

Research Article

Fast Fourier Transform Based Power Option Pricing with Stochastic Interest Rate, Volatility, and Jump Intensity

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Firstly, we present a more general and realistic double-exponential jump model with stochastic volatility, interest rate, and jump intensity. Using Feynman-Kac formula, we obtain a partial integrodifferential equation (PIDE), with respect to the moment generating function of log underlying asset price, which exists an affine solution. Then, we employ the fast Fourier Transform (FFT) method to obtain the approximate numerical solution of a power option which is conveniently designed with different risks or prices. Finally, we find the FFT method to compute that our option price has better stability, higher accuracy, and faster speed, compared to Monte Carlo approach.

1. Introduction

In financial markets, option pricing problem becomes important for market practitioners ranging from individuals to financial institutes. The classic methodology for option pricing is developed by Black and Scholes [1]. In their model, the stock price follows a geometric Brownian motion and the option pricing formula in no arbitrage. However, a large number of empirical examples show that the initial assumption of their model generates high pricing deviations between theory and practice, for example the volatility clustering in [2] and the incompleteness of markets in [3]. Due to those drawbacks, some market practitioners begin to propose new underlying asset processes to fit empirical facts and develop new option types to control and reduce risks.

Double-exponential jump model with stochastic volatility, interest rate, and jump intensity could overcome these drawbacks of the Black-Scholes model in [2–8]. For example, Santa-Clara and Yan [4, 5] find that innovations to two risks, respectively, denoted by diffusion volatility process and jump intensity process in their model, have affected the expected return in the stock market. The empirical results show that those two processes are largely uncorrelated and do not support models that make jump intensity vary with the level of diffusive volatility. Based on the results of their theory, we believe that two components of risk are more conducive to

the optimization of our model. An excellent contribution of our model is developing the model of Santa-Clara and Yan in [4, 5], combining stochastic volatility, interest rate, and mean-reverting jump intensity processes. Thus, our model better corresponds with the real market than other constant jump intensity models.

As for new option types, compared with other exotic options, power option with considerable operability is simpler and more convenient in practical financial market [9]. Unfortunately, research on power option pricing is literally nil according to our best knowledge. Therefore, in this paper, we define a power option and creatively study and explain how the power coefficient affects the valuation of option.

However, it is difficult to get an exact solution expression of option price. Several numerical methods have been proposed to solve it, including finite difference methods by Zhang and Wang in [10, 11], Monte Carlo simulation by Green in [12] and Glasserman in [13] and the FFT method by Carr and Madan in [14]. However, the finite difference method and Monte Carlo simulation are difficult to be applied in option pricing because they require substantially more computing time [14–17]. Compared to those approaches, the FFT is more effective and simpler to compute in [14–17]. Therefore, in this paper, we employ the FFT to compute the valuation of power option.

In conclusion, the prominent contribution in our paper is modeling a more general and realistic model with stochastic interest rate, volatility, and jump intensity. Defining the power option, discussing the effects of power coefficient, and using the FFT to obtain its numerical results are another three contributions.

This paper proceeds as follows. In Section 2, we build our model which links double-exponential jump, stochastic interest rate, volatility, and jump intensity. The characteristic function is derived in Section 3. In Section 4, we discuss the approximate solutions for power option pricing by the FFT. Numerical examples and discussions of the power coefficient are shown in Section 5. Conclusions are in Section 6.

2. Our Model

Let $(\Omega, \mathcal{F}_t, \mathbb{Q})$ be a probability space where \mathcal{F}_t is the filtration generated by the Brownian motion and jump process at time t , $0 \leq t \leq T$ and \mathbb{Q} is a risk-neutral probability. The underlying asset price $S(t)$ at time t is given by

$$\begin{aligned} dS(t)/S(t) &= (r(t) - \lambda(t)m)dt + \sqrt{V(t)}dW^s(t) \\ &\quad + (e^y - 1)dN^s(t), \quad S(0) = S_0 > 0, \\ dr(t) &= \kappa_r(\theta_r - r(t))dt \\ &\quad + \varepsilon_r\sqrt{r(t)}dW^r(t), \quad r(0) = r_0 > 0, \\ dV(t) &= \kappa_v(\theta_v - V(t))dt \\ &\quad + \varepsilon_v\sqrt{V(t)}dW^v(t), \quad V(0) = V_0 > 0, \\ d\lambda(t) &= \kappa_\lambda(\theta_\lambda - \lambda(t))dt \\ &\quad + \varepsilon_\lambda\sqrt{\lambda(t)}dW^\lambda(t), \quad \lambda(0) = \lambda_0 > 0, \end{aligned} \quad (1)$$

where $W^s(t)$, $W^r(t)$, $W^v(t)$, and $W^\lambda(t)$ are Brownian motions; the correlation coefficient of $W^s(t)$ and $W^v(t)$ is $\rho > 0$, and other motions are independent on each other; $N^s(t)$ is a Poisson process with stochastic intensity $\lambda(t)$, and jump size y is a random variable; $m = E^{\mathbb{Q}}[e^y - 1]$ is average jump amplitude; $r(t)$ is the stochastic interest rate and $V(t)$ is stochastic volatility of the underlying asset return; the parameters ε_r , ε_v , and ε_λ and the mean-reverting rates κ_r , κ_v , and κ_λ are positive constants; the constants θ_r , θ_v , and θ_λ are the long-term interest rate, volatility, and jump intensity, respectively.

We suppose that jump size y has an asymmetric double exponential distribution $f(y)$,

$$\begin{aligned} f(y) &= p \frac{1}{\eta_u} e^{(-1/\eta_u)y} 1_{y \geq 0} \\ &\quad + q \frac{1}{\eta_d} e^{(1/\eta_d)y} 1_{y < 0}, \quad 0 < \eta_u < 1, \eta_d > 0, \end{aligned} \quad (2)$$

where $p, q \geq 0$, $p + q = 1$ are probability of up-move jump and down-move jump, respectively. $1/\eta_u$ and $1/\eta_d$ are mean of positive jumps and negative jumps, respectively. Therefore, $m = p/(1 - \eta_u) + q/(\eta_d + 1) - 1$.

3. The Characteristic Function

Given the log asset price process in (1), it is possible to obtain the characteristic function $\phi(\cdot)$ of $X(\tau)$ at time $\tau = T - t$. Under the risk-neutral measure \mathbb{Q} , firstly, we define an explicit expression of the moment generating function (MGF) $M(\Phi, X, r, V, \lambda, \tau)$ of $X(\tau)$ at time τ ,

$$M(\Phi, x, r, V, \lambda, \tau) = E^{\mathbb{Q}} \left[e^{\Phi X(\tau)} \right] = e^{-r\tau} E^{\mathbb{Q}} \left[e^{r\tau} e^{\Phi X(\tau)} \right], \quad (3)$$

and the complex-valued characteristic function is given by $\phi(u) = M(iu)$.

From (3), we find that $M(\Phi, X, r, V, \lambda, \tau)$ can be interpreted as a contingent claim that pays off $e^{r\tau + \Phi X(\tau)}$ at time τ . Therefore, it is possible to get the MGF $M(\Phi, X, r, V, \lambda, \tau)$ and then obtain the characteristic function $\phi(u)$ in following theorem.

Theorem 1. *The characteristic function $\phi(u)$ of $X(\tau)$ under the measure \mathbb{Q} is given by*

$$\phi(u) = e^{iuX + iu(r-d)\tau + A(u, \tau) + B(u, \tau)V + C(u, \tau) + D(u, \tau)\lambda}, \quad (4)$$

where

$$\begin{aligned} A(u, \tau) &= -\frac{\kappa_v \theta_v}{\varepsilon_v^2} \left(\varphi_+ \tau + 2 \ln \left[\frac{\varphi_- + \varphi_+ e^{-\zeta \tau}}{2\zeta} \right] \right), \\ B(u, \tau) &= -(iu + u^2) \frac{1 - e^{-\zeta \tau}}{\varphi_- + \varphi_+ e^{-\zeta \tau}}, \\ C(u, \tau) &= -\frac{\kappa_\lambda \theta_\lambda}{\varepsilon_\lambda^2} \left(\psi_+ \tau + 2 \ln \left[\frac{\psi_- + \psi_+ e^{-\xi \tau}}{2\xi} \right] \right), \\ D(u, \tau) &= 2\Lambda(u) \frac{1 - e^{-\xi \tau}}{\psi_- + \psi_+ e^{-\xi \tau}}, \\ \varphi_\pm &= \mp (\kappa_v - iu\rho\varepsilon_v) + \zeta, \end{aligned} \quad (5)$$

$$\zeta = \sqrt{(\kappa_v - iu\rho\varepsilon_v)^2 + \varepsilon_v^2 (iu + u^2)},$$

$$\psi_\pm = \mp \kappa_\lambda + \xi,$$

$$\xi = \sqrt{\kappa_\lambda^2 - 2\varepsilon_\lambda^2 \Lambda(u)},$$

$$\begin{aligned} \Lambda(u) &= \frac{p}{1 - iu\eta_u} + \frac{q}{1 + iu\eta_d} \\ &\quad - 1 - iu \left(\frac{p}{1 - \eta_u} + \frac{q}{1 + \eta_d} - 1 \right). \end{aligned}$$

Proof. From (1), Feynman-Kac formula gives the following PIDE for the characteristic function,

$$\begin{aligned}
 & -M_\tau + \left(r - \frac{1}{2}V - \lambda m\right)M_x + \frac{1}{2}VM_{xx} + \kappa_v(\theta_v - V)M_v \\
 & + \frac{1}{2}\varepsilon_v^2VM_{vv} + \rho\varepsilon VM_{xv} + \kappa_\lambda(\theta_\lambda - \lambda)M_\lambda \\
 & + \frac{1}{2}\varepsilon_\lambda^2\lambda M_{\lambda\lambda} + \kappa_r(\theta_r - r)M_r + \frac{1}{2}\varepsilon_r^2rM_{rr} \\
 & + \lambda \int_{-\infty}^{\infty} [M(X+y) - M]\omega(y)dy = 0, \\
 & M(\Phi, X, r, V, \lambda, 0) = e^{\Phi X}.
 \end{aligned} \tag{6}$$

We conjecture that the solution of PDE (6) is

$$\begin{aligned}
 & M(\Phi, X, r, V, \lambda, \tau) \\
 & = e^{X\Phi + r\tau\Phi + A(\Phi, \tau) + B(\Phi, \tau)V + C(\Phi, \tau) + D(\Phi, \tau)\lambda + E(\Phi, \tau) + F(\Phi, \tau)r},
 \end{aligned} \tag{7}$$

with initial conditions $A(\Phi, 0) = 0, B(\Phi, 0) = 0, C(\Phi, 0) = 0, D(\Phi, 0) = 0, E(\Phi, 0) = 0,$ and $F(\Phi, 0) = 0.$

We consider the integral term in (6):

$$\lambda \int_{-\infty}^{\infty} [M(X+y) - M]\omega(y)dy = \lambda M(\Phi, X, r, V, \lambda, \tau)U(\Phi), \tag{8}$$

where $U(\Phi) := \int_{-\infty}^{\infty} (e^{\Phi y} - 1)\omega(y)dy = p/(1 - \Phi\eta_u) + q/(1 + \Phi\eta_d) - 1.$

Substituting (7) and (8) into (6) yields

$$\begin{aligned}
 & -A_\tau(\Phi, \tau) - B_\tau(\Phi, \tau)V - C_\tau(\Phi, \tau) - D_\tau(\Phi, \tau)\lambda \\
 & -E_\tau(\Phi, \tau) - F_\tau(\Phi, \tau)r - \frac{1}{2}V\Phi - \lambda m\Phi \\
 & + \frac{1}{2}V\Phi^2 + \kappa_v\theta_v B(\Phi, \tau) - \kappa_vVB(\Phi, \tau) \\
 & + \frac{1}{2}\varepsilon_v^2VB^2(\Phi, \tau) + \rho\varepsilon VB(\Phi, \tau) \\
 & + \kappa_r\theta_r F(\Phi, \tau) - \kappa_r rF(\Phi, \tau) + \frac{1}{2}\varepsilon_r^2rF^2(\Phi, \tau) \\
 & + \kappa_\lambda\theta_\lambda D(\Phi, \tau) - \kappa_\lambda\lambda D(\Phi, \tau) + \frac{1}{2}\varepsilon_\lambda^2\lambda D^2(\Phi, \tau) \\
 & + \lambda U(\Phi) = 0.
 \end{aligned} \tag{9}$$

If we denote $\Lambda(\Phi) := U(\Phi) - m\Phi = p/(1 - \Phi\eta_u) + q/(1 + \Phi\eta_d) - 1 - \Phi(p/(1 - \eta_u) + q/(1 + \eta_d) - 1),$ then (9) leads to a system of seven ordinary differential equations:

$$\begin{aligned}
 & A_\tau(\Phi, \tau) + C_\tau(\Phi, \tau) + E_\tau(\Phi, \tau) \\
 & = \kappa_v\theta_v B(\Phi, \tau) + \kappa_\lambda\theta_\lambda D(\Phi, \tau) + \kappa_r\theta_r F(\Phi, \tau),
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 & B_\tau(\Phi, \tau) = -\frac{1}{2}(\Phi - \Phi^2) - (\kappa_v - \rho\varepsilon\Phi) \\
 & \times B(\Phi, \tau) + \frac{1}{2}\varepsilon^2B^2(\Phi, \tau),
 \end{aligned} \tag{11}$$

$$D_\tau(\Phi, \tau) = -\kappa_\lambda D(\Phi, \tau) + \frac{1}{2}\varepsilon_\lambda^2D^2(\Phi, \tau) + \Lambda(\Phi), \tag{12}$$

$$F_\tau(\Phi, \tau) = -k_r F(\Phi, \tau) + \frac{1}{2}\varepsilon_r^2F^2(\Phi, \tau), \tag{13}$$

$$A_\tau(\Phi, \tau) = k_v\theta_v B(\Phi, \tau), \tag{14}$$

$$C_\tau(\Phi, \tau) = k_\lambda\theta_\lambda D(\Phi, \tau), \tag{15}$$

$$E_\tau(\Phi, \tau) = k_r\theta_r F(\Phi, \tau). \tag{16}$$

We consider the second ODE (11), in which the general solution can be derived for the Riccati equation. Making the substitution,

$$B(\Phi, \tau) = -\frac{2O'(\tau)}{\varepsilon^2O(\tau)}. \tag{17}$$

Hence,

$$O''(\tau) + (k_v - \rho\varepsilon\Phi)O'(\tau) - \frac{\varepsilon^2}{4}(\Phi - \Phi^2)O(\tau) = 0. \tag{18}$$

The ODE (11) has a general solution of the form

$$O(\tau) = C_1e^{(-1/2)\varphi_-\tau} + C_2e^{(1/2)\varphi_+\tau}, \tag{19}$$

where

$$\begin{aligned}
 & \varphi_\pm = \mp(k_v - \rho\varepsilon\Phi) + \varsigma, \\
 & \varsigma = \sqrt{(k_v - \rho\varepsilon\Phi)^2 + \varepsilon^2(\Phi - \Phi^2)}.
 \end{aligned} \tag{20}$$

According to (15) and initial condition $C(\Phi, 0) = 0,$ C_1 and C_2 are two constants to be determined from boundary conditions,

$$O(0) = C_1 + C_2, \tag{21}$$

$$O'(0) = -\frac{1}{2}\varphi_-C_1 + \frac{1}{2}\varphi_+C_2.$$

We obtain that the solutions of (21) is

$$\begin{aligned}
 & C_1 = \frac{\varphi_+O(0)}{2\varsigma}, \\
 & C_2 = \frac{\varphi_-O(0)}{2\varsigma}.
 \end{aligned} \tag{22}$$

Therefore, the exact solution of (11) is

$$B(\Phi, \tau) = -(\Phi - \Phi^2) \frac{1 - e^{-\zeta\tau}}{\varphi_- + \varphi_+ e^{-\zeta\tau}}. \quad (23)$$

Substituting (23) into (14) and using initial conditions $A(\Phi, 0) = 0$, we have

$$A(\Phi, \tau) = -\frac{\kappa_v \theta_v}{\varepsilon^2} \left(\varphi_+ \tau + 2 \ln \left[\frac{\varphi_- + \varphi_+ e^{-\zeta\tau}}{2\zeta} \right] \right). \quad (24)$$

The OEDs for $C(\Phi, \tau)$, $D(\Phi, \tau)$, $E(\Phi, \tau)$, and $F(\Phi, \tau)$ are solved by analogy. Hence

$$\begin{aligned} C(\Phi, \tau) &= -\frac{\kappa_\lambda \theta_\lambda}{\varepsilon^2} \left(\psi_+ \tau + 2 \ln \left[\frac{\psi_- + \psi_+ e^{-\xi\tau}}{2\xi} \right] \right), \\ D(\Phi, \tau) &= 2\Lambda(\Phi) \frac{1 - e^{-\xi\tau}}{\psi_- + \psi_+ e^{-\xi\tau}}, \\ E(\Phi, \tau) &= 0, \\ F(\Phi, \tau) &= 0, \\ \psi_\pm &= \mp \kappa_\lambda + \xi, \\ \xi &= \sqrt{\kappa_\lambda^2 - 2\varepsilon_\lambda^2 \Lambda(\Phi)}. \end{aligned} \quad (25)$$

Using $\phi(u) = M(iu)$, it is easy to get Theorem 1. \square

4. Power Option Pricing Using the Fast Fourier Transform

4.1. Power Option. Under the risk-neutral measure \mathbb{Q} , the price of the power call option $C(S, r, V, \lambda, t)$ at time t with strike price K and maturity date T is given by

$$C(S, r, V, \lambda, t) = E^{\mathbb{Q}} \left[e^{-\int_t^T r_u du} (S^\beta - K)^+ \right]. \quad (26)$$

As a European option, the valuation of power at initial time is

$$c(K, T, \beta) := C(S, r, V, \lambda, 0) = E^{\mathbb{Q}} \left[e^{-\int_0^T r_u du} (S^\beta - K)^+ \right]. \quad (27)$$

Remark 2. In particular, when $\beta = 1$, the power option degenerates into the vanilla option.

4.2. Fast Fourier Transform. Let k denote the log of the strike price K , and let $c(k)$ be the desired value of an initial power call option with strike price e^k . The risk-neutral density of the price $X(T)$ at time T is $q_T(X)$. From (27), the initial value of power call option is related to the risk-neutral density $q_T(X)$ by

$$c(k) = e^{-\int_0^T r_u du} \int_k^\infty (e^{\beta X(T)} - e^k) q_T(X) dX. \quad (28)$$

Here we employ the price of a zero coupon bond $Q(T)$ at time T to express the stochastic discounted process $e^{-\int_0^T r_u du}$

$$Q(T) = P(T) e^{-H(T)r_0}, \quad (29)$$

where

$$\begin{aligned} P(T) &= \left[\frac{2me^{(T/2)(\kappa_r + m)}}{(\kappa_r + m)(e^{mT} - 1) + 2m} \right]^{2\kappa_r \theta_r / \varepsilon_r^2}, \\ H(T) &= \frac{2(e^{mT} - 1)}{(\kappa_r + m)(e^{mT} - 1) + 2m}, \\ m &= \sqrt{\kappa_r^2 + 2\varepsilon_r^2}. \end{aligned} \quad (30)$$

Then (28) can be written as

$$c(k) = Q(T) \int_k^\infty (e^{\beta X(T)} - e^k) q_T(X) dX. \quad (31)$$

Note that $c(k)$ tends to S_0^β as k tends to $-\infty$, and hence the call pricing function (31) is not square integrable. According to Wong and Lo [15], we now consider the modified call price function $\bar{c}(k)$ defined by

$$\bar{c}(k) \equiv e^{\alpha k} c(k), \quad \text{for } \alpha > 0. \quad (32)$$

The Fourier transform of $\bar{c}(k)$ is defined by

$$\begin{aligned} \omega_T(u) &= \int_{-\infty}^\infty e^{iuk} \bar{c}(k) dk \\ &= \int_{-\infty}^\infty e^{iuk} e^{\alpha k} c(k) dk \\ &= \int_{-\infty}^\infty e^{iuk} e^{\alpha k} Q(T) \int_k^\infty (e^{\beta X} - e^k) q_T(X) dX dk \\ &= Q(T) \int_{-\infty}^\infty q_T(X) \int_{-\infty}^X (e^{\beta X + \alpha k} - e^{(1+\alpha)k}) e^{iuk} dk dX \\ &= Q(T) \int_{-\infty}^\infty e^{-rT} q_T(X) \left(\frac{e^{(\beta+\alpha+iu)X}}{\alpha+iu} - \frac{e^{(1+\alpha+iu)X}}{1+\alpha+iu} \right) dX \\ &= Q(T) \left[\frac{\phi_T(u - (\beta+\alpha)i)}{\alpha+iu} - \frac{\phi_T(u - (1+\alpha)i)}{1+\alpha+iu} \right]. \end{aligned} \quad (33)$$

Then the inverse transform of $\omega_T(u)$ is given by

$$\bar{c}(k) = \frac{1}{\pi} \int_0^\infty e^{-iuk} \omega_T(u) du. \quad (34)$$

Then

$$c(k) = \frac{e^{-\alpha k}}{\pi} \int_0^\infty e^{-iuk} \omega_T(u) du. \quad (35)$$

Using the Trapezoid rule for the integral in (35), the value of $c(k)$ is approximated as

$$c(k) \approx \frac{e^{-\alpha k}}{\pi} \sum_{j=0}^N e^{-iuk} \omega_T(u_j) \nu, \quad (36)$$

where $u_j = \nu(j-1)$

The FFT returns N values of k and for a regular spacing size of z where N is a power of 2. The value for k is

$$k_u = -b + z(a-1), \quad \text{for } a = 1, \dots, N, \quad (37)$$

which corresponds to log strike prices ranging from $-b$ to b , where $b = Nz/2$.

Substituting (37) into (36) yields

$$c(k_u) \approx \frac{e^{-\alpha k_u}}{\pi} \sum_{j=0}^N e^{-iz\nu(j-1)(a-1)} e^{ibu_j} \omega_T(u_j) \nu; \quad (38)$$

here we denote $\nu z = 2\pi/N$.

With Simpson's rule weightings, the value of European call option is

$$c(k_u) \approx \frac{e^{-\alpha k_u}}{\pi} \sum_{j=1}^N e^{-i(2\pi/N)(j-1)(a-1)} e^{ibu_j} \omega_T \times (u_j)^{\frac{\nu}{3}} [3 + (-1)^j - \ell_{j-1}], \quad (39)$$

where

$$\ell_n = \begin{cases} 1, & n = 0, \\ 0, & \text{otherwise.} \end{cases} \quad (40)$$

5. Numerical Examples and Discussion

In this section, we present several numerical experiments to compare numerical differences between the FFT and Monte Carlo simulation and discuss the effects of power coefficient. Now, we perform some Monte Carlo simulations for the endowment dynamics by using the discrete scheme of system (1) as follows:

$$\begin{aligned} \ln S(t + \Delta t) &= \ln S(t) + \left(r - d - \lambda(t)m - \frac{1}{2}V(t) \right) \Delta t \\ &\quad + \sqrt{V(t)} dW^s(t) \\ &\quad + (e^\nu - 1) \ln S(t) (N(t + \Delta t) - N(t)), \\ V(t + \Delta t) &= V(t) + \kappa_v (\theta_v - V(t)) \Delta t + \varepsilon_v \sqrt{V(t)} dW^v(t), \\ \lambda(t + \Delta t) &= \lambda(t) + \kappa_\lambda (\theta_\lambda - \lambda(t)) dt + \varepsilon_\lambda \sqrt{\lambda(t)} dW^\lambda(t), \\ r(t + \Delta t) &= r(t) + \kappa_r (\theta_r - V(t)) \Delta t + \varepsilon_r \sqrt{r(t)} dW^r(t), \end{aligned} \quad (41)$$

where $dW^s(t) = \varepsilon_1 \sqrt{\Delta t}$, $dW^v(t) = \rho \varepsilon_1 \sqrt{\Delta t}$, $dW^\lambda(t) = \varepsilon_2 \sqrt{\Delta t}$, and $dW^r(t) = \varepsilon_3 \sqrt{\Delta t}$; Δt is the time interval; $\varepsilon_1, \varepsilon_2,$

TABLE 1: Power option prices in $\beta = 0.98$ case: FFT versus Monte Carlo.

Strike price	FFT	Monte Carlo	% difference
90.2830	6.0740	6.0725	0.0246
92.1938	5.4487	5.448	0.0136
94.1451	4.8635	4.8624	0.0218
96.1377	4.3188	4.3212	-0.0544
98.1724	3.8143	3.8137	0.0161
100.2502	3.3498	3.3505	-0.0223
102.3720	2.9244	2.9242	0.0066
104.5387	2.5371	2.538	-0.035
106.7512	2.1867	2.1859	0.035
107.8750	2.0249	2.0235	0.0693
110.1581	1.7269	1.7255	0.0824

TABLE 2: Power option prices in $\beta = 1$ case: FFT versus Monte Carlo.

Strike price	FFT	Monte Carlo	% difference
90.2830	10.7797	10.7799	-0.002
92.1938	9.9356	9.9344	0.0117
94.1451	9.1277	9.1266	0.0126
96.1377	8.3574	8.3575	-0.0013
98.1724	7.6256	7.6263	-0.0095
100.2502	6.9334	6.936	-0.0373
102.3720	6.2811	6.2804	0.0106
104.5387	5.6691	5.6693	-0.0033
106.7512	5.0974	5.0973	0.0016
107.8750	4.8265	4.8246	0.0401
110.1581	4.3145	4.3141	0.0102

and ε_3 are samples from the standard normal distribution with correlation coefficient 0; $N(t) = j$, if $S_j \leq t \leq S_{j+1}$, $S_j = \sum_{k=1}^j \tau_k$, τ_k is the k th jump interval which obeys an exponential distribution with parameter λ .

The option prices are calculated by Monte Carlo simulation using

$$\begin{aligned} V(S(t), K, V(t), \lambda(t), r(t), t) \\ = E^{\mathbb{Q}} \left[\left(\frac{S_T^e + S_T^e}{2} - K \right)^+ \mid F_t \right]. \end{aligned} \quad (42)$$

In our numerical examples by the FFT, we take $N = 4098$ and $\nu = 600/N$ which lead to the log strike spacing of $z = \pi/300$ and the damping coefficient $\alpha = 1.18$. Other parameters are denoted as $r_0 = 0.05$, $\kappa_v = 0.3$, $\theta_v = 0.15$, $\varepsilon_v = 0.1$, $\kappa_\lambda = 5$, $\theta_\lambda = 0.6$, $\varepsilon_\lambda = 0.3$, $\kappa_r = 0.5$, $\theta_r = 0.05$, $\varepsilon_r = 0.1$, $\rho = -0.25$, $\eta_u = 0.03$, $\eta_d = 0.13$, $p = 0.4$, $q = 0.6$, $S_0 = 100$, $V_0 = 0.15$, $\lambda_0 = 3$, $T = 0.5$, $\kappa = 100$, and $\Delta t = 0.01$. The results are presented in Tables 1, 2, and 3.

Numerical examples presented that the FFT is faster than Monte Carlo method with different parameters. To different strike prices, the FFT takes around approximate 340 seconds to produce 4098 option prices in three numerical experiments. However, Monte Carlo technique takes around 1260 seconds. On the level of the accuracy, comparing with

TABLE 3: Power option prices in $\beta = 1.02$ case: FFT versus Monte Carlo.

Strike price	FFT	Monte Carlo	% difference
90.2830	15.9551	15.9556	-0.0031
92.1938	14.8717	14.8715	0.0012
94.1451	13.81993	13.8197	0.0014
96.1377	12.8024	12.8038	-0.0111
98.1724	11.8216	11.8219	-0.0025
100.2502	10.8797	10.8799	-0.0018
102.3720	9.9786	9.9802	-0.0159
104.5387	9.1201	9.1193	0.0088
106.7512	8.3053	8.306	-0.0084
107.8750	7.9147	7.9155	-0.0105
110.1581	7.1675	7.1673	0.0034

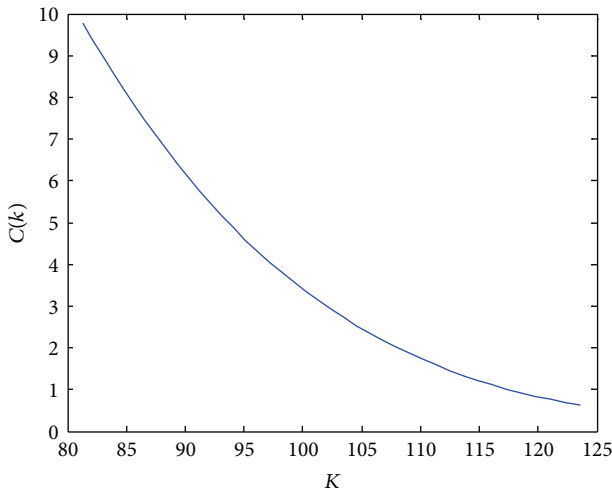


FIGURE 1: Power option prices in $\beta = 0.98$ case.

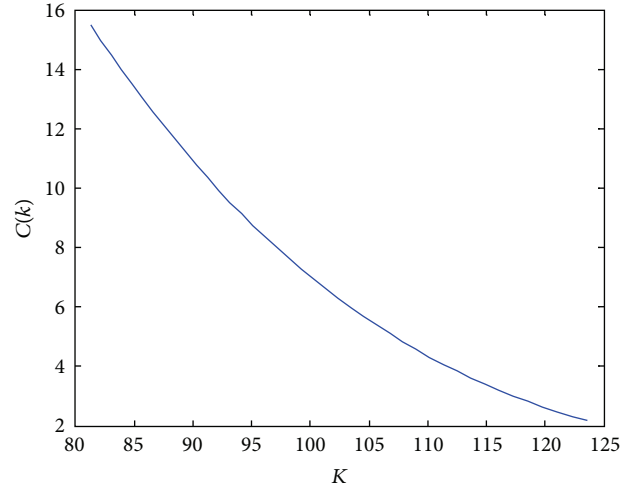


FIGURE 2: Power option prices in $\beta = 1$ case.

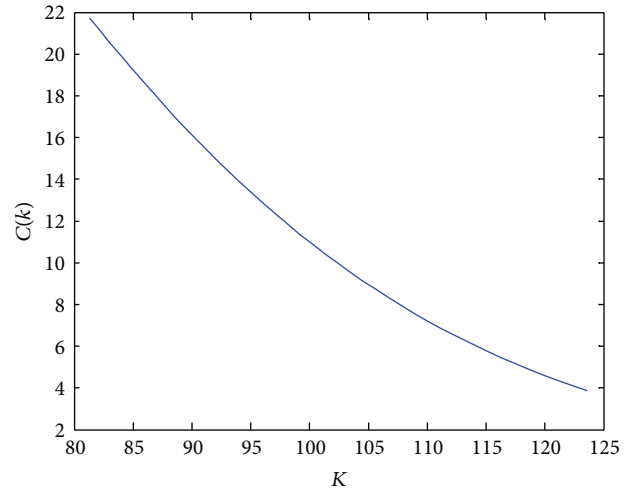


FIGURE 3: Power option prices in $\beta = 1.02$ case.

Monte Carlo, we show that the percentage differences of the FFT are less than 0.1% in Tables 1, 2, and 3. Therefore, the FFT is more accurate and faster than Monte Carlo.

To examine effects of the power coefficient β , we calculate valuations with $\beta = 0.98$, $\beta = 1$, and $\beta = 1.02$. Firstly, we can see that with the β rising, the gaps between S^β and K become greater in Figures 1, 2, and 3. Secondly, according to different β , we find that valuations of option are increasing with the growth of β . It is because the fluctuations and risks of the underlying asset price are enhancing with the growth of β , and in order to compensate risk, the option seller need gets higher price of option. It means that the seller can design different power options with different risks or prices by defining different the power coefficients. Therefore, power option is one of convenient options which are designed easily.

6. Conclusion

In this paper, we study a more general and realistic double exponential-jump model which links the stochastic interest rate, volatility, and jump intensity. If we decide the constant

interest rate, volatility, or jump intensity, our model can be degenerated into many previous models, for example, Black and Scholes' model in [1], Santa-Clara and Yan' model in [4, 5], Carr and Madan's model in [14], Zhang and Wang's model in [10, 11], and Zhang and Wang's model in [17].

Moreover, we obtain the characteristic function of the log asset price, which is important to not only take the FFT in [14–17] but also derive the option pricing formula, for example, in [18]. Then, in our paper, a power option is defined and its valuations are computed with the FFT by characteristic function. Through numerical examples, comparing the results produced from the FFT and the Monte Carlo simulations of power option valuation, we find that the FFT is approximate with Monte Carlo approach but has better stability, higher accuracy, and faster speed in pricing power option. In addition, we show that the seller can design different power options with different risks or prices by defining different the power coefficients.

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