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Research Article

Robust H_∞ Control for a Class of Discrete Time-Delay Stochastic Systems with Randomly Occurring Nonlinearities

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In this paper, we consider the robust H_∞ control problem for a class of discrete time-delay stochastic systems with randomly occurring nonlinearities. The parameter uncertainties enter all the system matrices; the stochastic disturbances are both state and control dependent, and the randomly occurring nonlinearities obey the sector boundedness conditions. The purpose of the problem addressed is to design a state feedback controller such that, for all admissible uncertainties, nonlinearities, and time delays, the closed-loop system is robustly asymptotically stable in the mean square, and a prescribed H_∞ disturbance rejection attenuation level is also guaranteed. By using the Lyapunov stability theory and stochastic analysis tools, a linear matrix inequality (LMI) approach is developed to derive sufficient conditions ensuring the existence of the desired controllers, where the conditions are dependent on the lower and upper bounds of the time-varying delays. The explicit parameterization of the desired controller gains is also given. Finally, a numerical example is exploited to show the usefulness of the results obtained.

1. Introduction

Various engineering systems, such as electrical networks, turbojet engines, microwave oscillators, nuclear reactors, and hydraulic systems, have the characteristics of time delay in signal transmissions. The existence of time delay is often a source of instability and poor performance. So far, the stability analysis and robust control for dynamic time-delay systems have attracted a number of researchers over the past years; see [1–4] and the references therein.

Since stochastic modeling has had extensive applications in control and communication problems, the stability analysis problem for linear stochastic time-delay systems has been studied by many authors. For example, in [5], the stability analysis problem for uncertain stochastic fuzzy systems with time delays has been considered. In [6], an LMI approach has been developed to cope with the robust H_∞ control problem for linear uncertain stochastic systems with state delay. In [7], the robust energy-to-peak filtering problem has been dealt with for uncertain stochastic time-delay systems. In [8], the robust integral sliding mode control problem has been studied for uncertain stochastic systems with time-varying delays, and the H_∞ performance has been analyzed in [9] for continuous-time stochastic systems with polytopic uncertainties.

On the other hand, it is well known that nonlinearity is ubiquitous and all pervading in the physical world and therefore nonlinear control has been an ever hot topic in the past few decades. It is worth mentioning that, among different descriptions of the nonlinearities, the so-called *sector nonlinearity* [10] has gained much attention for *deterministic* systems, and both the control analysis and model reduction problems have been investigated; see [11–13]. Recently, the control problem of nonlinear stochastic systems has stirred renewed research interests [14], and a variety of nonlinear stochastic systems have been investigated by different approaches, such as the minimax dynamic game approach [15], the input-to-state stabilization method [16, 17], the infinite-horizon risk-sensitive scheme [18], and the Lyapunov-based recursive design method [19]. Most recently, in [20–27], an H_∞-type theory has been developed for a large class of continuous- and discrete-time nonlinear stochastic systems. Notice that it is also quite common to describe nonlinearities as additive nonlinear disturbances. Such nonlinear disturbances may occur in a probabilistic way. For example, in a particular moment for networked control systems, the transmission channel for a large amount of packets may encounter severe network-induced congestions due to the bandwidth limitations, and the resulting phenomenon could be reflected by certain randomly occurring nonlinearities where the occurrence probability can be estimated via statistical tests. Recently some initial work has been reported on the dynamics for the systems with randomly occurring nonlinearities; see [27, 28] and the references therein. As far as we know, to date, however, the robust H_∞ control problem for stochastic systems with time delays and randomly occurring nonlinearities has not been fully investigated, especially for discrete-time cases, which motivates us to shorten such a gap in the present investigation.

The main contribution of this paper can be summarized as follows. (1) A general framework is established to cope with robust H_∞ control problem for a class of uncertain discrete-time stochastic systems involving randomly occurring sector nonlinearities and time-varying delays. (2) An effective LMI approach is proposed to design the state feedback controllers such that, for all the randomly occurring nonlinearities and time-delays, the overall uncertain closed-loop system is robustly asymptotically stable in the mean square and a prescribed H_∞ disturbance rejection attenuation level is guaranteed. We first establish the sufficient conditions for the uncertain nonlinear stochastic time-delay systems to be stable in the mean square and then derive the explicit expression of the desired controller gains. (3) Sector nonlinearity technique is utilized to reduce the conservativeness for the main results.

Notation. Throughout this paper, $\mathbb{N}$ and $\mathbb{N}^+$ stand for the natural numbers and the nonnegative integer set, respectively; $\mathbb{R}, \mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ denote, respectively, the set of real numbers, the n dimensional Euclidean space, and the set of all $n \times m$ real matrices. The superscript T represents the transpose for a matrix; the notation $X \geq Y$ (resp., $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (resp., positive definite). $|\cdot|$ denotes the standard Euclidean norm. If A is a square matrix, denote by $\lambda_{\max}(A)$ (resp., $\lambda_{\min}(A)$) the largest (resp., the smallest) eigenvalue of A. Matrices, if not explicitly specified, are assumed to have compatible dimensions. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise. In an underlying probability space $(\Omega, \mathcal{F}, \mathcal{P})$, $\mathbb{E}[\cdot]$ denotes the mean for a random variable, and $\text{Prob}\{\cdot\}$ means the occurrence probability of the event.

2. Problem Formulation

Consider, on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, the following uncertain discrete nonlinear stochastic system with time delays of the form:

$$\begin{aligned}
(\Sigma):\ & x(k+1) \\
&= A(k)x(k) + A_d(k)x(k-d(k)) \\
&\quad + \vartheta_k f(x(k)) + \zeta_k f_d(x(k-d(k))) + B_1(k)u(k) \\
&\quad + D_1(k)v(k) \\
&\quad + \big[G(k)x(k) + G_d(k)x(k-d(k)) \\
&\qquad + \vartheta_k f(x(k)) + \zeta_k f_d(x(k-d(k))) \\
&\qquad + B_2(k)u(k) + D_2(k)v(k)\big]w(k), \\
&\qquad y(k) = Cx(k) + Bu(k), \\
&x(j) = \phi(j), \quad j = -d_M, -d_M+1, \ldots, -1, 0,
\end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ is the control input; $y(k) \in \mathbb{R}^q$ is the controlled output; $w(k) \in \mathbb{R}$ is a white process noise with

$$\mathbb{E}[w(k)] = 0, \qquad \mathbb{E}\left[w^2(k)\right] = 1, \qquad \mathbb{E}\{w(i)w(j)\} = 0 \quad (i \neq j), \tag{2}$$

ϑ_k and ζ_k are mutually independent Bernoulli-distributed white sequences and also assumed to be independent of $w(k)$.

Let distribution laws of ϑ_k and ζ_k be given by

$$\begin{aligned}
\text{Prob}\{\vartheta_k = 1\} &= \mathbb{E}\{\vartheta_k\} = \beta_1, \\
\text{Prob}\{\vartheta_k = 0\} &= 1 - \mathbb{E}\{\vartheta_k\} = 1 - \beta_1, \\
\text{Prob}\{\zeta_k = 1\} &= \mathbb{E}\{\zeta_k\} = \beta_2, \\
\text{Prob}\{\zeta_k = 0\} &= 1 - \mathbb{E}\{\zeta_k\} = 1 - \beta_2.
\end{aligned} \tag{3}$$

It is well known that $\mathbb{E}\{\vartheta_k\} = \beta_1$, $\mathbb{E}\{\zeta_k\} = \beta_2$, and the variances of ϑ_k and ζ_k are, respectively, $\sigma_1 = \beta_1(1-\beta_1)$ and $\sigma_2 = \beta_2(1-\beta_2)$. Notice that ϑ_k and ζ_k characterize the random occurrence of nonlinear functions.

For the exogenous disturbance signal $v(k) \in \mathbb{R}^p$, it is assumed that $v(\cdot) \in l_{e_2}([0,\infty);\mathbb{R}^p)$, where $l_{e_2}([0,\infty);\mathbb{R}^p)$ is the space of nonanticipatory square-summable stochastic process $f(\cdot) = (f(k))_{k\in\mathbb{N}}$ with respect to $(\mathcal{F}_k)_{k\in\mathbb{N}}$ with the following norm:

$$\|f\|_{e_2} = \left\{\mathbb{E}\sum_{k=0}^{\infty}|f(k)|^2\right\}^{1/2} = \left\{\sum_{k=0}^{\infty}\mathbb{E}|f(k)|^2\right\}^{1/2}. \tag{4}$$

In the system (Σ), the positive integer $d(k)$ denotes the time-varying delay satisfying

$$d_m \leq d(k) \leq d_M, \quad k \in \mathbb{N}^+, \tag{5}$$

where d_m and d_M are known positive integers. The $\phi(j)$ $(j = -d_M, -d_M+1, \ldots, -1, 0)$ are the initial conditions independent of the process $\{w(\cdot)\}$.

In (Σ), C and B are known real constant matrices. The matrices $A(k)$, $A_d(k)$, $B_1(k)$, $D_1(k)$, $G(k)$, $G_d(k)$, $B_2(k)$, and $D_2(k)$ are time-varying matrices of the following form:

$$\begin{aligned} A(k) &= A + \Delta A(k), & A_d(k) &= A_d + \Delta A_d(k), \\ B_1(k) &= B_1 + \Delta B_1(k), & D_1(k) &= D_1 + \Delta D_1(k), \\ G(k) &= G + \Delta G(k), & G_d(k) &= G_d + \Delta G_d(k), \\ B_2(k) &= B_2 + \Delta B_2(k), & D_2(k) &= D_2 + \Delta D_2(k). \end{aligned} \tag{6}$$

Here, A, A_d, B_1, D_1, G, G_d, B_2, and D_2 are known real constant matrices; $\Delta A(k)$, $\Delta A_d(k)$, $\Delta B_1(k)$, $\Delta D_1(k)$, $\Delta G(k)$, $\Delta G_d(k)$, $\Delta B_2(k)$, and $\Delta D_2(k)$ are unknown matrices representing time-varying parameter uncertainties, which are assumed to satisfy the following admissible condition:

$$\begin{aligned} &\begin{bmatrix} \Delta A(k) & \Delta A_d(k) & \Delta B_1(k) & \Delta D_1(k) \\ \Delta G(k) & \Delta G_d(k) & \Delta B_2(k) & \Delta D_2(k) \end{bmatrix} \\ &\quad = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(k) \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix}, \end{aligned} \tag{7}$$

where M_i $(i = 1, 2)$ and N_i $(i = 1, 2, 3, 4)$ are known real constant matrices, and $F(k)$ is the unknown time-varying matrix-valued function subject to the following condition:

$$F^T(k) F(k) \leq I, \quad \forall k \in \mathbb{N}^+. \tag{8}$$

Furthermore, the vector-valued functions f and f_d satisfy the sector nonlinearity condition:

$$[f(x) - L_1 x]^T [f(x) - L_2 x] \leq 0, \quad \forall x \in \mathbb{R}^n, \tag{9}$$

$$[f_d(x) - U_1 x]^T [f_d(x) - U_2 x] \leq 0, \quad \forall x \in \mathbb{R}^n, \tag{10}$$

where $L_1, L_2, U_1, U_2 \in \mathbb{R}^{n \times n}$ are known real constant matrices.

Remark 1. It is customary that the nonlinear functions f, f_d are said to belong to sectors $[L_1, L_2]$, $[U_1, U_2]$, respectively [10]. The nonlinear descriptions in (9)-(10) are quite general descriptions that include the usual Lipschitz conditions as a special case. Note that both the control analysis and model reduction problems for systems with sector nonlinearities have been intensively studied; see for example [11–13].

Substituting the state feedback $u(k) = Kx(k)$ to system (Σ) gives the following closed-loop system:

$$\begin{aligned} (\Sigma_c): x(k+1) &= A_K(k) x(k) + A_d(k) x(k - d(k)) + \vartheta_k f(x(k)) \\ &\quad + \zeta_k f_d(x(k - d(k))) + D_1(k) v(k) \\ &\quad + [G_K(k) x(k) + G_d(k) x(k - d(k)) + \vartheta_k f(x(k)) \\ &\quad + \zeta_k f_d(x(k - d(k))) + D_2(k) v(k)] w(k), \\ y(k) &= C_K x(k), \\ x(k) &= \phi(k), \quad k = -d_M, -d_M + 1, \ldots, -1, 0, \end{aligned} \tag{11}$$

where

$$\begin{aligned} A_K(k) &= A(k) + B_1(k) K, \\ G_K(k) &= G(k) + B_2(k) K, \qquad C_K = C + BK. \end{aligned} \tag{12}$$

In this paper, we aim to solve the robust H_∞ control problem for the uncertain discrete nonlinear stochastic systems (Σ) with time-varying delays. More specifically, a state feedback controller of the form $u(k) = Kx(k)$ is to be designed such that the following two requirements are met simultaneously.

(1) The closed-loop system (Σ_c) with $v(k) = 0$ is mean-square asymptotically stable for all admissible uncertainties.

(2) Under zero initial condition, the closed-loop system satisfies $\|y\|_{e_2} \leq \gamma \|v\|_{e_2}$ for any nonzero $v(\cdot) \in l_{e_2}([0, +\infty); \mathbb{R}^{n \times m})$.

3. Main Results

In this section, we begin with the robust stabilization problem for the closed-loop system (Σ_c). A sufficient condition is derived in the form of an LMI in order to guarantee the robustly mean-square asymptotic stability for the closed-loop system (Σ_c) with $v(k) = 0$.

Theorem 2. *Let K be a given constant feedback gain matrix. The closed-loop system (Σ_c) with $v(t) = 0$ is robustly asymptotically stable in the mean square if there exist two positive definite matrices X, Q and two positive constant scalars ε_1, ε_2 such that the following LMI holds:*

$$\Psi < 0, \tag{13}$$

where

$$\Psi = \begin{bmatrix} \Upsilon & 0 & -X\check{L}_2 & 0 & \Theta_1 & \Theta_2 & X & 0 & XN_1^T & Y^T N_3^T \\ * & -Q & 0 & -X\check{U}_2 & XA_d^T & XG_d^T & 0 & X & XN_2^T & 0 \\ * & * & (2\sigma_1 - 1) I & 0 & \beta_1 I & \beta_1 I & 0 & 0 & 0 & 0 \\ * & * & * & (2\sigma_2 - 1) I & \beta_2 I & \beta_2 1 I & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_0 & \Gamma_0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \check{L}_1^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \check{U}_1^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix}, \tag{14}$$

$$\breve{L}_1 = \frac{\left(L_1^T L_2 + L_2^T L_1\right)}{2}; \qquad \breve{L}_2 = -\frac{\left(L_1^T + L_2^T\right)}{2}; \tag{15}$$

$$\breve{U}_1 = \frac{\left(U_1^T U_2 + U_2^T U_1\right)}{2}; \qquad \breve{U}_2 = -\frac{\left(U_1^T + U_2^T\right)}{2}; \tag{16}$$

$$Y = KX; \tag{17}$$

$$\Upsilon = -X + \left(d_M - d_m + 1\right) Q; \tag{18}$$

$$\Theta_1 = XA^T + Y^T B_1^T, \qquad \Theta_2 = XG^T + Y^T B_2^T, \tag{19}$$

$$\Xi_0 = -X + \left(\varepsilon_1 + \varepsilon_2\right) M_1 M_1^T, \tag{20}$$

$$\Gamma_0 = \left(\varepsilon_1 + \varepsilon_2\right) M_1 M_2^T, \tag{21}$$

$$\Pi_0 = -X + \left(\varepsilon_1 + \varepsilon_2\right) M_2 M_2^T. \tag{22}$$

Proof. For the stability analysis of the system (Σ_c), we construct the following Lyapunov-Krasovskii functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{23}$$

where

$$\begin{aligned} V_1(k) &= x^T(k) P x(k); \\ V_2(k) &= \sum_{i=k-d(k)}^{k-1} x^T(i) \widehat{Q} x(i); \\ V_3(k) &= \sum_{j=k-d_M+1}^{k-d_m} \sum_{i=j}^{k-1} x^T(i) \widehat{Q} x(i), \end{aligned} \tag{24}$$

with $P = X^{-1}$ and $\widehat{Q} = X^{-1} Q X^{-1}$.

By calculating the difference of $V(k)$ along the system (Σ_c) with $v(k) = 0$ and taking the mathematical expectation, we have

$$\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\{\Delta V_1(k)\} + \mathbb{E}\{\Delta V_2(k)\} + \mathbb{E}\{\Delta V_3(k)\}, \tag{25}$$

where

$$\begin{aligned} &\mathbb{E}\{\Delta V_1(k)\} \\ &\quad = \mathbb{E}\{V_1(k+1) - V_1(k)\} \\ &\quad = \mathbb{E}\Big\{\mathscr{F}_0^T(k) P \mathscr{F}_0(k) + \mathscr{G}_0^T(k) P \mathscr{G}_0(k) \\ &\qquad\quad + 2\sigma_1 f^T(x(k)) f(x(k)) \\ &\qquad\quad + 2\sigma_2 f_d^T(x(k-d(k))) f_d(x(k-d(k))) \\ &\qquad\quad - x^T(k) P x(k)\Big\}, \end{aligned} \tag{26}$$

with

$$\begin{aligned} \mathscr{F}_0(k) &= A_K(k) x(k) + A_d(k) x(k-d(k)) \\ &\quad + \beta_1 f(x(k)) + \beta_2 f_d(x(k-d(k))), \\ \mathscr{G}_0(k) &= G_K(k) x(k) + G_d(k) x(k-d(k)) + \beta_1 f(x(k)) \\ &\quad + \beta_2 f_d(x(k-d(k))), \end{aligned}$$

$$\begin{aligned} &\mathbb{E}\{\Delta V_2(k)\} \\ &\quad = \mathbb{E}\{V_2(k+1) - V_2(k)\} \\ &\quad = \mathbb{E}\left\{\sum_{i=k+1-d(k+1)}^{k} x^T(i)\widehat{Q}x(i) - \sum_{i=k-d(k)}^{k-1} x^T(i)\widehat{Q}x(i)\right\} \\ &\quad = \mathbb{E}\Bigg\{x^T(k)\widehat{Q}x(k) - x^T(k-d(k))\widehat{Q}x(k-d(k)) \\ &\qquad + \sum_{i=k-d(k+1)+1}^{k-1} x^T(i)\widehat{Q}x(i) \\ &\qquad - \sum_{i=k-d(k)+1}^{k-1} x^T(i)\widehat{Q}x(i)\Bigg\} \\ &\quad = \mathbb{E}\Bigg\{x^T(k)\widehat{Q}x(k) - x^T(k-d(k))\widehat{Q}x(k-d(k)) \\ &\qquad + \sum_{i=k-d_m+1}^{k-1} x^T(i)\widehat{Q}x(i) \\ &\qquad + \sum_{i=k-d(k)+1}^{k-d_m} x^T(i)\widehat{Q}x(i) \\ &\qquad - \sum_{i=k-d(k)+1}^{k-1} x^T(i)\widehat{Q}x(i)\Bigg\} \\ &\quad \le \mathbb{E}\Bigg\{x^T(k)\widehat{Q}x(k) - x^T(k-d(k))\widehat{Q}x(k-d(k)) \\ &\qquad + \sum_{i=k-d_M+1}^{k-d_m} x^T(i)\widehat{Q}x(i)\Bigg\}, \end{aligned}$$

$$\begin{aligned} &\mathbb{E}\{\Delta V_3(k)\} \\ &\quad = \mathbb{E}\{V_3(k+1) - V_3(k)\} \\ &\quad = \mathbb{E}\Bigg\{\sum_{j=k-d_M+2}^{k-d_m+1} \sum_{i=j}^{k} x^T(i)\widehat{Q}x(i) \\ &\qquad - \sum_{j=k-d_M+1}^{k-d_m} \sum_{i=j}^{k-1} x^T(i)\widehat{Q}x(i)\Bigg\} \\ &\quad = \mathbb{E}\Bigg\{\sum_{j=k-d_M+1}^{k-d_m} \sum_{i=j+1}^{k} x^T(i)\widehat{Q}x(i) \\ &\qquad - \sum_{j=k-d_M+1}^{k-d_m} \sum_{i=j}^{k-1} x^T(i)\widehat{Q}x(i)\Bigg\} \\ &\quad = \mathbb{E}\Bigg\{\sum_{j=k-d_M+1}^{k-d_m} \left(x^T(k)\widehat{Q}x(k) - x^T(j)\widehat{Q}x(j)\right)\Bigg\} \\ &\quad = \mathbb{E}\Bigg\{\left(d_M - d_m\right) x^T(k)\widehat{Q}x(k) \\ &\qquad - \sum_{i=k-d_M+1}^{k-d_m} x^T(i)\widehat{Q}x(i)\Bigg\}. \end{aligned} \tag{27}$$

Substituting (26)-(27) into (25) leads to

$$\begin{aligned}
&\mathbb{E}\{\Delta V(k)\} \\
&\quad\leq \mathbb{E}\Big\{\mathcal{F}_0^T(k) P \mathcal{F}_0(k) + \mathcal{G}_0^T(k) P \mathcal{G}_0(k) \\
&\qquad + x^T(k)\left[-P + (d_M - d_m + 1)\widehat{Q}\right] x(k) \\
&\qquad - x^T(k - d(k)) \widehat{Q} x(k - d(k)) \\
&\qquad + 2\sigma_1 f^T(x(k)) f(x(k)) \\
&\qquad + 2\sigma_2 f_d^T(x(k - d(k))) f_d(x(k - d(k)))\Big\} \\
&\quad= \mathbb{E}\Big\{\xi_0^T(k) \Psi_1(k) \xi_0(k) + \xi_0^T(k) \widehat{F}_0^T(k) P \widehat{F}_0(k) \xi_0(k) \\
&\qquad + \xi_0^T(k) \widehat{G}_0^T(k) P \widehat{G}_0(k) \xi_0(k)\Big\},
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
&\xi_0(k) \\
&\quad= \left[x^T(k)\ \ x^T(k - d(k))\ \ f^T(x(k))\ \ f_d^T(x(k - d(k)))\right]^T, \\
&\quad\widehat{F}_0(k) = \left[A_K(k)\ \ A_d(k - d(k))\ \ \beta_1 I\ \ \beta_2 I\right], \\
&\quad\widehat{G}_0(k) = \left[G_K(k)\ \ G_d(k - d(k))\ \ \beta_1 I\ \ \beta_2 I\right], \\
&\Psi_1(k) = \operatorname{diag}\left(-P + (d_M - d_m + 1)\widehat{Q}, -\widehat{Q}, 2\sigma_1 I, 2\sigma_1 I\right).
\end{aligned} \tag{29}$$

Notice that (9) is equivalent to

$$\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} \breve{L}_1 & \breve{L}_2 \\ \breve{L}_2^T & I \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \leq 0, \tag{30}$$

where $\breve{L}_1$, $\breve{L}_2$ are defined in (15) and, similarly, it follows from (10) that

$$\begin{bmatrix} x(k - d(k)) \\ f_d(x(k - d(k))) \end{bmatrix}^T \begin{bmatrix} \breve{U}_1 & \breve{U}_2 \\ \breve{U}_2^T & I \end{bmatrix} \begin{bmatrix} x(k - d(k)) \\ f_d(x(k - d(k))) \end{bmatrix} \leq 0, \tag{31}$$

where $\breve{U}_1$, $\breve{U}_2$ are defined in (16).

It follows from (28), (30), and (31) that

$$\begin{aligned}
&\mathbb{E}\{\Delta V(k)\} \\
&\quad\leq \mathbb{E}\Big\{\xi_0^T(k) \Psi_1(k) \xi_0(k) + \xi_0^T(k) \widehat{F}_0^T(k) P \widehat{F}_0(k) \xi_0(k) \\
&\qquad + \xi_0^T(k) \widehat{G}_0^T(k) P \widehat{G}_0(k) \xi_0(k)\Big\} \\
&\qquad - \mathbb{E}\Bigg\{\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} \breve{L}_1 & \breve{L}_2 \\ \breve{L}_2^T & I \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \\
&\qquad + \begin{bmatrix} x(k - d(k)) \\ f_d(x(k - d(k))) \end{bmatrix}^T \begin{bmatrix} \breve{U}_1 & \breve{U}_2 \\ \breve{U}_2^T & I \end{bmatrix} \\
&\qquad \times \begin{bmatrix} x(k - d(k)) \\ f_d(x(k - d(k))) \end{bmatrix}\Bigg\} \\
&\quad= \mathbb{E}\Big\{\xi_0^T(k)\left[\Psi_2(k) + \widehat{F}_0^T(k) P \widehat{F}_0(k)\right. \\
&\qquad \left. + \widehat{G}_0^T(k) P \widehat{G}_0(k)\right] \xi_0(k)\Big\},
\end{aligned} \tag{32}$$

where

$$\Psi_2(k) = \begin{bmatrix} -P + (d_M - d_m + 1)\widehat{Q} - \breve{L}_1 & 0 & \breve{L}_2 & 0 \\ 0 & -\widehat{Q} - \breve{U}_1 & 0 & \breve{U}_2 \\ \breve{L}_2^T & 0 & (2\sigma_1 - 1) I & 0 \\ 0 & \breve{U}_2^T & 0 & (2\sigma_2 - 1) I \end{bmatrix}. \tag{33}$$

From Lyapunov stability theory, in order to ensure the stability of the closed-loop system (Σ_c) with $v(k) = 0$, we just need to show

$$\Psi_2(k) + \widehat{F}_0^T(k) P \widehat{F}_0(k) + \widehat{G}_0^T(k) P \widehat{G}_0(k) < 0. \tag{34}$$

By *Schur complement*, (34) is equivalent to

$$\Psi_3(k) < 0, \tag{35}$$

where

$$\Psi_3(k) = \begin{bmatrix} -P + (d_M - d_m + 1)\widehat{Q} - \breve{L}_1 & 0 & -\breve{L}_2 & 0 & A_K^T(k) & G_K^T(k) \\ 0 & -\widehat{Q} - \breve{U}_1 & 0 & -\breve{U}_2 & A_d^T(k) & G_d^T(k) \\ -\breve{L}_2^T & 0 & (2\sigma_1 - 1) I & 0 & \beta_1 I & \beta_1 I \\ 0 & -\breve{U}_2^T & 0 & (2\sigma_2 - 1) I & \beta_2 I & \beta_2 I \\ A_K(k) & A_d(k) & \beta_1 I & \beta_2 I & -P^{-1} & 0 \\ G_K(k) & G_d(k) & \beta_1 I & \beta_2 I & 0 & -P^{-1} \end{bmatrix}. \tag{36}$$

Therefore, it remains to show $\Psi_3(k) < 0$. Let $\widehat{X} = \mathrm{diag}(X, X, I, I, I, I)$, and

$$\Psi_4(k) = \widehat{X}^T \Psi_3(k)\widehat{X} = \begin{bmatrix} \Upsilon - X\check{L}_1 X & 0 & -X\check{L}_2 & 0 & XA_K^T(k) & XG_K^T(k) \\ 0 & -Q - X\check{U}_1 X & 0 & -X\check{U}_2 & XA_d^T(k) & XG_d^T(k) \\ -\check{L}_2^T X & 0 & (2\sigma_1 - 1)I & 0 & \beta_1 I & \beta_1 I \\ 0 & -\check{U}_2^T X & 0 & (2\sigma_2 - 1)I & \beta_2 I & \beta_2 I \\ A_K(k)X & A_d(k)X & \beta_1 I & \beta_2 I & -X & 0 \\ G_K(k)X & G_d(k)X & \beta_1 I & \beta_2 I & 0 & -X \end{bmatrix}, \tag{37}$$

where Υ is defined in (18).

Obviously, $\Psi_3(k) < 0$ is equivalent to $\Psi_4(k) < 0$. Now, we can rewrite $\Psi_4(k)$ as follows:

$$\Psi_4(k) = \Psi_5(k) + \widetilde{X}_0^T \begin{bmatrix} -\check{L}_1 & 0 \\ 0 & -\check{U}_1 \end{bmatrix} \widetilde{X}_0, \tag{38}$$

where

$$\widetilde{X}_0 = \begin{bmatrix} X & 0 & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_5(k) = \begin{bmatrix} \Upsilon & 0 & -X\check{L}_2 & 0 & \Theta_1 & \Theta_2 \\ 0 & -Q & 0 & -X\check{U}_2 & XA_d^T(k) & XG_d^T(k) \\ -\check{L}_2^T X & 0 & (2\sigma_1 - 1)I & 0 & \beta_1 I & \beta_1 I \\ 0 & -\check{U}_2^T X & 0 & (2\sigma_2 - 1)I & \beta_2 I & \beta_2 I \\ \Theta_1^T & A_d(k)X & \beta_1 I & \beta_2 I & -X & 0 \\ \Theta_2^T & G_d(k)X & \beta_1 I & \beta_2 I & 0 & -X \end{bmatrix}, \tag{39}$$

where Θ_1 and Θ_2 are defined in (19).

According to *Schur complement*, $\Phi_4(k) < 0$ is equivalent to

$$\Psi_6(k) < 0, \tag{40}$$

where

$$\Psi_6(k) = \begin{bmatrix} \Upsilon & 0 & -X\check{L}_2 & 0 & \Theta_1 & \Theta_2 & X & 0 \\ 0 & -Q & 0 & -X\check{U}_2 & XA_d^T(k) & XG_d^T(k) & 0 & X \\ -\check{L}_2^T X & 0 & (2\sigma_1 - 1)I & 0 & \beta_1 I & \beta_1 I & 0 & 0 \\ 0 & -\check{U}_2^T X & 0 & (2\sigma_2 - 1)I & \beta_2 I & \beta_2 I & 0 & 0 \\ \Theta_1^T & A_d(k)X & \beta_1 I & \beta_2 I & -X & 0 & 0 & 0 \\ \Theta_2^T & G_d(k)X & \beta_1 I & \beta_2 I & 0 & -X & 0 & 0 \\ X & 0 & 0 & 0 & 0 & 0 & \check{L}_1^{-1} & 0 \\ 0 & X & 0 & 0 & 0, & 0 & 0 & \check{U}_1^{-1} \end{bmatrix}. \tag{41}$$

Noticing (7), we can rearrange $\Phi_6(k)$ as follows:

$$\Psi_6(k) = \Psi_6 + \Delta\Psi_6(k), \tag{42}$$

where

$$\Psi_6 = \begin{bmatrix} \Upsilon & 0 & -X\check{L}_2 & 0 & \Theta_1 & \Theta_2 & X & 0 \\ 0 & -Q & 0 & -X\check{U}_2 & XA_d^T & XG_d^T & 0 & X \\ -\check{L}_2^T X & 0 & (2\sigma_1 - 1)I & 0 & \beta_1 I & \beta_1 2I & 0 & 0 \\ 0 & -\check{U}_2^T X & 0 & (2\sigma_2 - 1)I & \beta_2 I & \beta_2 I & 0 & 0 \\ \Theta_1^T & A_d X & \beta_1 I & \beta_2 I & -X & 0 & 0 & 0 \\ \Theta_2^T & G_d X & \beta_1 I & \beta_2 I & 0 & -X & 0 & 0 \\ X & 0 & 0 & 0 & 0 & 0 & \check{L}_1^{-1} & 0 \\ 0 & X & 0 & 0 & 0 & 0 & 0 & \check{U}_1^{-1} \end{bmatrix}, \tag{43}$$

$$\begin{aligned}\Delta\Psi_6(k) &= \widehat{M}F(k)\widehat{N}_1 + \widehat{N}_1^T F^T(k)\widehat{M}^T \\ &\quad + \widehat{M}F(k)\widehat{N}_2 + \widehat{N}_2^T F^T(k)\widehat{M}^T,\end{aligned} \tag{44}$$

with

$$\begin{aligned}\widehat{M} &= \begin{bmatrix}0 & 0 & 0 & 0 & M_1^T & M_2^T & 0 & 0\end{bmatrix}^T, \\ \widehat{N}_1 &= \begin{bmatrix}N_1X & N_2X & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \\ \widehat{N}_2 &= \begin{bmatrix}N_3Y & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}.\end{aligned} \tag{45}$$

From (44), it follows

$$\Delta\Psi_6(k) \le \varepsilon_1\widehat{M}\widehat{M}^T + \varepsilon_2\widehat{M}\widehat{M}^T + \varepsilon_1^{-1}\widehat{N}_1^T\widehat{N}_1 + \varepsilon_2^{-1}\widehat{N}_2^T\widehat{N}_2, \tag{46}$$

and then we obtain from (42)–(46)

$$\Psi_6(k) \le \Psi_7(k) + \varepsilon_1^{-1}\widehat{N}_1^T\widehat{N}_1 + \varepsilon_2^{-1}\widehat{N}_2^T\widehat{N}_2, \tag{47}$$

where

$$\Psi_7(k) = \begin{bmatrix} \Upsilon & 0 & -X\check{L}_2 & 0 & \Theta_1 & \Theta_2 & X & 0 \\ 0 & -Q & 0 & -X\check{U}_2 & XA_d^T & XG_d^T & 0 & X \\ -\check{L}_2^T X & 0 & (2\sigma_1-1)I & 0 & \beta_1 I & \beta_1 I & 0 & 0 \\ 0 & -\check{U}_2^T X & 0 & (2\sigma_2-1)I & \beta_2 I & \beta_2 I & 0 & 0 \\ \Theta_1^T & A_dX & \beta_1 I & \beta_2 I & \Xi_0 & \Gamma_0 & 0 & 0 \\ \Theta_2^T & G_dX & \beta_1 I & \beta_2 I & \Gamma_0^T & \Pi_0 & 0 & 0 \\ X & 0 & 0 & 0 & 0 & 0 & \check{L}_1^{-1} & 0 \\ 0 & X & 0 & 0 & 0 & 0 & 0 & \check{U}_1^{-1} \end{bmatrix}, \tag{48}$$

and Ξ_0, Γ_0, and Π_0 are defined in (18)–(22).

Using *Schur complement* once again, it is not difficult to see that $\Psi < 0$ (i.e., the condition (13)) is equivalent to that the right-hand side of (47) is negative definite; that is, $\Psi_6(k) < 0$. Thus, we arrive at $\Psi_3(k) < 0$, which completes the proof of the theorem. □

Next we will analyze the H_∞ performance of the closed-loop system (Σ_c).

Theorem 3. *Let K be a given constant feedback gain matrix and γ a given positive constant. Then, the closed-loop system (Σ_c) is robustly mean-square asymptotically stable for $v(k) = 0$ and satisfies $\|y\|_{e_2} \le \gamma\|v\|_{e_2}$ under the zero initial condition for any nonzero $v(\cdot) \in l_{e_2}([0, +\infty); \mathbb{R}^{n\times m})$ if there exist two positive definite matrices X, Q and three scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ such that the following LMI holds*

$$\Phi_0 < 0, \tag{49}$$

where

$$\Phi_0 = \begin{bmatrix} \Upsilon & 0 & -X\check{L}_2 & 0 & 0 & \Theta_1 & \Theta_2 & X & 0 & \Theta_3 & XN_1^T & Y^TN_3 & 0 \\ * & -Q & 0 & -X\check{U}_2 & 0 & XA_d^T & XG_d^T & 0 & X & 0 & XN_2^T & 0 & 0 \\ * & * & (2\sigma_1-1)I & 0 & 0 & \beta_1 I & \beta_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & (2\sigma_2-1)I & 0 & \beta_2 I & \beta_2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & D_1^T & D_2^T & 0 & 0 & 0 & 0 & 0 & N_4^T \\ * & * & * & * & * & \Xi & \Gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \check{L}_1^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \check{U}_1^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix}, \tag{50}$$

$$\begin{aligned}\Theta_3 &= XC^T + Y^T B^T,\\ \Xi &= -X + (\varepsilon_1+\varepsilon_2+\varepsilon_3) M_1 M_1^T,\\ \Gamma &= (\varepsilon_1+\varepsilon_2+\varepsilon_3) M_1 M_2^T,\\ \Pi &= -X + (\varepsilon_1+\varepsilon_2+\varepsilon_3) M_2 M_2^T,\end{aligned} \tag{51}$$

and $\breve{L}_1$, $\breve{L}_2$, $\breve{U}_1$, $\breve{U}_2$, Y, Υ, Θ_1, *and* Θ_2 *are defined as in Theorem 2.*

Proof. It is not difficult to verify that $\Phi_0 < 0$ implies $\Psi < 0$ (i.e., (13)). According to Theorem 2, the closed-loop system (Σ_c) is robustly asymptotically stable in the mean square.

Next, we shall deal with the H_∞ performance of the closed-loop system. As in Theorem 2, define Lyapunov-Krasovskii functional candidate for the system (Σ_c) as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{52}$$

where

$$\begin{gathered} V_1(k) = x^T(k) P x(k), \qquad V_2(k) = \sum_{i=k-d(k)}^{k-1} x^T(i) \widehat{Q} x(i),\\ V_3(k) = \sum_{j=k-d_M+1}^{k-d_m} \sum_{i=j}^{k-1} x^T(i) \widehat{Q} x(i). \end{gathered} \tag{53}$$

And we also introduce

$$J(n) = \mathbb{E}\sum_{k=0}^{n} \left[y^T(k) y(k) - \gamma^2 v^T(k) v(k) \right], \tag{54}$$

where n is nonnegative integer. Under the zero initial condition, one has

$$\begin{aligned} J(n) = {}& \mathbb{E}\sum_{k=0}^{n} \left[y^T(k) y(k) - \gamma^2 v^T(k) v(k) + \Delta V(k) \right]\\ & - \mathbb{E}V(n+1). \end{aligned} \tag{55}$$

Along the similar lines to the proof of Theorem 2, we can derive that $J(n) \le 0$. Letting $n \to \infty$, we obtain $\|y\|_{e_2} \le \gamma\|v\|_{e_2}$, which concludes the proof of the theorem. □

Finally, let us consider the design of H_∞ control for the system (Σ). Based on Theorem 3, we have the following result.

Theorem 4. *Let $\gamma > 0$ be a given positive constant. Then, for the nonlinear stochastic system (Σ), there exists a state feedback controller such that the closed-loop system (Σ_c) is robustly mean-square asymptotically stable for $v(k) = 0$ and satisfies $\|y\|_{e_2} \le \gamma\|v\|_{e_2}$ under the zero initial condition for any nonzero $v(\cdot) \in l_{e_2}([0,+\infty);\mathbb{R}^{n\times m})$ if there exist two positive definite matrices X and Q, a matrix Y, and three scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$ such that the following LMI holds:*

$$\Phi < 0, \tag{56}$$

where

$$\Phi_0 = \begin{bmatrix} \Upsilon & 0 & -X\breve{L}_2 & 0 & 0 & \Theta_1 & \Theta_2 & X & 0 & \Theta_3 & XN_1^T & Y^T N_3 & 0\\ * & -Q & 0 & -X\breve{U}_2 & 0 & XA_d^T & XG_d^T & 0 & X & 0 & XN_2^T & 0 & 0\\ * & * & (2\sigma_1-1)I & 0 & 0 & \beta_1 I & \beta_1 I & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & (2\sigma_2-1)I & 0 & \beta_2 I & \beta_2 I & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\gamma^2 I & D_1^T & D_2^T & 0 & 0 & 0 & 0 & 0 & N_4^T\\ * & * & * & * & * & \Xi & \Gamma & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & \Pi & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & \breve{L}_1^{-1} & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & \breve{U}_1^{-1} & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & * & -I & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0\\ * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0\\ * & * & * & * & * & * & * & * & * & * & * & * & -\varepsilon_3 I \end{bmatrix}, \tag{57}$$

where $\breve{L}_1$, $\breve{L}_2$, $\breve{U}_1$, $\breve{U}_2$, Υ, Ξ, Γ, Π, Θ_1, Θ_2, and Θ_3 are defined as in Theorems 2 and 3. In this case, the state feedback gain matrix can be designed as

$$K = YX^{-1}. \tag{58}$$

Remark 5. In Theorem 4, the robust H_∞ controller design problem is reduced to the solvability of an LMI, which could be easily checked by utilizing the LMI Matlab toolbox. It is also worth pointing out that, in LMI framework, the sector nonlinearity condition is superior to commonly used Lipschitz condition in reducing the possible conservativeness.

4. Numerical Example

In this section, a numerical example is presented to demonstrate the usefulness of the developed method on the design of robust H_∞ control for the discrete uncertain nonlinear stochastic systems with time-varying delays.

Consider the system (Σ) with the following parameters:

$$A = \begin{bmatrix} 0.4 & 0.1 & 0 \\ 0 & -0.3 & 0.1 \\ 0.1 & 0 & -0.2 \end{bmatrix}, \qquad A_d = \begin{bmatrix} 0.2 & -0.1 & 0.1 \\ 0.1 & -0.2 & 0 \\ -0 & -0.2 & -0.1 \end{bmatrix},$$

$$E = H = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.2 & 0 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, \qquad E_d = H_d = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.1 & 0.2 & 0 \\ 0.1 & 0 & 0.1 \end{bmatrix},$$

$$G = \begin{bmatrix} -0.1 & 0.1 & 0 \\ -0 & 0.2 & 0.1 \\ -0.1 & -0.2 & 0.1 \end{bmatrix}, \qquad G_d = \begin{bmatrix} -0.1 & 0.2 & 0 \\ -0.1 & 0.2 & 0.1 \\ 0 & -0.1 & 0.2 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.2 & -0 & 0.1 \\ -0.2 & -0.1 & 0.1 \\ -0 & 0.2 & -0.1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & 0.1 \\ 0.2 & 1 \\ 0 & -0.1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -0.2 & 0 \\ -0.1 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & 0.2 \\ 0 & 0.3 \end{bmatrix},$$

$$L_1 = U_1 = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.3 & 0 \\ -0.1 & 0.1 & 0.3 \end{bmatrix},$$

$$L_2 = U_2 = \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0.1 & -0.3 & -0.1 \\ -0.1 & 0 & -0.3 \end{bmatrix}, \qquad M_1 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}, \qquad N_1 = N_2 = N_4 = N_5 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}^T,$$

$$N_3 = N_6 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}^T, \qquad d_m = 2,$$

$$d_M = 3, \qquad \beta_1 = 0.5, \qquad \beta_2 = 0.5. \tag{59}$$

The H_∞ performance level is taken as $\gamma = 0.9$. With the above parameters and by using the Matlab LMI Toolbox, we solve the LMI (56) and obtain

$$X = \begin{bmatrix} 5.7714 & 0.4304 & 2.2321 \\ 0.4304 & 4.1813 & 0.8207 \\ 2.2321 & 0.8207 & 13.0932 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.4729 & -0.1464 & 1.5125 \\ -0.1464 & 1.1697 & 0.7317 \\ 1.5125 & 0.7317 & 4.1582 \end{bmatrix}, \tag{60}$$

$$Y = \begin{bmatrix} 0.1037 & -1.5046 & 0.0842 \\ -0.7253 & -0.8760 & -2.1560 \end{bmatrix},$$

$$\varepsilon_1 = 10.0151, \qquad \varepsilon_2 = 4.8030,$$

$$\varepsilon_3 = 8.1230.$$

Therefore, the state feedback gain matrix can be designed as

$$K = YX^{-1} = \begin{bmatrix} 0.0364 & -0.3681 & 0.0233 \\ -0.0569 & -0.1754 & -0.1440 \end{bmatrix}. \tag{61}$$

5. Conclusions

In this paper, we have studied the robust H_∞ control problem for a class of uncertain discrete-time stochastic systems involving randomly occurring nonlinearities and time-varying delays. An effective linear matrix inequality (LMI) approach has been proposed to design the state feedback controllers such that the overall uncertain closed-loop system is robustly asymptotically stable in the mean square and a prescribed H_∞ disturbance rejection attenuation level is guaranteed. We have first investigated the sufficient conditions for the uncertain stochastic time-delay systems under consideration to be stable in the mean square and then derived the explicit expression of the desired controller gains. A numerical example has been provided to show the usefulness and effectiveness of the proposed design method. It should be pointed out that the main results of this paper can be extended to the case where the parameter uncertainties are of the polytopic type [9, 29] and the case where system measurements suffer from quantization effects.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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