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Research Article

Modulation Instability, Breathers, and Bound Solitons in an Erbium-Doped Fiber System with Higher-Order Effects

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We mainly investigate the generalized nonlinear Schrödinger-Maxwell-Bloch system which governs the propagation of optical solitons in nonlinear erbium-doped fibers with higher-order effects. We deduce Lax pair, analyze modulation instability conditions, construct the Darboux transformation, and derive the Akhmediev breathers, Ma-breathers, bound solitons, and two-breather solutions for this system. Considering the influences of higher-order effects, propagation properties of those solitons are discussed.

1. Introduction

In recent years, optical solitons have attracted many researchers for their potential applications in optical fiber transmission systems [1–3]. Based on the balance of the self-phase modulation and group velocity dispersion, the propagation of optical solitons in the picosecond regime is usually governed by the nonlinear Schrödinger [4] equation

$$iq_z + q_{tt} + 2\,|q|^2 q = 0, \tag{1}$$

where q denotes the slowly varying complex envelope of the wave. When considering propagation characters of the ultrashort pulses, (1) cannot describe the corresponding physical mechanism due to the absences of the fourth-order dispersion, higher-order nonlinearities, and self-steepening effects. Owing to the above three factors, the dynamic features of the ultrashort pulses can be depicted by the following generalized nonlinear Schrödinger equation (GNLS) [5, 6]:

$$\begin{aligned} &iq_z + q_{tt} + 2|q|^2 q \\ &\quad + \tau\left(q_{tttt} + 8|q|^2 q_{tt} + 2q^2 q_{tt}^* + 6q^* q_t^2\right. \\ &\quad \left. + 4|q_t|^2 q + 6|q|^4 q\right) = 0, \end{aligned} \tag{2}$$

where τ is a small dimensionless real parameter, and it is usually positive. In addition, (2) can also govern the nonlinear spin excitations in one-dimensional isotropic biquadratic Heisenberg ferromagnetic spin with the octupole-dipole interaction [7, 8].

In real optic fibers, the attenuation usually exists, in this context, erbium-doped fibers can minimize the attenuation [9]. The mathematical description of solitons propagating in erbium-doped fibers is the nonlinear Schrödinger-Maxwell-Bloch (NLS-MB) equations [10, 11]:

$$q_z = i\left(\frac{1}{2}q_{tt} + |q|^2 q\right) + 2p, \tag{3a}$$

$$p_t = 2i\omega p + q\eta, \tag{3b}$$

$$\eta_t = -\left(qp^* + pq^*\right), \tag{3c}$$

where subscripts z and t denote the partial derivatives with respect to the longitudinal distance and retarded time. ω is the frequency, the asterisk denotes the complex conjugate, $p = \nu_1\nu_2^*$, and $\eta = |\nu_2|^2 - |\nu_1|^2$ with ν_1 and ν_2 representing the wave functions in a two-level system [12, 13]. Many research achievements about system (3a), (3b), and (3c) have been obtained [14–16].

However, when taking the effects such as the fourth-order dispersion, higher-order nonlinearities, and self-steepening effects into account, the propagation of optical solitons in fibers doped with two-level resonant impurities like erbium

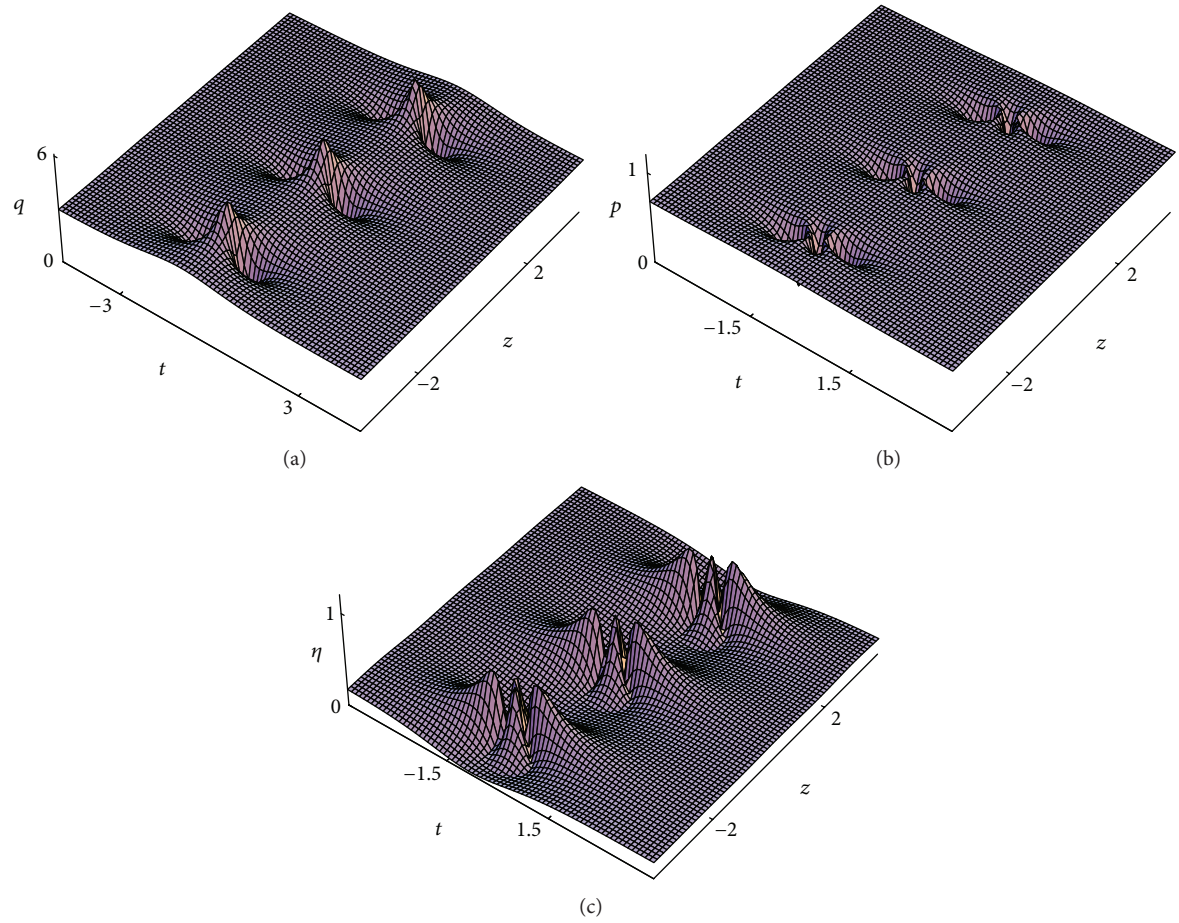

FIGURE 1: The parameters adopted here are $\kappa_s = 2.3$, $q_c = 1$, $\kappa_c = 3$, $\omega = 1$, and $\tau = 0$.

is usually described by the following generalized nonlinear Schrödinger-Maxwell-Bloch (GNLS-MB) system [9]:

$$\begin{aligned} q_z = {} & i\left(q_{tt} + 2|q|^2 q\right) \\ & + i\tau\left(q_{tttt} + 8|q|^2 q_{tt}\right. \\ & \quad + 2q^2 q_{tt}^* + 6q^* q_t^2 + 4|q_t|^2 q \\ & \quad \left. + 6|q|^4 q\right) + 2p, \end{aligned} \tag{4a}$$

$$p_t = 2i\omega p + q\eta, \tag{4b}$$

$$\eta_t = -\left(qp^* + pq^*\right). \tag{4c}$$

To our knowledge, investigations on system (4a), (4b), and (4c) have not been reported, and the aim of this paper is mainly to investigate the modulation instability conditions, generate the breather and bound solutions, and discuss the dynamic behaviors of those solutions for system (4a), (4b), and (4c).

The outline of this paper will be as follows: in Section 2, we will derive Lax pair and analyze the modulation instability conditions for system (4a), (4b), and (4c). In Section 3, by using the Darboux transformation, we will construct two types of one-breather solutions: Akhmediev breathers and Ma breathers on the nonzero continuous wave (cw) background. In Section 4, we will discuss analytically the interactions between neighboring bound solitons and two-breather solutions for system (4a), (4b), and (4c). Finally, our conclusions will be addressed in Section 5.

2. Lax Pair and Modulation Instability for System (4a), (4b), and (4c)

Employing the Ablowitz-Kaup-Newell-Segur formalism [17], we can derive the Lax pair for system (4a), (4b), and (4c) as

$$\Psi_t = U\Psi, \tag{5a}$$

$$\Psi_z = V\Psi, \tag{5b}$$

where $\Psi = (\psi_1, \psi_2)^T$ (T denotes the transpose of a matrix) and the matrices U and V have the form

$$U = -i\lambda U_1 + U_0, \tag{6a}$$

$$V = \lambda^4 V_4 + \lambda^3 V_3 + \lambda^2 V_2 + \lambda V_1 + V_0 + \frac{1}{\lambda + \omega} V_{-1}, \tag{6b}$$

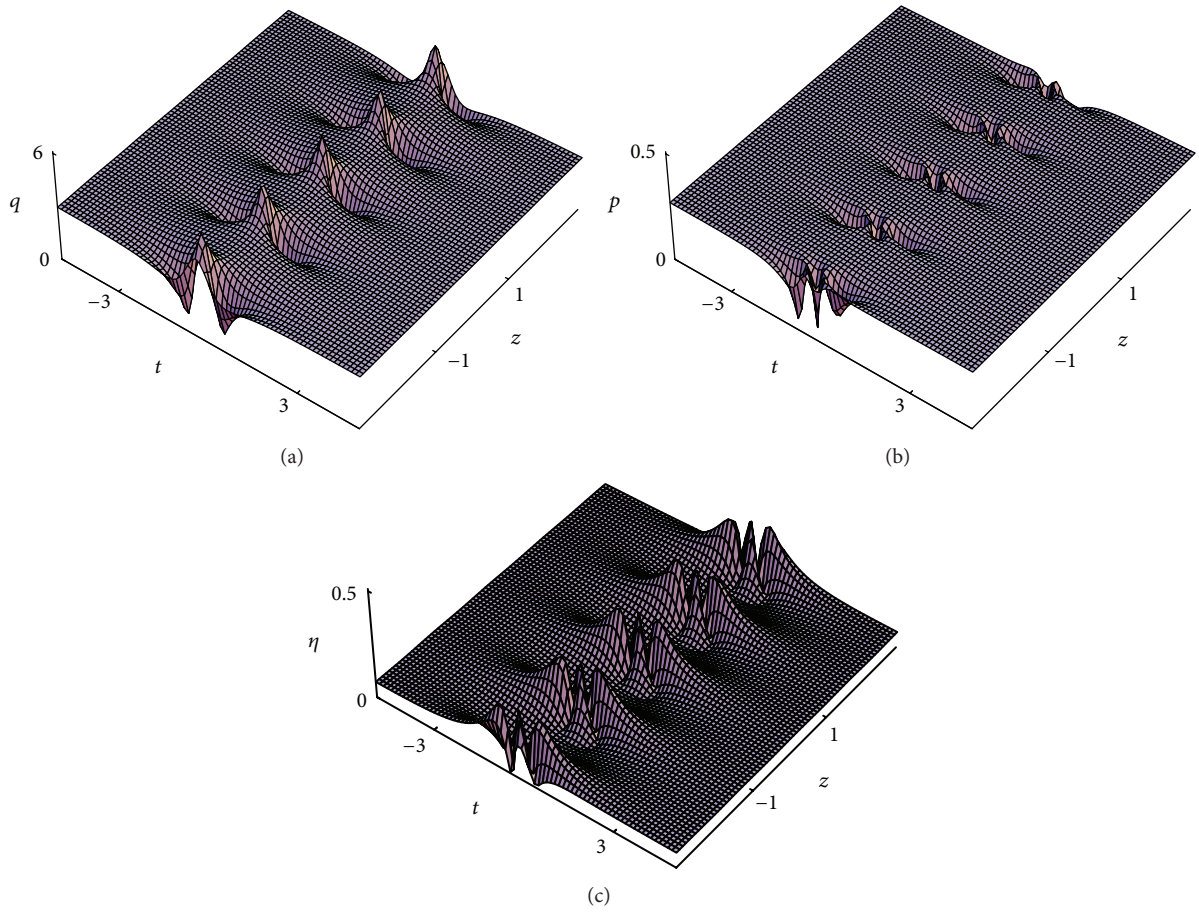

Figure 2: The parameters adopted here are $\kappa_s = 2.3$, $q_c = 1$, $\kappa_c = 3$, $\omega = 1$, and $\tau = 0.1$.

with λ as a spectral parameter,

$$U_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad U_0 = \begin{pmatrix} 0 & q \\ -q^* & 0 \end{pmatrix}, \qquad V_4 = 8i\tau U_1,$$

$$V_2 = \begin{pmatrix} -2i - 4i\tau|q|^2 & -4i\tau q_t \\ -4i\tau q_t^* & 2i + 4i\tau|q|^2 \end{pmatrix}, \qquad V_3 = -8\tau U_0,$$

$$V_1 = \begin{pmatrix} 2\tau q^* q_t - 2\tau q q_t^* & 2q + 4\tau|q|^2 q + 2\tau q_{tt} \\ -2q^* - 4\tau|q|^2 q^* - 2\tau q_{tt}^* & -2\tau q^* q_t + 2\tau q q_t^* \end{pmatrix}, \tag{7}$$

$$V_0 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & -a_{11} \end{pmatrix}, \qquad V_{-1} = i\begin{pmatrix} \eta & -p \\ -p^* & -\eta \end{pmatrix},$$

$$a_{11} = i|q|^2 + 3i|q|^4 - i\tau|q_t|^2 + i\tau q^* q_{tt} + i\tau q q_{tt}^*,$$

$$a_{12} = iq_t + 6i\tau|q|^2 q_t + i\tau q_{ttt},$$

$$a_{21} = iq_t^* + 6i\tau|q|^2 q_t^* + i\tau q_{ttt}^*.$$

Through symbolic computations, one can verify that system (4a), (4b), and (4c) can be concluded through the zero curvature equation $U_z - V_t + UV - VU = 0$.

Modulational instability (MI), which results from the interplay between nonlinearity and dispersive effects, refers to a parametric process in which a continuous or quasi-continuous wave undergoes a modulation of its amplitude or phase in the presence of weak perturbations [18–20]. The MI has some applications in condensate physics, plasma physics, hydrodynamics, nonlinear optics, and some other branches of physics [18]. Now by introducing the steady-state cw solutions for system (4a), (4b), and (4c) as $q = q_0 \exp 2i(\kappa + q_0^2 + 3q_0^4\tau)z$, $p = i\kappa q_0 \exp 2i(k + q_0^2 + 3q_0^4\tau)z$, $\eta = \omega\kappa$, here, with p_0 being the input power, we will discuss the modulation instability process for system (4a), (4b), and (4c).

We examine the MI process of the steady-state solutions by introducing the following perturbed solutions:

$$q = (q_0 + A_1) \exp 2i\left(\kappa + q_0^2 + 3q_0^4\tau\right)z, \tag{8a}$$

$$p = i(\kappa q_0 + A_2) \exp 2i\left(\kappa + q_0^2 + 3q_0^4\tau\right)z, \tag{8b}$$

$$\eta = \omega\kappa + B, \tag{8c}$$

where A_1, A_2, and B are weak perturbations with the assumed general expressions being $A_1 = u_1 \cos(Kz - \Omega t) + iv_1 \sin(Kz - \Omega t)$, $A_2 = u_2 \cos(Kz - \Omega t) + iv_2 \sin(Kz - \Omega t)$, $B = b\cos(Kz - \Omega t)$, where u_1, v_1, u_2, v_2, and b are real amplitudes of those perturbations, Ω denotes the real frequency of modulation

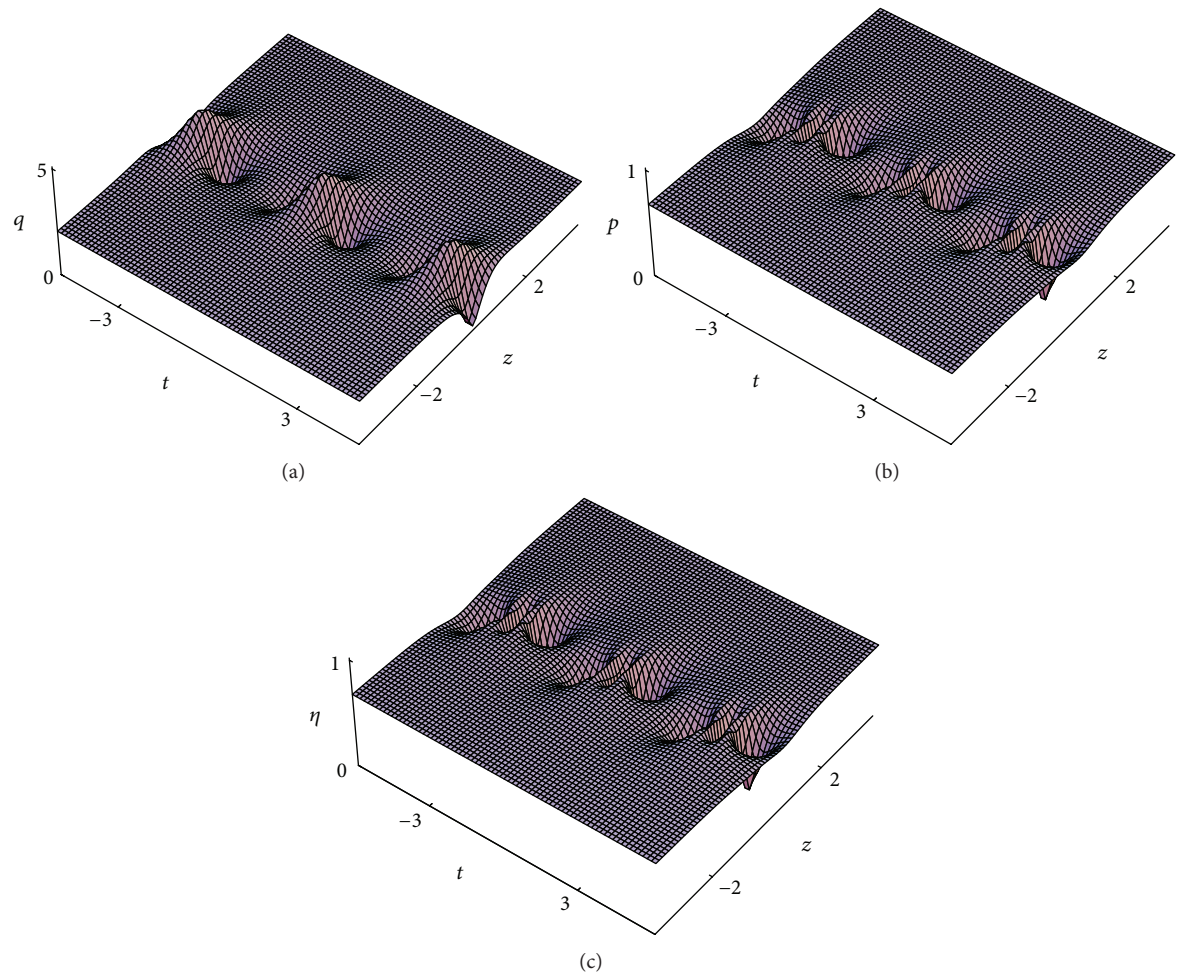

FIGURE 3: The parameters adopted here are $\kappa_s = 1.3$, $q_c = 1$, $\kappa_c = 3$, $\omega = 1$, and $\tau = 0.1$.

perturbations, and K is the real disturbance wave number. Inserting (8a), (8b), and (8c) into (4b) and (4c), we can obtain the linearized equations of u_1, v_1, u_2, v_2, and b, so u_2, v_2, and b can be solved as follows:

$$u_2 = \frac{2\omega\kappa\left(2u_1\omega - v_1\Omega\right)}{4q_0^2 + 4\omega^2 - \Omega^2}, \tag{9a}$$

$$v_2 = \frac{2\kappa\left(2q_0^2 v_1 + 2v_1\omega^2 - u_1\omega\Omega\right)}{4q_0^2 + 4\omega^2 - \Omega^2}, \tag{9b}$$

$$b = \frac{2\kappa q_0\left(v_1\Omega - 2u_1\omega\right)}{4q_0^2 + 4\omega^2 - \Omega^2}. \tag{9c}$$

Substituting (8a), (8b), (9a), and (9b) into (4a) and (4b), collecting the linear terms, we can derive two linear homogeneous equations for the perturbed unknown functions u_1 and v_1:

$$\left(4q_0^2 - \Omega^2\right) y_1 u_1 + y_2 v_1 = 0, \tag{10a}$$

$$y_2 u_1 - \Omega^2 y_1 v_1 = 0, \tag{10b}$$

with

$$y_1 = 2\kappa - \left(4q_0^2 + 4\omega^2 - \Omega^2\right)\left(1 + 6q_0^2\tau - \Omega^2\tau\right),$$
$$y_2 = 4\kappa\omega\Omega + K\left(4q_0^2 + 4\omega^2 - \Omega^2\right). \tag{11}$$

Equations (10a) and (10b) have nontrivial solutions if and only if the following determinant formed by the coefficients matrix vanishes, that is,

$$\begin{vmatrix} \left(4q_0^2 - \Omega^2\right) y_1 & y_2 \\ y_2 & -\Omega^2 y_1 \end{vmatrix} = 0. \tag{12}$$

Equation (12) leads to the dispersion relation of K and Ω which determines the modulation instability process of the steady-state cw solution as

$$K = \frac{4\kappa\omega\Omega + \left|\Omega y_1\right|\sqrt{\Omega^2 - 4q_0^2}}{\Omega^2 - 4q_0^2 - 4\omega^2}. \tag{13}$$

From (13), one can conclude that if $\Omega^2 - 4q_0^2 < 0$, the value of K will be complex; then the modulation instability will take place with $|\operatorname{Im} K|$ as the instability growth rate. So $\Omega^2 - 4q_0^2 < 0$ is the condition of the modulation instability for system (4a), (4b), and (4c).

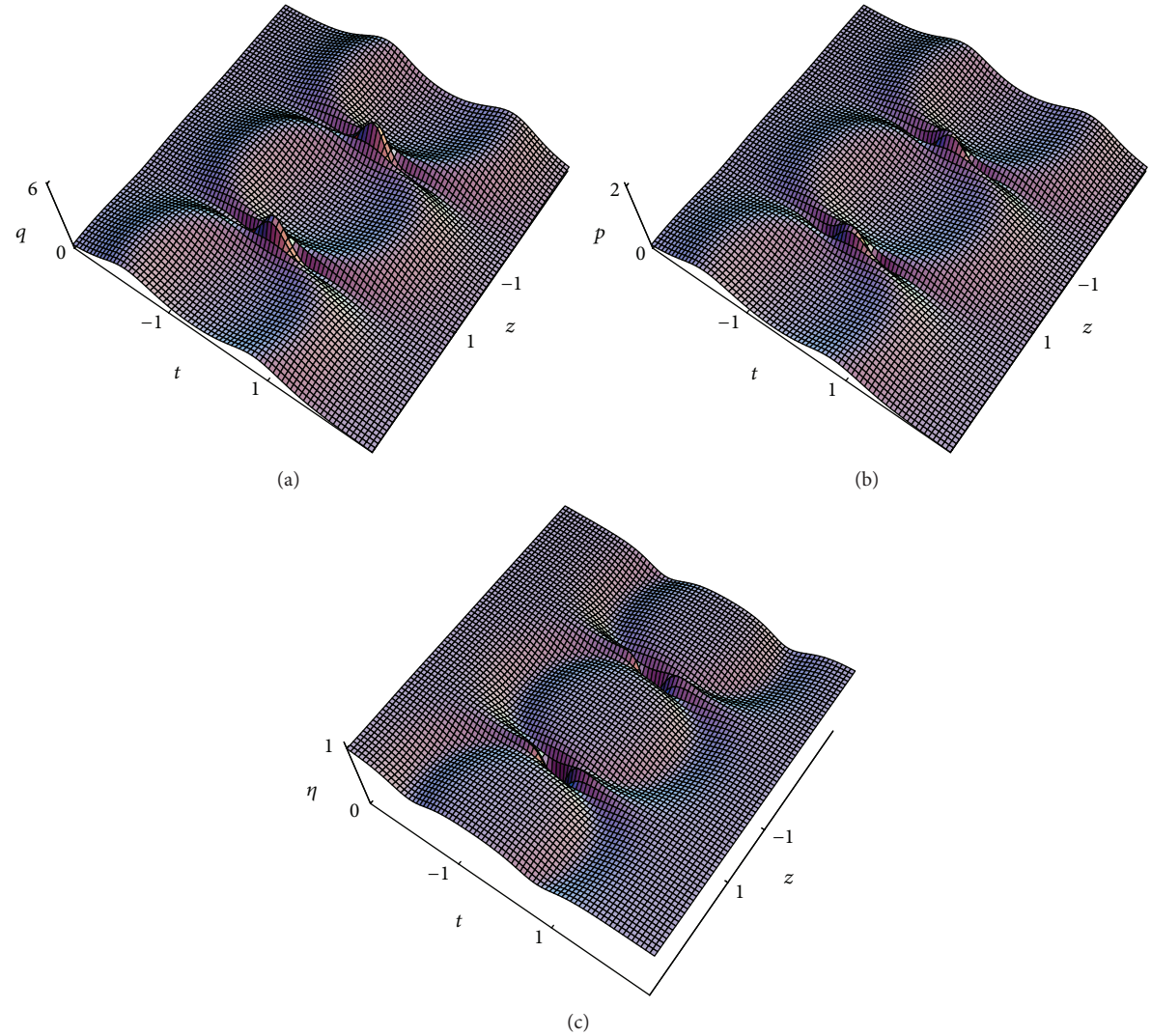

Figure 4: The parameters adopted here are $t_0 = 3$, $\omega = 3$, and $\tau = 0$.

3. One-Breather Solutions for System (4a), (4b), and (4c)

Based on Lax pair (5a) and (5b), we can construct the following Darboux transformation for system (4a), (4b), and (4c) as

$$q' = q - 2i\,\frac{(\lambda_1 - \lambda_1^*)\,\phi_1\phi_2^*}{|\phi_1|^2 + |\phi_2|^2}, \tag{14a}$$

$$p' = \frac{p\chi_1^2}{\chi_2\left(|\phi_1|^2 + |\phi_2|^2\right)^2} - \frac{p^*\,(\lambda_1 - \lambda_1^*)\,\phi_1^2\phi_2^{*2}}{\chi_2\left(|\phi_1|^2 + |\phi_2|^2\right)^2} + \frac{2\eta\chi_1\,(\lambda_1 - \lambda_1^*)\,\phi_1\phi_2^*}{\chi_2\left(|\phi_1|^2 + |\phi_2|^2\right)^2}, \tag{14b}$$

$$\eta' = \eta\left(1 + \frac{2\,(\lambda_1 - \lambda_1^*)\,|\phi_1|^2|\phi_2|^2}{\chi_2\left(|\phi_1|^2 + |\phi_2|^2\right)^2}\right) + \frac{p\,(\lambda_1 - \lambda_1^*)\,\chi_1\phi_2\phi_1^*}{\chi_2\left(|\phi_1|^2 + |\phi_2|^2\right)^2} - \frac{p^*\,(\lambda_1 - \lambda_1^*)\,\chi_1^*\phi_1\phi_2^*}{\chi_2\left(|\phi_1|^2 + |\phi_2|^2\right)^2}, \tag{14c}$$

with

$$\chi_1 = (\lambda_1 + \omega)\,|\phi_1|^2 + (\lambda_1^* + \omega)\,|\phi_2|^2,$$
$$\chi_2 = (\lambda_1 + \omega)\,(\lambda_1^* + \omega). \tag{15}$$

Considering the nonzero continuous wave background, we can take $q = q_c \exp i(\kappa_c z + \omega_c t)$, $p = ip_c \exp i(\kappa_c z + \omega_c t)$, and $\eta = \eta_c$ as the initial seeds, where q_c, p_c, η_c, κ_c, and ω_c are all real parameters. System (4a), (4b), and (4c) requires the following nonlinear dispersion relation:

$$p_c = -\frac{1}{2}\left(2q_c^2 - \kappa_c - \omega_c^2 + 6q_c^4\tau - 12q_c^2\omega_c^2\tau + \omega_c^4\tau\right), \tag{16a}$$

$$\eta_c = -\frac{1}{4}\,(2\omega - \omega_c) \times \left(2q_c^2 - \kappa_c - \omega_c^2 + 6q_c^4\tau - 12q_c^2\omega_c^2\tau + \omega_c^4\tau\right). \tag{16b}$$

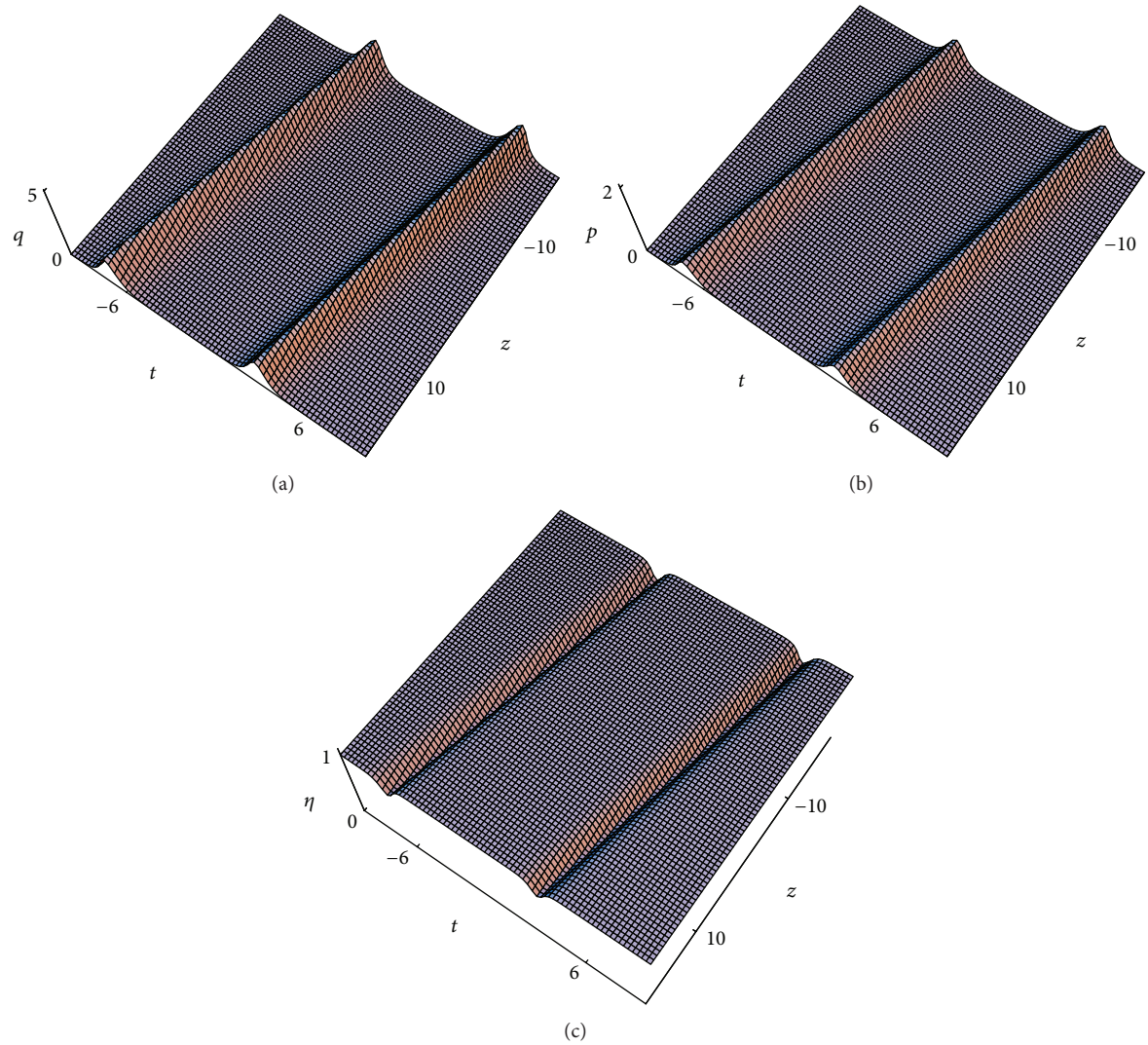

Figure 5: The parameters adopted here are $t_0 = 10$, $\omega = 3$, and $\tau = 0$.

By the method of separation of variables and the superposition principle, we can arrive at

$$\phi_1 = \left(c_1 \exp\theta_1 + c_2 \exp\theta_2\right) \exp i\left(\kappa_c z + \omega_c t\right), \tag{17a}$$

$$\phi_2 = c_3 \exp\theta_1 + c_4 \exp\theta_2, \tag{17b}$$

where

$$\begin{aligned}
\theta_1 &= \frac{t}{2}\left[-i\omega_c + \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_1^2 - 4\lambda_1\omega_c}\right] \\
&\quad + \frac{z}{2}\left[-i\kappa_c + \sqrt{-\kappa_c^2 + 4a^2 - 4b^2 - 4ia\kappa_c}\right], \\
\theta_2 &= \frac{t}{2}\left[-i\omega_c - \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_1^2 - 4\lambda_1\omega_c}\right] \\
&\quad + \frac{z}{2}\left[-i\kappa_c - \sqrt{-\kappa_c^2 + 4a^2 - 4b^2 - 4ia\kappa_c}\right],
\end{aligned} \tag{18}$$

with

$$\begin{aligned}
a &= 2i\lambda_1^2\left(-1 + 4\lambda_1^2\tau\right) \\
&\quad - iq_c^2\left(-1 + 4\lambda_1^2\tau - 4\lambda_1\omega_c\tau + 3\omega_c^2\tau\right) \\
&\quad + 3iq_c^4\tau + \frac{i\eta_c}{\lambda_1 + \omega}, \\
b &= q_c\left(4\lambda_1 q_c^2\tau + 2\lambda_1 - \omega_c - 4\lambda_1^2\omega_c\tau - 2\lambda_1\omega_c^2\tau\right. \\
&\quad \left. + \omega_c^3\tau - 8\lambda_1^3\tau - 6q_c^2\omega_c\tau\right) + \frac{p_c}{\lambda_1 + \omega},
\end{aligned} \tag{19}$$

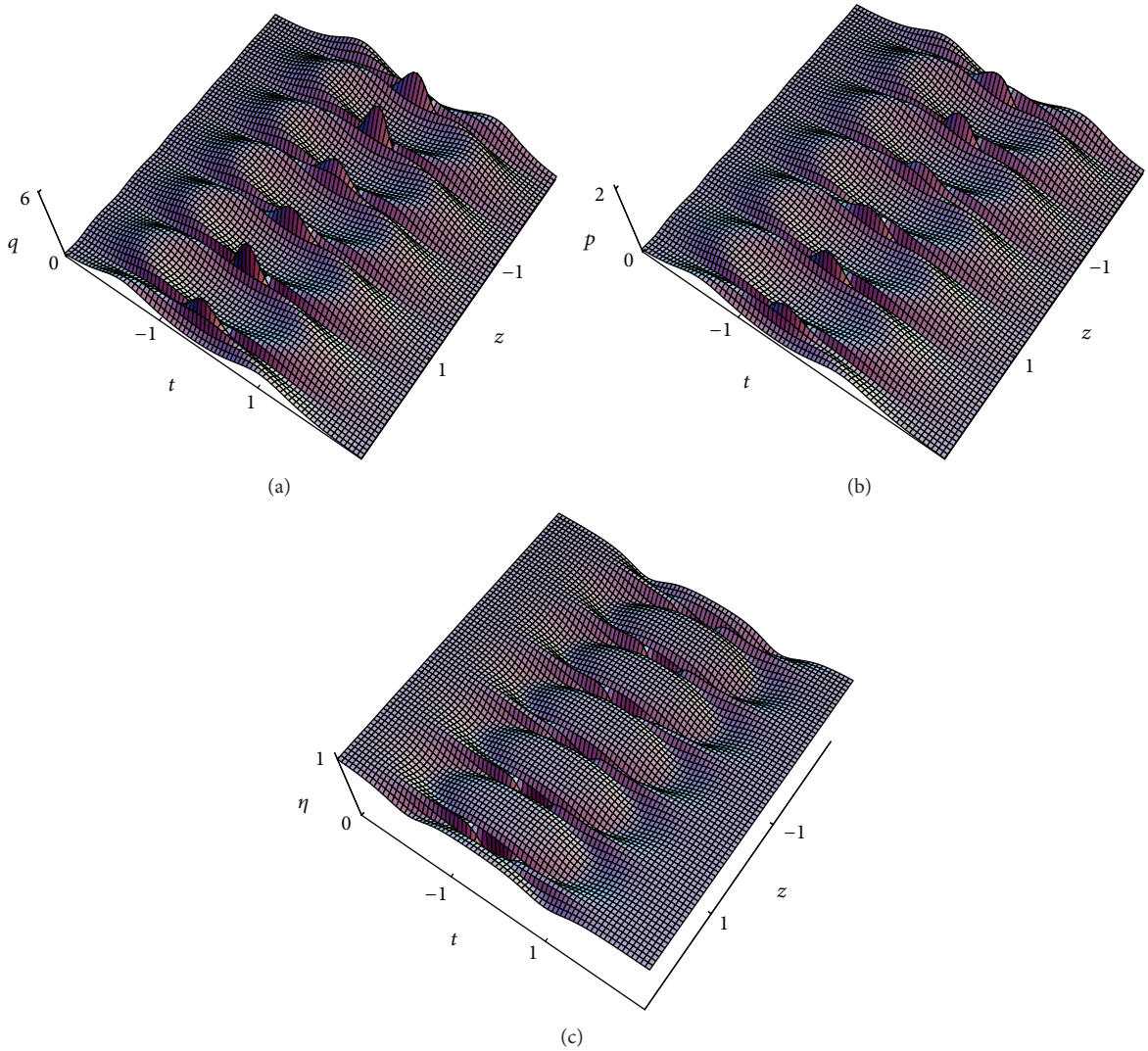

Figure 6: The parameters adopted here are $t_0 = 3$, $\omega = 3$, and $\tau = 5$.

and c_1, c_2, c_3, and c_4 are complex constants satisfied as

$$\begin{aligned} c_3 &= \frac{1}{q_c}\left[i\omega_c + i\lambda_1 + \frac{1}{2}\right. \\ &\qquad \left.\times\left(-i\omega_c + \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_1^2 - 4\lambda_1\omega_c}\right)\right]c_1, \\ &= (L + iM)\,c_1, \\ c_2 &= \frac{q_c}{(1/2)\left(-i\omega_c + \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_1^2 - 4\lambda_1\omega_c}\right) + i\omega_c + i\lambda_1}\,c_4 \\ &= (L + iM)\,c_4. \end{aligned} \tag{20}$$

Suppose that

$$\begin{aligned} \lambda_1 &= \frac{1}{2}\left(A_s + i\kappa_s\right), \qquad c_4 = c_1, \\ \zeta_1 + i\eta_1 &= \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_1^2 - 4\lambda_1\omega_c}, \\ \zeta_2 + i\eta_2 &= \sqrt{-\kappa_c^2 + 4a^2 - 4b^2 - 4ia\kappa_c}, \end{aligned} \tag{21}$$

with A_s, κ_s, ζ_1, η_1, ζ_2, and η_2 being real numbers. Now, substituting (17a) and (17b) into (14a), (14b), and (14c), we can obtain the solutions for system (4a), (4b), and (4c) as

$$q = \frac{G_1}{F_1}\exp i\left(\kappa_c z + \omega_c t\right), \tag{22a}$$

$$p = \frac{G_2}{F_2}\exp i\left(\kappa_c z + \omega_c t\right), \tag{22b}$$

$$\eta = \frac{G_3}{F_2}, \tag{22c}$$

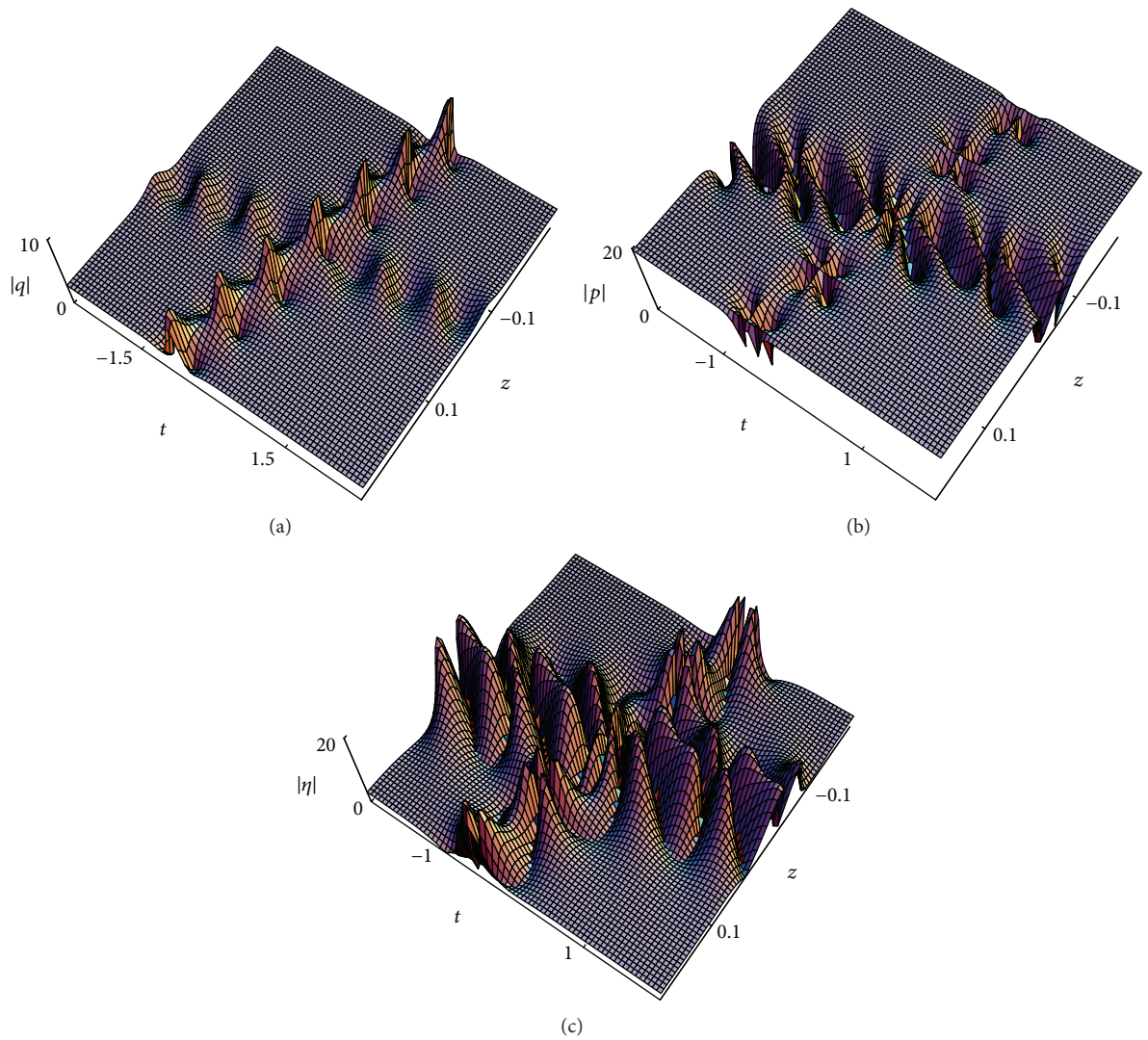

FIGURE 7: The parameters adopted here are $\lambda_1 = i$, $\lambda_2 = 4i$, $q_c = 3$, $\omega_c = 2$, $\kappa_c = 1$, $\omega = 0.5$, and $\tau = 0.1$.

where

$$\begin{aligned}
G_1 &= q_c - 2i\left(\lambda_1 - \lambda_1^*\right)\left[e^{\theta_1} + e^{\theta_2}\left(L + iM\right)\right] \\
&\quad \times \left[e^{\theta_2^*} + e^{\theta_1^*}\left(L - iM\right)\right], \\
F_1 &= e^{\theta_1^*}\left[2Le^{\theta_2} + \left(1 + L^2 + M^2\right)e^{\theta_1}\right] \\
&\quad + e^{\theta_2^*}\left[2Le^{\theta_1} + \left(1 + L^2 + M^2\right)e^{\theta_2}\right], \\
G_2 &= ip_c\left[\mu^2 + \sigma_1^2\sigma_2^{*2}\left(\lambda_1 - \lambda_1^*\right)^2\right] \\
&\quad + 2\eta_0\sigma_1\sigma_2^*\left(\lambda_1 - \lambda_1^*\right)\mu, \\
F_2 &= \left(\lambda_1 + \omega\right)\left(\lambda_1^* + \omega\right) \\
&\quad \times \left[4L\cosh\left(\theta_1 + \theta_2^*\right) + 2\left(L^2 + M^2 + 1\right)\cosh\left(\theta_1 + \theta_1^*\right)\right], \\
G_3 &= ip_c\left(\lambda_1 - \lambda_1^*\right)\left[\sigma_1^*\sigma_2\mu + \sigma_1\sigma_2^*\mu^*\right] \\
&\quad + \eta_c\left[\left(\lambda_1 + \omega\right)\left(\lambda_1^* + \omega\right)\left(\sigma_1\sigma_1^* + \sigma_2\sigma_2^*\right)^2\right. \\
&\quad \left. + 2\sigma_1\sigma_1^*\sigma_2\sigma_2^*\left(\lambda_1 - \lambda_1^*\right)^2\right],
\end{aligned} \tag{23}$$

with

$$\begin{aligned}
&\sigma_1 = e^{\theta_1} + e^{\theta_2}\left(L + iM\right), \qquad \sigma_2 = e^{\theta_2} + e^{\theta_1}\left(L + iM\right), \\
&\mu = \sigma_1\sigma_1^*\left(\lambda_1 + \omega\right) + \sigma_2\sigma_2^*\left(\lambda_1^* + \omega\right).
\end{aligned} \tag{24}$$

Now, we mainly discuss soliton solutions from three different cases.

Case 1. In the case of $q_c = 0$, that is to say, the initial seeds for (4a), (4b), and (4c) are $q_c = 0$, $p_c = 0$, and $\eta_c = 1$, solutions (22a), (22b), and (22c) reduce to one-soliton solutions.

Case 2. In the case of $q_c \neq 0$, $A_s = \omega_c = 0$, and $\kappa_s^2 > 4q_c^2$, symbolic computation results in the following:

$$\begin{aligned}
&\zeta_1 = \sqrt{-4q_c^2 + \kappa_s^2}, \qquad \eta_1 = 0, \\
&\zeta_2 = \frac{\zeta_1 h_1}{4\omega^2 + \kappa_s^2}, \qquad \eta_2 = \frac{\zeta_1 h_2}{4\omega^2 + \kappa_s^2},
\end{aligned} \tag{25}$$

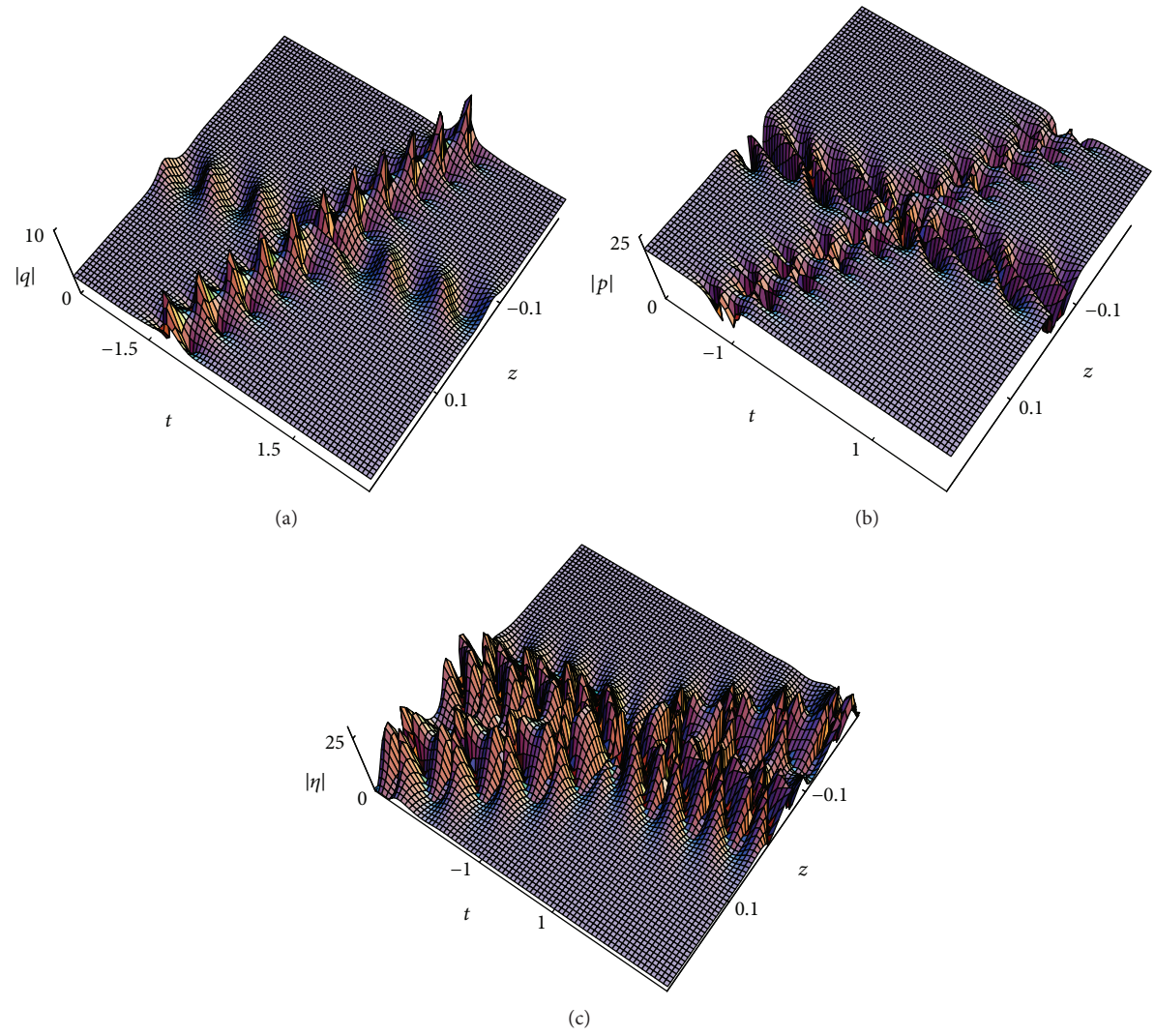

FIGURE 8: The parameters adopted here are $\lambda_1 = i$, $\lambda_2 = 2i$, $q_c = 3$, $\omega_c = 2$, $\kappa_c = 1$, $\omega = 0.5$, and $\tau = 0.1$.

with

$$h_1 = 2q_c^2 - \kappa_c + \kappa_s^2 + 6\tau q_c^4 + 2\tau q_c^2 \kappa_s^2 + \tau \kappa_s^4,$$
$$h_2 = -2\omega\kappa_s - 4\omega\tau\kappa_s q_c^2 - 2\omega\tau\kappa_s^3. \tag{26}$$

Substituting (25) into (17a) and (17b), we can reduce solutions (22a), (22b), and (22c) to the breathers displayed in Figures 1 and 2.

From Figure 1, we can observe that the main features of propagations of those breathers- are periodic in the space coordinate and aperiodic in the time coordinate, so the solitons shown in Figure 1 are Akhmediev breathers [21–23]. In addition, we can observe that Figures 1(a) and 1(c) depict bright breathers, and Figure 1(b) shows the dark one.

Comparing Figure 2 with Figure 1, one can see that in Figure 2, under the influence of the increasing values of the parameter τ, the number of peaks on the same space interval is increasing when τ goes up from 0 to 0.1. So, we can conclude that the parameter τ controls the period of the Akhmediev breathers.

Case 3. In the case of $q_c \neq 0$, $A_s = \omega_c = 0$, and $\kappa_s^2 < 4q_c^2$, through direct computation, one can obtain the following:

$$\zeta_1 = 0, \qquad \eta_1 = \sqrt{4q_c^2 - \kappa_s^2},$$
$$\zeta_2 = \eta_1 \frac{h_1\kappa_s - 2h_2\omega}{4\omega^2 + \kappa_s^2}, \qquad \eta_2 = \eta_1 \frac{2h_1\omega + h_2\kappa_s}{4\omega^2 + \kappa_s^2}. \tag{27}$$

Substituting (27) into (17a) and (17b), solutions (22a), (22b), and (22c) become other breathers as displayed in Figure 3.

From Figure 3, one can observe that those breathers are periodic in the time coordinate and aperiodic in the space coordinate; that is, those are Ma-breather solitons [24–27], and for functions q and η, the solutions are bright solitons while for function p the solution is the dark one. In addition, one can find that the separations between adjacent peaks

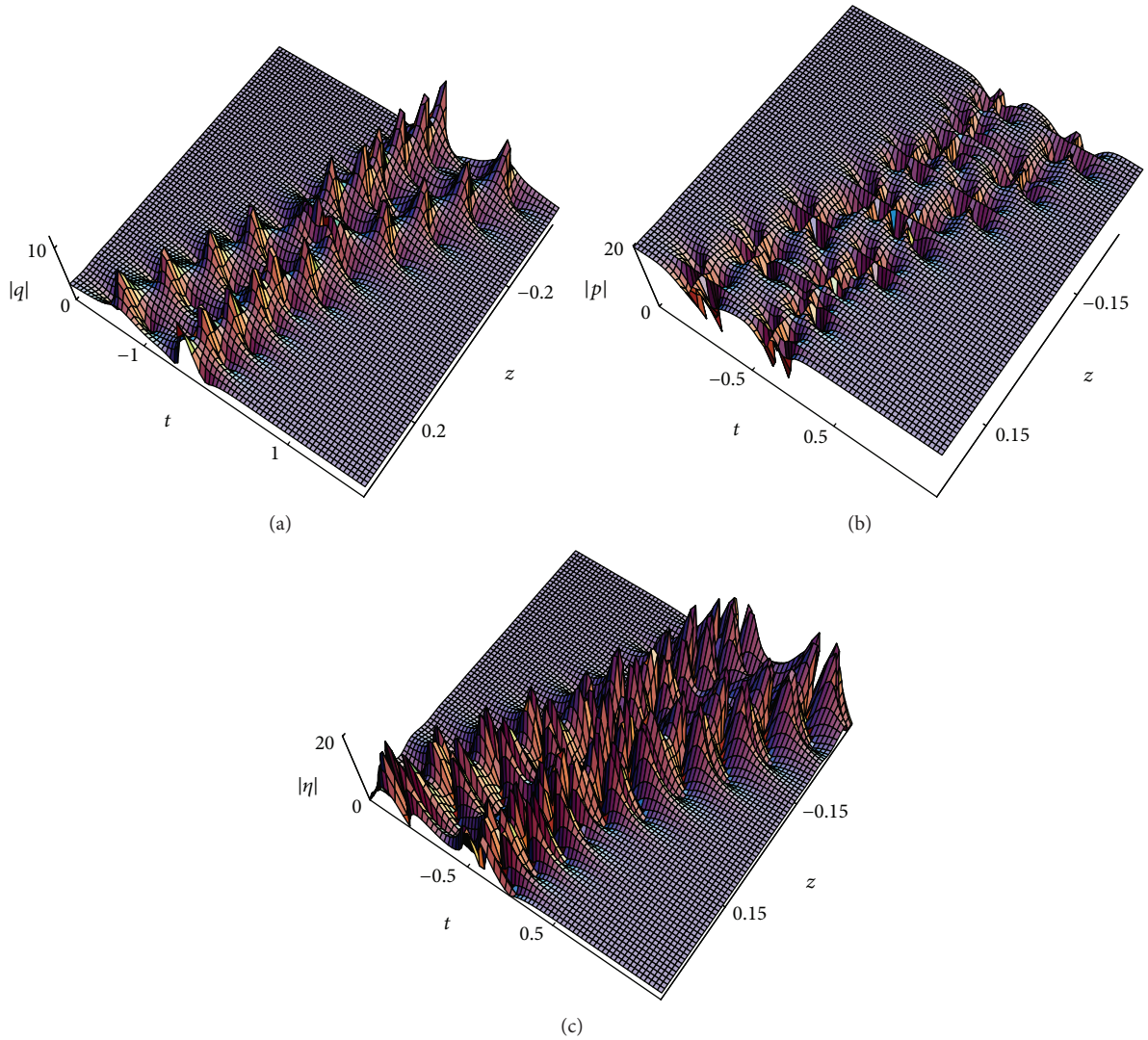

FIGURE 9: The parameters adopted here are $\lambda_1 = 5i$, $\lambda_2 = 4i$, $q_c = 3$, $\omega_c = 2$, $\kappa_c = 1$, $\omega = 0.5$, and $\tau = 0.1$.

in Figure 3 gradually increase as $\kappa_s \to 2q_c$ and eventually reduce into the rogue waves, the properties of which have been discussed in [28–31].

4. Dynamic Features of Two-Soliton Solutions for System (4a), (4b), and (4c)

In this section, we will construct two-soliton solutions for system (4a), (4b), and (4c). Taking the same seeds in Section 2, that is, $q = q_c \exp i(\kappa_c z + \omega_c t)$, $p = ip_c \exp i(\kappa_c z + \omega_c t)$, $\eta = \eta_c$, and iterating the DT twice, one can obtain

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[\lambda_2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - H\Lambda H^{-1} \right] \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \tag{28}$$

where

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_1^* \end{pmatrix}, \qquad H = \begin{pmatrix} \phi_1 & \phi_2^* \\ \phi_2 & -\phi_1^* \end{pmatrix},$$
$$\varphi_1 = d_1 e^{\theta_3} + d_2 e^{-\theta_4}, \qquad \varphi_2 = d_3 e^{\theta_3} + d_4 e^{-\theta_4}, \tag{29}$$

with

$$\begin{aligned}
\theta_3 &= \frac{t}{2}\left[-i\omega_c + \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_2^2 - 4\lambda_2\omega_c}\right] \\
&\quad + \frac{z}{2}\left[-i\kappa_c + \sqrt{-\kappa_c^2 + 4a'^2 - 4b'^2 - 4ia'\kappa_c}\right], \\
\theta_4 &= \frac{t}{2}\left[-i\omega_c - \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_2^2 - 4\lambda_2\omega_c}\right] \\
&\quad + \frac{z}{2}\left[-i\kappa_c - \sqrt{-\kappa_c^2 + 4a'^2 - 4b'^2 - 4ia'\kappa_c}\right], \\
a' &= 2i\lambda_2^2\left(-1 + 4\lambda_2^2\tau\right) - iq_c^2 \\
&\quad \times \left(-1 + 4\lambda_2^2\tau - 4\lambda_2\omega_c\tau + 3\omega_c^2\tau\right) + 3iq_c^4\tau \\
&\quad + \frac{i\eta_c}{\lambda_2 + \omega}, \\
b' &= q_c\left(4\lambda_2 q_c^2\tau + 2\lambda_2 - \omega_c - 4\lambda_2^2\omega_c\tau\right. \\
&\quad \left. - 2\lambda_2\omega_c^2\tau + \omega_c^3\tau - 8\lambda_2^3\tau - 6q_c^2\omega_c\tau\right) \\
&\quad + \frac{p_c}{\lambda_2 + \omega},
\end{aligned} \tag{30}$$

λ_2 is another eigenvalue for Lax pair (5a) and (5b), and d_1, d_2, d_3, and d_4 are complex constants satisfied as

$$\begin{aligned}
d_3 &= \frac{1}{q_c}\left[i\omega_c + i\lambda_2 + \frac{1}{2}\right. \\
&\quad \left. \times \left(-i\omega_c + \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_2^2 - 4\lambda_2\omega_c}\right)\right] d_1, \\
d_2 &= \frac{q_c}{(1/2)\left(-i\omega_c + \sqrt{-\omega_c^2 - 4q_c^2 - 4\lambda_2^2 - 4\lambda_2\omega_c}\right) + i\omega_c + i\lambda_2} \\
&\quad \times d_4.
\end{aligned} \tag{31}$$

Thus, one can derive the expressions of two-soliton solutions for system (4a), (4b), and (4c) as

$$q^{(2)} = q' - 2i\,\frac{(\lambda_2 - \lambda_2^*)\,\psi_1\psi_2^*}{|\psi_1|^2 + |\psi_2|^2}, \tag{32a}$$

$$\begin{aligned}
p^{(2)} &= \frac{p'\chi_1'^2}{\chi_2'\left(|\psi_1|^2 + |\psi_2|^2\right)^2} - \frac{p'^*\,(\lambda_2 - \lambda_2^*)\,\psi_1^2\psi_2^{*2}}{\chi_2'\left(|\psi_1|^2 + |\psi_2|^2\right)^2} \\
&\quad + \frac{2\eta'\chi_1'\,(\lambda_2 - \lambda_2^*)\,\psi_1\psi_2^*}{\chi_2'\left(|\psi_1|^2 + |\psi_2|^2\right)^2},
\end{aligned} \tag{32b}$$

$$\begin{aligned}
\eta^{(2)} &= \eta'\left(1 + \frac{2\,(\lambda_2 - \lambda_2^*)\,|\psi_1|^2|\psi_2|^2}{\chi_2'\left(|\psi_1|^2 + |\psi_2|^2\right)^2}\right) \\
&\quad + \frac{p'\,(\lambda_2 - \lambda_2^*)\,\chi_1'\psi_2\psi_1^*}{\chi_2'\left(|\psi_1|^2 + |\psi_2|^2\right)^2} - \frac{p'^*\,(\lambda_2 - \lambda_2^*)\,\chi_1'^*\psi_1\psi_2^*}{\chi_2'\left(|\psi_1|^2 + |\psi_2|^2\right)^2},
\end{aligned} \tag{32c}$$

with

$$\begin{aligned}
\chi_1' &= (\lambda_2 + \omega)\,|\psi_1|^2 + (\lambda_2^* + \omega)\,|\psi_2|^2, \\
\chi_2' &= (\lambda_2 + \omega)\,(\lambda_2^* + \omega).
\end{aligned} \tag{33}$$

Now, we will analyze (32a), (32b), and (32c) under two different cases.

4.1. Interaction Characters of Bound Solitons. In this section, we will investigate the interaction between neighboring solitons for system (4a), (4b), and (4c). Taking $q_c = 0$, $p_c = 0$, $\eta_c = 1$ and iterating the DT twice, one can generate two-soliton solutions with two spectral parameters $\lambda_1 = \vartheta_1 + i\varrho_1$ and $\lambda_2 = \vartheta_2 - i\varrho_2$ for system (4a), (4b), and (4c). Assuming the input launching pulses as $q(0,t) = \mathrm{sech}(t - t_0) + \mathrm{sech}(t + t_0)$, here t_0 denotes the soliton separation. Supposing that $\vartheta_1 = \vartheta_2 = 0$, we can derive that

$$\varrho_{1,2} = 1 + \frac{2t_0}{\sinh(2t_0)} \pm \mathrm{sech}\, t_0. \tag{34}$$

Under the circumstances $t_0 = 3$ and $\tau = 0$, that is, the higher-order dispersive effects are absent, the two-soliton solutions can turn into bound solitons under suitable parameters chosen as shown in Figure 4. We can observe that main features are that the mutual attractions and repulsions repeat periodically when the bound solitons propagate, and this phenomenon may be harmful for optical soliton communication [32–34].

In fact we can suppress the periodical mutual attractions and repulsions through increasing the initial pulse separation, that is, the value of t_0. As portrayed in Figure 5, when $t_0 = 10$, one can find that the mutual attractions and repulsions between two bound solitons disappear, and the two solitons propagate in parallel without any effect on each other even if the propagation distance grows long enough.

Figure 6 depicts the effects of the higher-order dispersive terms on the propagations of the solitons. As the pictures show, the periods of the bound solitons can be suppressed when τ decreases.

So we can conclude that the higher-order dispersive terms can control the propagation periods of the bound soliton.

4.2. Interaction Characters of Two-Breather Solutions. In this section, we will construct two-breather solutions for system (4a), (4b), and (4c). Taking $q_c \neq 0$, $p_c \neq 0$, $\eta_c \neq 0$, $\omega_c = 0$ and iterating the DT twice, one can get two-breather solutions for system (4a), (4b), and (4c). When $|\lambda_1| < |q_c|$ and $|\lambda_2| > |q_c|$, the Akhmediev breathers and Ma-breathers can coexist as portrayed in Figure 7. Main features of the interaction between two breathers in Figure 7 are that they interact perpendicularly and the shapes, amplitudes, and pulse widths of the two breathers all remain invariant, so the interactions are elastic.

When $|\lambda_1| < |q_c|$ and $|\lambda_2| < |q_c|$, the two-breather solutions that evolve from two Ma-breathers come into being as portrayed in Figure 8. And the interactions between these two-breathers in Figure 8 are also elastic.

When $|\lambda_1| > |q_c|$ and $|\lambda_2| > |q_c|$, the two-breather solutions that evolve from two Akhmediev-breathers take place as depicted in Figure 9. And the interactions between these two-breathers in Figure 9 are also elastic.

5. Conclusions

Our main attention has been focused on system (4a), (4b), and (4c) which describes the propagation of optical solitons in nonlinear erbium-doped fibers with higher-order effects. Lax pair and modulation instability conditions for this system have been investigated. Two types of breathers (Akhmediev breathers and Ma-breathers), bound soliton solutions, and two-breather solutions have been constructed by Darboux transformation. Propagation properties of those solitons under the influences of higher-order effects have been discussed.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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