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Research Article

Nonlinear H_∞ Optimal Control Scheme for an Underwater Vehicle with Regional Function Formulation

Zool H. Ismail[1] and Matthew W. Dunnigan[2]

[1] *Centre for Artificial Intelligence & Robotics, Universiti Teknologi Malaysia, Jalan Semarak, 54100 Kuala Lumpur, Malaysia*
[2] *School of Engineering and Physical Sciences, Heriot-Watt University, Edinburgh EH14 4AS, UK*

Correspondence should be addressed to Zool H. Ismail; zool@fke.utm.my

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A conventional region control technique cannot meet the demands for an accurate tracking performance in view of its inability to accommodate highly nonlinear system dynamics, imprecise hydrodynamic coefficients, and external disturbances. In this paper, a robust technique is presented for an Autonomous Underwater Vehicle (AUV) with region tracking function. Within this control scheme, nonlinear H_∞ and region based control schemes are used. A Lyapunov-like function is presented for stability analysis of the proposed control law. Numerical simulations are presented to demonstrate the performance of the proposed tracking control of the AUV. It is shown that the proposed control law is robust against parameter uncertainties, external disturbances, and nonlinearities and it leads to uniform ultimate boundedness of the region tracking error.

1. Introduction

A valuable robotic system for the ocean environment is known as an Autonomous Underwater Vehicle (AUV). It has been used for many years in the oil and gas industry to obtain detailed maps of the ocean floor as well as to supervise pipeline activities [1]. The ongoing research on AUVs has given attention to the improvement of navigation and tracking control schemes. The conventional control methodologies are not the most suitable choice and they cannot guarantee the required tracking performance since an underwater vehicle exhibits inherent highly nonlinear system dynamics, imprecise hydrodynamic coefficients, and external disturbances. On the other hand, sliding mode control, due to its robustness against modelling inaccuracies and external disturbances, has been demonstrated to be a very attractive approach to cope with these problems [2–6]. However, a well-known drawback of conventional sliding mode controllers is the chattering effect. Therefore, to overcome the undesired effects of the control chattering, the authors in [7, 8] proposed a saturation function rather than a sign function. This substitution can minimize or, when desired, even completely eliminate chattering, but the trajectory tracking error is uniformly ultimately bounded (UUB), which in fact means that a steady-state error will always remain. In order to enhance the tracking performance inside the boundary layer, some adaptive strategy should be used for uncertainty/disturbance compensation.

Recently, a nonlinear H_∞ optimal control scheme was adopted for an underwater robotic system as an external tracking control loop and a disturbance observer was used as an internal disturbance compensation loop [9, 10]. The resultant control obtained by combining these two controls is then derived. A brief review of H_∞ optimality control has been presented in [11]. Moreover, the disturbance observer [12] is chosen, so that the L_2-gain conditions of the nonlinear H_∞ optimal control are relaxed, the magnitude of extended disturbances is reduced, and the robustness of the resulting control is improved without increasing the control input beyond that of the nonlinear H_∞ optimal control alone. By using this control, the underwater robotic system can successfully follow the given trajectories, even when uncertainties and disturbances exist. An adaptive region tracking control was presented in [13] for an AUV where a region is used rather than a point due to minimizing the control effort

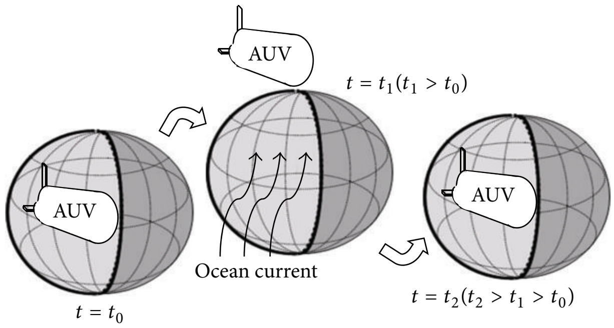

Figure 1: An illustration of an underwater robotic system which performs tracking task in a spherical region.

to track the region. Note that the total potential energy of the desired region is a summation of the potential energy associated with each region. Inspired from [13], some related research works such as in [14, 15] have been carried out to ensure that the marine robotic systems can cope with the underwater conditions and missions.

In this paper, a nonlinear H_∞ optimal control with region tracking function is proposed for an underwater vehicle. The proposed dynamic region control, where it is formulated in task space, aims to reduce the energy consumed by vehicle thrusters. Within the region function formulation, the controller activates and sends commands to the thrusters only when the AUV is outside the desired region, and hence it significantly reduces energy consumption. However, the disturbances such as ocean currents may pull the underwater vehicle out of its desired region. This is likely to occur when the AUV navigates near to the boundary as illustrated in Figure 1. Hence, a nonlinear H_∞ optimal control is proposed in this paper to counteract this problem. The performance of conventional region tracking control and region function adopted with nonlinear H_∞ optimal control law can be observed with respect to the existence of unidirectional and bounded ocean current. The rest of the paper is organized as follows: Section 2 describes the kinematic and dynamic properties of an AUV. In Section 3, the nonlinear H_∞ optimal control with region function formulation is briefly explained. The stability analysis using a Lyapunov-like function is also given in this section. In Section 4, numerical simulation results are provided to demonstrate the performance of the proposed control. Finally, the paper is concluded with some remarks in Section 5.

2. Kinematic and Dynamic Model of an AUV

2.1. Kinematic Model. The relationship between inertial and body-fixed vehicle velocity can be described using the Jacobian matrix $J(\eta_2)$ in the following form:

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} J_1(\eta_2) & 0_{3\times3} \\ 0_{3\times3} & J_2(\eta_2) \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \Longleftrightarrow \dot{\eta} = J(\eta_2)\,\nu, \quad (1)$$

where $\eta_1 = [x \ y \ z]^T \in \mathbb{R}^3$ and $\eta_2 = [\phi \ \theta \ \psi]^T \in \mathbb{R}^3$ denote the position and the orientation of the vehicle, respectively, expressed in the inertial fixed frame. J_1 and J_2 are the transformation matrices expressed in terms of the Euler angles. The linear and angular velocity vectors, $\nu_1 = [u \ v \ w]^T \in \mathbb{R}^3$ and $\nu_2 = [p \ q \ r]^T \in \mathbb{R}^3$, respectively, are described in terms of the body-fixed frame.

2.2. Dynamic Model. Let the velocity state vector with respect to the body-fixed frame be defined by $\nu \in \mathbb{R}^6$; and the underwater vehicle dynamic equation can be expressed in closed form as [16]

$$M\dot{\nu} + C(\nu)\,\nu + g(\eta) + F_{\text{ext}} = \tau, \quad (2)$$

where M and $C(\nu)$ represent the inertia matrix and the Coriolis and centripetal forces matrix including the effects of added mass and hydrodynamic damping by body motion and $g(\eta)$ is the restoring force. F_{ext} contains the effects of external disturbances and the effects of added mass and hydrodynamic damping by body motion in static water. The dynamic (2) preserves the following properties [16, 17].

Property 1. The inertia matrix M is symmetric and positive definite such that $M = M^T > 0$ and $\gamma I \le M \le \Upsilon I$.

Property 2. $C(\nu)$ is the skew-symmetric matrix such that $C(\nu) = -C^T(\nu)$.

In the Property 1, γ and Υ denote the minimum and maximum eigenvalues of the inertia matrix, respectively. The matrix I is the identity matrix that has suitable dimension.

3. Nonlinear H_∞ Optimal Control Law with Region Formulation

In the region-based control framework, the desired moving target is specified by a region at the desired trajectory. A robust nonlinear H_∞ optimal control for AUV proposed in this paper is formulated as follows.

First, the vehicle needs to converge into a region with specific shape. The objective function for this region is defined by the following:

$$f(\delta\eta_B) \le 0, \quad (3)$$

where $\delta\eta_B = B(\eta - \eta_d) \in \mathbb{R}^6$ are the continuous first partial derivatives of the dynamic region; $\eta_d(t)$ is the time-varying reference point inside the geometric shape and $B(t)$ is a time-varying and nonsingular scaling factor. It is assumed that $\eta_d(t)$ and $B(t)$ are bounded functions of time. To achieve the scaling formation, that is, if the scaling factor increases, then the size of a desired region also increases, a nonsingular matrix is defined as follows:

$$B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, \quad (4)$$

where B_1 is the scaling matrix of η_1 and B_2 is the scaling matrix of η_2. This function is useful when the AUV needs to adapt the moving region, depending on the situation and environment.

The corresponding potential energy function for the desired region described in (3) can be specified as

$$P(\delta\eta_B) = \begin{cases} 0, & f(\delta\eta_B) \le 0 \\ \dfrac{k_p}{2} f^2(\delta\eta_B), & f(\delta\eta_B) > 0, \end{cases} \tag{5}$$

such that

$$P(\delta\eta_B) = \frac{k_p}{2}\left[\max\left(0, f(\delta\eta_B)\right)\right]^2, \tag{6}$$

where k_p is a positive scalar. Differentiating (6) with respect to $\delta\eta_B$ gives

$$\left(\frac{\partial P(\delta\eta_B)}{\partial\eta_B}\right)^T = k_p \max\left(0, f(\delta\eta_B)\right)\left(\frac{\partial f(\delta\eta_B)}{\partial\eta_B}\right)^T. \tag{7}$$

Now, let (7) be represented as the region error $\widetilde{e}_B$ in the following form:

$$\widetilde{e}_B = \max\left(0, f(\delta\eta_B)\right)\left(\frac{\partial f(\delta\eta_B)}{\partial\eta_B}\right)^T. \tag{8}$$

If B is set to an identity matrix, then a useful vector v_r is defined as

$$v_r = J^{-1}\dot{\eta}_d - \alpha J^{-1}\widetilde{e}_B - \beta J^{-1}\int \widetilde{e}_B\, dt, \tag{9}$$

where α and β are arbitrary positive constants. The matrix J^{-1} represents the inverse of the Jacobian matrix. From the arguments of trigonometric functions, this matrix is bounded. Based on the structure of (8) and (9) and the subsequent stability analysis, a filtered tracking error vector for an underwater vehicle is defined as

$$r(t) = v - J^{-1}\dot{\eta}_d + \alpha J^{-1}\widetilde{e}_B + \beta J^{-1}\int \widetilde{e}_B\, dt. \tag{10}$$

From the definition of r in (10), the control law for an AUV can be proposed in the following form:

$$\tau = -K_v r + \widehat{M}\dot{v}_r + \widehat{C}(v)\, v_r + \widehat{g}(\eta), \tag{11}$$

where $K_v = K + (1/\kappa^2)I$; K and I are the positive definite matrix and identity matrix, respectively. $\widehat{M}$, $\widehat{C}(v)$, and $\widehat{g}(\eta)$ are the nominal matrices and vectors of M, $C(v)$, and $g(\eta)$, respectively. The derivative of v_r in (9) is given as

$$\begin{aligned} \dot{v}_r = {} & \dot{J}^{-1}\dot{\eta}_d + J^{-1}\ddot{\eta}_d - \alpha\dot{J}^{-1}\widetilde{e}_B - \alpha J^{-1}\dot{\widetilde{e}}_B \\ & - \beta\dot{J}^{-1}\int \widetilde{e}_B\, dt - \beta J^{-1}\widetilde{e}_B, \end{aligned} \tag{12}$$

where $\dot{\eta}_d(t)$, $\ddot{\eta}_d(t)$, and $\dot{J}^{-1}(t)$ are all assumed to be bounded functions of time. Substituting (11) into (2) produces a closed-loop dynamic equation for $r(t)$ as follows:

$$M\dot{r} + C(v)\, r + K_v r - \omega = 0, \tag{13}$$

where ω is the extended disturbance vector which is defined in the following form:

$$\omega = \widetilde{M}\dot{v}_r + \widetilde{C}(v)\, v_r + \widetilde{g}(\eta) - F_{\text{ext}}, \tag{14}$$

where $\widetilde{(\cdot)} = \widehat{(\cdot)} - (\cdot)$ denotes the parameter estimation error. The modeling error acts as a disturbance in (14) when the AUV is in motion. Note that κ is the L_2 gain for disturbance attenuation satisfying the following condition:

$$\int_0^T z^T z\, dt = \int_0^T \omega^T\omega\, dt, \tag{15}$$

where $z^T z$ is defined as the weighted sum of the quadratic forms of the error states and the control input. Since nonlinear H_∞ optimal control scheme is based on feedback tracking errors, $z^T z$ can be approximated up to magnitude of these errors.

Remark 1. Equation (13) can be represented in state space such that the nonlinear H_∞ optimality satisfies [18]

$$\int_0^T \left\{x^T Q x + u^T R u\right\} dt = \kappa^2 \int_0^T \omega^T\omega\, dt \tag{16}$$

with $\alpha^2 > 2\beta$. x and u in (16) denote the state and input variables, respectively. Meanwhile, the matrices Q and R are state weighting and input weighting matrices, respectively, and they are determined by inverse optimal problem with respect to specific L_2 attenuation gain, κ.

Theorem 2. *Let the filtered tracking error vector r be upper bounded as the following form:*

$$\|r\| \le \sqrt{\frac{\Upsilon}{\gamma}\frac{\kappa^2}{\sqrt{2k_m\kappa^2 + 1}}}\|\omega\|_\infty, \tag{17}$$

where k_m is scalar constant and $\|\omega\|_\infty$ denotes an infinity norm of ω for a given time interval. Then, the control law (11) *above is continuous and the closed-loop system is uniformly ultimately bounded (u.u.b) as defined in [11].*

Proof. The following nonnegative function is introduced to analyze the stability of the proposed control law:

$$V = \frac{1}{2} r^T M r. \tag{18}$$

Differentiating V with respect to time and utilizing (10) and (14), a closed-loop dynamic (13) yields

$$\dot{V} = -r^T C(v)\, r - r^T K_v r + r^T\omega. \tag{19}$$

Simplifying (19) leads to

$$\begin{aligned} \dot{V} &= -r^T K_v r + r^T\omega \\ &= -r^T\left(K + \left(\frac{1}{\kappa^2}\right)I\right)r + r^T\omega \\ &= -r^T\left(K + \left(\frac{1}{2\kappa^2}\right)I\right)r \\ &\quad - \frac{\kappa^2}{2}\left|\frac{1}{\kappa^2}r - \omega\right|^2 + \frac{\kappa^2}{2}\|\omega\|_\infty^2, \end{aligned} \tag{20}$$

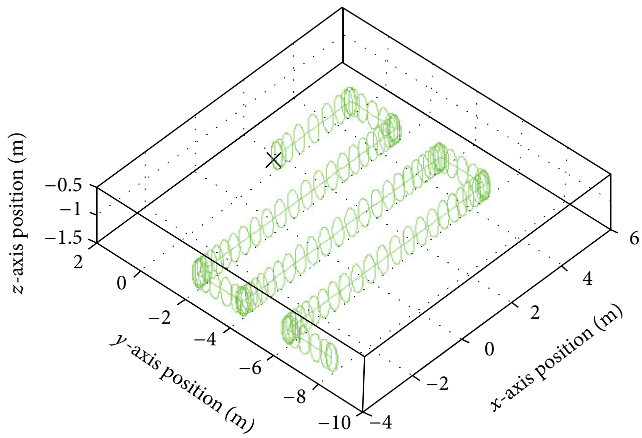

FIGURE 2: The desired lawnmower trajectory where "×" marks the initial position of the AUV.

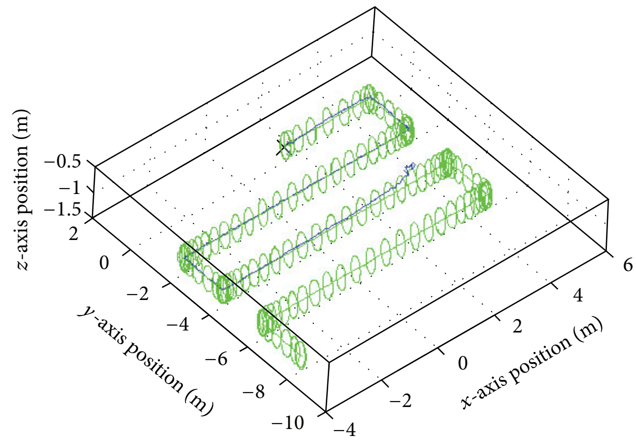

FIGURE 3: Three-dimensional view for conventional region tracking control.

TABLE 1: Simulation result: vehicle forces for four vertical thrusters (N).

	Set-point tracking control	Nonlinear H_∞ optimal controller
Thrusters 1	45.20	42.20
Thrusters 2	71.38	66.80
Thrusters 3	45.38	42.78
Thrusters 4	66.96	65.69
Total input	116.97	111.30

where Property 2 is used. Let k_m be the minimum diagonal element of gain matrix K utilizing the worst case disturbance [11] to yield the following inequality:

$$\dot{V} \leq -\left(k_m + \frac{1}{2\kappa^2}\right)\|r\|^2 + \frac{\kappa^2}{2}\|\omega\|_\infty^2. \tag{21}$$

From (21), it is necessary to choose sufficiently large value of k_m to ensure the negative definiteness of $\dot{V}$. Therefore, implying the results and terminology of [19], the ultimate boundedness of $\|r\|$ can be obtained as in (17). □

Remark 3. It is assumed that the norm of extended disturbance which includes tracking errors is not deviated largely, when a control input (11) is used in (2). Thus, the control gain can be changed according to (17), so that the satisfactory performance of proposed control law with region function formulation can be achieved.

4. Simulation Results

In this section, simulation studies are carried out to assess the effectiveness of the proposed nonlinear H_∞ optimal control law with region function formulation for an underwater vehicle. The performance of conventional tracking control and the proposed technique is observed concerning two cases: the first case is the conventional region tracking control and the second case is where the region function is adopted with nonlinear H_∞ optimal control law. Both control laws are observed with respect to the existence of random disturbances and bounded ocean current. The ODIN AUV [20, 21] that is known as a near-spherical omnidirectional vehicle equipped with four horizontal thrusters and four vertical thrusters is chosen as the Autonomous Underwater Vehicle model in these numerical simulations. The following inequality function is defined as

$$\begin{aligned} f(\delta\eta_B) = {} & s_x(x-x_0)^2 + s_y(y-y_0)^2 \\ & + s_z(z-z_0)^2 + s_\phi(\phi-\phi_0)^2 \\ & + s_\theta(\theta-\theta_0)^2 + s_\psi(\psi-\psi_0)^2 \leq \kappa_r^2, \end{aligned} \tag{22}$$

where the element of $\{s_x, s_y, s_z, s_\phi, s_\theta, s_\psi\}$ is the component of the time-varying scaling matrix B and κ_r is a scalar tolerance. In these simulations, the matrix B is defined as the identity matrix and κ_r is set to 0.25. Note that (22) can also be represented as the root mean square error for all axes. In Table 1, the norm values of required forces for four vertical thrusters are presented. The total control input is included to signify the overall energy needed for the system to maintain at depth −1.2 meter. Notice that when the proposed controller is utilized, the energy requirement is reduced as compared with set-point tracking method.

The underwater vehicle is required to track a predefined trajectory as illustrated in Figure 2 where the green (cross-section) path is the horizontal basis position initialized at the position $[1.5 \;\; 0 \;\; -1.2]^T$ m. Moreover, the vehicle is initialized at the same position $\eta_1(0) = [1.5 \;\; 0 \;\; -1.2]^T$ m while its attitude is kept constant during simulation and the initial values are $\eta_2(0) = [0 \;\; 0 \;\; 0]^T$ degrees. From Figures 3, 4, 5, and 6, it has been shown that the proposed control scheme

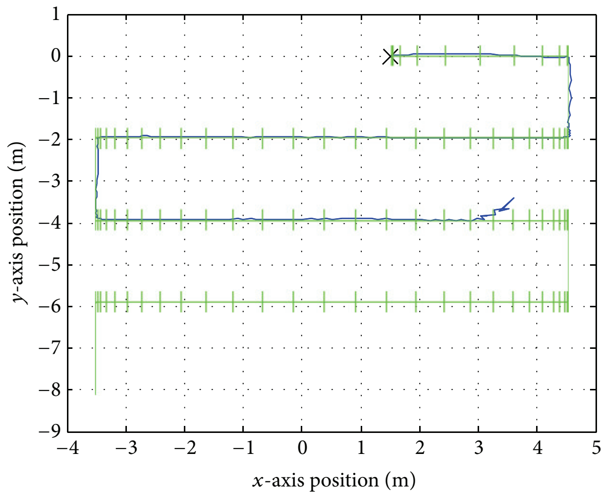

Figure 4: Planar view for conventional region tracking control.

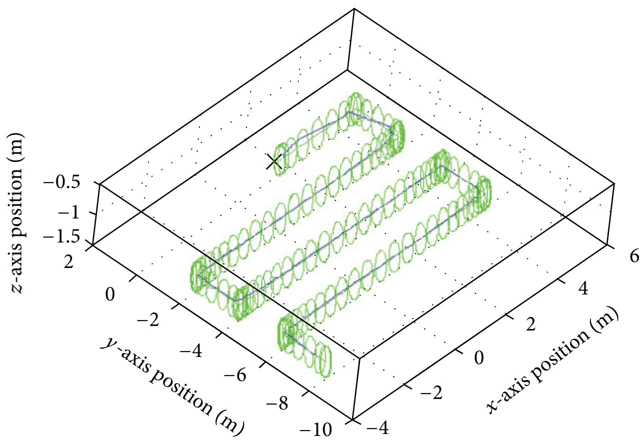

Figure 5: Three-dimensional view for the nonlinear H_∞ optimal control law with region function formulation.

exhibited a more robust tracking performance than the conventional region control, when parameter uncertainties, current effects, and disturbances exist. In Figure 4, the acute fluctuations in the early stages of the simulation were mainly caused by parameter uncertainties in the restoring force and moment. However, as long as the AUV is inside the desired region, the control input is turned off, and when the disturbances pull the vehicle out, the control input is applied to navigate the AUV back into the region.

5. Conclusion

A new nonlinear H_∞ optimal control law with region function formulation for a hovering underwater vehicle with four horizontal and four vertical thrusters has been presented in this paper. Two cases have been considered: the first case is the conventional region tracking control and the second case is where the region function is adopted with the nonlinear H_∞ optimal control law. Both control laws are observed with respect to the existence of unidirectional and bounded

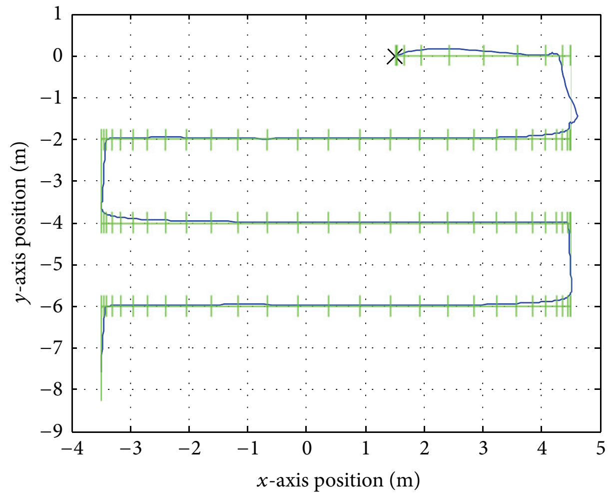

Figure 6: Planar view for nonlinear H_∞ optimal control law with region function formulation.

ocean currents. Although the underwater disturbances exist during task execution, the AUV is still able to track a desired moving region. A Lyapunov-like function has been proposed for stability analysis. Simulation results have been presented to demonstrate the performance of the proposed controller.

Appendix

An omnidirectional intelligent navigator (ODIN) is a near-spherical AUV designed in the University of Hawaii. The dynamic model of ODIN is given by [20, 21]

$$[M_{\mathrm{RB}} + M_{\mathrm{A}}]\,\dot{\nu} + [C_{\mathrm{RB}}(\nu) + C_{\mathrm{A}}(\nu)]\,\nu + D(\nu)\,\nu + g(\eta) = \tau, \tag{A.1}$$

where the subscripts RB and A represent the rigid body and added mass terms of the relevant parameters, respectively. The numerical values for the matrices of the vehicle dynamic equation (A.1) are given as

$$M_{\mathrm{RB}} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & 0 \\ 0 & m & 0 & -mz_G & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_G & 0 & I_{xx} & 0 & 0 \\ mz_G & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix}, \tag{A.2}$$

where $I_{xx} = I_{yy} = I_{zz} = I = (8/15)\pi\rho_v r_{\mathrm{ODIN}}^5$ are the moments of inertia about the principle axes.

Consider

$$M_{\mathrm{A}}=\begin{bmatrix} X_{ud} & 0 & 0 & 0 & 0 & 0\\ 0 & Y_{vd} & 0 & 0 & 0 & 0\\ 0 & 0 & Z_{wd} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_{\mathrm{RB}}(\nu)=\begin{bmatrix} 0 & 0 & 0 & mz_Gu_6 & mu_3 & -mu_2\\ 0 & 0 & 0 & -mu_3 & mz_Gu_6 & mu_1\\ 0 & 0 & 0 & m(u_2-z_Gu_4) & -m(u_1+z_Gu_5) & 0\\ -mz_Gu_6 & mu_3 & -m(u_2-z_Gu_4) & 0 & Iu_6 & -Iu_5\\ -mu_3 & -mz_Gu_6 & m(u_1+z_Gu_5) & -Iu_6 & 0 & Iu_4\\ mu_2 & -mu_1 & 0 & Iu_5 & -Iu_4 & 0 \end{bmatrix},$$

$$C_{\mathrm{A}}(\nu)=\begin{bmatrix} 0 & 0 & 0 & 0 & mu_3 & -mu_2\\ 0 & 0 & 0 & -mu_3 & 0 & mu_1\\ 0 & 0 & 0 & mu_2 & -mu_1 & 0\\ 0 & mu_3 & -mu_2 & 0 & 0 & 0\\ -mu_3 & 0 & mu_1 & 0 & 0 & 0\\ mu_2 & -mu_1 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{A.3}$$

$$D(\nu)=\begin{bmatrix} -d_{t1}|u_1| & 0 & 0 & 0 & 0 & 0\\ 0 & -d_{t1}|u_2| & 0 & 0 & 0 & 0\\ 0 & 0 & -d_{t1}|u_3| & 0 & 0 & 0\\ 0 & 0 & 0 & -d_{r1}|u_4|-d_{r2} & 0 & 0\\ 0 & 0 & 0 & 0 & -d_{r1}|u_5|-d_{r2} & 0\\ 0 & 0 & 0 & 0 & 0 & -d_{r1}|u_6|-d_{r2} \end{bmatrix},$$

$$g(\eta)=\begin{bmatrix} \left(mg-\frac{4}{3}\pi r^3\rho g\right)\sin(\theta)\\ -\left(mg-\frac{4}{3}\pi r^3\rho g\right)\cos(\theta)\sin(\phi)\\ -\left(mg-\frac{4}{3}\pi r^3\rho g\right)\cos(\theta)\cos(\phi)\\ z_Gmg\cos(\theta)\sin(\phi)\\ z_Gmg\sin(\theta)\\ 0 \end{bmatrix}.$$

Provided that $r_{\mathrm{ODIN}} = 0.31$ m is the radius of ODIN, $m = 125.0$ kg is the mass of ODIN, $z_G = 0.05$ m is the distance of the center of gravity from the geometric center, $\rho_v = 965\,\mathrm{kg/m^3}$ is the average density of the ODIN AUV, $\rho = 1000\,\mathrm{kg/m^3}$ is the density of fresh water, and $g = 9.81\,\mathrm{m/s^2}$. The hydrodynamic derivatives are given by $X_{ud} = Y_{vd} = Z_{wd} = (2/3)\pi\rho r_{\mathrm{ODIN}}^3$, the translational quadratic damping factor $d_{t1} = -248\,\mathrm{N(s/m)^2}$, the angular quadratic damping factor $d_{r1} = -280\,\mathrm{Ns^2/m}$, and the angular linear damping factor $d_{r2} = -230\,\mathrm{Ns^2/m}$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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