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Research Article

Permanence and Extinction for a Nonautonomous Malaria Transmission Model with Distributed Time Delay

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We study the permanence, extinction, and global asymptotic stability for a nonautonomous malaria transmission model with distributed time delay. We establish some sufficient conditions on the permanence and extinction of the disease by using inequality analytical techniques. By a Lyapunov functional method, we also obtain some sufficient conditions for global asymptotic stability of this model. A numerical analysis is given to explain the analytical findings.

1. Introduction and the Model

There have been lots of researches about SEIRS, SIRS models, in which the infectious diseases spread in a single population [1, 2]. In recent years, the study of diseases spreading among multiple populations has increased gradually, such as avian influenza and malaria. Malaria remains one of the most prevalent and lethal human infectious diseases in the world. Malaria is a protozoan infection of red blood cells caused in human by four species of the genus *Plasmodium* (*Plasmodium falciparum, Plasmodium vivax, Plasmodium ovale, and Plasmodium malariae*). The malaria parasites are generally transmitted to the human host through the bite of an infected female anopheline mosquito.

There has been a great deal of work about using mathematical models to study malaria [3–5]. However, considering malaria often occurs in most tropical and some subtropical regions of the world [6], environmental and climatic factors play an important role in the geographical distribution and transmission of malaria [7]. Malaria fluctuates over time and often exhibits seasonal behaviors, especially in the northern areas. Therefore, it is meaningful and essential to take account of malaria model with periodic environment [8]. However, up to now, there have been few results about malaria model with periodic environment. In [9], a malaria transmission model with periodic birth rate and age structure for the vector population was presented by Lou and Zhao, and they further showed that $\mathscr{R}_0$ is the threshold value determining the extinction and the uniform persistence of the disease. Later, they used these analytic results to study the malaria transmission cases in KwaZulu-Natal Province, South Africa.

Motivated by the work of [8, 10–12], studied a malaria transmission model with periodic environment. By applying the way of computing the basic reproduction number for a wide class of compartmental epidemic models in periodic environments given by Wang and Zhao [13], Lei Wang, Zhidong Teng, and Tailei Zhang calculated the basic reproduction number and indicated it was the threshold value determining the extinction and the uniform persistence of the disease. They studied the following model:

$$\begin{aligned} S_R'(t) &= r(t) S_R(t)\left(1-\frac{S_R}{k(t)}\right) - \alpha(t) S_R I_H, \\ I_R'(t) &= \alpha(t) S_R I_H - d(t) I_R, \\ S_H'(t) &= \lambda - \beta(t) S_H I_R - \delta S_H + \gamma R_H, \\ I_H'(t) &= \beta(t) S_H I_R - (\delta+\mu+\sigma) I_H, \\ R_H'(t) &= \mu I_H - \delta R_H - \gamma R_H. \end{aligned} \tag{1}$$

In [14], the researcher constructed a mathematical model to interpret the spread of wild avian influenza from the birds to the humans, after the emergence of mutant avian influenza, with nonautonomous ordinary differential equations and distributed time delay due to the intracellular delay between initial infection of a cell and the release of new virus particles. The researcher studied the following model:

$$\begin{aligned}
X'(t) &= c(t) - b(t)X(t) - \omega(t)X(t)\int_0^h Y(t-s)\,\mathrm{d}\eta(s),\\
Y'(t) &= \omega(t)X(t)\int_0^h Y(t-s)\,\mathrm{d}\eta(s) - \{b(t)+m(t)\}Y(t),\\
S'(t) &= \lambda(t) - \mu(t)S(t) - \beta_1(t)S(t)\int_0^h Y(t-s)\,\mathrm{d}\eta(s)\\
&\quad - \beta_2(t)S(t)\int_0^h H(t-s)\,\mathrm{d}\eta(s),\\
B'(t) &= \beta_1(t)S(t)\int_0^h Y(t-s)\,\mathrm{d}\eta(s)\\
&\quad - \{\mu(t)+d_1(t)+\xi(t)\}B(t),\\
H'(t) &= \beta_2(t)S(t)\int_0^h H(t-s)\,\mathrm{d}\eta(s) + \xi(t)B(t)\\
&\quad - \{\mu(t)+d_2(t)+\gamma(t)\}H(t),\\
R'(t) &= \gamma(t)H(t) - \mu(t)R(t).
\end{aligned} \tag{2}$$

Motivated by system (2), considering the intracellular delay between initial infection of a cell biting by an infected female anopheline mosquito and the release of new virus particles, we study the system (1) with distributed time delay, which can be more reasonable. We construct the following model:

$$\begin{aligned}
S_R'(t) &= c(t) - \alpha(t)S_R\int_0^h I_H(t-s)\,\mathrm{d}\eta(s) - b(t)S_R,\\
I_R'(t) &= \alpha(t)S_R\int_0^h I_H(t-s)\,\mathrm{d}\eta(s) - d(t)I_R - b(t)I_R,\\
S_H'(t) &= \lambda(t) - \beta(t)S_H\int_0^h I_R(t-s)\,\mathrm{d}\eta(s) - \delta(t)S_H\\
&\quad + \gamma(t)R_H,\\
I_H'(t) &= \beta(t)S_H\int_0^h I_R(t-s)\,\mathrm{d}\eta(s)\\
&\quad - (\delta(t)+\mu(t)+\sigma(t))I_H,\\
R_H'(t) &= \mu(t)I_H - \delta(t)R_H - \gamma(t)R_H.
\end{aligned} \tag{3}$$

Here $N_R(t) = S_R(t) + I_R(t)$ and $N_H(t) = S_H(t) + I_H(t) + R_H(t)$ denote the total number of mosquito and human population, respectively, at time t; $S_R(t)$, $I_R(t)$, $S_H(t)$, $I_H(t)$, and $R_H(t)$ represent the densities (or fractions) of susceptible mosquitoes, infected mosquitoes, susceptible humans, infective humans, and recovered humans, respectively, at time t. Due to the mosquito's short lifespan, it cannot recover from the infection. Consequently, we only divide the total mosquito population into two classes: the susceptible and the infected.

The quantities $c(t)$, $\alpha(t)$, $b(t)$, $d(t)$, $\lambda(t)$, $\beta(t)$, $\delta(t)$, $\gamma(t)$, $\mu(t)$, and $\sigma(t)$ are as follows:

$c(t)$: the instantaneous growth rate function of the mosquito population;

$b(t)$: the instantaneous natural death rate function of the mosquito population;

$d(t)$: the instantaneous additional death rate function of the mosquito population;

$\alpha(t)$: the transmission rate function from human to mosquito when susceptible mosquitoes contact infective humans and the rate of transmission is of the form

$$\alpha(t)S_R\int_0^h I_H(t-s)\,\mathrm{d}\eta(s); \tag{4}$$

$\lambda(t)$: the instantaneous immigration rate function of the human population;

$\delta(t)$: the instantaneous natural death rate function of the human population;

$\gamma(t)$: the instantaneous rate function of which the recovered becomes susceptible again;

$\mu(t)$: the instantaneous recovered rate function of the human population;

$\sigma(t)$: the instantaneous disease-induced death rate function of the human population;

$\beta(t)$: the transmission rate function from mosquito to human when susceptible humans take contact with infected mosquitoes and the rate of transmission is of the form:

$$\beta(t)S_H\int_0^h I_R(t-s)\,\mathrm{d}\eta(s). \tag{5}$$

The nonnegative constant h is the time delay. The function $\eta(s) : [0,h] \to [0,\infty)$ is nondecreasing and has bounded variation such that

$$\int_0^h \mathrm{d}\eta(s) = \eta(h) - \eta(0) = 1. \tag{6}$$

The time delay is due to intracellular delay between initial infection of a cell and the release of new virions. Those infected at time $t-s$ become infectious at time s $(0 \le s \le h)$ later with different probabilities. Additionally, comparing with the human, the mosquito's life is much too shorter, so we suppose that

$$b(t) + d(t) \gg \delta(t) + \mu(t) + \sigma(t). \tag{7}$$

This paper is organized as follows. In Section 2, we establish some sufficient conditions on the permanence and extinction of the disease. In Section 3, we analyze global asymptotical stability of the disease. Some numerical simulations are given in Section 4. And finally, in Section 5, we come to a conclusion.

2. Permanence and Extinction

In this section, we first introduce the following assumptions for system (3): Functions $c(t)$, $\alpha(t)$, $b(t)$, $d(t)$, $\lambda(t)$, $\beta(t)$, $\delta(t)$, $\gamma(t)$, $\mu(t)$, and $\sigma(t)$ are positive continuous bounded and have positive lower bounds.

The initial conditions of (3) are given as

$$S_R(\theta)=\varphi_1(\theta),\qquad I_R(\theta)=\varphi_2(\theta),\qquad S_H(\theta)=\varphi_3(\theta),$$
$$I_H(\theta)=\varphi_4(\theta),\qquad R_H(\theta)=\varphi_5(\theta),\quad -h\le\theta\le 0, \tag{8}$$

where $\varphi=(\varphi_1,\varphi_2,\varphi_3,\varphi_4,\varphi_5)^T\in C$ such that $\varphi_i(\theta)\ge 0$ $(i=1,2,3,4,5)$, $\forall\theta\in[-h,0]$. C denotes the Banach space $C([-h,0],R^5)$ of continuous functions mapping the interval $[-h,0]$ into R^5 and the norm of an element φ in C is designated by $\|\varphi\|=\sup_{-h\le\theta\le 0}\{|\varphi_1(\theta)|,|\varphi_2(\theta)|,|\varphi_3(\theta)|,|\varphi_4(\theta)|,|\varphi_5(\theta)|\}$. For a biological meaning, we further assume that $\varphi_i(0)>0$ $(i=1,2,3,4,5)$.

Lemma 1 (see [15]). *If the functions $c(t)$, $\alpha(t)$, $b(t)$, $d(t)$, $\lambda(t)$, $\beta(t)$, $\delta(t)$, $\gamma(t)$, $\mu(t)$, and $\sigma(t)$ are continuous and bounded on $[0,+\infty)$, then there exists a unique solution of the system* (3) *with initial conditions* (8) *defined on* $[0,+\infty)$.

Firstly we discuss the permanence of the system (3).

For a continuous and bounded function $f(t)$ defined on $[0,+\infty)$, we introduce the following signs:

$$f^l=\inf_{t\ge0}f(t),\qquad f^u=\sup_{t\ge0}f(t). \tag{9}$$

Definition 2 (see [16]). The system (3) is said to be permanent, that is, the long-term survival (will not vanish in time) of all components of the system (3), if there are positive constants v_i and M_i $(i=1,2,3,4,5)$ such that

$$\begin{aligned}
v_1&\le\liminf_{t\to\infty}S_R(t)\le\limsup_{t\to\infty}S_R(t)\le M_1,\\
v_2&\le\liminf_{t\to\infty}I_R(t)\le\limsup_{t\to\infty}I_R(t)\le M_2,\\
v_3&\le\liminf_{t\to\infty}S_H(t)\le\limsup_{t\to\infty}S_H(t)\le M_3,\\
v_4&\le\liminf_{t\to\infty}I_H(t)\le\limsup_{t\to\infty}I_H(t)\le M_4,\\
v_5&\le\liminf_{t\to\infty}R_H(t)\le\limsup_{t\to\infty}R_H(t)\le M_5,
\end{aligned} \tag{10}$$

hold for any solution $(S_R(t),I_R(t),S_H(t),I_H(t),R_H(t))$ of (3) with initial conditions of type (8). Here v_i and M_i $(i=1,2,3,4,5)$ are independent of (8).

Theorem 3. *Set $r^*=(\beta^l/(d+b)^u)(\lambda^l/\delta^u)$, $r_*=(\alpha^l/(\delta+\mu+\sigma)^u)(c^l/b^u)$, and $r_0=(\alpha^l/(d+b)^u)(c^l/b^u)$. The system* (3) *with initial conditions* (8) *is permanent provided that $r^*\ge r_*$ and $r_0>1$.*

Proof. We will give the following Propositions 4–8 to complete the proof of this theorem. □

Proposition 4. *The solution $(S_R(t),I_R(t),S_H(t),I_H(t),R_H(t))$ of* (3) *with initial conditions* (8) *is positive for all $t\ge0$, and*

$$\limsup_{t\to+\infty}N_R(t)\le\frac{c^u}{b^l};\qquad \limsup_{t\to+\infty}N_H(t)\le\frac{\lambda^u}{\delta^l}. \tag{11}$$

Proof. Since the functions $c(t)$, $\alpha(t)$, $b(t)$, $d(t)$, $\lambda(t)$, $\beta(t)$, $\delta(t)$, $\gamma(t)$, $\mu(t)$, and $\sigma(t)$ are continuous and bounded on $[0,+\infty)$, the solution of (3) with initial conditions (8) exists and is unique on $[0,+\infty)$. Now,

$$\begin{aligned}
S_R(t)&=S_R(0)\exp\left[-\int_0^t\left\{\alpha(\theta)\int_0^h I_H(\theta-s)\,d\eta(s)+b(\theta)\right\}d\theta\right]\\
&\quad+\int_0^t c(u)\exp\left[\int_t^u\left\{\alpha(\theta)\int_0^h I_H(\theta-s)\,d\eta(s)\right.\right.\\
&\qquad\left.\left.+\,b(\theta)\right\}d\theta\,du\right]>0,\quad t\ge0.
\end{aligned} \tag{12}$$

Next, we show that $I_R(t)>0$ for all $t\ge0$. Otherwise there exists a $t_1\in(0,+\infty)$ such that $I_R(t_1)=0$ and $I_R(t)>0$ for all $t\in[0,t_1)$. We claim that $I_H(t)>0$ for all $t\in[0,t_1)$. If this is not true, then there exists a $t_2\in[0,t_1)$ such that $I_H(t_2)=0$ and $I_H(t)>0$ for all $t\in[0,t_2)$. From the fourth equation of system (3), we have

$$\begin{aligned}
I_H(t_2)&=I_H(0)\exp\left\{-\int_0^{t_2}(\delta(\theta)+\mu(\theta)+\sigma(\theta))\,d\theta\right\}\\
&\quad+\int_0^{t_2}\int_0^h\beta(u)S_H(u)I_R(u-s)\,d\eta(s)\\
&\qquad\times\exp\left\{\int_{t_2}^u(\delta(\theta)+\mu(\theta)+\sigma(\theta))\,d\theta\right\}du>0,
\end{aligned} \tag{13}$$

which contradicts with $I_H(t_2)=0$. Therefore, $I_H(t)>0$ for all $t\in[0,t_1)$. Integrating the second equation of system (3) from 0 to t_1, we have

$$\begin{aligned}
I_R(t_1)&=I_R(0)\exp\left\{-\int_0^{t_1}(d(\theta)+b(\theta))\,d\theta\right\}
\end{aligned}$$

$$+\int_0^{t_1}\int_0^h \alpha(u) S_R(u) I_H(u-s)\,\mathrm{d}\eta(s) \times \exp\left\{\int_{t_1}^{u} (d(\theta)+b(\theta))\,\mathrm{d}\theta\right\}\mathrm{d}u > 0, \quad (14)$$

which contradicts with $I_R(t_1)=0$. Therefore, $I_R(t)>0$ for all $t\geq 0$. Thus $I_H(t)>0$.

From the fifth equation of system (3), we have

$$R_H(t) = R_H(0)\exp\left\{-\int_0^t (\delta(s)+\gamma(s))\,\mathrm{d}s\right\} + \int_0^t \mu(u) I_H(u)\exp\left\{\int_t^u (\delta(s)+\gamma(s))\,\mathrm{d}s\right\}\mathrm{d}u > 0, \quad t\geq 0. \quad (15)$$

Last, from the third equation of system (3), we have

$$S_H(t) = S_H(0)\exp\left\{-\int_0^t\left(\beta(\theta)\int_0^h I_R(\theta-s)\,\mathrm{d}\eta(s)+\delta(\theta)\right)\mathrm{d}\theta\right\} + \int_0^t (\lambda(u)+\gamma(u)R_H(u)) \times \exp\left\{\int_t^u\left(\beta(\theta)\int_0^h I_R(\theta-s)\,\mathrm{d}\eta(s) + \delta(\theta)\right)\mathrm{d}\theta\right\}\mathrm{d}u > 0, \quad t\geq 0. \quad (16)$$

Therefore, $S_R(t)>0, I_R(t)>0, S_H(t)>0, I_H(t)>0, R_H(t)>0$ for all $t\geq 0$. Thus, $\forall t\in[0,\infty)$,

$$N_R'(t) \leq c(t) - b(t)N_R(t) - d(t)I_R(t) \leq c^u - b^l N_R(t), \quad \Longrightarrow \limsup_{t\to+\infty} N_R(t) \leq \frac{c^u}{b^l}. \quad (17)$$

Similarly,

$$N_H'(t) \leq \lambda(t) - \delta(t)N_H(t) - \sigma(t)I_H(t) \leq \lambda^u - \delta^l N_H(t), \quad \Longrightarrow \limsup_{t\to+\infty} N_H(t) \leq \frac{\lambda^u}{\delta^l}. \quad (18)$$

This completes the proof. □

Proposition 5. *The solution* $(S_R(t), I_R(t), S_H(t), I_H(t), R_H(t))$ *of* (3) *with initial conditions* (8) *satisfies*

$$\liminf_{t\to\infty} S_R(t) \geq \frac{c^l\delta^l}{\alpha^u\lambda^u+\delta^l b^u} \equiv \nu_1 > 0. \quad (19)$$

Proof. By Proposition 4, for any $\epsilon>0$ (no matter however small), there exists a $t_1>0$, such that

$$I_H(t) \leq \frac{\lambda^u}{\delta^l}+\epsilon, \quad t\geq t_1. \quad (20)$$

Therefore, from the first equation of system (3), when $t\geq t_1+h$,

$$S_R'(t) \geq c(t) - \left[\alpha(t)\left(\frac{\lambda^u}{\delta^l}+\epsilon\right)+b(t)\right]S_R(t) \geq c^l - \left[\alpha^u\left(\frac{\lambda^u}{\delta^l}+\epsilon\right)+b^u\right]S_R(t), \quad \Longrightarrow \liminf_{t\to\infty} S_R(t) \geq \frac{c^l}{\alpha^u(\lambda^u/\delta^l+\epsilon)+b^u}. \quad (21)$$

Since $\epsilon>0$ can be made arbitrarily small, the result of this proposition is valid. This completes the proof. □

Proposition 6. *Set* $I(t) = I_R(t) + I_H(t)$, *assume that* $r^* \geq r_*$ *and* $r_0 > 1$, *and then for any solution* $(S_R(t), I_R(t), S_H(t), I_H(t), R_H(t))$ *of* (3) *with initial conditions* (8) *we have*

$$\liminf_{t\to\infty} I(t) \geq \eta e^{-[(d+b)^u+(\delta+\mu+\sigma)^u](h+\rho)} \equiv \nu_2 > 0, \quad (22)$$

where $\eta>0$ *and* $\rho>0$ *will be given in the proof.*

Proof. Since $r_0>1$ and it is obvious that $c^l/G \to c^l/b^u$, as $\eta\to 0$, where $G=b^u+\eta\alpha^u$. Then, there exists two positive constants η and ρ such that

$$\frac{c^l}{G}\{1-\exp(-G\rho)\}\frac{\alpha^l}{(d+b)^u} > 1. \quad (23)$$

From that condition (7), we can get $(d+b)^u > (\delta+\mu+\sigma)^u$. Then, we have

$$\frac{c^l}{G}\{1-\exp(-G\rho)\}\frac{\alpha^l}{(\delta+\mu+\sigma)^u} > 1. \quad (24)$$

Similarly, since $r^*\geq r_*>1$, it is obvious that $\lambda^l/H \to \lambda^l/\delta^u$, as $\eta\to 0$ where $H=\delta^u+\eta\beta^u$. Then, for the positive constants η and ρ, we have

$$\frac{\lambda^l}{H}\{1-\exp(-H\rho)\}\frac{\beta^l}{(d+b)^u} > 1. \quad (25)$$

Firstly, from the second and the fourth equation of (3) together [17]. We get the following equivalent system:

$$\begin{aligned}
S_R'(t) &= c(t) - \alpha(t) S_R \int_0^h I_H(t-s)\,\mathrm{d}\eta(s) - b(t) S_R, \\
I'(t) &= \alpha(t) S_R \int_0^h I_H(t-s)\,\mathrm{d}\eta(s) - (d(t)+b(t)) I_R \\
&\quad + \beta(t) S_H \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\
&\quad - (\delta(t)+\mu(t)+\sigma(t)) I_H, \\
S_H'(t) &= \lambda(t) - \beta(t) S_H \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\
&\quad - \delta(t) S_H + \gamma(t) R_H, \\
R_H'(t) &= \mu(t) I_H - \delta(t) R_H - \gamma(t) R_H.
\end{aligned} \tag{26}$$

Let us consider the following differential function $V(t)$:

$$\begin{aligned}
V(t) &= I(t) + \int_0^h \int_{t-s}^t \alpha(u+s) S_R(u+s) I_H(u)\,\mathrm{d}u\,\mathrm{d}\eta(s) \\
&\quad + \int_0^h \int_{t-s}^t \beta(u+s) S_H(u+s) I_R(u)\,\mathrm{d}u\,\mathrm{d}\eta(s).
\end{aligned} \tag{27}$$

The derivative $V(t)$ along solution of (26) is

$$\begin{aligned}
&V'(t) \\
&= \alpha(t) S_R \int_0^h I_H(t-s)\,\mathrm{d}\eta(s) - (d(t)+b(t)) I_R \\
&\quad + \beta(t) S_H \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\
&\quad - (\delta(t)+\mu(t)+\sigma(t)) I_H \\
&\quad + \int_0^h \alpha(t+s) S_R(t+s) I_H(t)\,\mathrm{d}\eta(s) \\
&\quad - \int_0^h \alpha(t) S_R(t) I_H(t-s)\,\mathrm{d}\eta(s) \\
&\quad + \int_0^h \beta(t+s) S_H(t+s) I_R(t)\,\mathrm{d}\eta(s) \\
&\quad - \int_0^h \beta(t) S_H(t) I_R(t-s)\,\mathrm{d}\eta(s) \\
&= \left[\int_0^h \alpha(t+s) S_R(t+s)\,\mathrm{d}\eta(s) \right.\\
&\qquad \left. - (\delta(t)+\mu(t)+\sigma(t))\right] I_H(t) \\
&\quad + \left[\int_0^h \beta(t+s) S_H(t+s)\,\mathrm{d}\eta(s) - (d(t)+b(t))\right] I_R(t).
\end{aligned} \tag{28}$$

We claim that it is impossible that $I(t) \le \eta$, $\forall t \ge t_1$ (t_1 is any nonnegative constant). Suppose the contrary; then as $t \ge t_1 + h$,

$$S_R'(t) \ge c(t) - (\eta\alpha(t) + b(t)) S_R(t) \ge c^l - G S_R(t). \tag{29}$$

For $t > t_1 + h$, integrating the above inequality from $t_1 + h$ to t, we obtain

$$\begin{aligned}
S_R(t) &\ge S_R(t_1+h) \exp\left(\int_t^{t_1+h} G\,\mathrm{d}s\right) \\
&\quad + \int_{t_1+h}^t c^l \exp\left(\int_t^s G\,\mathrm{d}\theta\right)\mathrm{d}s \\
&\ge \int_{t_1+h}^t c^l \exp\left(\int_0^s G\,\mathrm{d}\theta - \int_0^t G\,\mathrm{d}\theta\right)\mathrm{d}s \\
&= \frac{\int_{t_1+h}^t c^l \exp\left(\int_0^s G\,\mathrm{d}\theta\right)\mathrm{d}s}{\exp\left(\int_0^t G\,\mathrm{d}\theta\right)} \\
&= \left(\frac{c^l}{\exp(Gt)}\right) \int_{t_1+h}^t \exp(Gs)\,\mathrm{d}s.
\end{aligned} \tag{30}$$

Hence

$$\begin{aligned}
S_R(t) &\ge \left(\frac{c^l}{G}\right) \frac{\exp(Gt) - \exp(G(t_1+h))}{\exp(Gt)} \\
&= \left(\frac{c^l}{G}\right) [1 - \exp(-G(t - t_1 - h))].
\end{aligned} \tag{31}$$

Therefore, $S_R(t) \ge (c^l/G)[1 - \exp(-G\rho)] \equiv S_R^{\triangle}$, $\forall t \ge t_1 + h + \rho \equiv t_2$.

Similarly, from the third equation of system (26), we have

$$S_H(t) \ge \left(\frac{\lambda^l}{H}\right)[1 - \exp(-H\rho)] \equiv S_H^{\triangle}, \quad \forall t \ge t_2. \tag{32}$$

Notice that $r_0 > 1$; from the previous discussion, we have $\alpha^l S_R^{\triangle}/(d+b)^u > 1$. And from $r^* \ge r_* > 1$, we can get $\beta^l S_H^{\triangle}/(d+b)^u \ge \alpha^l S_R^{\triangle}/(\delta+\mu+\sigma)^u > 1$. Since $(d+b)^u > (\delta+\mu+\sigma)^u$, we have

$$\beta^l S_H^{\triangle} - (d+b)^u \ge \alpha^l S_R^{\triangle} - (\delta+\mu+\sigma)^u > 0. \tag{33}$$

Thus

$$\begin{aligned} V'(t) &\geq \left[\alpha^l S_R^{\triangle} - (\delta+\mu+\sigma)^u\right] I_H(t) \\ &\quad + \left[\beta^l S_H^{\triangle} - (d+b)^u\right] I_R(t) \\ &\geq \left[\alpha^l S_R^{\triangle} - (\delta+\mu+\sigma)^u\right] \left[I_H(t) + I_R(t)\right] \\ &= (\delta+\mu+\sigma)^u \left[\frac{\alpha^l S_R^{\triangle}}{(\delta+\mu+\sigma)^u} - 1\right] I(t), \quad \forall t \geq t_2. \end{aligned} \tag{34}$$

Let us take $\underline{i} = \min_{t_2 \leq t \leq t_2+h} I(t)$. Next we will prove that $I(t) \geq \underline{i}$, $\forall t \geq t_2$.

Suppose that it is not true; then there exists $T > 0$, such that $I(t) \geq \underline{i}$, for all $t \in [t_2, t_2+h+T]$, $I(t_2+h+T) = \underline{i}$, and $I'(t_2+h+T) \leq 0$. On the other hand, by the second equation of (26), as $t = t_2+h+T$, we have

$$\begin{aligned} I'(t) &\geq \alpha^l S_R^{\triangle} \int_0^h I_H(t-s)\,\mathrm{d}\eta(s) - (d+b)^u I_R \\ &\quad + \beta^l S_H^{\triangle} \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) - (\delta+\mu+\sigma)^u I_H \\ &= \alpha^l S_R^{\triangle} \int_0^h I_H(t-s)\,\mathrm{d}\eta(s) + \alpha^l S_R^{\triangle} \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\ &\quad - \alpha^l S_R^{\triangle} \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) + \beta^l S_H^{\triangle} \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\ &\quad - (d+b)^u I_R - (d+b)^u I_H + (d+b)^u I_H \\ &\quad - (\delta+\mu+\sigma)^u I_H \\ &= \alpha^l S_R^{\triangle} \int_0^h I(t-s)\,\mathrm{d}\eta(s) - (d+b)^u I(t) \\ &\quad + \left(\beta^l S_H^{\triangle} - \alpha^l S_R^{\triangle}\right) \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\ &\quad + \left[(d+b)^u - (\delta+\mu+\sigma)^u\right] I_H. \end{aligned} \tag{35}$$

Since $\beta^l S_H^{\triangle} - (d+b)^u \geq \alpha^l S_R^{\triangle} - (\delta+\mu+\sigma)^u > 0$, so $\beta^l S_H^{\triangle} - \alpha^l S_R^{\triangle} \geq (d+b)^u - (\delta+\mu+\sigma)^u > 0$.

Thus

$$I'(t) \geq (d+b)^u \left[\frac{\alpha^l S_R^{\triangle}}{(d+b)^u} - 1\right] \underline{i} > 0, \tag{36}$$

since from $\alpha^l S_R^{\triangle}/(d+b)^u > 1$. This is a contradiction. Hence, $I(t) \geq \underline{i}$, $\forall t \geq t_2$. Consequently,

$$V'(t) \geq (\delta+\mu+\sigma)^u \left[\frac{\alpha^l S_R^{\triangle}}{(\delta+\mu+\sigma)^u} - 1\right] \underline{i} > 0, \quad \forall t \geq t_2, \tag{37}$$

which implies $V(t) \to \infty$ as $t \to \infty$. From Proposition 4, $V(t)$ is bounded. This is a contradiction. Hence the claim is proved. From this claim, we will discuss the following two possibilities:

(1) $I(t) \geq \eta$ for all large t;

(2) $I(t)$ oscillates about η for all large t.

Finally, we will show that $I(t) \geq \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)}$ for sufficiently large t. Evidently, we only need to consider case (2).

Let t_1 and t_2 be sufficiently large times satisfying $I(t_1) = I(t_2) = \eta$, $I(t) < \eta$ as $t \in (t_1, t_2)$.

If $t_2 - t_1 \leq h + \rho$, since $I'(t) \geq -(d+u)^u I_R - (\delta+\mu+\sigma)^u I_H \geq -[(d+u)^u + (\delta+\mu+\sigma)^u] I(t)$ and $I(t_1) = \eta$, integrating the above inequality from t_1 to t, according to the comparison theorem, $I(t) \geq \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)}$, $\forall t \in [t_1, t_2]$. If $t_2 - t_1 > h + \rho$, then it is obvious that $I(t) \geq \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)}$ for all $t \in [t_1, t_1+h+\rho]$. From the above discussion, we see that $S_R(t) \geq S_R^{\triangle}$, $S_H(t) \geq S_H^{\triangle}$, $\forall t \in [t_1+h+\rho, t_2]$; we will show that $I(t) \geq \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)}$, $\forall t \in [t_1+h+\rho, t_2]$. If it is not true, then there exists a $T^* \geq 0$, such that $I(t) \geq \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)}$, $\forall t \in [t_1, t_1+h+\rho+T^*]$, $I(t_1+h+\rho+T^*) = \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)}$, and $I'(t_1+h+\rho+T^*) \leq 0$.

Using the second equation of system (26), as $t = t_1+h+\rho+T^*$, we have

$$\begin{aligned} I'(t) &\geq \alpha^l S_R^{\triangle} \int_0^h I(t-s)\,\mathrm{d}\eta(s) - (d+b)^u I(t) \\ &\quad + \left(\beta^l S_H^{\triangle} - \alpha^l S_R^{\triangle}\right) \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\ &\quad + \left[(d+b)^u - (\delta+\mu+\sigma)^u\right] I_H \\ &\geq (d+b)^u \left[\frac{\alpha^l S_R^{\triangle}}{(d+b)^u} - 1\right] \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)} > 0, \end{aligned} \tag{38}$$

We get the last inequality by use of $\alpha^l S_R^{\triangle}/(d+b)^u > 1$. This is a contradiction. Therefore, $I(t) \geq \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)}$, $\forall t \in [t_1, t_2]$. Hence

$$\liminf_{t \to \infty} I(t) \geq \eta e^{-[(d+b)^u + (\delta+\mu+\sigma)^u](h+\rho)} \equiv \nu_2 > 0. \tag{39}$$

This completes the proof of Proposition 6. □

Proposition 7. *The solution* $(S_R(t), I_R(t), S_H(t), I_H(t), R_H(t))$ *of* (3) *with initial conditions* (8) *satisfies*

$$\liminf_{t \to \infty} S_H(t) \geq \frac{\lambda^l b^l}{\beta^u c^u + \delta^u b^l} \equiv \nu_3 > 0. \tag{40}$$

Proof. From the third equation of system (26), we have

$$S_H'(t) \geq \lambda(t) - \beta(t) S_H \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) - \delta(t) S_H. \tag{41}$$

From Proposition 4, for any $\epsilon > 0$ (no matter however small), there exists a $t_1 > 0$ such that

$$I_R(t) \leq \frac{c^u}{b^l} + \epsilon, \tag{42}$$

as $t \geq t_1$. Thus when $t \geq t_1 + h$,

$$\begin{aligned} S_H'(t) &\geq \lambda(t) - \left\{\beta(t)\left(\frac{c^u}{b^l} + \epsilon\right) + \delta(t)\right\} S_H(t) \\ &\geq \lambda^l - \left\{\beta^u\left(\frac{c^u}{b^l} + \epsilon\right) + \delta^u\right\} S_H(t), \\ \Longrightarrow \liminf_{t\to\infty} S_H(t) &\geq \frac{\lambda^l}{\beta^u\left(c^u/b^l + \epsilon\right) + \delta^u}. \end{aligned} \tag{43}$$

Since $\epsilon > 0$ can be made arbitrarily small, the result of this proposition is valid. This completes the proof. □

Proposition 8. *Assume that $r^* \geq r_*$ and $r_0 > 1$; then for any solution $(S_R(t), I_R(t), S_H(t), I_H(t), R_H(t))$ of* (3) *with initial conditions* (8)*, we have*

$$\liminf_{t\to\infty} R_H(t) \geq \frac{\mu^l}{\delta^u + \gamma^u}\left[v_2 - \frac{c^u}{b^l}\right] \equiv v_4 > 0. \tag{44}$$

Proof. From the fourth equation of system (26), we have

$$R_H'(t) = \mu(t) I(t) - \mu(t) I_R(t) - \delta(t) R_H - \gamma(t) R_H. \tag{45}$$

From Proposition 4, for any $\epsilon > 0$, there exists a $t_1 > 0$ such that

$$I_R(t) \leq \frac{c^u}{b^l} + \epsilon, \quad t \geq t_1. \tag{46}$$

From Proposition 6, when $t \geq t_1$, we have

$$\begin{aligned} R_H'(t) &\geq \mu(t) v_2 - \mu(t)\left(\frac{c^u}{b^l} + \epsilon\right) - \left(\delta(t) + \gamma(t)\right) R_H \\ &= \mu(t)\left[v_2 - \left(\frac{c^u}{b^l} + \epsilon\right)\right] - \left(\delta(t) + \gamma(t)\right) R_H \\ &\geq \mu^l\left[v_2 - \left(\frac{c^u}{b^l} + \epsilon\right)\right] - \left(\delta^u + \gamma^u\right) R_H(t). \end{aligned} \tag{47}$$

So, when $v_2 > c^u/b^l$, according to the comparison theorem and the arbitrariness of ϵ, we have

$$\liminf_{t\to\infty} R_H(t) \geq \frac{\mu^l}{\delta^u + \gamma^u}\left[v_2 - \frac{c^u}{b^l}\right] > 0. \tag{48}$$

This completes the proof. □

Thus, the system (3) with initial conditions (8) is permanent provided that $r^* \geq r_*$ and $r_0 > 1$.

Remark 9. In this paper, we only find the inferior limit of $I(t)$ for system (26), that is, the inferior limit of $I_R(t) + I_H(t)$. We cannot find the inferior limits of $I_R(t)$ and $I_H(t)$ for system (3), respectively. Even though we can also obtain the permanence of system (3), as $\liminf_{t\to\infty} I(t) \geq v_2 > 0$, there exist the following three possibilities:

(1) $\liminf_{t\to\infty} I_R(t) > 0$ and $\liminf_{t\to\infty} I_H(t) > 0$: in this case, it is obvious that system (3) is permanent;

(2) $\liminf_{t\to\infty} I_R(t) > 0$ and $\liminf_{t\to\infty} I_H(t) = 0$: in this case, infected mosquitoes exist all the time; then as long as the effective infection occurs between infected mosquitoes and susceptible humans, the human population will be infected ultimately, so system (3) is permanent;

(3) $\liminf_{t\to\infty} I_R(t) = 0$ and $\liminf_{t\to\infty} I_H(t) > 0$: in this case, infective humans exist all the time; then as long as the effective infection occurs between infective humans and susceptible mosquitoes, the mosquito population will be infected ultimately, so system (3) is permanent. In fact, we only pay attention to the human population of system (3). We do not care whether mosquito population is infected or not.

Next, we will use the following lemma to discuss the extinction of the epidemic.

Lemma 10 (see [14]). *Consider an autonomous delay differential equation*

$$x'(t) = a_1 \int_0^h x(t-s)\, d\eta(s) - a_2 x(t), \tag{49}$$

where a_1, a_2 are two constants. If $0 \leq a_1 < a_2$, then for any solution $x(t)$ with initial condition $\varphi(\theta) \geq 0, \theta \in [-h, 0]$, we have

$$\lim_{t\to\infty} x(t) = 0. \tag{50}$$

Theorem 11. *Set $R^* = (\beta^u/(d+b)^l)(\lambda^u/\delta^l), R_* = (\alpha^u/(\delta+\mu+\sigma)^l)(c^u/b^l)$. If $R^* < R_* < 1$, then $\lim_{t\to\infty} I(t) = 0$; that is, the disease in system* (3) *will be extinct.*

Proof. Note that

$$\begin{aligned} I'(t) &= \alpha(t) S_R \int_0^h I_H(t-s)\, d\eta(s) - (d(t) + b(t)) I_R \\ &\quad + \beta(t) S_H \int_0^h I_R(t-s)\, d\eta(s) \\ &\quad - (\delta(t) + \mu(t) + \sigma(t)) I_H \\ &\leq \alpha^u \overline{S}_R \int_0^h I_H(t-s)\, d\eta(s) - (d+b)^l I_R \\ &\quad + \beta^u \overline{S}_H \int_0^h I_R(t-s)\, d\eta(s) - (\delta+\mu+\sigma)^l I_H \\ &= \alpha^u \overline{S}_R \int_0^h I_H(t-s)\, d\eta(s) + \alpha^u \overline{S}_R \int_0^h I_R(t-s)\, d\eta(s) \end{aligned}$$

$$\begin{aligned}
& - (\delta+\mu+\sigma)^l I_H - (\delta+\mu+\sigma)^l I_R \\
& - \alpha^u \overline{S}_R \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) + \beta^u \overline{S}_H \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) \\
& - (d+b)^l I_R + (\delta+\mu+\sigma)^l I_R \\
= {}& (\delta+\mu+\sigma)^l \left[\frac{\alpha^u \overline{S}_R}{(\delta+\mu+\sigma)^l} \int_0^h I(t-s)\,\mathrm{d}\eta(s) - I(t) \right] \\
& + \left[(d+b)^l - (\delta+\mu+\sigma)^l \right] \\
& \times \left[\frac{\beta^u \overline{S}_H - \alpha^u \overline{S}_R}{(d+b)^l - (\delta+\mu+\sigma)^l} \int_0^h I_R(t-s)\,\mathrm{d}\eta(s) - I_R \right],
\end{aligned} \tag{51}$$

where $\overline{S}_H, \overline{S}_R$ are some upper bounds of S_H, S_R, respectively, and will be given later.

From Proposition 4, there exists a $t_1 > 0$ and a sufficiently small $\epsilon > 0$, such that

$$S_H(t) \le \frac{\lambda^u}{\delta^l} + \epsilon \triangleq \overline{S}_H, \quad S_R(t) \le \frac{c^u}{b^l} + \epsilon \triangleq \overline{S}_R, \quad t \ge t_1. \tag{52}$$

As $R^* < R_* < 1$, we have

$$\frac{\beta^u \overline{S}_H}{(d+b)^l} < \frac{\alpha^u \overline{S}_R}{(\delta+\mu+\sigma)^l} < 1. \tag{53}$$

And from condition (7), we can obtain $(d+b)^l > (\delta+\mu+\sigma)^l$. So from (53) we have

$$\beta^u \overline{S}_H - (d+b)^l < \alpha^u \overline{S}_R - (\delta+\mu+\sigma)^l < 0. \tag{54}$$

Thus

$$\beta^u \overline{S}_H - \alpha^u \overline{S}_R < (d+b)^l - (\delta+\mu+\sigma)^l. \tag{55}$$

Next we will discuss the extinction in two cases.

(1) If $\beta^u \overline{S}_H - \alpha^u \overline{S}_R \le 0$, then the second part of (51) is negative, so we have

$$\begin{aligned}
& I'(t) \\
& \le (\delta+\mu+\sigma)^l \left[\frac{\alpha^u \overline{S}_R}{(\delta+\mu+\sigma)^l} \int_0^h I(t-s)\,\mathrm{d}\eta(s) - I(t) \right].
\end{aligned} \tag{56}$$

Using the Lemma 10 and the comparison theorem, we come to $\alpha^u \overline{S}_R/(\delta+\mu+\sigma)^l < 1$; that is, $R_* < 1$, $\lim_{t\to\infty} I(t) = 0$.

(2) If $\beta^u \overline{S}_H - \alpha^u \overline{S}_R > 0$, then (55) becomes

$$0 < \frac{\beta^u \overline{S}_H - \alpha^u \overline{S}_R}{(d+b)^l - (\delta+\mu+\sigma)^l} < 1. \tag{57}$$

Now we denote the second part of (51) as $Y'(t)$; that is,

$$\begin{aligned}
& Y'(t) \\
& = \left[(d+b)^l - (\delta+\mu+\sigma)^l \right] \\
& \quad \times \left[\frac{\beta^u \overline{S}_H - \alpha^u \overline{S}_R}{(d+b)^l - (\delta+\mu+\sigma)^l} \int_0^h Y(t-s)\,\mathrm{d}\eta(s) - Y(t) \right].
\end{aligned} \tag{58}$$

Using the Lemma 10, we have $\lim_{t\to\infty} Y(t) = 0$. Thus when $t \to \infty$, (51) becomes

$$\begin{aligned}
& I'(t) \\
& \le (\delta+\mu+\sigma)^l \left[\frac{\alpha^u \overline{S}_R}{(\delta+\mu+\sigma)^l} \int_0^h I(t-s)\,\mathrm{d}\eta(s) - I(t) \right].
\end{aligned} \tag{59}$$

Using the Lemma 10 and the comparison theorem again, we come to $\alpha^u \overline{S}_R/(\delta+\mu+\sigma)^l < 1$; that is, $R_* < 1$, $\lim_{t\to\infty} I(t) = 0$. This completes the proof. □

3. Global Asymptotic Stability

In this section, we derive sufficient conditions for the global asymptotic stability of system (3) with initial conditions (8).

Definition 12 (see [16]). System (3) with initial conditions (8) is said to be globally asymptotically stable if

$$\begin{aligned}
& \lim_{t\to\infty} \left| S_{R1}(t) - S_{R2}(t) \right| = 0, \\
& \lim_{t\to\infty} \left| I_{R1}(t) - I_{R2}(t) \right| = 0, \\
& \lim_{t\to\infty} \left| S_{H1}(t) - S_{H2}(t) \right| = 0, \\
& \lim_{t\to\infty} \left| I_{H1}(t) - I_{H2}(t) \right| = 0, \\
& \lim_{t\to\infty} \left| R_{H1}(t) - R_{H2}(t) \right| = 0
\end{aligned} \tag{60}$$

hold for two solutions $(S_{R1}(t), I_{R1}(t), S_{H1}(t), I_{H1}(t), R_{H1}(t))$ and $(S_{R2}(t), I_{R2}(t), S_{H2}(t), I_{H2}(t), R_{H2}(t))$ of (3) with initial conditions of type (8).

Assume that $(S_R(t), I_R(t), S_H(t), I_H(t), R_H(t))$ is a solution of (3). By the uniform boundedness of solutions of (3), there is a $L > 0$ (in fact, $L = \max\{c^u/b^l, \lambda^u/\delta^l\} + \epsilon$, where $\epsilon > 0$ can be made arbitrary small) independent of initial conditions (8) such that

$$\begin{aligned}
& 0 \le S_R(t) \le L, \quad 0 \le I_R(t) \le L, \quad 0 \le S_H(t) \le L, \\
& \qquad 0 \le I_H(t) \le L, \quad 0 \le R_H(t) \le L,
\end{aligned} \tag{61}$$

for large enough t. Without loss of generality, we may assume that

$$0 \le S_R(t) \le L, \quad 0 \le I_R(t) \le L, \quad 0 \le S_H(t) \le L, \quad 0 \le I_H(t) \le L, \quad 0 \le R_H(t) \le L, \quad \forall t \ge 0. \tag{62}$$

Theorem 13. *If there exist $c_1, c_2, c_3 > 0$, such that the functions $B_i(t)$ $(i = 1,2,3,4,5)$ are nonnegative on $[0,\infty)$ and for any interval sequence $\{[a_i, b_i]\}_1^{\infty}$, $[a_i, b_i] \cap [a_j, b_j] = \emptyset$, and $b_i - a_i = b_j - a_j > 0$, for all $i, j = 1,2,3,\ldots$, $i \ne j$, one has $\sum_{k=1}^{\infty} \int_{a_k}^{b_k} B_i(t)\mathrm{d}t = \infty$, then system (3) with initial conditions (8) is globally asymptotically stable. Here*

$$\begin{aligned}
B_1(t) &= c_1 b(t) - c_2 L\alpha(t), \\
B_2(t) &= c_2(d(t) + b(t)) - (c_1 + c_2) L \int_0^h \beta(t+s)\,\mathrm{d}\eta(s), \\
B_3(t) &= c_2\delta(t) - c_1 L\beta(t), \\
B_4(t) &= c_1(\delta(t) + \sigma(t)) + (c_1 - c_3)\mu(t) \\
&\quad + (c_1 + c_2) L \int_0^h \alpha(t+s)\,\mathrm{d}\eta(s), \\
B_5(t) &= c_3\delta(t) + (c_3 - c_2)\gamma(t).
\end{aligned} \tag{63}$$

Proof. Assume that $(S_{R1}(t), I_{R1}(t), S_{H1}(t), I_{H1}(t), R_{H1}(t))$ and $(S_{R2}(t), I_{R2}(t), S_{H2}(t), I_{H2}(t), R_{H2}(t))$ are any two solutions of system (3) with initial conditions of type (8).

The right-upper derivatives of $|S_{R1}(t) - S_{R2}(t)|$, $|I_{R1}(t) - I_{R2}(t)|$, $|S_{H1}(t) - S_{H2}(t)|$, $|I_{H1}(t) - I_{H2}(t)|$, $|R_{H1}(t) - R_{H2}(t)|$ along the solution of system (3) and (8) are given below:

$$\begin{aligned}
&D^+ |S_{R1}(t) - S_{R2}(t)| \\
&\quad = \operatorname{sgn}(S_{R1}(t) - S_{R2}(t)) \\
&\qquad \times \Big\{ -\alpha(t) S_{R1}(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad - b(t) S_{R1}(t) + \alpha(t) S_{R2}(t) \int_0^h I_{H2}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad + b(t) S_{R2}(t) \Big\} \\
&\quad = \operatorname{sgn}(S_{R1}(t) - S_{R2}(t)) \\
&\qquad \times \Big\{ -\alpha(t) S_{R1}(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad + \alpha(t) S_{R2}(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad - \alpha(t) S_{R2}(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad + \alpha(t) S_{R2}(t) \int_0^h I_{H2}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad - b(t)(S_{R1}(t) - S_{R2}(t)) \Big\} \\
&\quad = \operatorname{sgn}(S_{R1}(t) - S_{R2}(t)) \\
&\qquad \times \Big\{ -\alpha(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s)(S_{R1}(t) - S_{R2}(t)) \\
&\qquad\quad + \alpha(t) S_{R2}(t) \int_0^h (I_{H2}(t-s) - I_{H1}(t-s))\,\mathrm{d}\eta(s) \\
&\qquad\quad - b(t)(S_{R1}(t) - S_{R2}(t)) \Big\},
\end{aligned} \tag{64}$$

so

$$\begin{aligned}
&D^+ |S_{R1}(t) - S_{R2}(t)| \\
&\quad \le \alpha(t) L \int_0^h |I_{H1}(t-s) - I_{H2}(t-s)|\,\mathrm{d}\eta(s) \\
&\qquad - b(t)|S_{R1}(t) - S_{R2}(t)|,
\end{aligned} \tag{65}$$

$$\begin{aligned}
&D^+ |I_{R1}(t) - I_{R2}(t)| \\
&\quad = \operatorname{sgn}(I_{R1}(t) - I_{R2}(t)) \\
&\qquad \times \Big\{ \alpha(t) S_{R1}(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad - \alpha(t) S_{R2}(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad + \alpha(t) S_{R2}(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad - \alpha(t) S_{R2}(t) \int_0^h I_{H2}(t-s)\,\mathrm{d}\eta(s) \\
&\qquad\quad - (d(t) + b(t))(I_{R1}(t) - I_{R2}(t)) \Big\} \\
&\quad = \operatorname{sgn}(I_{R1}(t) - I_{R2}(t)) \\
&\qquad \times \Big\{ \alpha(t) \int_0^h I_{H1}(t-s)\,\mathrm{d}\eta(s)(S_{R1}(t) - S_{R2}(t)) \\
&\qquad\quad + \alpha(t) S_{R2}(t) \int_0^h (I_{H1}(t-s) - I_{H2}(t-s))\,\mathrm{d}\eta(s) \\
&\qquad\quad - (d(t) + b(t))(I_{R1}(t) - I_{R2}(t)) \Big\},
\end{aligned} \tag{66}$$

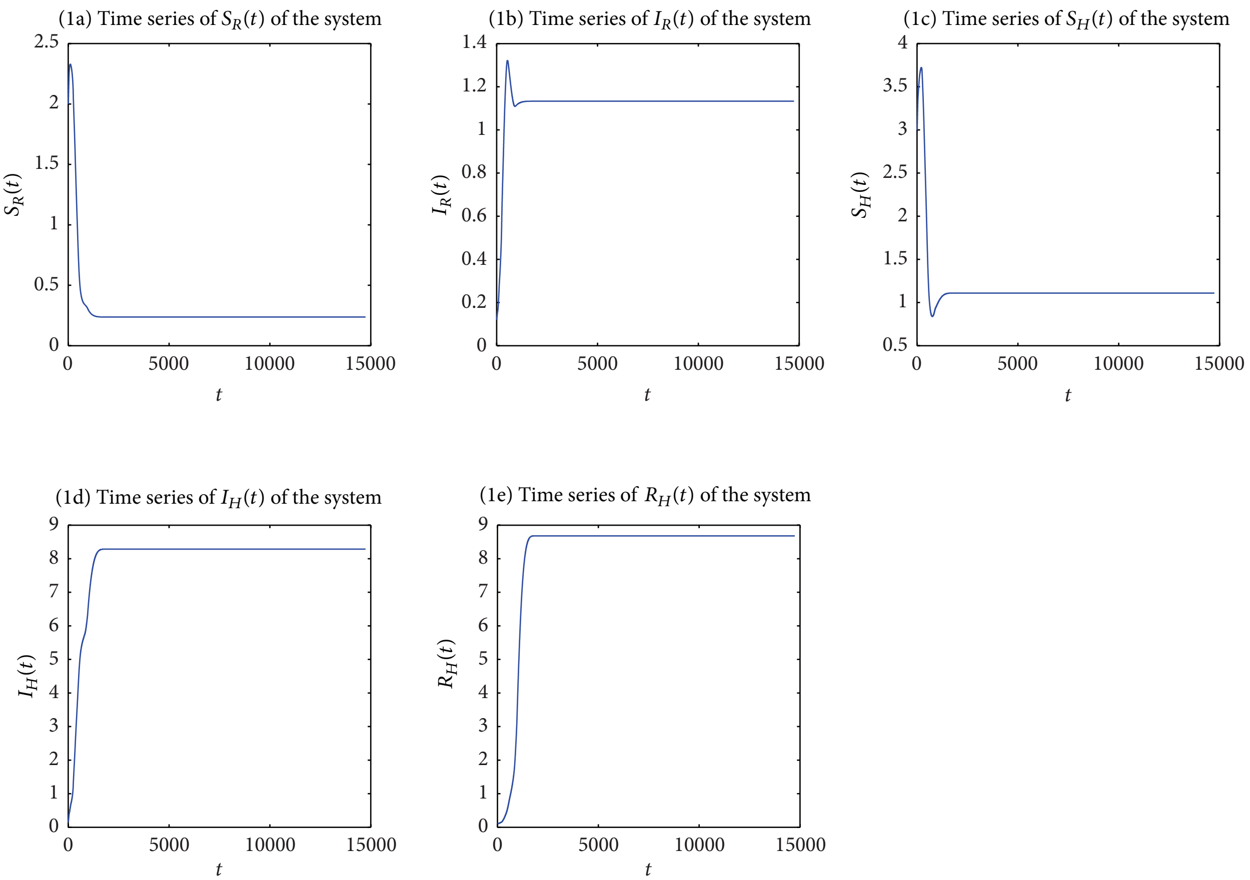

Figure 1: (a)–(e) show that system (3) with initial conditions $(S_R(0), I_R(0), S_H(0), I_H(0), R_H(0)) = (2, 0.12, 3, 0.15, 0.1)$ is permanent.

so

$$\begin{aligned} D^{+}\left|I_{R1}(t)-I_{R2}(t)\right| &\leq \alpha(t) L\left|S_{R1}(t)-S_{R2}(t)\right| \\ &\quad+\alpha(t) L \int_0^h\left|I_{H1}(t-s)-I_{H2}(t-s)\right| \mathrm{d}\eta(s) \\ &\quad-(d(t)+b(t))\left|I_{R1}(t)-I_{R2}(t)\right|, \end{aligned} \tag{67}$$

$$\begin{aligned} D^{+}&\left|S_{H1}(t)-S_{H2}(t)\right| \\ &=\operatorname{sgn}\left(S_{H1}(t)-S_{H2}(t)\right) \\ &\quad\times\Big\{-\beta(t) S_{H1}(t) \int_0^h I_{R1}(t-s) \mathrm{d}\eta(s) \\ &\qquad+\beta(t) S_{H2}(t) \int_0^h I_{R1}(t-s) \mathrm{d}\eta(s) \\ &\qquad-\beta(t) S_{H2}(t) \int_0^h I_{R1}(t-s) \mathrm{d}\eta(s) \\ &\qquad+\beta(t) S_{H2}(t) \int_0^h I_{R2}(t-s) \mathrm{d}\eta(s) \\ &\qquad-\delta(t)\left(S_{H1}-S_{H2}\right)+\gamma(t)\left(R_{H1}-R_{H2}\right)\Big\} \\ &=\operatorname{sgn}\left(S_{H1}(t)-S_{H2}(t)\right) \\ &\quad\times\Big\{-\beta(t) \int_0^h I_{R1}(t-s) \mathrm{d}\eta(s)\left(S_{H1}-S_{H2}\right) \\ &\qquad+\beta(t) S_{H2}(t) \int_0^h\left(I_{R2}(t-s)-I_{R1}(t-s)\right) \mathrm{d}\eta(s) \\ &\qquad-\delta(t)\left(S_{H1}-S_{H2}\right)+\gamma(t)\left(R_{H1}-R_{H2}\right)\Big\}, \end{aligned} \tag{68}$$

so

$$\begin{aligned} D^{+}\left|S_{H1}(t)-S_{H2}(t)\right| &\leq \beta(t) L \int_0^h\left|I_{R1}(t-s)-I_{R2}(t-s)\right| \mathrm{d}\eta(s) \\ &\quad-\delta(t)\left|S_{H1}-S_{H2}\right|+\gamma(t)\left|R_{H1}-R_{H2}\right|, \end{aligned} \tag{69}$$

$$\begin{aligned} D^{+}&\left|I_{H1}(t)-I_{H2}(t)\right| \\ &=\operatorname{sgn}\left(I_{H1}(t)-I_{H2}(t)\right) \end{aligned}$$

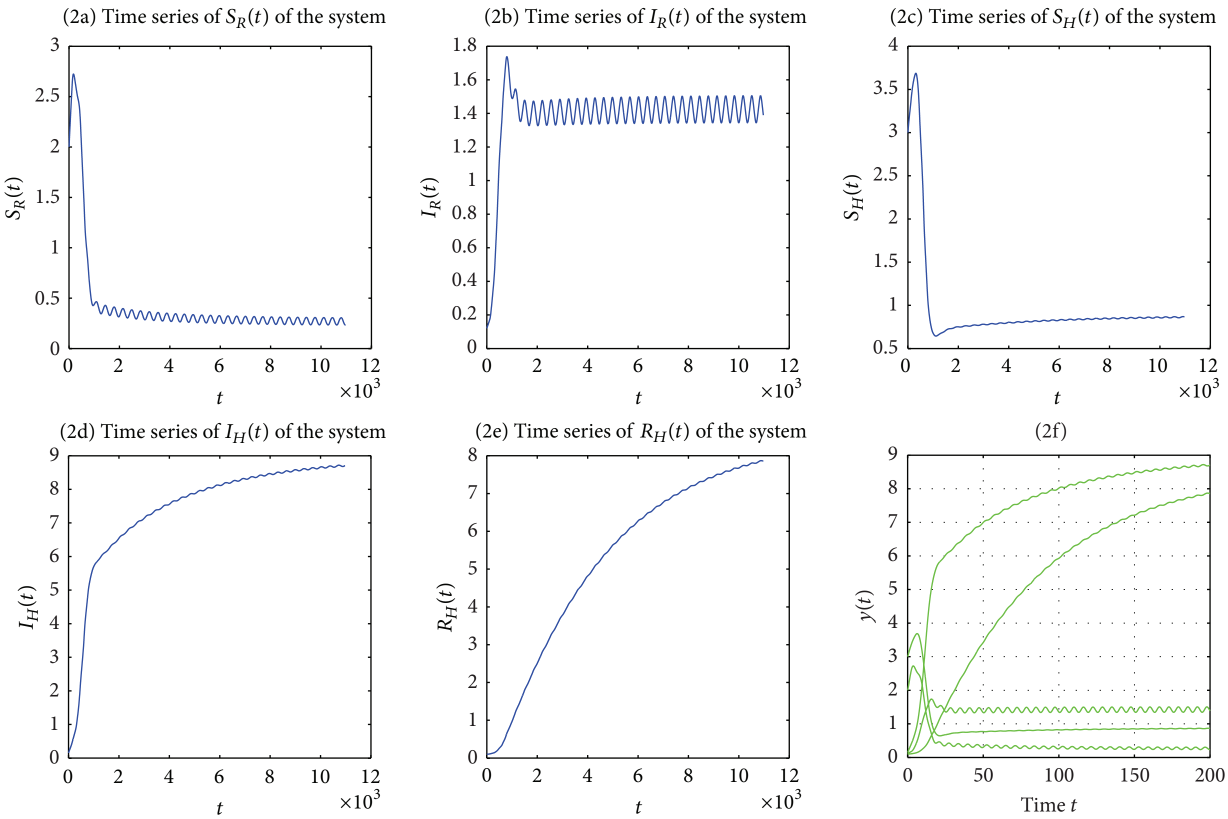

FIGURE 2: (a)–(e) show that system (3) with initial conditions $(S_R(0), I_R(0), S_H(0), I_H(0), R_H(0)) = (2, 0.12, 3, 0.15, 0.1)$ is permanent.

$$\begin{aligned}
&\times \left\{\beta(t) S_{H1} \int_0^h I_{R1}(t-s)\,d\eta(s) - \beta(t) S_{H2} \int_0^h I_{R1}(t-s)\,d\eta(s) + \beta(t) S_{H2} \int_0^h I_{R1}(t-s)\,d\eta(s) - \beta(t) S_{H2}(t) \int_0^h I_{R2}(t-s)\,d\eta(s) - (\delta(t)+\mu(t)+\sigma(t))(I_{H1}-I_{H2})\right\} \\
&= \operatorname{sgn}(I_{H1}(t) - I_{H2}(t)) \\
&\times \left\{\beta(t) \int_0^h I_{R1}(t-s)\,d\eta(s)(S_{H1}-S_{H2}) + \beta(t) S_{H2} \int_0^h (I_{R1}(t-s) - I_{R2}(t-s))\,d\eta(s) - (\delta(t)+\mu(t)+\sigma(t))(I_{H1}-I_{H2})\right\},
\end{aligned} \tag{70}$$

so

$$\begin{aligned}
D^+ |I_{H1}(t) - I_{H2}(t)| &\le \beta(t) L |S_{H1} - S_{H2}| + \beta(t) L \int_0^h |I_{R1}(t-s) - I_{R2}(t-s)|\,d\eta(s) - (\delta(t)+\mu(t)+\sigma(t)) |I_{H1} - I_{H2}|,
\end{aligned} \tag{71}$$

$$\begin{aligned}
D^+ |R_{H1}(t) - R_{H2}(t)| &= \operatorname{sgn}(R_{H1}(t) - R_{H2}(t)) \times \{\mu(t)(I_{H1}-I_{H2}) - (\delta(t)+\gamma(t))(R_{H1}-R_{H2})\},
\end{aligned} \tag{72}$$

so

$$D^+ |R_{H1}(t) - R_{H2}(t)| \le \mu(t) |I_{H1} - I_{H2}| - (\delta(t)+\gamma(t)) |R_{H1} - R_{H2}|. \tag{73}$$

Define $V_1(t) = |S_{R1}(t) - S_{R2}(t)| + |I_{H1}(t) - I_{H2}(t)|$, $V_2(t) = |I_{R1}(t) - I_{R2}(t)| + |S_{H1}(t) - S_{H2}(t)|$, $V_3(t) = |R_{H1}(t) - R_{H2}(t)|$.

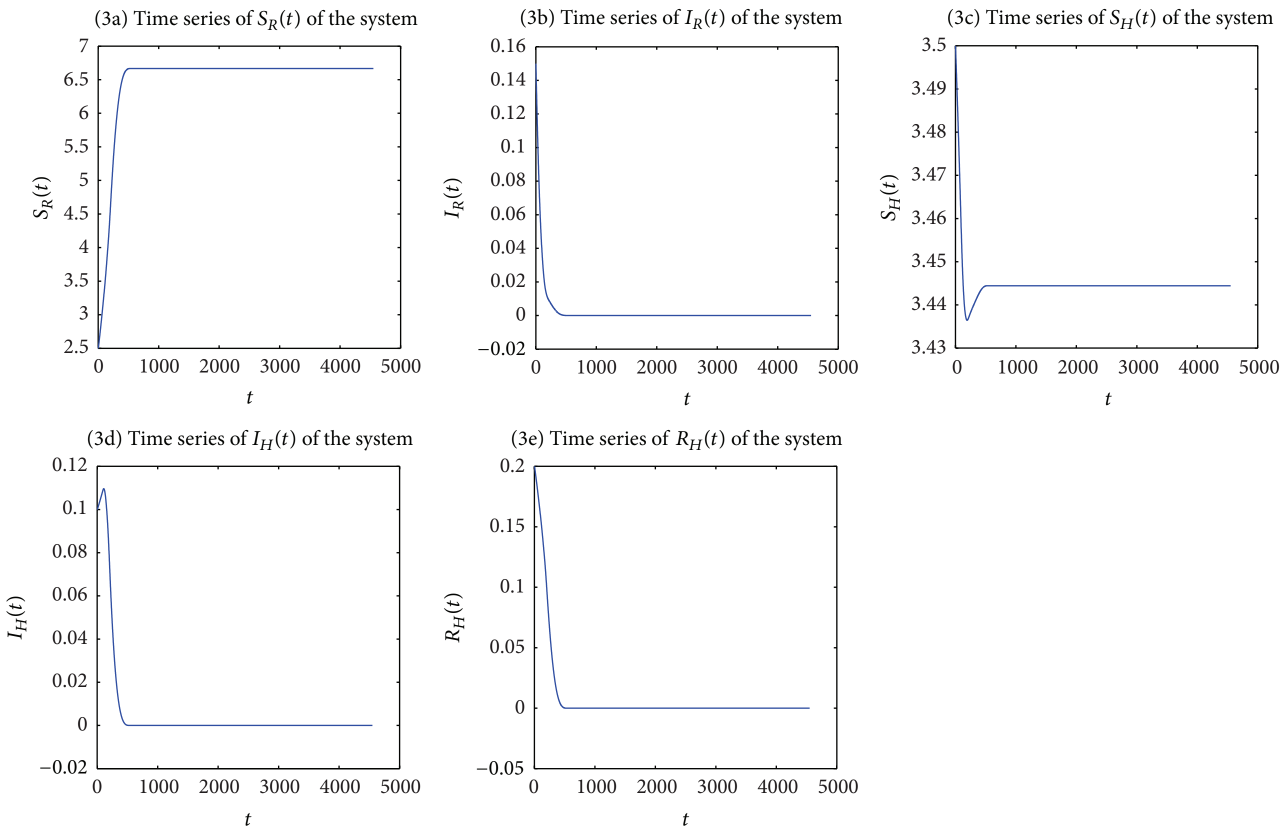

FIGURE 3: (a)–(e) show that the disease in system (3) with initial conditions $(S_R(0), I_R(0), S_H(0), I_H(0), R_H(0)) = (2.5, 0.15, 3.5, 0.1, 0.2)$ will be extinct.

Calculating the right-upper derivatives of $V_1(t), V_2(t), V_3(t)$ along the solution of system (3) and (8), we have

$$\begin{aligned} D^+V_1(t) &\le -b(t)\left|S_{R1}(t)-S_{R2}(t)\right| + \beta(t)L\left|S_{H1}-S_{H2}\right| \\ &\quad - (\delta(t)+\mu(t)+\sigma(t))\left|I_{H1}-I_{H2}\right| \\ &\quad + \alpha(t)L\int_0^h \left|I_{H1}(t-s)-I_{H2}(t-s)\right| d\eta(s) \\ &\quad + \beta(t)L\int_0^h \left|I_{R1}(t-s)-I_{R2}(t-s)\right| d\eta(s), \\ D^+V_2(t) &\le \alpha(t)L\left|S_{R1}(t)-S_{R2}(t)\right| - (d(t)+b(t))\left|I_{R1}(t)-I_{R2}(t)\right| \\ &\quad - \delta(t)\left|S_{H1}-S_{H2}\right| + \gamma(t)\left|R_{H1}-R_{H2}\right| \\ &\quad + \alpha(t)L\int_0^h \left|I_{H1}(t-s)-I_{H2}(t-s)\right| d\eta(s) \\ &\quad + \beta(t)L\int_0^h \left|I_{R1}(t-s)-I_{R2}(t-s)\right| d\eta(s), \\ D^+V_3(t) &\le \mu(t)\left|I_{H1}-I_{H2}\right| - (\delta(t)+\gamma(t))\left|R_{H1}-R_{H2}\right|. \end{aligned} \tag{74}$$

Define $V_4(t)$ as

$$\begin{aligned} V_4(t) &= \int_0^h \int_{t-s}^t \alpha(u+s)L\left|I_{H1}(u)-I_{H2}(u)\right| du\, d\eta(s) \\ &\quad + \int_0^h \int_{t-s}^t \beta(u+s)L\left|I_{R1}(u)-I_{R2}(u)\right| du\, d\eta(s). \end{aligned} \tag{75}$$

The right-upper derivative of $V_4(t)$ along the solution of system (3) and (8) is given below:

$$\begin{aligned} D^+V_4(t) &= L\left|I_{H1}(t)-I_{H2}(t)\right| \int_0^h \alpha(t+s)\, d\eta(s) \\ &\quad - \alpha(t)L\int_0^h \left|I_{H1}(t-s)-I_{H2}(t-s)\right| d\eta(s) \\ &\quad + L\left|I_{R1}(t)-I_{R2}(t)\right| \int_0^h \beta(t+s)\, d\eta(s) \\ &\quad - \beta(t)L\int_0^h \left|I_{R1}(t-s)-I_{R2}(t-s)\right| d\eta(s). \end{aligned} \tag{76}$$

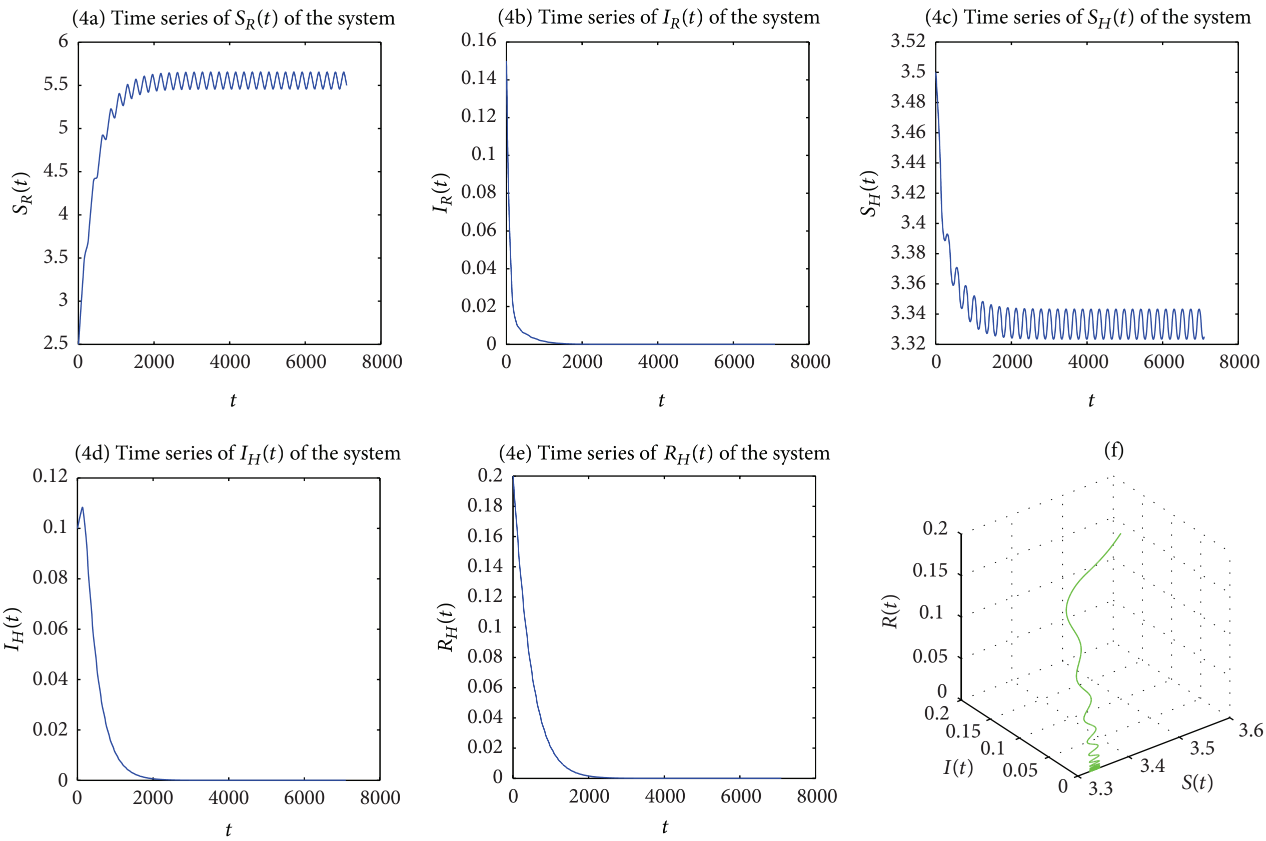

FIGURE 4: (a)–(e) show that the disease in system (3) with initial conditions $(S_R(0), I_R(0), S_H(0), I_H(0), R_H(0)) = (2.5, 0.15, 3.5, 0.1, 0.2)$ will be extinct.

Let $V(t) = c_1V_1(t) + c_2V_2(t) + c_3V_3(t) + (c_1 + c_2)V_4(t)$; then by using (74) and (76), we have

$$\begin{aligned} D^+V(t) &\le -B_1(t)\left|S_{R1}(t) - S_{R2}(t)\right| - B_2(t)\left|I_{R1}(t) - I_{R2}(t)\right| \\ &\quad - B_3(t)\left|S_{H1} - S_{H2}\right| - B_4(t)\left|I_{H1}(t-s) - I_{H2}(t-s)\right| \\ &\quad - B_5(t)\left|R_{H1} - R_{H2}\right|, \quad \forall t \ge h, \end{aligned} \tag{77}$$

where $B_i(t)$ $(i = 1, 2, 3, 4, 5)$ are defined in (63).

Integrating (77) from h to t, we have

$$\begin{aligned} \int_h^t \Big\{ & B_1(t)\left|S_{R1}(t) - S_{R2}(t)\right| + B_2(t)\left|I_{R1}(t) - I_{R2}(t)\right| \\ & + B_3(t)\left|S_{H1} - S_{H2}\right| + B_4(t)\left|I_{H1}(t-s) - I_{H2}(t-s)\right| \\ & + B_5(t)\left|R_{H1} - R_{H2}\right| \Big\} dt \le V(h) - V(t). \end{aligned} \tag{78}$$

So

$$\begin{aligned} \int_h^t \Big\{ & B_1(t)\left|S_{R1}(t) - S_{R2}(t)\right| + B_2(t)\left|I_{R1}(t) - I_{R2}(t)\right| \\ & + B_3(t)\left|S_{H1} - S_{H2}\right| + B_4(t)\left|I_{H1}(t-s) - I_{H2}(t-s)\right| \\ & + B_5(t)\left|R_{H1} - R_{H2}\right| \Big\} dt < \infty. \end{aligned} \tag{79}$$

By assumptions about $B_i(t)$ and the boundedness of $(S_{R1}(t), I_{R1}(t), S_{H1}(t), I_{H1}(t), R_{H1}(t))$ and $(S_{R2}(t), I_{R2}(t), S_{H2}(t), I_{H2}(t), R_{H2}(t))$ on $[0, \infty]$, we obtain from system (3) that $|S_{R1}(t) - S_{R2}(t)|$, $|I_{R1}(t) - I_{R2}(t)|$, $|S_{H1}(t) - S_{H2}(t)|$, $|I_{H1}(t) - I_{H2}(t)|$, and $|R_{H1}(t) - R_{H2}(t)|$ are bounded and uniformly continuous on $[0, \infty)$.

It follows from (79) that

$$\lim_{t\to\infty} \left|S_{R1}(t) - S_{R2}(t)\right| = 0,$$
$$\lim_{t\to\infty} \left|I_{R1}(t) - I_{R2}(t)\right| = 0,$$
$$\lim_{t\to\infty} \left|S_{H1}(t) - S_{H2}(t)\right| = 0,$$

$$\lim_{t\to\infty}\left|I_{H1}(t)-I_{H2}(t)\right|=0,$$
$$\lim_{t\to\infty}\left|R_{H1}(t)-R_{H2}(t)\right|=0. \tag{80}$$

This shows that system (3) with initial conditions (8) is globally asymptotically stable. This completes the proof. □

Corollary 14. *If there exist $c_1, c_2, c_3 > 0$ such that*

$$\begin{aligned}
&\liminf_{t\to\infty}\{c_1 b(t)-c_2 L\alpha(t)\}>0,\\
&\liminf_{t\to\infty}\left\{c_2(d(t)+b(t)) - (c_1+c_2)L\int_0^h \beta(t+s)\,\mathrm{d}\eta(s)\right\}>0,\\
&\liminf_{t\to\infty}\{c_2\delta(t)-c_1 L\beta(t)\}>0,\\
&\liminf_{t\to\infty}\left\{c_1(\delta(t)+\sigma(t))+(c_1-c_3)\mu(t) + (c_1+c_2)L\int_0^h \alpha(t+s)\,\mathrm{d}\eta(s)\right\}>0,\\
&\liminf_{t\to\infty}\{c_3\delta(t)+(c_3-c_2)\gamma(t)\}>0,
\end{aligned} \tag{81}$$

then system (3) *with initial conditions* (8) *is globally asymptotically stable.*

Theorem 15. *Specially, if system* (3) *is ω-periodic and there are positive constants v_i and M_i $(i = 1, 2, 3, 4, 5)$ such that*

$$\begin{aligned}
&v_1 \le \liminf_{t\to\infty} S_R(t) \le \limsup_{t\to\infty} S_R(t) \le M_1,\\
&v_2 \le \liminf_{t\to\infty} I_R(t) \le \limsup_{t\to\infty} I_R(t) \le M_2,\\
&v_3 \le \liminf_{t\to\infty} S_H(t) \le \limsup_{t\to\infty} S_H(t) \le M_3,\\
&v_4 \le \liminf_{t\to\infty} I_H(t) \le \limsup_{t\to\infty} I_H(t) \le M_4,\\
&v_5 \le \liminf_{t\to\infty} R_H(t) \le \limsup_{t\to\infty} R_H(t) \le M_5
\end{aligned} \tag{82}$$

hold for any solution $(S_R(t), I_R(t), S_H(t), I_H(t), R_H(t))$ of (3) *with initial conditions* (8)*, then system* (3) *has positive periodic solution with period ω.*

Corollary 16. *If system* (3) *is ω-periodic and the conditions in Theorems 3 and 13 are valid, then there exists a unique positive ω-periodic solution which is globally asymptotically stable.*

4. Numerical Simulations

To demonstrate the theoretical results obtained in this paper, we will give some numerical simulations.

Firstly, for system (3) we consider the special case; that is, the parameter values are constants as $\lambda(t) = 0.29$, $\beta(t) = 0.29$, $\alpha(t) = 0.19$, $\mu(t) = 0.022$, $c(t) = 0.4$, $d(t) = 0.22$, $\sigma(t) = 0.011$, $b(t) = 0.11$, $\gamma(t) = 0.01$, and $\delta(t) = 0.011$. The delay $h = \Pi/2$ and $\eta(s) = s/h$. It is easy to verify $r^* \ge r_*, r_0 > 1$; the conditions in Theorem 3 are all valid. So according to Theorem 3, we know that the system (3) is permanent. Figure 1 shows trajectories of $S_R(t)$, $I_R(t)$, $S_H(t)$, $I_H(t)$, and $R_H(t)$, respectively.

Now, we consider the general case; that is, we take account of malaria model with periodic environment, we chose the parameter values as $\lambda(t) = 0.3 + 0.01\cos t$, $\beta(t) = 0.3 + 0.01\sin t$, $\alpha(t) = 0.2 + 0.01\cos t$, $\mu(t) = 0.02 + 0.002\sin t$, $c(t) = 0.5 + 0.1\sin t$, $d(t) = 0.2 + 0.02\sin t$, $\sigma(t) = 0.012 + 0.001\sin t$, $b(t) = 0.1 + 0.01\sin t$, $\gamma(t) = 0.01 + 0.0001\cos t$, $\delta(t) = 0.01 + 0.001\sin t$, and the conditions in Theorem 11 are all valid. Figure 2 shows trajectories of $S_R(t)$, $I_R(t)$, $S_H(t)$, $I_H(t)$, and $R_H(t)$, respectively.

Next, we also consider the special case firstly; we chose the parameter values as $\lambda(t) = 0.31$, $\beta(t) = 0.031$, $\alpha(t) = 0.011$, $\mu(t) = 0.018$, $c(t) = 0.6$, $d(t) = 0.48$, $\sigma(t) = 0.011$, $b(t) = 0.09$, $\gamma(t) = 0.01$, and $\delta(t) = 0.09$. The delay $h = \Pi/2$ and $\eta(s) = s/h$. It is easy to verify $R^* < R_* < 1$; the conditions in Theorem 11 are all valid. So according to Theorem 11, we know that the disease in system (3) will be extinct. Figure 3 shows trajectories of $S_R(t)$, $I_R(t)$, $S_H(t)$, $I_H(t)$, and $R_H(t)$, respectively.

Now, we consider the general case; we chose the parameter values as $\lambda(t) = 0.3 + 0.01\cos t$, $\beta(t) = 0.03 + 0.001\sin t$, $\alpha(t) = 0.01 + 0.001\cos t$, $\mu(t) = 0.02 + 0.002\sin t$, $c(t) = 0.5 + 0.1\sin t$, $d(t) = 0.5 + 0.02\sin t$, $\sigma(t) = 0.012 + 0.001\sin t$, $b(t) = 0.1 + 0.01\sin t$, $\gamma(t) = 0.01 + 0.0001\cos t$, and $\delta(t) = 0.1 + 0.01\sin t$ and the conditions in Theorem 11 are all valid. Figure 4 shows trajectories of $S_R(t)$, $I_R(t)$, $S_H(t)$, $I_H(t)$, and $R_H(t)$, respectively.

5. Conclusions

In this paper, we study the permanence, extinction, and global asymptotic stability for a nonautonomous malaria transmission model with distributed time delay, that is, (3). We establish some sufficient conditions on the permanence and extinction of the disease by using inequality analytical techniques. When $r^* \ge r_*$ and $r_0 > 1$, the system is permanent. When $R^* < R_* < 1$, the disease in system will be extinct. By a Lyapunov functional method, according to the definition of global asymptotic stability, we also obtain some sufficient conditions for global asymptotic stability of this model. This method has been used in many references [11, 13, 14].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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