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Review Article

Basic Developments of Quality Characteristics Monitoring

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Process control tools are a widely used approach in many operations and production processes. Process control chart ranks as one of the most important theories used in these disciplines. This paper reviewed the bias of quality characteristics monitoring. Specifically, this study tries to provide a comprehensive understanding of theories of process control. The text starts with a theoretical review of statistical process control theories and follows by a technical introduction to developed tools for process control.

1. Introduction

Statistical process control (SPC) is a collection of seven tools which is useful in improving the quality level by decreasing the variability and increasing the stability of the process. The most well-known tool of SPC is the control charts. Control chart is a graphical tool based on the measurement data obtained in the course of time from the process. Based on the nature of the data obtained from the process, two broad categories of control charts existed; namely, variable and attribute control charts. If the quality characteristics of the product items could be measured as a numerical scale such as weight and height, variable control chart is appropriate. On the other hand, if the quality characteristics could not be measured in numerical scale such as color and softness, attribute control chart could be utilized. By comparing these two types of control charts, we can conclude that, firstly, variable control charts need a smaller sample size than attribute control charts to construct. Secondly, in variable control charts, assignable cause could be detected sooner than attribute control charts. Thirdly, the cost and time for constructing an attribute control chart are less than a variable control chart, and finally, in attribute control charts, we could monitor more than one quality characteristic at the same time in one control chart. In the following, we technically review the attribute control charts.

2. Attribute Control Chart

Attribute control charts consist of four different control charts. If the production items are categorized into two groups based on the specification limits, the beyond statistical distribution is binomial, and each item is known as confirm or nonconfirm with the specification limits. In this case, proportion of nonconforming items (p chart) and number of nonconforming items (np chart) are appropriate. If the number of defect in a period of production time or in one production item is considered, the beyond statistical distribution is poison, and the suitable control charts are known as c chart and u chart. In the current research, we concentrated on the p chart.

2.1. The Attribute Control Charts Literature. Selecting the proper sample size for constructing the attribute control charts is so important. According to Ryan and Schwertman [1] the adequate sample size should be selected to ensure that the normality assumption is not violated. This difficulty gets more important when the proportion of nonconforming is small, because in this case the sample size should be large enough to have at least one item in the categories of nonconforming items. However, a large sample size is too hard to collect in some situations where the output rate of the process is small, and also it is time consuming and costly. To overcome this difficulty, Schwertman and Ryan [2] proposed dual a *np* chart which consists of two charts. The first chart has a tighter control limit which requires a smaller sample size, and the second one is a CUSUM chart.

For overcoming the large sample size, Chen [3] also proposed an alternative approach. He suggested two charts which are based on discrete probability integral and arcsine transformations.

Nelson [4] also proposed an alternative approach. He suggested counting the number of conforming items between two consecutive nonconforming items. He assumed that this observation has an exponential distribution; so, by using a transformation to a normal distribution, we could monitor the process.

Several researchers discussed another topic which is the speed of detecting an abnormal shift in proportion of nonconforming items. To detect an abnormal shift like variable control charts, CUSUM chart is a good alternative approach. Reynolds and Stoumbos [5] proposed two different CUSUM charts. One is based on binomial distribution, and the second one is based on a Bernolli variable.

3. Control Charts for Categorical Data

One of the major areas in SPC is monitoring the proportion of the nonconforming units in the production processes. One of the usual control charts for such cases is the *p* chart. Instead of classifying the production units into two groups (conforming and nonconforming), suppose that they have been classified into more than two groups. As an example, they are classified into three groups: minor defect, major defect, and absent of defect. If the produced unit has a minor defect, it can be repaired by low cost and attempts. But if it has a major defect, it can be repaired by lots of cost, or it must be discarded.

If the produced units classify into more than two groups, categorical control charts could be used. In the following, categorical control charts are explained in detail.

3.1. Generalized p Chart. Suppose that Π_1, Π_2, Π_3 are the proportion of the process. This case is comparable with the *p* chart situation. Case I is when the proportions are known before. Case II is when the proportions are unknown before and at first; in phase I when the process is supposed to be in control, they must be estimated.

For monitoring a multinomial distribution, independent samples should be collected during the process. Suppose that X_{i1}, X_{i2}, X_{i3} show the number of observations in category 1, 2, and 3, respectively, in period i. Base period is shown with $i = 0$. n_i is the sample size for monitoring period i.

First, consider case I where the proportions are known before. A statistical standard approach for solving such a problem is using Pearson's goodness of fit statistic as follow [6]:

$$Y_i^2 = \sum_{j=1}^{3} \frac{\left(X_{ij} - n_i \Pi_j\right)^2}{n_i \Pi_j}, \qquad (1)$$

where the process is in the state of in control and Y_i^2 has chi-square distribution with two degree of freedom.

The control chart based on (1) has an upper control limit which is determined with a percentile of chi-square distribution.

It should be noted that in processes with c categories, the upper control limit of summation in (1) should be c, and the statistic Y_i^2 has a chi-square distribution with c-1 degree of freedom.

Now, consider the second problem (case II). The goodness of fit test is not appropriate here. An appropriate statistical approach is a consistency test between base period and other periods of the process [7]. This statistic for period i is as (2).

Consider the following:

$$\begin{aligned} Z_i^2 &= \sum_{k=i,0} \sum_{j=1}^{3} \frac{n_k \left(X_{kj}/n_k - \left(X_{ij} + X_{0j}\right) / \left(n_i + n_0\right)\right)^2}{\left(X_{ij} + X_{0j}\right) / \left(n_i + n_0\right)} \\ &= n_i n_0 \sum_{j=1}^{3} \frac{\left(P_{ij} + P_{0j}\right)^2}{X_{ij} + X_{0j}}, \end{aligned} \qquad (2)$$

where $P_{kj} = X_{kj}/n_k$ is the ratio of each sample. If $n_i \to \infty$ where n_0/n_i is limited and greater than zero, so that Z_i^2 has a chi-square distribution with two degrees of freedom. Therefore, in case I, the control chart for this case also has an upper control limit equal to an appropriate percentile of chi-square distribution.

There is no theoretical rule for sufficient sample size for using chi-square distribution in such a case. Some rules of thumb exist to determine enough sample size. The most famous rule was proposed by Cochran [8]. He declared that the twenty percent of the frequency of each category should be greater than 5, and the expected frequency of each categories should be greater than one.

3.2. Grouped Observations. Even when the quality characteristic is variable, it is more economical to classify it into k categories than to measure it exactly. As measuring a variable characteristic need, cost and time, using gauge for quality inspection is suggested. As Steiner et al. [9] mentioned: "usually quality data are gathering in grouped manner."

3.3. Fuzzy Control Charts. Based on the nature of the quality characteristics, two broad categories of control charts are developed, namely, variable and attribute control charts. Variable control charts are used to monitor continuous characteristics of the products such as length, weight, and voltage which are measurable on numerical scales. However, it is not always possible to express the quality characteristics on a numerical scale. For these characteristics such as appearance, softness, and color, control charts for attribute are used. Control chart for proportion nonconforming is one of the attribute control charts. In this chart, each product unit is classified as "conforming" or "nonconforming," depending upon whether or not they meet specifications. Then, by using the principles of Shewhart control charts, this chart called p-chart is formed. But as Raz and Wang [10, 11] also mentioned, the binary classification into "conforming" and "nonconforming" used in p-chart might not be appropriate in many situations where there might be a number of intermediate levels. In this case, for measuring the quality-related characteristics, it is necessary to use several intermediate levels besides conforming and nonconforming. For example, the quality of product can be classified by one of the following terms: "perfect," "good," "medium," "poor," and "fair," depending on deviation from specifications. Data obtained in this way are called categorical data, and we can use multinomial distribution instead of binary distribution. Several statistical researches have been done in this area. The early research goes back to Duncan [6, 7], who introduced a chi-square control chart for monitoring a multinomial process with categorical data. Later, this type of control chart is discussed further by Marcucci [12] and Nelson [13]. Marcucci introduced a statistical approach for a case, where the proportion of each category is not known before.

But the problem still exists. As we know, the quality level of each product is determined by the quality inspectors, and they do this task mentally. For example, one product might be classified into *perfect* category by an inspector but classified into *good* category by another inspector. It means that determining the quality level of the product mentally by the inspectors is in an uncertainty situation. As Yager and Zadeh [14] also indicated that in fact the main problem is vagueness that corresponds to the mental affect. Fuzzy set theory could be used because of the uncertainty situation and vague environment. In case of monitoring attribute data by using fuzzy set theory, several researches exist. Raz and Wang [10, 11] proposed an approach based on fuzzy set theory for monitoring attribute processes when quality characteristics are classified into mutually exclusive categories. Kanagawa et al. [15] present a control chart based on the probability density function existing behind the linguistic data, continuing the Raz and Wang approach. These approaches are discussed by Laviolette et al. [16], Almond [17], and Kandel et al. [18] and reviewed by Woodall et al. [19] and Taleb and Limam [20]. Later, Gülbay et al. [21–23] proposed an α-level fuzzy control chart for attributes in order to reflect the vagueness of data and tightness of inspection. In the following, the most famous research in the area of fuzzy attribute control charts will be illustrated in detail.

3.3.1. The Raz and Wang Approach. Constructing a control chart involves determining the center line (CL), upper control limit (UCL), and lower control limit (LCL). This is calculated based on the random sample from the process. When linguistic data are used, it is necessary to state the related fuzzy set by a representative value. In the following, several approaches to determine a representative value for a fuzzy set are explained, and after that probabilistic and fuzzy membership approach will be presented.

Representative Value. To keep the standard format of the Shewhart control chart, it is necessary to transfer the associated fuzzy set to a crisp value which we call representative value. This transformation could be done in different ways. In the following, four methods which are similar in the principle to central tendency in statistics are represented. It must be mentioned that there is no theoretical baseline to select between these four methods, and the selection is completely arbitrary. In the following definitions, F is the fuzzy subset, x is the base variable, and $\mu_F(x)$ is the membership function.

(1) The *fuzzy mode*, f_{mode}, is the value of the base variable where the membership is equal to 1:

$$\mu_F\left(f_{\text{mode}}\right) = 1\left(f_{\text{mode}} = \left\{x \mid \mu_f(x) = 1\right\},\ \forall x \in F\right). \quad (3)$$

The fuzzy mode is unique if $\mu_F(x)$ is unimodal.

In the special case where $\widetilde{A} = (a, b, c)$ is a triangular fuzzy number, the fuzzy mode is equal to b; so, we could have

$$f_{\text{mode}} = b. \quad (4)$$

(2) The *α-level fuzzy midrange*, $f_{\text{mr}}(\alpha)$, is the average of the endpoint of an α-level cut. An α-level cut of F, denoted by F_a, is a nonfuzzy subset of the base variable x containing all the values with a membership function value greater than or equal to α. Thus,

$$F_\alpha = \left\{x \mid \mu_F(x) \geq \alpha\right\}. \quad (5)$$

Note that the fuzzy mode is a special case of the α-level fuzzy midrange with $\alpha = 1$.

Suppose that $\widetilde{A}$ is a triangular fuzzy number. Applying α-cut of fuzzy set, the values of a^α and c^α are determined as follows:

$$\begin{aligned} a^\alpha &= a + \alpha(b - a), \\ c^\alpha &= c - \alpha(c - b). \end{aligned} \quad (6)$$

So, α-level fuzzy midrange for a triangular fuzzy number could be calculated as follows:

$$\begin{aligned} f_{\text{mr}}(\alpha) &= \frac{a^\alpha + c^\alpha}{2} \\ \Longrightarrow f_{\text{mr}}(\alpha) &= \frac{(a + c) + \alpha[(b - a) - (c - b)]}{2}. \end{aligned} \quad (7)$$

(3) The *fuzzy median*, f_{med}, is the point which divides the area under the membership function into two equal regions, satisfying the following equation:

$$\begin{aligned} \int_{-\infty}^{f_{\text{med}}} \mu_F(x)\, dx &= \int_{f_{\text{med}}}^{+\infty} \mu_F(x)\, dx \\ &= \frac{1}{2} \int_{-\infty}^{+\infty} \mu_F(x)\, dx. \end{aligned} \quad (8)$$

(4) The *fuzzy average*, f_{avg}, is defined by Zadeh [24] as follows:

$$f_{\text{avg}} = Av(x : F) = \frac{\int_0^1 x\mu_F(x)\, dx}{\int_0^1 \mu_F(x)\, dx}. \quad (9)$$

Generally, two first approaches are simpler in calculation, especially when the membership function was nonlinear.

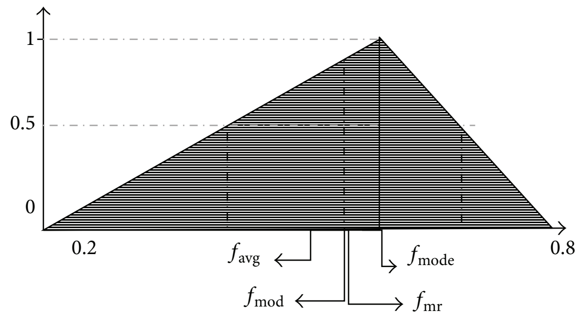

Figure 1: Representative value.

However, when the membership function is too nonsymmetrical, the result of fuzzy mode might be bias. Fuzzy midrange is more flexible, because a different level for α could be selected. When in addition to the place of membership function, the shape of the membership function is important; then, the best choice would be fuzzy average, because it has been calculated from a wide principle.

For comparison, consider a fuzzy set like $\widetilde{A}$ as follows:

$$\mu_A(x) = \begin{cases} 0 & x \leq 0.2 \\ 2.5x - 0.5 & 0.2 \leq x \leq 0.6 \\ -5x + 4 & 0.6 \leq x \leq 0.8 \\ 0 & x \geq 0.8. \end{cases} \quad (10)$$

Representative value for $\widetilde{A}$ would be $f_{\text{mode}} = 0.6$, $f_{\text{med}} = 0.546$, $f_{\text{mr}} = 0.55$, and $f_{\text{avg}} = 0.533$; Figure 1 shows these results as well.

Representing a Sample. A sample could involve several observations which are selected for the inspection. Each observation is classified with a linguistic term and related to a known membership function. These separate linguistic terms need to combine to become a representative value for the sample. This combination of the observation could be done both before and after transferring the linguistic terms to representative values.

In the first case, related fuzzy sets to linguistic terms in a sample should be added together and then divided into the number of sample observations. This operation is done based on the fuzzy mathematics. The result would be a fuzzy set which might not be similar to any of the preliminary terms but is the representative of the quality of that sample. Then, a numerical value as a representative could be calculated by one of the four transformation techniques which were explained in the previous section. Suppose that t linguistic terms existed which were shown by L_i $(i = 1, 2, \ldots, t)$. For each linguistic value, a related fuzzy set such as F_i with a membership function like $\mu_i(x_i)$ is defined. Consider a sample like S with n observation $S = \{(F_1, k_1), (F_2, k_2), \ldots, (F_t, k_t)\}$, where k_i is the number of items that classify to linguistic value L_I by quality inspectors and $k_1 + k_2 + \cdots + k_t = n$. The fuzzy set which is the mean of a sample fuzzy set is shown by MFs. The membership function of MFs is $\mu_s(x_s)$ as follows [25]:

$$\mu_S(x_S) = \operatorname*{Max}_{x_S = (k_1 x_1 + k_2 x_2 + \cdots + k_t x_t)/n} \times \{\operatorname{Min}[\mu_1(x_1), \mu_2(x_2), \ldots, \mu_t(x_t)]\}. \quad (11)$$

The representative value of the sample could be calculated by one of the transformation approach on the $\mu_s(x_s)$.

If the mean of the sample constructs after transferring the linguistic value to representative value, the calculation would be easier. The representative value of the F_i is shown by r_i. The sample mean, M, as the mean of the r_i could be calculated as follows:

$$M = \frac{(r_1 k_1 + r_2 k_2 + \cdots + r_t k_t)}{n}. \quad (12)$$

The first approach keeps fuzziness more than the second approach with the need of more calculation especially when we have a nonlinear membership function. In the following, an example is provided to show both approaches.

Consider a linguistic variable for the evaluation of the quality characteristic of a product with a set of terms such as *perfect, good, medium, poor,* and *bad*. Base variable is a level of quality which standardized in the interval $[0, 1]$. Zero shows the best quality and 1 shows the lower quality. Membership functions associated with each linguistic term are as follows.

$$\mu_{\text{perfect}}(x) = \begin{cases} 1 - 4x & 0 \leq x \leq 0.25 \\ 0 & x \geq 0.25, \end{cases}$$

$$\mu_{\text{good}}(x) = \begin{cases} 4x & 0 \leq x \leq 0.25 \\ 2 - 4x & 0.25 \leq x \leq 0.5 \\ 0 & x \geq 0.5, \end{cases}$$

$$\mu_{\text{medium}}(x) = \begin{cases} o & x \leq 0.25;\ x \geq 0.75 \\ 4x - 1 & 0.25 \leq x \leq 0.5 \\ 3 - 4x & 0.5 \leq x \leq 0.75, \end{cases} \quad (13)$$

$$\mu_{\text{poor}}(x) = \begin{cases} 0 & x \leq 0.5 \\ 4x - 2 & 0.5 \leq x \leq 0.75 \\ 4 - 4x & 0.75 \leq x \leq 1, \end{cases}$$

$$\mu_{\text{bad}}(x) = \begin{cases} 0 & x \leq 0.75 \\ 4x - 3 & 0.75 \leq x \leq 1. \end{cases}$$

These membership functions are depicted in Figure 2.

Consider a sample with 10 observations as

$$S = \{(F_{\text{perfect}}, 3), (F_{\text{good}}, 2), (F_{\text{medium}}, 2), (F_{\text{poor}}, 2), (F_{\text{bad}}, 1)\}. \quad (14)$$

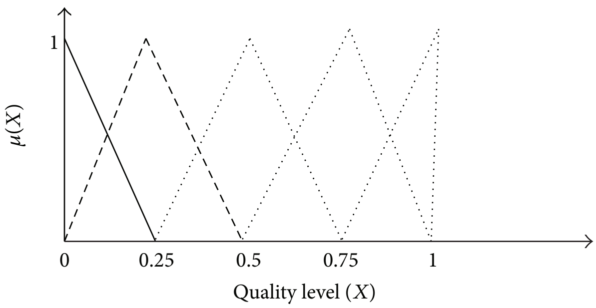

Figure 2: Membership functions for 5 linguistic terms.

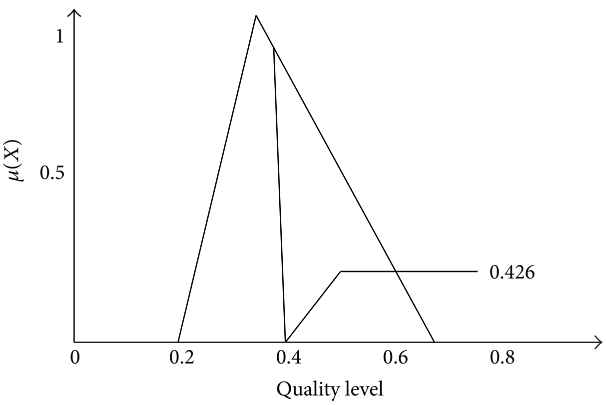

Figure 3: Combined membership function of the sample.

By combining these 10 observations based on the first approach, fuzzy set associated with the sample mean could be defined by the following membership function:

$$\mu_S(x) = \begin{cases} 0 & 0 \le x \le 0.2 \\ 5x - 1 & 0.2 \le x \le 0.4 \\ 2.333 - 3.333x & 0.4 \le x \le 0.7 \\ 0 & x \ge 0.7. \end{cases} \quad (15)$$

Figure 3 also shows this membership function. The representative value by using fuzzy median for this set would be 0.426.

By using the second approach, first, the representative value for each linguistic term must be calculated. By using the fuzzy median, we have

$$r_{\text{perfect}} = 0.146, \quad r_{\text{good}} = 0.25,$$
$$r_{\text{medium}} = 0.5, \quad r_{\text{poor}} = 0.75, \quad r_{\text{bad}} = 0.854. \quad (16)$$

Finally, sample mean could be calculated as follows:

$$(0.146 \times 3 + 0.25 \times 2 + 0.5 \times 2 + 0.75 \times 2 + 0.854 \times 1)\, 10^{-1} = 0.429. \quad (17)$$

Calculation of the Center Line. Normally, center line could be calculated as the average of the sample mean. Here, also both approaches could be used. Suppose m sample with n observations, then CL would be as follows:

$$\text{CL} = \frac{\sum_{j=1}^{m} M_j}{m}, \quad (18)$$

where M_j is the sample mean of the jth sample.

In the following, for determining the control limits, two approaches are explained, namely, probabilistic approach and membership approach.

Probabilistic Control Limits. In the traditional control charts, control limits were determined with a coefficient of the standard deviation of the process. So, here also we need an estimation of the standard deviation. For m sample with n observations, standard deviation is shown by SD_j for jth sample and calculated as follows:

$$\text{SD}_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{t} k_{ij}\left(r_j - M_j\right)^2}, \quad (19)$$

where t is the number of linguistic terms, r_i is the representative value associated with linguistic term L_i, and M_j is the jth sample mean. The mean of m standard deviation was shown by MSD and calculated as follows:

$$\text{MSD} = \frac{1}{m}\sum_{j=1}^{m} \text{SD}_j. \quad (20)$$

Suppose that sample distribution is approximately normal, or sample size is large enough ($n > 25$). Then, for calculating the control limits, we could use the standard method. We have

$$\text{Probabilistic LCL} = \text{Max}\left\{0, \left(\text{CL} - A_3 \cdot \text{MSD}\right)\right\},$$
$$\text{Probabilistic UCL} = \text{Min}\left\{1, \left(\text{CL} + A_3 \cdot \text{MSD}\right)\right\},$$
$$A_3 = \frac{3}{C_4\sqrt{n}} C_4 = \sqrt{\frac{2\left((n-2)/2\right)!}{n-1\left((n-3)/2\right)!}}. \quad (21)$$

The coefficient A_3 and C_4 and table of other coefficient values could be found in Montgomery [26] and any other standard references.

Membership Control Limits. In contrast to the traditional control charts which are constructed based on the probability distribution of the sample mean, membership control limits are based on the membership function. In the following, constructing the membership control limits would be explained.

Consider a convex fuzzy set, and suppose that x_m is the fuzzy mode of the membership function. We could define an inverse membership function which consists of two parts. One part which is in the left side of x_m and is shown by $x_l(\alpha)$, and another part which is in the right side of x_m and shown by $x_r(\alpha)$. The inverse membership function is defined as follows: $x_l(\alpha)$ is the minimum value of the base variable x in which the membership value of them is equal to α, and $x_r(\alpha)$ is the maximum value of the base variable x in which

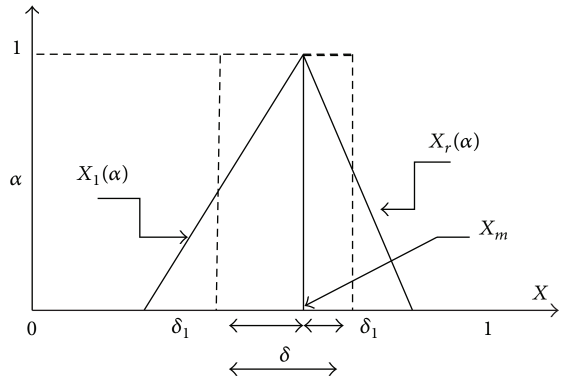

Figure 4: Deviation of mean for a fuzzy set.

the membership value of them is equal to a. In other words, $x_l(\alpha)$ and $x_r(\alpha)$ are the endpoints of α-cut. Now, a value for the deviation of fuzzy set which is called mean deviation and shown as δ could be calculated as follows by using the summation, the deviation of left mean, δ_l, and deviation of right mean, δ_r [25]:

$$\delta_l = \int_{\alpha=0}^{1} [x_m - x_l(\alpha)] \cdot d\alpha,$$
$$\delta_r = \int_{\alpha=0}^{1} [x_r(\alpha) - x_m] \cdot d\alpha, \tag{22}$$

where δ_l and δ_r are left and right deviations of mean, respectively. Their values are equal to the area under the membership function at the left and right side of the mode point of fuzzy set. For fuzzy set $\widetilde{A}$, mean deviation $\delta(A)$ could be calculated as follows:

$$\begin{aligned}\delta(A) = \delta_l(A) + \delta_r(A) &= \int_{\alpha=0}^{1} [x_m - x_l(\alpha)] \cdot d\alpha \\ &+ \int_{\alpha=0}^{1} [x_r(\alpha) - x_m] \\ &= \int_{\alpha=0}^{1} [x_r(\alpha) - x_l(\alpha)] \cdot d\alpha.\end{aligned} \tag{23}$$

In this equation, α is the level of membership. In fact, deviation of a fuzzy set, is a numerical value which stated by the dimension of the base variable. Figure 4 presents the deviation of mean for a fuzzy set.

Suppose that we have m sample with n observations. At first, the fuzzy mean of each sample must be calculated by using fuzzy mathematics, and then, the grand mean of m sample must calculated. For determining the control limits by using the previous equation, at first, the deviation of grand mean should be calculated. The control limits a known distance from the center line. This distance is equal to a coefficient of deviation of the grand mean. We could have

$$\text{Membership LCL} = \text{Max}\{0, (\text{CL} - K\delta)\},$$
$$\text{Membership UCL} = \text{Min}\{1, (\text{CL} + K\delta)\}, \tag{24}$$

where k is a coefficient which shows the distance from center line. The value of k could be determined by using the Monte-Carlo simulation when we suppose that type I error is fixed.

3.3.2. The Kanagawa Approach. Raz and Wang [10, 11] proposed a general approach for designing control chart for monitoring the mean of the process. This approach is based on the normal assumption and just monitors the mean of the process. Kanagawa et al. [15] proposed an approach for estimating the probability density function beyond the linguistic data, and by using it, they design control charts for monitoring both the mean and the variation of the process.

Probability Density Function for Linguistic Data. The objective is to design a control chart for monitoring the variation as well as the mean of a process by using the probability density function (p.d.f). The mentioned probability density function which is beyond the linguistic variables generates the linguistic data randomly and based on the mental judgment of the inspectors.

Suppose that for standard data in the interval [0, 1], p.d.f could be determined based on the Gram-Charlier series:

$$f(x) = \phi(x)[1 + \alpha_1 H_1(x) + \alpha_2 H_2(x) + \cdots], \tag{25}$$

where $\phi(x)$ is a standard normal probability density function and H_r is the Hermite polynomial with the degree of r:

$$\begin{aligned} H_1(x) &= x \\ H_2(x) &= x^2 - 1 \\ H_3(x) &= x^3 - 3x \\ &\vdots \end{aligned} \tag{26}$$

The relationship between α_r and β_r is

$$\begin{aligned} \alpha_1 &= \beta_1 \\ \alpha_2 &= \frac{\beta_2 - 1}{2} \\ \alpha_3 &= \frac{\beta_3 - 3\beta_1}{6} \\ &\vdots \end{aligned} \tag{27}$$

Also, the relationship between β_r and K_r would be

$$\begin{aligned} K_1 &= \beta_1 \\ K_2 &= \beta_2 - \beta_1^2 \\ K_3 &= \beta_3 - 3\beta_2\beta_1 + 2\beta_1^3 \\ &\vdots \end{aligned} \tag{28}$$

Linguistic data could be considered as fuzzy data. So, based on the Gülbay and Kahraman [23] definition, the probability of a linguistic data like L_i happening is

$$\Pr\{L_i\} = \int_R \mu_i(x) f(x)\,dx. \tag{29}$$

When the p.d.f and membership function of the linguistic variable are known, previous equations helps to calculate the probability of happening the event of base variable X in the interval $[x, x+dx]$ with condition of happening the evidence of L_i, as follows:

$$\Pr(X \mid L_i)\,dx = \frac{\mu_i(x) f(x)\,dx}{\Pr(L_i)}. \tag{30}$$

In addition, if k_i is known, β_r could be calculated as follows:

$$\beta_r = \frac{1}{n}\sum_{i=1}^{t}\int_{-\infty}^{+\infty} k_i x^r \Pr(X \mid L_i)\,dx. \tag{31}$$

Based on Kanagawa et al. [15], by using the Gram-Charlier series with degree r ($r = 1, 2, \ldots, t$) and by using the following algorithm, we could estimate the p.d.f.

Step 1. By using fuzzy mode, the value of scalar number of the membership function associated with each linguistic value $X_1, X_2, \ldots, X_t$ must be determined. Then, $k_r^{(0)}$ is calculated as follows:

$$\beta_r^{(0)} = \frac{1}{n}\sum_{i=1}^{t} k_i x_i^r. \tag{32}$$

Continuously, by using (32) other torques must be determined.

Step 2. The torque which is determined in Step 1 inserts into the p.d.f.

Step 3. The values of $f(x)$ insert into (31) and update the torque.

Step 4. Repeat Steps 2 and 3 until giving the following condition:

$$\beta_r^{(0)} = \frac{1}{n}\sum_{i=1}^{t} k_i x_i^r. \tag{33}$$

Now, by using this assumption where is $X_1, X_2, \ldots, X_n$ are independent random variables from $f(x)$, α upper percent of normal distribution by using the Cornish-Fisher development method would be

$$\begin{aligned} Z_\alpha = u_\alpha &+ \frac{K_3/K_2^{3/2}}{6\sqrt{n}}\left(u_\alpha^2 - 1\right) \\ &+ \frac{K_4/K_2^2}{24n}\left(u_\alpha^3 - 3u_\alpha\right) \\ &- \frac{K_3^2/K_2^3}{36n}\left(2u_\alpha^3 - 5u_\alpha\right) + \cdots, \end{aligned} \tag{34}$$

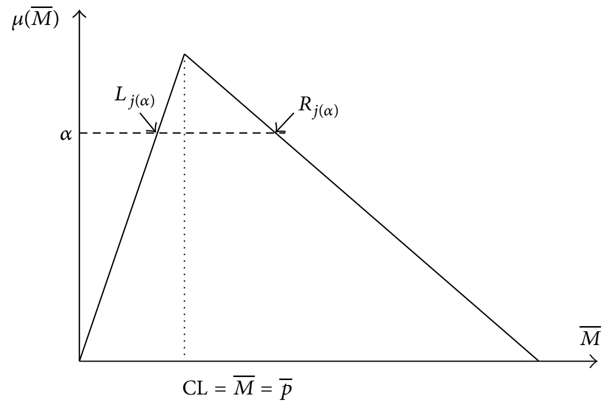

Figure 5: TFN for M and M_j for sample j.

where

$$Z = \frac{\overline{X} - K_1}{(K_2/n)^{1/2}}, \tag{35}$$

and u_α shows α upper percent of normal distribution with mean equal to zero and variance equal to 1.

3.3.3. α-Level Fuzzy Control Chart. As mentioned before in crisp state, control limits for the proportion of nonconforming could be calculated as (1). In fuzzy state, sample mean M_j and center line CL could be calculated as follows:

$$\begin{aligned} \text{CL} = \overline{M}_j &= \sum_{j=1}^{m} M_j, \\ M_j &= \frac{\sum_{j=1}^{m} k_{ij} r_i}{n_j}, \quad i = 1, 2, \ldots, t. \end{aligned} \tag{36}$$

As CL is a fuzzy set, it could be stated by a triangular fuzzy number (TFN), where its fuzzy mode is equal to CL. Figure 5 depicted CL as a TFN.

4. Conclusion

To conclude, this study has technically reviewed control charts. The author in this paper covered the first phase of developments in the context of control charts. In the second phase, most of the works are based on hybrid charts as well as works which are focusing on the use of more productive charting methods [27]. The second part starts by 2000s. Clearly, developments of phase two charts are all based on pure charts which are in phase one and have been reviewed in this paper. The contribution of this study was to review pure control charts to show start points to direct further studies. Further researches could continue reviewing the developments of control charts in second phase, as well as using the pure charts of the first phase to modify the chart's productivity. The author is continuing this study to modify the current available control charts, using fuzzy theory approach.

Conflict of Interests

The author declares no possible conflict of interests.

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