Research Article

Some Properties on Estrada Index of Folded Hypercubes Networks

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Let G be a simple graph with n vertices and let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of its adjacency matrix; the Estrada index $EE(G)$ of the graph G is defined as the sum of the terms e^{λ_i}, $i = 1, 2, \ldots, n$. The n-dimensional folded hypercube networks FQ_n are an important and attractive variant of the n-dimensional hypercube networks Q_n, which are obtained from Q_n by adding an edge between any pair of vertices complementary edges. In this paper, we establish the explicit formulae for calculating the Estrada index of the folded hypercubes networks FQ_n by deducing the characteristic polynomial of the adjacency matrix in spectral graph theory. Moreover, some lower and upper bounds for the Estrada index of the folded hypercubes networks FQ_n are proposed.

1. Introduction

Complex networks have become an important area of multidisciplinary research involving mathematics, physics, social sciences, biology, and other theoretical and applied sciences. It is well known that interconnection networks play an important role in parallel communication systems. An interconnection network is usually modelled by a connected graph $G = (V, E)$, where V denotes the set of processors and E denotes the set of communication links between processors in networks. Let G be a graph with vertices labelled $1, 2, \ldots, n$. The adjacency matrix $A(G)$ of G is an $n \times n$ matrix with the (i, j)-entry equal to 1 if vertices i and j are adjacent and 0 otherwise. The spectrum of G is the spectrum of its adjacency matrix and consists of the numbers $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. In this work we are concerned with finite undirected connected simple graphs (networks). For the underlying graph theoretical definitions and notations we follow [1].

The energy of the graph G [2] is defined as

$$E(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{1}$$

Another graph-spectrum-based invariant, recently put forward by Ernesto Estrada, is defined as

$$EE = EE(G) = \sum_{i=1}^{n} e^{\lambda_i}. \tag{2}$$

This graph invariant appeared for the first time in the year 2000, in a paper by Estrada [3], dealing with the folding of protein molecules. Estrada and Rodríguez-Velázquez showed that EE provides a measure of the centrality of complex (communication, social, metabolic, etc.) networks [4, 5].

Denote by $M_k = M_k(G) = \sum_{i=1}^{n} (\lambda_i)^k$ the kth spectral moment of the graph G. From the Taylor expansion of e^x, we have the following important relation between the Estrada index and the spectral moments of G:

$$EE(G) = \sum_{k=0}^{\infty} \frac{M_k(G)}{k!}. \tag{3}$$

At this point one should recall [4] that $M_k(G)$ is equal to the number of self-returning walks of length k of the graph

G. The first few spectral moments of an (n, m)-graph with m edges and t triangles satisfy the following relations [4]:

$$M_0 = \sum_{i=1}^{n}(\lambda_i)^0 = n; \qquad M_1 = \sum_{i=1}^{n}(\lambda_i)^1 = 0;$$

$$M_2 = \sum_{i=1}^{n}(\lambda_i)^2 = 2m; \qquad M_3 = \sum_{i=1}^{n}(\lambda_i)^3 = 6t. \tag{4}$$

For $1 \le i \le n$, let d_i be the degree of vertex v_i in *G*. The first Zagreb index [6] of the graph *G* is defined as $Zg(G) = \sum_{i=1}^{n} d_i^2$:

$$M_4 = \sum_{i=1}^{n}(\lambda_i)^4 = 2Zg(G) - 2m + 8q;$$

$$M_5 = \sum_{i=1}^{n}(\lambda_i)^5 = 30t + 10p + 10r, \tag{5}$$

where *p* and *q* are the numbers of pentagons and quadrangles in *G*, and *r* is the number of subgraphs consisting of a triangle with a pendent vertex attached [7].

The hypercubes Q_n is one of the most popular and efficient interconnection networks due to its many excellent performances for some practical applications. There is a large amount of literature on the properties of hypercubes networks [8–11]. As an important variant of Q_n, the folded hypercubes networks FQ_n, proposed by Amawy and Latifi [8], are the graphs obtained from Q_n by adding an edge between any pair of vertices complementary addresses. The folded hypercubes FQ_n obtained considerable attention due to its perfect properties, such as symmetry, regular structure, strong connectivity, small diameter, and many of its properties which have been explored [12–19].

The remainder of the present paper is organized as follows. In Section 2, we present some basic notations and some preliminaries in our discussion. The proofs of our main results are in Section 3 and some conclusions are given in Section 4, respectively.

2. Notations and Some Preliminaries

In this section, we introduce some basic properties which will be used in the proofs of our main results.

Let $P_{FQ_n}(x)$ be the characteristic polynomial of the adjacency matrix of the folded hypercube FQ_n; the following results were shown in [12].

Lemma 1 (see [12]). *The characteristic polynomial of the adjacency matrix of the FQ_n ($n \ge 3$) is*

$$P(FQ_n; \lambda) = [\lambda - (n-7)][\lambda - (n-3)]^3 P(FQ_{n-1}; \lambda - 1)$$

$$\times \prod_{i=2}^{n-2} P(FQ_{n-i}; \lambda - (i-4)). \tag{6}$$

Lemma 2 (see [12]). *For FQ_n with $n \ge 3$, the spectrum of adjacency matrix is as follows:*
   (1) *If $n \equiv 0 \pmod 2$,*

$$Spec(FQ_n) = \begin{pmatrix} -n+1 & -n+5 & -n+9 & \cdots & n-7 & n-3 & n+1 \\ C_n^0 + C_n^1 & C_n^2 + C_n^3 & C_n^4 + C_n^5 & \cdots & C_n^{n-4} + C_n^{n-3} & C_n^{n-2} + C_n^{n-1} & C_n^n \end{pmatrix}, \tag{7}$$

   (2) *if $n \equiv 1 \pmod 2$,*

$$Spec(FQ_n) = \begin{pmatrix} -n-1 & -n+3 & -n+7 & \cdots & n-7 & n-3 & n+1 \\ C_n^0 & C_n^1 + C_n^2 & C_n^3 + C_n^4 & \cdots & C_n^{n-4} + C_n^{n-3} & C_n^{n-2} + C_n^{n-1} & C_n^n \end{pmatrix}, \tag{8}$$

where C_n^i are the binomial coefficients and the elements in the first and second rows are the eigenvalues of the adjacency matrix of FQ_n and the corresponding multiplicities, respectively.

Lemma 3 (see [20]). *The eigenvalues of a bipartite graph satisfy the pairing property: $\lambda_{n-i+1} = \lambda_i$, $i = 1, 2, \ldots, n$. Therefore, if the graph G is bipartite and if η_0 is nullity (the multiplicity of its eigenvalue zero), then*

$$EE(G) = \eta_0 + 2\sum_{+} \cosh(i), \tag{9}$$

where cosh *stands for the hyperbolic cosine* $\cosh(x) = (e^x + e^{-x})/2$, *whereas* $\sum_+$ *denotes summation over all positive eigenvalues of the corresponding graph.*

Lemma 4 (see [21]). *Let G be a graph with m edges. For $k \ge 4$,*

$$M_{k+2} \ge M_k, \tag{10}$$

with equality for all even $k \ge 4$ if and only if G consists of m copies of K_2 and possibly isolated vertices and with equality for all odd $k \ge 5$ if and only if G is a bipartite graph.

The following lemma is an immediate result of the previous lemma.

Lemma 5 (see [22]). *Let G be an (n, m) graph with m edges. For $k \geq 4$,*

$$\sum_{i=1}^{n}(2\lambda_i)^{k+2} \geq 4\sum_{i=1}^{n}(2\lambda_i)^k, \qquad (11)$$

with equality for all even $k \geq 4$ if and only if G consists of m copies of K_2 and possibly isolated vertices and with equality for all odd $k \geq 5$ if and only if G is a bipartite graph.

Lemma 6 (see [23]). *Let G be a regular graph of degree $r \neq 0$ and of order n. Then its Estrada index is bounded by*

$$e^r + (n-1)e^{-r/(n-1)} \leq EE(G) < n - 2 + e^r + e^{\sqrt{r(n-r)-1}}. \qquad (12)$$

Equality holds if and only if $\lambda_2 = \lambda_3 = \cdots = \lambda_n = -r/(n-1)$.

Lemma 7 (see [23]). *The Estrada index $EE(G)$ and the graph energy $E(G)$ satisfy the following inequality:*

$$\frac{1}{2}E(G)(e-1) + n - n_+ \leq EE(G) \leq n - 1 + e^{E(G)/2}, \qquad (13)$$

and equalities on both sides hold if and only if $E(G) = 0$.

3. Main Results

3.1. The Estrada Index of Folded Hypercubes Networks FQ_n. In this section, we present some explicit formulae for calculating the Estrada index of FQ_n. For convenience, we assume that $C_n^i = 0$ if $i < 0$ or $i > n$.

Theorem 8. *For any FQ_n with $n \geq 3$, then*

(1) $EE(FQ_n) = \sum_{i=0}^{n/2}(C_n^{2i} + C_n^{2i+1})e^{4i-n+1}$, $i = 0, 1, \ldots, n/2$, *if $n \equiv 0 \pmod 2$;*

(2) $EE(FQ_n) = \sum_{i=0}^{n/2}(C_n^{2i-1} + C_n^{2i})e^{4i-n-1}$, $i = 0, 1, \ldots, (n+1)/2$, *if $n \equiv 1 \pmod 2$,*

where the $4i - n + 1$ and $4i - n - 1$ $(i = 0, 1, \ldots, n/2$ or $(n+1)/2)$ are the eigenvalues of the adjacent matrix of FQ_n and C_n^i denotes the binomial coefficients.

Proof. By Lemma 1, the characteristic polynomial of the adjacent matrix of FQ_n is

$$P(FQ_n; \lambda) = [\lambda - (n-7)][\lambda - (n-3)]^3 P(FQ_{n-1}; \lambda - 1)$$
$$\times \prod_{i=2}^{n-2} P(FQ_{n-i}; \lambda - (i-4)). \qquad (14)$$

Through calculating eigenvalues of characteristic polynomial and its multiplicities, we obtained that

(1) if $n \equiv 0 \pmod 2$, FQ_n have $n/2 + 1$ different eigenvalues $4i - n + 1$, with the multiplicities $C_n^{2i} + C_n^{2i+1}$, where $i = 0, 1, \ldots, n/2$;

(2) if $n \equiv 1 \pmod 2$, FQ_n have $(n+1)/2$ different eigenvalues $4i - n - 1$, with the multiplicities $C_n^{2i-1} + C_n^{2i}$, where $i = 0, 1, \ldots, (n+1)/2$.

Combining with the definition of the Estrada index, we derived the result of Theorem 8. $\qquad \square$

3.2. Some Bounds for the Estrada Index of Folded Hypercubes Networks FQ_n. It is well known that FQ_n have 2^n vertices. Let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq \lambda_{n+1} \geq \cdots \geq \lambda_{2^n}$ be the eigenvalues of FQ_n with nonincreasing order. In order to obtain the bounds for the Estrada index of FQ_n, we prove some results by utilizing the arithmetic and geometric mean inequality; in our proof, some techniques in [22] are referred to.

Theorem 9. *For any FQ_n with $n \geq 2$, one has*

$$\sqrt{4^n + (n+1)2^{n+1} + 8t + [\cosh(2) - 3]M_4 + \left[\cosh(2) - \frac{10}{3}\right]M_5}$$
$$< EE(FQ_n), \qquad (15)$$

where $M_4 = 2Zg(G) - 2m + 8q$, $M_5 = 30t + 10p + 10r$, p and q are the numbers of pentagons and quadrangles in FQ_n, and r is the number of subgraphs consisting of a triangle with a pendent vertex attached.

Proof. In order to obtain the lower bounds for the Estrada index, consider that

$$EE^2(FQ_n) = \sum_{i=1}^{2^n} e^{2\lambda_i} + 2\sum_{i<j} e^{\lambda_i} e^{\lambda_j}. \qquad (16)$$

Noting that $M_0 = 2^n$, $M_1 = 0$, $M_2 = (n+1)2^{n-1}$, and $M_3 = 6t$, we obtain

$$\sum_{i=1}^{2^n} e^{2\lambda_i} = \sum_{i=1}^{2^n} \sum_{k \geq 0} \frac{(2\lambda_i)^k}{k!}$$

$$= 2^n + (n+1)2^{n+1} + 8t + \sum_{i=1}^{2^n} \sum_{k \geq 4} \frac{(2\lambda_i)^k}{k!}$$

$$= 2^n + (n+1)2^{n+1} + 8t + \sum_{k \geq 2} \frac{1}{(2k)!}\sum_{i=1}^{2^n}(2\lambda_i)^{2k}$$

$$+ \sum_{k \geq 2} \frac{1}{(2k+1)!}\sum_{i=1}^{2^n}(2\lambda_i)^{2k+1}. \qquad (17)$$

By Lemma 5,

$$\sum_{i=1}^{n}(2\lambda_i)^{k+2} \geq 4\sum_{i=1}^{n}(2\lambda_i)^k, \qquad (18)$$

we can get that

$$\sum_{i=1}^{2^n} e^{2\lambda_i} \geq 2^n + (n+1)\, 2^{n+1} + 8t$$

$$+ \sum_{k\geq 2} \frac{1}{(2k)!} \sum_{i=1}^{2^n} 2^{2k-4} (2\lambda_i)^4$$

$$+ \sum_{k\geq 2} \frac{1}{(2k+1)!} \sum_{i=1}^{2^n} 2^{2k-4} (2\lambda_i)^5 \tag{19}$$

$$= 2^n + (n+1)\, 2^{n+1} + 8t + [\cosh(2) - 3]\, M_4$$

$$+ \left[\cosh(2) - \frac{10}{3} \right] M_5,$$

where $M_4 = 2Zg(G) - 2m + 8q$, $M_5 = 30t + 10p + 10r$, p and q are the numbers of pentagons and quadrangles in FQ_n, and r is the number of subgraphs consisting of a triangle with a pendent vertex attached.

As for the terms $2\sum_{i<j} e^{\lambda_i} e^{\lambda_j}$, by the arithmetic and geometric mean inequality and the fact that $M_1 = 0$,

$$2\sum_{i<j} e^{\lambda_i} e^{\lambda_j} \geq 2^n (2^n - 1) \left(\prod_{i<j} e^{\lambda_i} e^{\lambda_j} \right)^{2/2^n(2^n-1)}$$

$$= 2^n (2^n - 1) \left[\left(\prod_{i=1}^{2^n} e^{\lambda_i} \right)^{2^n-1} \right]^{2/2^n(2^n-1)} \tag{20}$$

$$= 2^n (2^n - 1) \left(e^{M_1} \right)^{2/2^n}$$

$$= 2^n (2^n - 1),$$

where the equality holds if and only if $\lambda_1 = \cdots = \lambda_{2^n}$.

Combining with equalities (19) and (20),

$$\sqrt{4^n + (n+1)\, 2^{n+1} + 8t + [\cosh(2)-3]\, M_4 + \left[\cosh(2)-\frac{10}{3}\right] M_5}$$

$$\leq EE(FQ_n), \tag{21}$$

where $M_4 = 2Zg(G) - 2m + 8q$, $M_5 = 30t + 10p + 10r$, p and q are the numbers of pentagons and quadrangles in FQ_n, and r is the number of subgraphs consisting of a triangle with a pendent vertex attached.

Notice that the equality of (21) holds if and only if the equalities of (19) and (20) hold; that is, the equality holds if and only if $\lambda_1 = \cdots = \lambda_{2^n}$, which is impossible for any FQ_n with $n \geq 2$. Therefore, this implies the results of Theorem 9. □

We now consider the upper bound for the Estrada index of FQ_n as follows.

Theorem 10. *For any FQ_n with $n \geq 2$, one has*

$$EE(FQ_n) < 2^n - 1 + e^{\sqrt{(n+1)2^n}}. \tag{22}$$

Proof. According to the definition of Estrada index we get

$$EE(FQ_n) = 2^n + \sum_{i=1}^{2^n} \sum_{k\geq 1} \frac{\lambda_i^k}{k!} \leq 2^n + \sum_{i=1}^{2^n} \sum_{k\geq 1} \frac{|\lambda_i|^k}{k!} \tag{23}$$

$$= 2^n + \sum_{k\geq 1} \frac{1}{k!} \sum_{i=1}^{2^n} \left[(\lambda_i)^2 \right]^{k/2}.$$

Notice the inequality

$$\sum_{i=1}^{2^n} \left[(\lambda_i)^2 \right]^{k/2} \leq \left[\sum_{i=1}^{2^n} (\lambda_i)^2 \right]^{k/2}; \tag{24}$$

substituting inequality (24) into (23) we obtain that

$$EE(FQ_n) \leq 2^n + \sum_{k\geq 1} \frac{1}{k!} \left[\sum_{i=1}^{2^n} (\lambda_i)^2 \right]^{k/2}$$

$$= 2^n - 1 + \sum_{k\geq 0} \frac{1}{k!} \left[\sum_{i=1}^{2^n} (\lambda_i)^2 \right]^{k/2}. \tag{25}$$

Since the equality holds in FQ_n,

$$\sum_{i=1}^{2^n} (\lambda_i)^2 = (n+1)\, 2^n. \tag{26}$$

Hence,

$$EE(FQ_n) \leq 2^n - 1 + \sum_{k\geq 0} \frac{1}{k!} \left[(n+1)\, 2^n \right]^{k/2}$$

$$= 2^n - 1 + \sum_{k\geq 0} \frac{\sqrt{(n+1)2^n}^k}{k!} \tag{27}$$

$$= 2^n - 1 + e^{\sqrt{(n+1)2^n}}.$$

It is evident that equality of (25) will be attained if and only if the graph FQ_n has no nonzero eigenvalues, which, in turn, happens only in the case of the edgeless graph $\overline{K_n}$; it is impossible for any FQ_n with $n \geq 2$ that directly leads to the inequality in (27).

Hence, we can obtain the upper bound for the Estrada index of FQ_n:

$$EE(FQ_n) < 2^n - 1 + e^{\sqrt{(n+1)2^n}}. \tag{28}$$

The proof of Theorem 10 is completed. □

Remark 11. In [23], it was proved that

$$e^r + (n-1)\, e^{-r/(n-1)} \leq EE(G) < n - 2 + e^r + e^{\sqrt{r(n-r)-1}}, \tag{29}$$

with equality, holds if and only if $\lambda_2 = \lambda_3 = \cdots = \lambda_n = -r/(n-1)$.

Notice that the spectral radius of FQ_n is $\lambda_1 = n + 1$ and $r = n + 1$; applying Lemma 6, we also give the lower and upper bounds connecting $EE(FQ_n)$ and its spectral radius by simple computations, where the equality is impossible for any FQ_n; hence

$$e^{n+1} + (2^n - 1) e^{(-n-1)/(2^n-1)}$$
$$< EE(FQ_n) < 2^n - 2 + e^{n+1} + e^{\sqrt{(n+1)[2^n-(n+1)]-1}}. \tag{30}$$

3.3. Some Properties on Estrada Index Involving Energy of FQ_n.

In this section, we investigate the relations between the Estrada index and the energy of FQ_n. We firstly prove the lower bounds involving energy for the Estrada index of FQ_n; in Theorem 12 proof, some techniques in [23] are referred to.

Theorem 12. For any FQ_n with $n \geq 2$, one has

$$\frac{1}{2} (e - 1) E(FQ_n) + (2^n - n_i) < EE(FQ_n). \tag{31}$$

Proof. Assume that n_i denote the number of positive eigenvalues; we begin with the definition of Estrada index $EE(FQ_n)$:

$$EE(FQ_n) = \sum_{i=1}^{2^n} e_i^\lambda = \sum_{\lambda_i \leq 0} e_i^\lambda + \sum_{\lambda_i > 0} e_i^\lambda. \tag{32}$$

Since $e^x \geq 1 + x$, with equality, holds if and only if $x = 0$, we have

$$\sum_{\lambda_i \leq 0} e_i^\lambda \geq \sum_{\lambda_i \leq 0} (1 + \lambda_i) = (2^n - n_i) + \left(\lambda_{n_i+1} + \cdots + \lambda_n\right). \tag{33}$$

The other underlying inequality is $e^x \geq ex$ and equality holds if and only if $x = 1$; we get

$$\sum_{\lambda_i > 0} e_i^\lambda \geq \sum_{\lambda_i > 0} e\lambda_i = e\left(\lambda_1 + \lambda_2 \cdots + \lambda_{n_i}\right). \tag{34}$$

Substituting the inequalities (33) and (34) into (32),

$$EE(FQ_n) \geq (2^n - n_i) + \left(\lambda_{n_i+1} + \cdots + \lambda_n\right)$$
$$+ e\left(\lambda_1 + \lambda_2 \cdots + \lambda_{n_i}\right)$$
$$= (2^n - n_i) + \left(\lambda_1 + \lambda_2 \cdots + \lambda_{n_i} + \lambda_{n_i+1} + \cdots + \lambda_n\right)$$
$$+ (e - 1)\left(\lambda_1 + \lambda_2 \cdots + \lambda_{n_i}\right)$$
$$= (2^n - n_i) + (e - 1)\left(\lambda_1 + \lambda_2 \cdots + \lambda_{n_i}\right). \tag{35}$$

Note that

$$\lambda_1 + \lambda_2 \cdots + \lambda_{n_i} = \frac{1}{2} E(FQ_n). \tag{36}$$

From the above inequalities (35) and (36), we arrive at

$$\frac{1}{2} (e - 1) E(FQ_n) + (2^n - n_i) \leq EE(FQ_n), \tag{37}$$

with equality if and only if FQ_n is an empty graph with 2^n vertices, which is impossible.

Hence,

$$\frac{1}{2} (e - 1) E(FQ_n) + (2^n - n_i) < EE(FQ_n), \tag{38}$$

as desired. □

We now derive the upper bounds involving energy for the Estrada index of FQ_n.

Theorem 13. For any FQ_n with $n \geq 2$, one has

$$EE(FQ_n) < E(FQ_n) + 2^n - 1 - \sqrt{(n+1) 2^n} + e^{\sqrt{(n+1)2^n}}. \tag{39}$$

Proof. We consider that

$$EE(FQ_n) = \sum_{i=1}^n e_i^\lambda = 2^n + \sum_{i=1}^{2^n} \sum_{k \geq 1} \frac{\lambda_i^k}{k!}$$
$$\leq 2^n + \sum_{i=1}^{2^n} \sum_{k \geq 1} \frac{|\lambda_i|^k}{k!}. \tag{40}$$

Taking into account the definition of graph energy equation (1), we obtain

$$EE(FQ_n) \leq 2^n + E(FQ_n) + \sum_{i=1}^{2^n} \sum_{k \geq 2} \frac{|\lambda_i|^k}{k!}$$
$$= 2^n + E(FQ_n) + \sum_{k \geq 2} \frac{1}{k!} \sum_{i=1}^{2^n} \left[(\lambda_i)^2\right]^{k/2}. \tag{41}$$

In light of the inequality (24) holds for integer $k \geq 2$, we obtain that

$$EE(FQ_n) \leq 2^n + E(FQ_n) + \sum_{k \geq 2} \frac{1}{k!} \left[\sum_{i=1}^{2^n} (\lambda_i)^2\right]^{k/2}$$
$$= 2^n + E(FQ_n) - 1 - \sqrt{(n+1) 2^n}$$
$$+ \sum_{k \geq 0} \frac{1}{k!} \left[\sum_{i=1}^{2^n} (\lambda_i)^2\right]^{k/2}. \tag{42}$$

Substituting (26) into (42), we get

$$EE(FQ_n) \leq 2^n + E(FQ_n) - 1 - \sqrt{(n+1) 2^n}$$
$$+ \sum_{k \geq 0} \frac{1}{k!} \left[(n+1) 2^n\right]^{k/2}$$
$$= 2^n + E(FQ_n) - 1 - \sqrt{(n+1) 2^n} + e^{\sqrt{(n+1)2^n}}, \tag{43}$$

with equality if and only if FQ_n is an empty graph with 2^n vertices, which is impossible.

From the above argument, we get the result of Theorem 13. □

4. Conclusions

The main purpose of this paper is to investigate the Estrada index of FQ_n with $n \geq 2$; we established the explicit formulae for calculating the Estrada index of FQ_n by deducing the characteristic polynomial of the adjacency matrix in spectral graph theory.

Moreover, some lower and upper bounds for Estrada index of FQ_n were proposed by utilizing the arithmetic and geometric mean inequality. The lower and upper bounds for the Estrada index involving energy of FQ_n were also obtained.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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