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Research Article

Nonlinear Response of Strong Nonlinear System Arisen in Polymer Cushion

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A dynamic model is proposed for a polymer foam-based nonlinear cushioning system. An accurate analytical solution for the nonlinear free vibration of the system is derived by applying He's variational iteration method, and conditions for resonance are obtained, which should be avoided in the cushioning design.

1. Introduction

Packaged products can be potentially damaged by dropping. In order to prevent any damage, a product and a cushioning packaging are always included in a packaging system [1, 2], and it is very important to investigate the condition for resonance. However, the oscillation in the packaging system is of inherent nonlinearity [3–5], and it remains a problem to obtain the resonance condition for nonlinear packaging system. Polymer foams, especially EPS (expanded polystyrene), are widely used for cushion or protective packaging, and the governing equations can be expressed as

$$\begin{gathered} m\ddot{x}+ \beta_3 th(\beta_1 x) + \beta_4 \tan(\beta_2 x) \\ + \beta_5 \tan^3(\beta_2 x) = 0, \\ x(0) = 0, \\ \dot{x}(0) = \sqrt{2gh}. \end{gathered} \tag{1}$$

Here, the coefficient m denotes the mass of the packaged product, while β_i denote, respectively, the characteristic constants of polymer foams which could be obtained by compression test, and h is the dropping height.

By introducing these parameters: $T_0 = \sqrt{m/\beta_1\beta_3}$, $L = 1/\beta_1$ and let $X = x/L$, $T = t/T_0$, $\lambda_1 = \beta_2/\beta_1$, $\lambda_2 = \beta_4/\beta_3$, and $\lambda_3 = \beta_5/\beta_3$, (1) can be written in the following forms

$$\begin{gathered} \ddot{X} + thX + \lambda_2 \tan(\lambda_1 X) \\ + \lambda_3 \tan^3(\lambda_1 X) = 0, \\ X(0) = 0, \\ \dot{X}(0) = V = \frac{T_0}{L}\sqrt{2gh} = \sqrt{\frac{2\beta_1 mgh}{\beta_3}}. \end{gathered} \tag{2}$$

By using Taylor series for $\sin X$ and $\tan X$, (2) can be equivalently written as

$$\begin{gathered} \ddot{X} + \omega_{01}^2 X + \left(-\frac{1}{3} + \lambda_3\lambda_1^3 + \frac{1}{3}\lambda_2\lambda_1^3\right)X^3 \\ + \left(\frac{2}{15} + \frac{2}{15}\lambda_2\lambda_1^5 + \lambda_3\lambda_1^5\right)X^5 + \frac{11}{15}\lambda_3\lambda_1^7 X^7 = 0, \\ X(0) = 0, \\ \dot{X}(0) = V = \frac{T_0}{L}\sqrt{2gh} = \sqrt{\frac{2\beta_1 mgh}{\beta_3}}, \end{gathered} \tag{3}$$

where

$$\omega_{01} = \sqrt{1 + \lambda_1 \lambda_2}. \tag{4}$$

2. Variational Iteration Method

The variational iteration method [6–13] has been widely applied in solving many different kinds of nonlinear equations [6–16], and is especially effective in solving nonlinear vibration problems with approximations [17–20]. Applying the variational iteration method [6–13], the following iteration formulae can be constructed:

$$\begin{aligned} X_1 = {} & X_0 + \frac{1}{\omega_{01}} \\ & \times \int_0^t \sin \omega_{01} (s - t) \left\{ \ddot{X}_0 + \omega_{01}^2 X_0 \right. \\ & \qquad + \left(-\frac{1}{3} + \lambda_3 \lambda_1^3 + \frac{1}{3} \lambda_2 \lambda_1^3 \right) X_0^3 \\ & \qquad + \left(\frac{2}{15} + \frac{2}{15} \lambda_2 \lambda_1^5 + \lambda_3 \lambda_1^5 \right) X_0^5 \\ & \qquad \left. + \frac{11}{15} \lambda_3 \lambda_1^7 X_0^7 \right\} ds. \end{aligned} \tag{5}$$

Beginning with the initial solutions,

$$X_0 = A \sin (\Omega t). \tag{6}$$

We have

$$\begin{aligned} X_1 = {} & A \sin (\Omega t) - \frac{1}{\omega_{01} (\Omega^2 - \omega_{01}^2)} \\ & \times \left(aA + \frac{3}{4} bA^3 + \frac{5}{256} cA^5 - \frac{637}{1024} dA^7 \right) \\ & (\Omega \sin (\omega_{01} t) + \omega_{01} \sin (\Omega t)) \\ & - \frac{1}{4\omega_{01} (9\Omega^2 - \omega_{01}^2)} \left(bA^3 + \frac{5}{64} cA^5 - \frac{189}{256} dA^7 \right) \\ & (3\Omega \sin (\omega_{01} t) + \omega_{01} \sin (3\Omega t)) \\ & + \frac{1}{16\omega_{01} (25\Omega^2 - \omega_{01}^2)} \left(cA^5 + \frac{7}{4} dA^7 \right) \\ & (5\Omega \sin (\omega_{01} t) + \omega_{01} \sin (5\Omega t)) \\ & - \frac{dA^7}{64\omega_{01} (49\Omega^2 - \omega_{01}^2)} \\ & \times (7\Omega \sin (\omega_{01} t) + \omega_{01} \sin (7\Omega t)), \end{aligned} \tag{7}$$

where

$$\begin{aligned} a &= 1 + \lambda_1 \lambda_2, \\ b &= -\frac{1}{3} + \frac{1}{3} \lambda_2 \lambda_1^3 + \lambda_3 \lambda_1^3, \\ c &= \frac{2}{15} + \frac{2}{15} \lambda_2 \lambda_1^5 + \lambda_3 \lambda_1^5, \\ d &= \frac{11}{15} \lambda_3 \lambda_1^7. \end{aligned} \tag{8}$$

3. Resonance

The resonance can be expected when one of the following conditions is met:

$$\begin{aligned} \Omega &= \omega_{01}, \\ \Omega &= \frac{1}{3} \omega_{01}, \\ \Omega &= \frac{1}{5} \omega_{01}, \\ \Omega &= \frac{1}{7} \omega_{01}. \end{aligned} \tag{9}$$

These conditions should be avoided during the cushioning packaging design procedure.

4. Conclusion

The conditions for resonance, which should be avoided in the cushioning packaging design procedure, can be easily obtained using the variational iteration method.

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