_3 = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{Q}_{1i}), \qquad \lambda_4 = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{Q}_{2i}),$$

$$\lambda_5 = \lambda_{\max}(\widetilde{Q}), \qquad \lambda_6 = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{\mathcal{X}}_i),$$

$$\lambda_7 = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{\mathcal{X}}), \qquad \lambda_8 = \lambda_{\max}(\widetilde{Y}_1),$$

$$\lambda_9 = \lambda_{\max}(\widetilde{Y}_2), \qquad \lambda_{10} = \max_{i\in\mathcal{N}}\lambda_{\max}(\widetilde{Z}_i),$$

$$\lambda_{11} = \lambda_{\max}(\widetilde{Z}), \qquad \lambda_{12} = \lambda_{\max}(H),$$

$$\widetilde{P}_i = R^{-(1/2)}P_iR^{-(1/2)},$$

$$\widetilde{Q}_{li} = R^{-(1/2)}Q_{li}R^{-(1/2)} \quad (l=1,2),$$

$$\widetilde{Q} = R^{-(1/2)}QR^{-(1/2)},$$

$$\widetilde{\mathcal{X}}_i = R^{-(1/2)}\mathcal{X}_iR^{-(1/2)} \quad (l=1,2),$$

$$\widetilde{X} = R^{-(1/2)}XR^{-(1/2)},$$

$$\widetilde{Y}_s = R^{-(1/2)}Y_sR^{-(1/2)} \quad (s=1,2),$$

$$\widetilde{Z}_i = R^{-(1/2)}Z_iR^{-(1/2)},$$

$$\widetilde{Z} = R^{-(1/2)}ZR^{-(1/2)}. \tag{22}$$

Proof. Firstly, a novel process is defined in this paper as follows:

$$x_t(s) = x(t+s), \quad s\in[-h,0]. \tag{23}$$

Then, the following Lyapunov-Krasovskii functional is considered:

$$V(x_t,r_t,t) = \sum_{l=1}^{5}V_l(x_t,r_t,t), \tag{24}$$

where

$$V_1(x_t,r_t,t) = x(t)^{\top}e^{\delta t}P_{r_t}x(t),$$

$$V_2(x_t,r_t,t) = \int_{t-h}^{t}e^{\delta(s+h)}x^{\top}(s)Q_{1r_t}x(s)\,ds + \int_{t-\tau_{r_t}(t)}^{t}e^{\delta(s+h)}x^{\top}(s)Q_{2r_t}x(s)\,ds + \int_{-h}^{0}\int_{t+\theta}^{t}e^{\delta(s+h)}x^{\top}(s)Qx(s)\,ds\,d\theta,$$

$$V_3(x_t,r_t,t) = \int_{-h}^{0}\int_{t+\theta}^{t}e^{\delta(s-\theta)}\eta^{\top}(s)\mathcal{X}_{r_t}\eta(s)\,ds\,d\theta + \int_{-h}^{0}\int_{\theta}^{0}\int_{t+v}^{t}e^{\delta(s-\theta)}\eta^{\top}(s)\mathcal{X}\eta(s)\,ds\,dv\,d\theta,$$

$$V_4(x_t,r_t,t) = \int_{-h}^{0}\int_{t+\theta}^{t}e^{\delta(s-\theta)}x^{\top}(s)Y_1x(s)\,ds\,d\theta + \int_{-h}^{0}\int_{t+\theta}^{t}e^{\delta(s-\theta)}\dot{x}^{\top}(s)Y_2\dot{x}(s)\,ds\,d\theta,$$

$$V_5(x_t,r_t,t) = \int_{-h}^{0}\int_{\theta}^{0}\int_{t+v}^{t}e^{\delta(s-\theta)}\dot{x}^{\top}(s)Z_{r_t}\dot{x}(s)\,ds\,dv\,d\theta + \int_{-h}^{0}\int_{\varsigma}^{0}\int_{\theta}^{0}\int_{t+v}^{t}e^{\delta(s-\theta)}\dot{x}^{\top}(s) \times Z\dot{x}(s)\,ds\,dv\,d\theta\,d\varsigma, \tag{25}$$

where $\eta(t) = [x^{\top}(t), \dot{x}^{\top}(t)]^{\top}$.

Letting i represent the time t, that is, $r_t = i\in\mathcal{N}$, one has

$$\begin{aligned}\pounds V_1(x_t,i,t) &= \delta e^{\delta t}x^{\top}(t)P_ix(t) + 2e^{\delta t}x^{\top}(t)P_i\dot{x}(t) + e^{\delta t}x^{\top}(t)\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)x(t)\\ &= e^{\delta t}x^{\top}(t)\left(\sum_{j=1}^{N}\pi_{ij}P_j + \delta P_i\right)x(t) + 2e^{\delta t}x^{\top}(t)P_i \times (A_ix(t) + A_{\tau i}x(t-\tau_i(t)) + D_i\omega(t)).\end{aligned} \tag{26}$$

Noting $\pi_{ij}\geq 0$ for $j\neq i$ and $\pi_{ii}\leq 0$, one has

$$\begin{aligned}\pounds V_2(x_t,i,t) &= e^{\delta t}x^{\top}(t)\left(e^{\delta h}Q_{1i} + e^{\delta h}Q_{2i} + hQ\right)x(t) - e^{\delta t}x^{\top}(t-h)Q_{1i}x(t-h)\\ &\quad - (1-\dot{\tau}_i(t))e^{\delta(t+h-\tau_i(t))}x^{\top} \times (t-\tau_i(t))Q_{2i}x(t-\tau_i(t))\\ &\quad + \int_{t-h}^{t}e^{\delta s}x^{\top}(s)\left(e^{\delta h}\sum_{j=1}^{N}\pi_{ij}Q_{1j} - Q\right)x(s)\,ds\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t} e^{\delta(s+h)}x^{\top}(s)\,Q_{2j}x(s)\,ds\\
&\le e^{\delta t}x^{\top}(t)\left(e^{\delta h}Q_{1i}+e^{\delta h}Q_{2i}+hQ\right)x(t)\\
&\quad -e^{\delta t}x^{\top}(t-h)\,Q_{1i}x(t-h)\\
&\quad -\left(1-\dot{\tau}_i(t)\right)e^{\delta t}x^{\top}\left(t-\tau_i(t)\right)Q_{2i}x\left(t-\tau_i(t)\right)\\
&\quad +\int_{t-h}^{t} e^{\delta s}x^{\top}(s)\left(e^{\delta h}\sum_{j=1}^{N}\pi_{ij}Q_{1j}+e^{\delta h}\right.\\
&\qquad\qquad \left.\times\sum_{j=1,j\neq i}^{N}\pi_{ij}Q_{2j}-Q\right)\\
&\qquad \times x(s)\,ds.
\end{aligned}
\tag{27}
$$

It follows from (15) and (28) that

$$
\begin{aligned}
£V_2\left(x_t,i,t\right)&=e^{\delta t}x^{\top}(t)\left(e^{\delta h}Q_{1i}+e^{\delta h}Q_{2i}+hQ\right)x(t)\\
&\quad -e^{\delta t}x^{\top}(t-h)\,Q_{1i}x(t-h)\\
&\quad -\left(1-\dot{\tau}_i(t)\right)e^{\delta t}x^{\top}\left(t-\tau_i(t)\right)Q_{2i}x\left(t-\tau_i(t)\right),
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
£V_3\left(x_t,i,t\right)=&\int_{-h}^{0}\int_{t+\theta}^{t} e^{\delta(s-\theta)}\eta^{\top}(s)\left(\sum_{j=1}^{N}\pi_{ij}\mathcal{X}_{ij}-\mathcal{X}\right)\\
&\times\eta(s)\,ds\,d\theta\\
&+e^{\delta t}\eta^{\top}\mathcal{X}_i\eta(t)\int_{-h}^{0}e^{-\delta v}dv\\
&-e^{\delta t}\int_{t-h}^{t}\eta^{\top}(s)\,\mathcal{X}_i\eta(s)\,ds\\
&+e^{\delta t}\eta^{\top}(t)\,\mathcal{X}\eta(t)\int_{-h}^{0}\int_{v}^{0}e^{-\delta v}d\theta\,dv.
\end{aligned}
\tag{29}
$$

By employing Lemma 4, we can obtain that

$$
\begin{aligned}
&-\int_{t-h}^{t}\eta^{\top}(s)\,\mathcal{X}_i\eta(s)\,ds\\
&=-\int_{t-\tau_i(t)}^{t}\eta^{\top}(s)\,\mathcal{X}_i\eta(s)\,ds-\int_{t-h}^{t-\tau_i(t)}\eta^{\top}(s)\,\mathcal{X}_i\eta(s)\,ds\\
&\le-\frac{h}{\tau_i(t)}\left[\int_{t-\tau_i(t)}^{t}\eta(s)\,ds\right]^{\top}\frac{\mathcal{X}_i}{h}\left[\int_{t-\tau_i(t)}^{t}\eta(s)\,ds\right]\\
&\quad-\frac{h}{h-\tau_i(t)}\left[\int_{t-h}^{t-\tau_i(t)}\eta(s)\,ds\right]^{\top}\frac{\mathcal{X}_i}{h}\left[\int_{t-h}^{t-\tau_i(t)}\eta(s)\,ds\right]\\
&\le-\begin{bmatrix}\int_{t-\tau_i(t)}^{t}\eta(s)\,ds\\ \int_{t-h}^{t-\tau_i(t)}\eta(s)\,ds\end{bmatrix}^{\top}\begin{bmatrix}\dfrac{\mathcal{X}_i}{h} & \mathcal{S}_i\\ * & \dfrac{\mathcal{X}_i}{h}\end{bmatrix}\begin{bmatrix}\int_{t-\tau_i(t)}^{t}\eta(s)\,ds\\ \int_{t-h}^{t-\tau_i(t)}\eta(s)\,ds\end{bmatrix}.
\end{aligned}
\tag{30}
$$

It follows from (30) and (31) that

$$
\begin{aligned}
V_3\left(x_t,i,t\right)\le{}& e^{\delta t}\eta^{\top}(t)\\
&\times\left(\frac{e^{\delta h}-1}{\delta}\mathcal{X}_i+\frac{e^{\delta h}-\delta he^{\delta h}-1}{\delta^2}\mathcal{X}\right)\eta(t)\\
&-e^{\delta t}\begin{bmatrix}\int_{t-\tau_i(t)}^{t}\eta(s)\,ds\\ \int_{t-h}^{t-\tau_i(t)}\eta(s)\,ds\end{bmatrix}^{\top}\begin{bmatrix}\dfrac{\mathcal{X}_i}{h} & \mathcal{S}_i\\ * & \dfrac{\mathcal{X}_i}{h}\end{bmatrix}\\
&\times\begin{bmatrix}\int_{t-\tau_i(t)}^{t}\eta(s)\,ds\\ \int_{t-h}^{t-\tau_i(t)}\eta(s)\,ds\end{bmatrix}.
\end{aligned}
\tag{31}
$$

Consider

$$
\begin{aligned}
£V_4\left(x_t,i,t\right)={}&hx^{\top}(t)\,Y_1x(t)\\
&-\int_{t-h}^{t}x^{\top}(s)\,Y_1x(s)\,ds+h\dot{x}^{\top}(t)\,Y_2\dot{x}(t)\\
&-\int_{t-h}^{t}\dot{x}^{\top}(s)\,Y_2\dot{x}(s)\,ds.
\end{aligned}
\tag{32}
$$

Moreover, the following two zero equalities with any symmetric matrices W_1 and W_2 are considered:

$$
\begin{aligned}
0={}&x^{\top}(t)\,W_1x(t)-x^{\top}\left(t-\tau_i(t)\right)W_1x\left(t-\tau_i(t)\right)\\
&-2\int_{t-\tau_i(t)}^{t}x^{\top}(s)\,W_1\dot{x}(s)\,ds,
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
0={}&x^{\top}\left(t-\tau_i(t)\right)W_2x\left(t-\tau_i(t)\right)\\
&-x^{\top}(t-h)\,W_2x(t-h)\\
&-2\int_{t-h}^{t-\tau_i(t)}x^{\top}(s)\,W_2\dot{x}(s)\,ds.
\end{aligned}
\tag{34}
$$

With the above two zero equalities (34) and (35), an upper bound of $£V_4(x_t, i, t)$ is

$$\begin{aligned} £_4(x_t, i, t) &= x^\top(t)(hY_1 + W_1)x(t) + h\dot{x}^\top(t)Y_2\dot{x}(t) \\ &+ x^\top(t-\tau_i(t))(W_2 - W_1)x(t-\tau_i(t)) \\ &- x^\top(t-h)W_2x(t-h) \\ &- \int_{t-\tau_i(t)}^{t}\begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix}^\top\begin{bmatrix} Y_1 & W_1 \\ * & Y_2\end{bmatrix}\begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix} \\ &- \int_{t-h}^{t-\tau_i(t)}\begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix}^\top\begin{bmatrix} Y_1 & W_1 \\ * & Y_2\end{bmatrix}\begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix}. \end{aligned} \quad (35)$$

From (19), (20), and (36), one can obtain

$$\begin{aligned} £_4(x_t, i, t) &= x^\top(t)(hY_1 + W_1)x(t) + h\dot{x}^\top(t)Y_2\dot{x}(t) \\ &+ x^\top(t-\tau_i(t))(W_2 - W_1)x(t-\tau_i(t)) \\ &- x^\top(t-h)W_2x(t-h). \end{aligned} \quad (36)$$

Now, $£V_5(x_t, i, t)$ is obtained as follows:

$$\begin{aligned} &£V_5(x_t, i, t) \\ &\quad= \int_{-h}^{0}\int_{\theta}^{0}\int_{t+v}^{t} e^{\delta(s-\theta)}\dot{x}^\top(s)\left(\sum_{j=1}^{N}\pi_{ij}Z_j - Z\right) \\ &\qquad\times\dot{x}(s)\,ds\,dv\,d\theta \\ &\quad+ e^{\delta t}\dot{x}^\top(t)Z_i\dot{x}(t)\int_{-h}^{0}\int_{\theta}^{0}e^{-\delta v}dv\,d\theta \\ &\quad- e^{\delta t}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^\top(s)Z_i\dot{x}(s)\,ds\,d\theta \\ &\quad+ e^{\delta t}\dot{x}^\top(t)Z\dot{x}(t)\int_{-h}^{0}\int_{\varsigma}^{0}\int_{\theta}^{0}e^{-\delta\theta}dv\,d\theta\,d\varsigma. \end{aligned} \quad (37)$$

By using Lemma 5, one has

$$\begin{aligned} &-\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^\top(s)Z_i\dot{x}(s)\,ds\,d\theta \\ &\quad\le -\frac{2}{h^2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^\top(s)\,ds\,d\theta Z_i\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)\,ds\,d\theta \\ &\quad= -2\left[x(t) - \frac{1}{h}\int_{t-\tau_i(t)}^{t}x(s)\,ds - \frac{1}{h}\int_{t-h}^{t-\tau_i(t)}x(s)\,ds\right]^\top Z_i \\ &\qquad\times\left[x(t) - \frac{1}{h}\int_{t-\tau_i(t)}^{t}x(s)\,ds - \frac{1}{h}\int_{t-h}^{t-\tau_i(t)}x(s)\,ds\right]. \end{aligned} \quad (38)$$

Together with (38) and (39), it implies that

$$\begin{aligned} £V_5(x_t, i, t) &\le e^{\delta t}\dot{x}^\top(t)\frac{e^{\delta h} - \delta h - 1}{\delta^2}Z_i\dot{x}(t) \\ &+ e^{\delta t}\dot{x}^\top(t)\frac{h\delta^2e^{\delta h} + e^{\delta h} + \delta h + 1}{\delta^3}Z\dot{x}(t) \\ &- 2e^{\delta t}\left[x(t) - \frac{1}{h}\int_{t-\tau_i(t)}^{t}x(s)\,ds - \frac{1}{h}\int_{t-h}^{t-\tau_i(t)}x(s)\,ds\right]^\top Z_i \\ &\times\left[x(t) - \frac{1}{h}\int_{t-\tau_i(t)}^{t}x(s)\,ds - \frac{1}{h}\int_{t-h}^{t-\tau_i(t)}x(s)\,ds\right]. \end{aligned} \quad (39)$$

From (26)–(40), we can eventually obtain

$$£V(x_t, r_t, t) - \delta\omega^\top(t)H\omega(t) \le e^{\delta t}\xi^\top(t)\Xi_i\xi(t), \quad (40)$$

where

$$\begin{aligned} \xi^\top(t) = \Bigg[&x^\top(t), x^\top(t-\tau_i(t)), x^\top(t-h), \int_{t-\tau_i(t)}^{t}x^\top(s)\,ds, \\ &\int_{t-h}^{t-\tau_i(t)}x^\top(s)\,ds, \omega^\top(t)\Bigg]. \end{aligned} \quad (41)$$

It follows from (45) that

$$\mathbb{E}\{£V(x_t, r_t, t)\} \le \mathbb{E}[\eta V(x_t, r_t, t)] + \delta\omega^\top(t)H\omega(t). \quad (42)$$

Multiplying the above inequality by $e^{-\eta t}$ yields that

$$\mathbb{E}\{£[e^{-\eta t}V(x_t, r_t, t)]\} \le e^{-\eta t}\delta\omega^\top(t)H\omega(t). \quad (43)$$

Integrating the inequality from 0 to t, we have

$$\begin{aligned} &e^{-\eta t}\mathbb{E}[V(x_t, r_t, t)] - \mathbb{E}[V(x_0, r_0, 0)] \\ &\quad\le \delta\int_{0}^{t}e^{-\eta s}\omega^\top(s)H\omega(s)\,ds. \end{aligned} \quad (44)$$

Denoting $\widetilde{P}_i = R^{-(1/2)}P_iR^{-(1/2)}$, $\widetilde{Q}_i = R^{-(1/2)}Q_iR^{-(1/2)}$, $\widetilde{Q} = R^{-(1/2)}QR^{-(1/2)}$, $\widetilde{\mathscr{X}}_i = R^{-(1/2)}\mathscr{X}_iR^{-(1/2)}$, $\widetilde{\mathscr{X}} = R^{-(1/2)}\mathscr{X}R^{-(1/2)}$, $\widetilde{Y}_i = R^{-(1/2)}Y_iR^{-(1/2)}$, $\widetilde{Y} = R^{-(1/2)}YR^{-(1/2)}$, $\widetilde{Z}_i = R^{-(1/2)}Z_iR^{-(1/2)}$, and $\widetilde{Z} = R^{-(1/2)}ZR^{-(1/2)}$ yields that

$$\begin{aligned} &\mathbb{E}[V(x_0, r_0, 0)] \\ &\quad\le \max_{i\in\mathscr{N}}\lambda_{\max}(\widetilde{P}_i)x^\top(0)Rx(0) \\ &\qquad+ \left(\max_{i\in\mathscr{N}}\lambda_{\max}(Q_{1i}) + \max_{i\in\mathscr{N}}\lambda_{\max}(Q_{2i})\right)e^{\delta h} \end{aligned}$$

$$
\begin{aligned}
&\times \int_{-h}^{0} e^{\delta s} x^{\top}(s) R x(s)\, ds \\
&+ e^{\delta h} \lambda_{\max}(Q) \int_{-h}^{0} \int_{\theta}^{0} e^{\delta s} x^{\top}(s) R x(s)\, ds \\
&+ e^{\delta h} \max_{i \in \mathcal{N}} \lambda_{\max}(\mathcal{X}_i) \int_{-h}^{0} \int_{\theta}^{0} e^{-\delta\theta} \eta^{\top}(s) R \eta(s)\, ds\, d\theta \\
&+ e^{\delta h} \lambda_{\max}(\mathcal{X}) \int_{-h}^{0} \int_{\theta}^{0} \int_{\upsilon}^{0} e^{-\delta\theta} \eta^{\top}(s) R \eta(s)\, ds\, d\theta\, d\upsilon \\
&+ (\lambda_{\max}(Y_1) + \lambda_{\max}(Y_2)) e^{\delta h} \\
&\times \int_{-h}^{0} \int_{\theta}^{0} e^{-\delta\theta} x^{\top}(s) R x(s)\, ds\, d\theta \\
&+ e^{\delta h} \max_{i \in \mathcal{N}} \lambda_{\max}(Z_i) \int_{-h}^{0} \int_{\theta}^{0} \int_{\upsilon}^{0} e^{-\delta\theta} x^{\top}(s) R x(s)\, ds\, d\theta \\
&+ e^{\delta h} \lambda_{\max}(Z) \int_{-h}^{0} \int_{\varsigma}^{0} \int_{\theta}^{0} \int_{\upsilon}^{0} e^{-\delta\theta} x^{\top}(s) R x(s)\, ds\, d\theta\, d\varsigma \\
\le & \Big\{ \max_{i \in \mathcal{N}} \lambda_{\max}(\widetilde{P}_i) + h e^{\delta h} \\
&\quad \times \Big(\max_{i \in \mathcal{N}} \lambda_{\max}\Big(Q_{1i} + \max_{i \in \mathcal{N}} \lambda_{\max}(Q_{2i})\Big)\Big) \\
&\quad + h^2 e^{\delta h} \lambda_{\max}(Z) + h^2 e^{\delta h} \max_{i \in \mathcal{N}} \lambda_{\max}(\widetilde{\mathcal{X}}_i) \\
&\quad + \frac{1}{2} h^3 e^{\delta h} \lambda_{\max}(\widetilde{\mathcal{X}}) + h^2 e^{\delta h} (\lambda_{\max}(Y_1) + \lambda_{\max}(Y_2)) \\
&\quad + \frac{1}{2} h^3 e^{\delta h} \max_{i \in \mathcal{N}} \lambda_{\max}(Z_i) + \frac{1}{6} h^4 e^{\delta h} \lambda_{\max}(Z) \Big\} \\
&\quad \times \sup_{-h \le s \le 0} \{x^{\top}(s) R x(s), \dot{x}^{\top}(s) R \dot{x}(s)\} = c_1 \Lambda.
\end{aligned}
\tag{45}
$$

For scalars $\eta > 0$ and $T \ge t \ge 0$, (46) turns out to be

$$
\begin{aligned}
\mathbb{E}[V(x_t, r_t, t)] &\le \mathbb{E}\left[e^{\eta t} V(x_0, r_0, 0)\right] \\
&\quad + e^{\eta t} \delta \int_{0}^{t} e^{-\eta s} \omega^{\top}(s) H \omega(s)\, ds \\
&\le e^{\eta T} c_1 \Lambda + d\delta e^{\eta T} \lambda_{\max}(H) \int_{0}^{T} e^{-\eta s} ds \\
&\le e^{\eta T} \left\{ c_1 \Lambda + d\delta \lambda_{12} \frac{1}{\eta} \left(1 - e^{-\eta T}\right) \right\}.
\end{aligned}
\tag{46}
$$

To illustrate the bounded, (26) takes the following form:

$$
\begin{aligned}
&\mathbb{E}[V(x_t, r_t, t)] \ge \mathbb{E}\left[x^{\top}(t) e^{\lambda t} P_i x(t)\right] \\
&\ge \max_{i \in \mathcal{N}} \lambda_{\min}(P_i) \mathbb{E}\left[x^{\top}(t) R x(t)\right] = \lambda_1 \mathbb{E}\left[x^{\top}(t) R x(t)\right].
\end{aligned}
\tag{47}
$$

From inequalities (46)–(48), one has

$$
\mathbb{E}\left[x^{\top}(t) R x(t)\right] \le \frac{e^{\eta T}}{\lambda_1} \left\{ c_1 \Lambda + d\delta \lambda_{12} \frac{1}{\eta} \left(1 - e^{-\eta T}\right) \right\}.
\tag{48}
$$

Finally, inequalities (24) and (49) guarantee that

$$
\mathbb{E}\left[x^{\top}(t) R x(t)\right] < c_2.
\tag{49}
$$

Therefore, the Markovian jump system (1) is finite-time stochastic bounded with respect to (c_1, c_2, d, R, T). □

Remark 7. It should be noted that $\tau_i(t)$ and $\dot{\tau}_i(t)$ may, respectively, get the different upper bound due to the fact that condition (6) holds. However, $\tau_i(t)$ and $\dot{\tau}_i(t)$ always lead to conservativeness for $\tau_i(t) \le h = \max\{h_i, i \in \mathcal{N}\}$ and $\dot{\tau}_i(t) \le \mu = \max\{\mu_i, i \in \mathcal{N}\}$ in [14–18], and this case can be improved with employing the different Lyapunov-Krasovskii functional (26).

Remark 8. It should be pointed out that, in Theorem 6, the novelty of the Lyapunov functional (26) lies in the following: (i) triple-integral terms $V_3(x_t, r_t, t)$ and $V_5(x_t, r_t, t)$ and four-integral term $V_5(x_t, r_t, t)$ are introduced and (ii) the distinct Lyapunov matrices $(P_i, Q_{1i}, Q_{2i}, \mathcal{X}_i, Z_i)$ are chosen for different system modes i $(i = 1, 2, \ldots, N)$.

For the condition $r_t = i$, the Markovian jump system given in this paper is followed by

$$
\begin{aligned}
\dot{x}(t) &= \overline{A}_i x(t) + A_{\tau i} x\left(t - \tau_{r_t}(t)\right) + D_i \omega(t), \\
z(t) &= C_i x(t) + C_{\tau i} x\left(t - \tau_i(t)\right) + F_i \omega(t),
\end{aligned}
\tag{50}
$$

where

$$
\overline{A}_i = A_i + B_i K_i.
\tag{51}
$$

Theorem 9. *System* (53) *is finite-time stochastic bounded with respect to* (c_1, c_2, d, R, T) *with a disturbance attenuation, if there exist matrices*

$$
\begin{gathered}
P_i > 0, \quad Q_{li} > 0 \quad (l = 1, 2), \quad Q > 0, \\
\mathcal{X}_i = \begin{bmatrix} X_{1i} & X_{2i} \\ X_{3i} & X_{4i} \end{bmatrix} > 0, \quad \mathcal{X} = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix} > 0, \\
Y_s > 0 \quad (s = 1, 2), \quad Z_i > 0, \quad Z > 0,
\end{gathered}
\tag{52}
$$

scalars $c_1 < c_2$, $T > 0$, $\lambda_s > 0$, $(s = 1, 2, \ldots, 12)$, $\eta > 0$ *and* $\Lambda > 0$, *such that for all* $i, j \in \mathcal{N}$, (15)–(20) *and the following inequalities hold:*

$$\Sigma_i = \begin{bmatrix} \Sigma_{11i} & \Xi_{12i} & S_{4i} & \Xi_{14i} & \Xi_{15i} & -S_{3i} + \dfrac{Z_i + Z_i^{\top}}{h} & P_i D_i & C_i^{\top} \\ * & \Xi_{22i} & -S_{4i} + \dfrac{X_{4i}}{h} & 0 & \dfrac{X_{3i}}{2h} + \dfrac{X_{2i}^{\top}}{2h} & S_{3i} - S_{2i}^{\top} - \dfrac{X_{3i}}{2h} - \dfrac{X_{2i}^{\top}}{2h} & 0 & C_{\tau i}^{\top} \\ * & * & -Q_{1i} - \dfrac{X_{4i}}{h} - W_2 & 0 & 0 & S_{2i}^{\top} + \dfrac{X_{3i}}{2h} + \dfrac{X_{2i}^{\top}}{2h} & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55i} & -S_{1i} - \dfrac{Z_i}{h^2} - \dfrac{Z_i^{\top}}{h^2} & 0 & 0 \\ * & * & * & * & * & -\dfrac{X_{1i}}{h} - \dfrac{Z_i}{h^2} - \dfrac{Z_i^{\top}}{h^2} & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & F_i^{\top} \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{53}$$

$$c_1 \Lambda + d\gamma^2 \frac{1}{\eta}\left(1 - e^{-\eta T}\right) < \lambda_1 e^{-\eta T} c_2, \tag{54}$$

where

$$\begin{aligned} \Sigma_{11i} &= \sum_{j=1}^{N} \pi_{ij} P_j + \delta P_i + P_i \overline{A}_i + \overline{A}_i^{\top} P_i \\ &\quad + e^{\delta h}\left(Q_{1i} + Q_{2i}\right) + hQ + \frac{e^{\delta h - 1}}{\delta} X_{1i} \\ &\quad + \frac{e^{\delta h} - \delta h e^{\delta h} - 1}{\delta^2} X_1 - \frac{X_{4i}}{h} + hY_1 + W_1 - Z_i - Z_i^{\top}. \end{aligned} \tag{55}$$

Proof. Considering the Lyapunov-Krasovskii functional in Theorem 6 and from Schur's Lemma, it turns out to be

$$\begin{aligned} &£V\left(x_t, i, t\right) + z^{\top}(t) z(t) - \gamma^2 \omega^{\top}(t) \omega(t) \\ &\quad \le \xi^{\top}(t) \Theta_i\left(\mu_{pi}, h_{qi}\right) \xi(t). \end{aligned} \tag{56}$$

Thanks to (54), we have

$$\begin{aligned} \mathbb{E}\left\{£V\left(x_t, i, t\right)\right\} &\le \mathbb{E}\left[\eta V\left(x_t, i, t\right)\right] \\ &\quad + \gamma^2 \omega^{\top}(t) \omega(t) - \mathbb{E}\left[z^{\top}(t) z(t)\right]. \end{aligned} \tag{57}$$

Multiplying the (58) by $e^{-\eta t}$, (58) can be written as

$$\begin{aligned} &\mathbb{E}\left\{£\left[e^{-\eta t} V\left(x_t, i, t\right)\right]\right\} \\ &\quad \le e^{-\eta t}\left[\gamma^2 \omega^{\top}(t) \omega(t) - z^{\top}(t) z(t)\right]. \end{aligned} \tag{58}$$

Under the condition of zero initial and $\mathbb{E}[V(x_t, i, t)] > 0$, one has

$$\begin{aligned} &\int_0^T e^{-\eta t}\left[\gamma^2 \omega^{\top}(t) \omega(t) - z^{\top}(t) z(t)\right] dt \\ &\quad \le \mathbb{E}\left\{\int_0^T £\left[e^{-\eta t} V\left(x_t, i, t\right)\right] dt\right\} \le V\left(x_0, r_0, 0\right) = 0. \end{aligned} \tag{59}$$

Using the Dynkin formula, it results that

$$\mathbb{E}\left[\int_0^T e^{-\eta v} z^{\top}(v) z(v)\, dv\right] \le \gamma^2 \mathbb{E}\left[\int_0^T e^{-\eta v} \omega^{\top}(v) \omega(v)\, dv\right]. \tag{60}$$

Finally, it is easy to obtains that

$$\mathbb{E}\left[\int_0^T z^{\top}(v) z(v)\, dv\right] \le \gamma^2 e^{\eta T} \mathbb{E}\left[\int_0^T \omega^{\top}(v) \omega(v)\, dv\right]. \tag{61}$$

Therefore, the Markovian jump system (53) is finite-time stochastic bounded with an performance γ. □

4. Finite-Time H_∞ Control

Theorem 10. *System* (53) *is finite-time stochastic bounded with respects to* (c_1, c_2, d, R, T) *with an disturbance attenuation, if there exists matrices*

$$P_i > 0, \quad \overline{P}_i, Q_{li} > 0 \quad (l = 1, 2), \quad \overline{K}_i, Q > 0,$$

$$\mathcal{X}_i = \begin{bmatrix} X_{1i} & X_{2i} \\ X_{3i} & X_{4i} \end{bmatrix} > 0, \quad \mathcal{X} = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix} > 0, \tag{62}$$

$$Y_s > 0 \quad (s = 1, 2), \quad Z_i > 0, \quad Z > 0,$$

scalars $c_1 < c_2$, $T > 0$, $\lambda_s > 0$, $(s = 1, 2, \ldots, 12)$, $\eta > 0$ *and* $\Lambda > 0$, *such that for all* $i, j \in \mathcal{N}$, (15)–(20) *and the following inequalities hold:*

$$\begin{bmatrix} \Theta_{11i} & \Xi_{12i} & S_{4i} & \Xi_{14i} & \Xi_{15i} & -S_{3i}+\dfrac{Z_i+Z_i^{\top}}{h} & P_iD_i & C_i^{\top} \\ * & \Xi_{22i} & -S_{4i}+\dfrac{X_{4i}}{h} & 0 & \dfrac{X_{3i}}{2h}+\dfrac{X_{2i}^{\top}}{2h} & S_{3i}-S_{2i}^{\top}-\dfrac{X_{3i}}{2h}-\dfrac{X_{2i}^{\top}}{2h} & 0 & C_{\tau i}^{\top} \\ * & * & -Q_{1i}-\dfrac{X_{4i}}{h}-W_2 & 0 & 0 & S_{2i}^{\top}+\dfrac{X_{3i}}{2h}+\dfrac{X_{2i}^{\top}}{2h} & 0 & 0 \\ * & * & * & \Xi_{44i} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{55i} & -S_{1i}-\dfrac{Z_i}{h^2}-\dfrac{Z_i^{\top}}{h^2} & 0 & 0 \\ * & * & * & * & * & -\dfrac{X_{1i}}{h}-\dfrac{Z_i}{h^2}-\dfrac{Z_i^{\top}}{h^2} & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{63}$$

$$P_iB_i = B_i\overline{P}_i \tag{64}$$

$$c_1\Lambda + d\gamma^2\frac{1}{\eta}\left(1-e^{-\eta T}\right) < \lambda_1 e^{-\eta T}c_2, \tag{65}$$

where

$$\begin{aligned} \Theta_{11i} = {} & \sum_{j=1}^{N}\pi_{ij}P_j + \delta P_i + P_iA_i + B_i\overline{K}_i \\ & + A_i^{\top}P_i + \overline{K}_i^{\top}B_i^{\top} + e^{\delta h}\left(Q_{1i}+Q_{2i}\right) \\ & + hQ + \frac{e^{\delta h-1}}{\delta}X_{1i} + \frac{e^{\delta h}-\delta h e^{\delta h}-1}{\delta^2}X_1 \\ & - \frac{X_{4i}}{h} + hY_1 + W_1 - Z_i - Z_i^{\top}, \end{aligned} \tag{66}$$

and the state feedback gain matrices considered in this paper could be designed as follows:

$$K_i = \overline{P}_i^{-1}\overline{K}_i, \quad \forall i = 1, 2, \ldots, N. \tag{67}$$

Proof. This proof can be completed in view of Theorem 9 with $P_iB_i = B_i\overline{P}_i$ and $\overline{P}_iK_i = \overline{K}_i$. □

5. Illustrative Example

Example 1. Considering the following example with parameters

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.9 & 0.5 \\ -0.32 & -0.8 \end{bmatrix}, & A_{\tau 1} &= \begin{bmatrix} -0.5 & -0.3 \\ 0.3 & -0.2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} -1.05 & 0.8 \\ -0.15 & -1.3 \end{bmatrix}, & C_1 &= \begin{bmatrix} 0.6 & -0.4 \\ 0.35 & -0.41 \end{bmatrix}, \end{aligned} \tag{68}$$

the transition probabilities matrix is given as follows:

$$\Omega = \begin{bmatrix} -0.2 & 0.2 \\ 0.8 & -0.8 \end{bmatrix}. \tag{69}$$

Given the different upper bounds of h and δ, the results of the maximum upper bound of decay rates δ and maximum values of h for different time delays are obtained in Tables 1 and 2, respectively. This example indicates fully that the method proposed in the paper plays an important role in reducing conservatism. It can be also seen that our results in this paper show significant improvement over the results obtained in [11, 12]. This clearly shows that our results have less conservatism in the above two cases.

Example 2. Consider the Markovian jump system (1) where

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.8 & 1.5 \\ 2 & 3 \end{bmatrix}, & A_{\tau 1} &= \begin{bmatrix} -0.45 & 1 \\ -0.5 & 2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} -1 & 0.2 \\ 0.5 & -0.1 \end{bmatrix}, & D_1 &= \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, & C_{\tau 1} &= \begin{bmatrix} 0.03 & 0 \\ 0.01 & 0.02 \end{bmatrix}, \\ E_1 &= \begin{bmatrix} 0.02 & 0 \\ 0.01 & 0.01 \end{bmatrix}, & D_1 &= \begin{bmatrix} 0.01 \\ 0.001 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -2 & 1.2 \\ 1 & 4 \end{bmatrix}, & A_{\tau 2} &= \begin{bmatrix} -1 & 1.2 \\ 0 & -0.5 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -1 & 1 \\ 0.5 & -2 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.1 & 0.02 \\ 0 & 0.1 \end{bmatrix}, & C_{\tau 2} &= \begin{bmatrix} 0.02 & 0 \\ 0.1 & 0.02 \end{bmatrix}, \\ E_2 &= \begin{bmatrix} 0.04 & 0 \\ 0.1 & 0.01 \end{bmatrix}, & F_2 &= \begin{bmatrix} 0.04 \\ 0.01 \end{bmatrix}, \end{aligned} \tag{70}$$

and corresponding transition rate matrix is

$$\Omega = \begin{bmatrix} -1.2 & 1.2 \\ 1 & -1 \end{bmatrix}. \tag{71}$$

Table 1: Comparison of the upper bounds of the decay rate for different delays.

	$h = 0.2$	$h = 0.5$	$h = 0.8$	$h = 1$	$h = 1.2$
[12]	1.2683	1.0064	0.7962	0.6838	0.5900
[11]	1.3618	1.1769	0.9420	0.7694	0.6261
Theorem 6	1.3622	1.1771	0.9426	0.7696	0.6263

Table 2: Comparison of the allowable values of time delay h for different decay rates.

	$\delta = 0.6$	$\delta = 0.8$	$\delta = 1$	$\delta = 1.2$	$\delta = 1.4$
[12]	1.1768	0.7938	0.5081	0.2731	0.0657
[11]	1.2435	0.9626	0.7368	0.4651	0.1302
Theorem 6	1.2441	0.9630	0.7372	0.4655	0.1304

Assuming that $R = I, T = 2, c_1 = 1$, and $d = 0.01$, by suing LMI toolbox, Theorem 10 provides the following controller gains:

$$K_1 = \begin{bmatrix} -11.5351 & -8.1210 \\ 13.2230 & 10.5612 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -21.2123 & 15.5613 \\ -23.2318 & 16.4518 \end{bmatrix}. \tag{72}$$

6. Conclusions

We have presented the problems of finite-time stochastic H_∞ performance analysis of continuous-time systems with random abrupt changes in this paper. By using a different Lyapunov-Krasovskii functional, several sufficient conditions are provided to ensure the Markovian jump system is finite-time stochastic bounded. The controller gains can be dealt with by LMIs toolbox and optimization techniques. At last, two numerical examples are proposed to illustrate the effective and advantage of the developed theories.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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