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Erratum

Erratum to "Positive Solution to a Fractional Boundary Value Problem"

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In the paper entitled "Positive solution to a fractional boundray value problems," the following problem (P1) is studied:

$$ {}^{c}D_{0^+}^{q}u(t) = f\left(t, u(t), {}^{c}D_{0^+}^{\sigma}u(t)\right), \quad 0 < t < 1, \tag{1.1}$$

$$ u(0) = u''(0) = 0, \qquad u'(1) = \alpha u''(1), \tag{1.2}$$

where $f : [0,1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a given function, $2 < q < 3$, and $1 < \sigma < 2$. Remarking that all the calculuses in this paper are done for $0 < \sigma < 1$ and that if we take $1 < \sigma < 2$, then ${}^{c}D_{0^+}^{\sigma}Tu = (1/\Gamma(2-\sigma))\int_0^t ((Tu)''(s)/(t-s)^{\sigma-1})ds$ and the second derivative with respect to t of $G(t,s)$ is discontinuous for $s = t$, consequently we cannot apply this method to establish the existence and positivity of solution. For this reason, we correct the study of problem (P1) by taking $0 < \sigma < 1$, and then the following corrections are needed.

(1) In page 3, in Lemma 2.3, we should correct ${}^{c}D_{0^+}^{\alpha}t^{\beta-1} = (\Gamma(\beta)/\Gamma(\beta-\alpha))t^{\beta-\alpha-1}$, $\beta > n$.

(2) Equation (2.6) must be

$$ u(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(t,s)\, y(s)\, ds. \tag{2.6}$$

The Green function in (2.7) is

$$ G(t,s) = \begin{cases} \dfrac{(1-s)^{3-q}(t-s)^{q-1}}{(q-1)(q-2)} + \alpha t - \dfrac{t(1-s)}{q-2}, & s < t, \\ \alpha t - \dfrac{t(1-s)}{q-2}, & t \le s. \end{cases} \tag{2.7}$$

(3) Equation (2.11) becomes

$$ \begin{aligned} u(t) = {} & \frac{1}{\Gamma(q-2)} \times \int_0^t \left[\frac{(t-s)^{q-1}}{(q-1)(q-2)} + \frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \right] \times y(s)\, ds \\ & + \frac{1}{\Gamma(q-2)} \int_t^1 \left[\frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \right] y(s)\, ds. \end{aligned} \tag{2.11}$$

Equation (2.12) must be

$$ u(t) = \frac{1}{\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(t,s)\, y(s)\, ds. \tag{2.12}$$

(4) Equation (3.1) must be

$$ Tu(t) = \frac{1}{\Gamma(q-2)} \times \int_0^1 \frac{1}{(1-s)^{3-q}} G(t,s)\, f\left(s, u(s), {}^{c}D_{0^+}^{\sigma}u(s)\right) ds. \tag{3.1}$$

In Theorem 3.2, the condition (3.5) must be

$$ C_g + C_h < \frac{1}{2}, \qquad A_g + A_h < \frac{\Gamma(2-\sigma)}{2}. \tag{3.5}$$

(5) Equation (3.12) must be

$$|Tu - Tv| < \frac{1}{2}\|u - v\|, \tag{3.12}$$

and (3.13) becomes

$${}^{c}D_{0^+}^{\sigma}Tu - {}^{c}D_{0^+}^{\sigma}Tv = \frac{1}{\Gamma(1-\sigma)}\int_0^t \frac{(Tu)'(s) - (Tv)'(s)}{(t-s)^{\sigma}}ds. \tag{3.13}$$

(6) Equation (3.14) is

$$G_1(t,s) = \frac{\partial G(t,s)}{\partial t} = \begin{cases} \dfrac{(1-s)^{3-q}(t-s)^{q-2}}{(q-2)} + \alpha - \dfrac{(1-s)}{q-2}, & s < t, \\ \alpha - \dfrac{(1-s)}{q-2}, & t \le s. \end{cases} \tag{3.14}$$

(7) Equation (3.15) is as follows:

$$\begin{aligned} &{}^{c}D_{0^+}^{\sigma}Tu - {}^{c}D_{0^+}^{\sigma}Tv \\ &\quad = \frac{1}{\Gamma(q-2)\Gamma(1-\sigma)} \\ &\qquad \times \int_0^t \int_0^1 (t-s)^{-\sigma}\frac{1}{(1-r)^{3-q}}G_1(s,r) \\ &\qquad\quad \times \left(f\left(r, u(r), {}^{c}D_{0^+}^{\sigma}u(r)\right)\right. \\ &\qquad\qquad \left.- f\left(r, v(r), {}^{c}D_{0^+}^{\sigma}v(r)\right)\right) dr\, ds. \end{aligned} \tag{3.15}$$

Equation (3.16) is as follows:

$$\begin{aligned} &\left|{}^{c}D_{0^+}^{\sigma}Tu - {}^{c}D_{0^+}^{\sigma}Tv\right| \\ &\quad \le \frac{\max|u - v|}{\Gamma(q-2)\Gamma(1-\sigma)} \\ &\qquad \times \int_0^t \int_0^1 (t-s)^{-\sigma}\frac{1}{(1-r)^{3-q}}\left|G_1(s,r)\right| g(r)\, dr\, ds \\ &\qquad + \frac{\max\left|{}^{c}D_{0^+}^{\sigma}u - {}^{c}D_{0^+}^{\sigma}v\right|}{\Gamma(q-2)\Gamma(1-\sigma)} \\ &\qquad \times \int_0^t \int_0^1 (t-s)^{-\sigma}\frac{1}{(1-r)^{3-q}}\left|G_1(s,r)\right| h(r)\, dr\, ds. \end{aligned} \tag{3.16}$$

Equation (3.17) is as follows:

$$\begin{aligned} &\int_0^1 \frac{1}{(1-r)^{3-q}}\left|G_1(s,r)\right| g(r)\, dr \\ &\quad \le \Gamma(q-2)\left(2I_{0^+}^{q-1}g(1) + |\alpha|\, I_{0^+}^{q-2}g(1)\right). \end{aligned} \tag{3.17}$$

(8) Equation (3.18) is as follows:

$$\left|{}^{c}D_{0^+}^{\sigma}Tu - {}^{c}D_{0^+}^{\sigma}Tv\right| \le \|u - v\| \frac{1}{\Gamma(2-\sigma)}\left(A_g + A_h\right). \tag{3.18}$$

Equation (3.19) becomes

$$\left|{}^{c}D_{0^+}^{\sigma}Tu - {}^{c}D_{0^+}^{\sigma}Tv\right| \le \frac{1}{2}\|u - v\|. \tag{3.19}$$

Equation (3.21) is as follows:

$$\text{a.e } (t, x, \overline{x}) \in [0,1] \times \mathbb{R}^2. \tag{3.21}$$

Equation (3.22) is as follows:

$$\left(\psi(r) + \phi(r) + 1\right)\left(\frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)}\right) < r. \tag{3.22}$$

Equation (3.29) becomes

$$\left|{}^{c}D_{0^+}^{\sigma}Tu\right| \le \frac{C_2}{\Gamma(2-\sigma)}\left(\psi(r) + \phi(r) + 1\right). \tag{3.29}$$

Equation (3.30) is

$$\|Tu\| = \left(\psi(r) + \phi(r) + 1\right)\left(\frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)}\right). \tag{3.30}$$

Equation (3.32) is as follows:

$$\begin{aligned} &\left|Tu(t_1) - Tu(t_2)\right| \\ &\quad \le \frac{C}{\Gamma(q-2)} \\ &\qquad \times \int_0^{t_1} \frac{(t_2 - s)^{q-1} - (t_1 - s)^{q-1}}{(q-1)(q-2)} \\ &\qquad\qquad + (t_2 - t_1)\left(\frac{|\alpha|}{(1-s)^{3-q}} - \frac{(1-s)^{q-2}}{q-2}\right)ds \\ &\qquad + \int_{t_1}^{t_2} \frac{(t_2 - s)^{q-1}}{(q-1)(q-2)} \\ &\qquad\qquad + (t_2 - t_1)\left(\frac{|\alpha|}{(1-s)^{3-q}} - \frac{(1-s)^{q-2}}{q-2}\right)ds \\ &\qquad + \int_{t_2}^{1} (t_1 - t_2)\left(\frac{|\alpha|}{(1-s)^{3-q}} - \frac{(1-s)^{q-2}}{q-2}\right)ds. \end{aligned} \tag{3.32}$$

Equation (3.33) is as follows:

$$\begin{aligned}
&|Tu(t_1) - Tu(t_2)| \\
&\quad \le \frac{C}{\Gamma(q-2)} \\
&\qquad \times \Bigg[(t_2 - t_1) \\
&\qquad\quad \times \int_0^{t_1} \frac{1}{(q-2)} \\
&\qquad\qquad + \left(\frac{|\alpha|}{(1-s)^{3-q}} + \frac{(1-s)^{q-2}}{q-2} \right) ds \\
&\qquad\quad + \int_{t_1}^{t_2} \frac{(t_2-s)^{q-1}}{(q-2)(q-1)} \\
&\qquad\qquad + (t_2 - t_1) \left(\frac{|\alpha|}{(1-s)^{3-q}} + \frac{(1-s)^{q-2}}{q-2} \right) ds \\
&\qquad\quad + (t_1 - t_2) \int_{t_2}^{1} \left(\frac{|\alpha|}{(1-s)^{3-q}} + \frac{(1-s)^{q-2}}{q-2} \right) ds \Bigg].
\end{aligned} \tag{3.33}$$

Equation (3.34) is as follows:

$$\begin{aligned}
&|Tu(t_1) - Tu(t_2)| \\
&\quad \le \frac{C(t_2 - t_1)}{\Gamma(q-2)} \frac{4 + 3|\alpha|}{(q-2)} \\
&\qquad + \frac{C}{\Gamma(q-2)} \int_{t_1}^{t_2} \frac{(t_2 - s)^{q-1}}{(q-2)(q-1)} ds.
\end{aligned} \tag{3.34}$$

Equation (3.35) is as follows:

$$\begin{aligned}
&\left| {}^cD_{0^+}^{\sigma} Tu(t_1) - {}^cD_{0^+}^{\sigma} Tu(t_2) \right| \\
&\quad \le \frac{1}{\Gamma(1-\sigma)} \int_0^{t_1} \left((t_1 - s)^{-\sigma} - (t_2 - s)^{-\sigma} \right) \\
&\qquad\qquad \times \left| (Tu(s))' \right| ds \\
&\qquad + \frac{1}{\Gamma(1-\sigma)} \int_{t_1}^{t_2} (t_2 - s)^{-\sigma} \left| (Tu(s))' \right| ds.
\end{aligned} \tag{3.35}$$

Equation (3.37) is as follows:

$$\begin{aligned}
&\left| {}^cD_{0^+}^{\sigma} Tu(t_1) - {}^cD_{0^+}^{\sigma} Tu(t_2) \right| \\
&\quad \le \Big([\psi(r) + \phi(r) + 1] \\
&\qquad \times C_2 \left[2(t_2 - t_1)^{1-\sigma} + t_2^{1-\sigma} - t_1^{1-\sigma} \right] \Big) \\
&\qquad \times (\Gamma(2-\sigma))^{-1}.
\end{aligned} \tag{3.37}$$

Equation (3.39) is as follows:

$$\left| {}^cD_{0^+}^{\sigma} u(t) \right| \le \frac{C_2}{\Gamma(2-\sigma)} (\psi(r) + \phi(r) + 1). \tag{3.39}$$

Equation (3.40) is as follows:

$$\|u\| \le (\psi(r) + \phi(r) + 1) \left(\frac{C_1}{\Gamma(q-2)} + \frac{C_2}{\Gamma(2-\sigma)} \right) < r. \tag{3.40}$$

(9) (H2) $0 < \int_0^1 (1/(1-s)^{3-q}) G(s,s) a(s) ds < \infty$.
In Lemma 4.1, we have $\alpha \ge 1/(q-2)$ and (4.1) becomes: If $t, s \in [\tau, 1]$, $\tau > 0$, then

$$\begin{aligned}
&0 < \tau G(s,s) \le G(t,s) \le \frac{2}{\tau^2} G(s,s), \\
&0 < G_1(s,s) \le G_1(t,s) \le \frac{2}{\tau} G_1(s,s).
\end{aligned} \tag{4.1}$$

Equation (4.2) should be

$$\begin{aligned}
&\frac{\alpha t}{(1-s)^{3-q}} - \frac{t(1-s)^{q-2}}{q-2} \\
&\quad = \frac{t}{(q-2)(1-s)^{3-q}} [(q-2)\alpha - 1 + s]
\end{aligned} \tag{4.2}$$

which is positive if $\alpha \ge 1/(q-2)$.

Equation (4.3): let $t, s \in [\tau, 1]$; it is easy to see that $G(s,s) \ne 0$, and then we have

$$\begin{aligned}
\frac{G(t,s)}{G(s,s)} &= \frac{(t-s)^{q-1}(1-s)^{3-q}}{(q-1)s[(q-2)\alpha - 1 + s]} + \frac{t}{s} \\
&\le \frac{1 + (1-s)^2}{s^2} \\
&\le \frac{2}{\tau^2}, \quad 0 < \tau \le s \le t \le 1, \\
\frac{G(t,s)}{G(s,s)} &= \frac{t}{s} \le \frac{2}{\tau} \le \frac{2}{\tau^2}, \quad 0 < \tau \le t \le s \le 1.
\end{aligned} \tag{4.3}$$

Finally, since $G(s,s)$ is nonnegative, we obtain $0 < \tau G(s,s) \le G(t,s) \le (2/\tau^2) G(s,s)$.

(10) In Lemma 4.3, put $\alpha \ge 1/(q-2)$ and inequality (4.5) becomes

$$\min_{t \in [\tau, 1]} \left(u(t) + {}^cD_{0^+}^{\sigma} u(t) \right) \ge \frac{\tau^3}{2} \|u\|. \tag{4.5}$$

Equation (4.7) is as follows:

$$\begin{aligned}
u(t) &\le \frac{2}{\tau^2 \Gamma(q-2)} \\
&\quad \times \int_0^1 \frac{1}{(1-s)^{3-q}} G(s,s) a(s) f_1\left(u(s), {}^cD_{0^+}^{\sigma} u(s)\right) ds.
\end{aligned} \tag{4.7}$$

Equation (4.8) is as follows:

$$\begin{aligned}{}^{c}D_{0^+}^{\sigma}u(t) &= \frac{1}{\Gamma(q-2)\Gamma(1-\sigma)} \\ &\quad\times\int_0^t\int_0^1 (t-s)^{-\sigma}\frac{1}{(1-r)^{3-q}}G_1(s,r) \\ &\quad\times a(r)\, f_1\left(u(r), {}^{c}D_{0^+}^{\sigma}u(r)\right) ds\, dr \\ &\le \frac{2}{\tau\Gamma(q-2)\Gamma(2-\sigma)} \\ &\quad\times\int_0^1 \frac{1}{(1-r)^{3-q}}G_1(r,r)\, a(r) \\ &\quad\times f_1\left(u(r), {}^{c}D_{0^+}^{\sigma}u(r)\right) dr.\end{aligned} \tag{4.8}$$

Equation (4.9) is as follows:

$$\begin{aligned}\|u\| &\le \frac{2}{\tau^2\Gamma(q-2)} \\ &\quad\times\int_0^1 \frac{1}{(1-s)^{3-q}}\left[G(s,s)+\frac{G_1(s,s)}{\Gamma(2-\sigma)}\right] \\ &\quad\times a(s)\, f_1\left(u(s), {}^{c}D_{0^+}^{\sigma}u(s)\right) ds.\end{aligned} \tag{4.9}$$

Equation (4.10) is as follows:

$$\begin{aligned}&\int_0^1 \frac{1}{(1-s)^{3-q}}\left[G(s,s)+\frac{G_1(s,s)}{\Gamma(2-\sigma)}\right] \\ &\quad\times a(s)\, f_1\left(u(s), {}^{c}D_{0^+}^{\sigma}u(s)\right) ds \\ &\quad\ge \frac{\tau^2\Gamma(q-2)}{2}\|u\|.\end{aligned} \tag{4.10}$$

Equation (4.11): in view of the left hand side of (4.1), we obtain for all $t \in [\tau, 1]$

$$\begin{aligned}u(t) &\ge \frac{\tau}{\Gamma(q-2)} \\ &\quad\times\int_0^1 \frac{1}{(1-s)^{3-q}}G(s,s)\, a(s)\, f_1\left(u(s), {}^{c}D_{0^+}^{\sigma}u(s)\right) ds.\end{aligned} \tag{4.11}$$

Equation (4.12) is as follows:

$$\begin{aligned}{}^{c}D_{0^+}^{\sigma}u(t) &\ge \frac{1}{\Gamma(q-2)\Gamma(2-\sigma)} \\ &\quad\times\int_0^1 \frac{1}{(1-r)^{3-q}}G_1(r,r)\, a(r) \\ &\quad\times f_1\left(u(r), {}^{c}D_{0^+}^{\sigma}u(r)\right) dr.\end{aligned} \tag{4.12}$$

Equation (4.13) is as follows:

$$\begin{aligned}&\min_{t\in[\tau,1]}\left(u(t)+{}^{c}D_{0^+}^{\sigma}u(t)\right) \\ &\quad\ge \frac{\tau}{\Gamma(q-2)}\int_0^1 \frac{1}{(1-s)^{3-q}} \\ &\quad\times\left[G(s,s)+\frac{G_1(s,s)}{\Gamma(2-\sigma)}\right] \\ &\quad\times a(s)\, f_1\left(u(s), {}^{c}D_{0^+}^{\sigma}u(s)\right) ds.\end{aligned} \tag{4.13}$$

Equation (4.14) is as follows:

$$\min_{t\in[\tau,1]}\left(u(t)+{}^{c}D_{0^+}^{\sigma}u(t)\right) \ge \frac{\tau^3}{2}\|u\|. \tag{4.14}$$

Equation (4.17) is as follows:

$$K = \left\{u \in E^+, \min_{t\in[\tau,1]}\left(u(t)+{}^{c}D_{0^+}^{\sigma}u(t)\right) \ge \frac{\tau^3}{2}\|u\|\right\}. \tag{4.17}$$

Equation (4.19) is as follows:

$$\begin{aligned}Tu(t) &= \frac{1}{\Gamma(q-2)} \\ &\quad\times\int_0^1 \frac{1}{(1-s)^{3-q}} \\ &\quad\times G(t,s)\, a(s)\, f_1\left(u(s), {}^{c}D_{0^+}^{\sigma}u(s)\right) ds \\ &\le \frac{2\varepsilon\|u\|}{\tau^2\Gamma(q-2)}\int_0^1 \frac{1}{(1-s)^{3-q}}G(s,s)\, a(s)\, ds.\end{aligned} \tag{4.19}$$

Equation (4.20) is as follows:

$$\begin{aligned}&{}^{c}D_{0^+}^{\sigma}Tu(t) \\ &\quad= \frac{1}{\Gamma(q-2)\Gamma(1-\sigma)} \\ &\quad\times\int_0^t\int_0^1 \frac{1}{(1-r)^{3-q}}(t-s)^{-\sigma}G_1(s,r) \\ &\quad\times a(r)\, f_1\left(u(r), {}^{c}D_{0^+}^{\sigma}u(r)\right) ds\, dr \\ &\quad\le \frac{2}{\tau\Gamma(q-2)\Gamma(2-\sigma)} \\ &\quad\times\int_0^1 \frac{1}{(1-r)^{3-q}}G_1(r,r)\, a(r) \\ &\quad\times\left(|u(r)|+\left|{}^{c}D_{0^+}^{\sigma}u(r)\right|\right) dr \\ &\quad\le \frac{2\varepsilon\|u\|}{\tau^2\Gamma(q-2)\Gamma(2-\sigma)}\int_0^1 \frac{1}{(1-s)^{3-q}}G_1(s,s)\, a(s)\, ds.\end{aligned} \tag{4.20}$$

Equation (4.21) is as follows:

$$\|Tu\| \leq \frac{2\varepsilon \|u\|}{\tau^2\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} \times \left[G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)}\right] a(s)\,ds. \quad (4.21)$$

Equation (4.22) is as follows:

$$\varepsilon \leq \tau^2\Gamma(q-2) \times \left(2\int_0^1 \frac{1}{(1-s)^{3-q}} \times \left[G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)}\right] a(s)\,ds\right)^{-1}. \quad (4.22)$$

Let

$$R = \max\left\{2R_1, \frac{2R_2}{\tau^3}\right\}. \quad (1)$$

Equation (4.23) is as follows:

$$\min_{t\in[\tau,1]} \left(u(t) + {}^cD_{0^+}^{\sigma}u(t)\right) \geq \frac{\tau^3}{2}\|u\| = \frac{\tau^3}{2}R \geq R_2. \quad (4.23)$$

Using the left hand side of (4.1) and Lemma 4.1, we obtain (4.24):

$$Tu(t) \geq \frac{\tau^4 M\|u\|}{2\Gamma(q-2)} \int_0^1 \frac{1}{(1-s)^{3-q}} G(s,s)\,a(s)\,ds. \quad (4.24)$$

Equation (4.25) is as follows:

$${}^cD_{0^+}^{\sigma}Tu(t) \geq \frac{\tau^3 M\|u\|}{2\Gamma(q-2)\Gamma(2-\sigma)} \times \int_0^1 \frac{1}{(1-s)^{3-q}} G_1(s,s)\,a(s)\,ds. \quad (4.25)$$

Equation (4.26) is as follows:

$$Tu(t) + {}^cD_{0^+}^{\sigma}Tu(t) \geq \frac{\tau^4 M\|u\|}{2\Gamma(q-2)} \times \int_0^1 \frac{1}{(1-s)^{3-q}} \left[G(s,s) + \frac{G_1(s,s)}{\Gamma(2-\sigma)}\right] \times a(s)\,ds. \quad (4.26)$$

Equation (4.27) is as follows:

$$M \geq 2\Gamma(q-2) \times \left(\tau^4 \int_0^1 \left[\left(1/(1-s)^{3-q}\right) G(s,s) + G_1(s,s)/\Gamma(2-\sigma)\right] a(s)\,ds\right)^{-1}. \quad (4.27)$$

(11) In Example 4.6, if we choose $\sigma = 1/4 < 1$; then we get the same results with

$$C_g + C_h = 0.49821 < \frac{1}{2}, \qquad A_g + A_h = 0.42552 < \frac{\Gamma(2-\sigma)}{2} = 0.459. \quad (2)$$

In Example 4.7, choose $\sigma = 1/5$, $\psi(x) = (x/10)^2 + 1$, and $\phi(\overline{x}) = \ln(1+\overline{x}^2)/9 + 1$; then we get the same results.

Remark 1. One can study the problem (P1) for $1 < \sigma < 2$ and the function f depending only on t and u instead of $f(t, u(t), {}^cD_{0^+}^{\sigma}u(t))$.